

1. Pronaći opće rješenje ODJ $4y'' - 4y' = 2x + 3$ i provjeriti dobiveno rješenje. 15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = \sin x + \cos y$. 15
3. Pronaći ravninu koja dira graf funkcije $f(x, y) = y\sqrt{x} - y^2 - x + 6y$ povučenu u točki $(4, 1, z_0)$ tog grafa. 15
4. $\int_0^2 2x^2 \cos x \, dx = ?$ 20
5. $\int_0^1 \frac{2x}{x^2 - 4} = ?$ 15
6. Izračunati površinu područja omeđenog krivuljama $y^2 - 2y - 2 + x = 0$ i $x + y + 1 = 0$. 20

Ukupno:

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f	$\frac{df}{dx}$	Tablica nekih integrala		
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\ln x$	$\frac{1}{x}$	$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x \, dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$	$\int \frac{dx}{x} = \ln x + C$	$\int \cot x \, dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
e^x	e^x	$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\alpha^x (\alpha > 0)$	$\alpha^x \ln \alpha$	$\int a^x \, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\sin x$	$\cos x$	$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\cos x$	$-\sin x$	$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	
$\tan x$	$\frac{1}{\cos^2 x}$			
$\cot x$	$-\frac{1}{\sin^2 x}$			
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$			
$\arctan x$	$\frac{1}{1+x^2}$			

① $4y'' - 4y' = 2x + 3$

$4r^2 - 4r = 0$

$r_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 0 \cdot 4}}{2 \cdot 4}$

$r_{1,2} = \frac{4 \pm 4}{8} \quad r_1 = 1 \quad r_2 = 0$

$g(x) = 2x + 3$

$g(x) = e^{\lambda x} (P_m(x) \cos(\beta x) + Q_m(x) \sin(\beta x))$
 $= e^{0 \cdot x} ((2x + 3) \cos(0 \cdot x) + 0 \cdot \sin(0 \cdot x))$
 $\lambda = 0 \quad m = 1 \quad \beta = 0 \quad n = 0$

$N = 1 \quad \lambda + \beta i = 0 \quad k = 1$

$y_p = x^k e^{\lambda x} (S_n(x) \cos(\beta x) + T_n(x) \sin(\beta x))$

$y_p = x^1 e^{0 \cdot x} ((Ax + B) \cos(0 \cdot x) + (Cx + D) \sin(0 \cdot x))$

$y_p = x(Ax + B)$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$y_p = Ax^2 + Bx$

$C_1 e^{r_1 x} + C_2 e^{r_2 x}$
 $y_H = C_1 e^{1x} + C_2 e^{0x}$
 $= C_1 e^x + C_2$

$$4 \cdot (2A) - 4(2Ax + B) = 2x + 3$$

$$8A - 8Ax - 4B = 2x + 3$$

$$4 \neq x: \quad -8A = 2 \quad | :(-8)$$

$$A = -\frac{1}{4}$$

$$\text{danas: } 8A - 4B = 3$$

$$8 \cdot \left(-\frac{1}{4}\right) - 4B = 3$$

$$-2 - 4B = 3$$

$$-4B = 3 + 2$$

$$-4B = 5 \quad | :(-4)$$

$$B = -\frac{5}{4}$$

$$y_p = Ax^2 + Bx$$

$$y_p = -\frac{1}{4}x^2 - \frac{5}{4}x$$

$$y = y_H + y_p$$

$$y = C_1 x + C_2 - \frac{1}{4}x^2 - \frac{5}{4}x$$

$$(2) \quad f(x,y) = \sin x + \cos y$$

$$\frac{\partial f}{\partial x} = \cos x$$

$$\frac{\partial f}{\partial y} = -\sin y$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

$$-\sin y = 0$$

$$y = 0 + k\pi, \quad k \in \mathbb{Z}$$

$T\left(\frac{\pi}{2}, 0\right) \rightarrow$ STACIONARNA TOČKA

$$\frac{\partial^2 f}{\partial x^2} = -\sin x = A$$

$$\frac{\partial^2 f}{\partial y^2} = -\cos y = C$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0 = B$$

PROPUSTILI STE
LOKALNI MINIMUM
 $\checkmark T\left(\frac{3\pi}{2}, \pi\right)$

$$\Delta = AC - B^2 = \left(-\sin\left(\frac{\pi}{2}\right)\right) \cdot \left(-\cos 0\right) - 0 = -1 \cdot (-1) - 0 = 1$$

$$\Delta(T) > 0$$

$$A = -1 < 0$$

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$$f\left(\frac{\pi}{2}, 0\right) = \sin\left(\frac{\pi}{2}\right) + \cos 0 = 1 + 1 = 2$$

$T\left(\frac{\pi}{2}, 0\right) \rightarrow$ LOKALNI MAXIMUM

\rightarrow točka u kojoj maksimum postiže svoju vrijednost.

$$T\left(\frac{\pi}{2}, 0, 2\right)$$

$$(3) \quad f(x,y) = y\sqrt{x} - y^2 - x + 6y$$

$$T(4, 1, z_0) \rightarrow 1 \cdot \sqrt{4} - 1 - 4 + 6 \cdot 1 = z_0$$

$$z_0 = z_0$$

$$T(4, 1, 3)$$

$$\frac{\partial f}{\partial x} = \frac{y}{2\sqrt{x}} - 1$$

$$f_x(t) = \frac{1}{2\sqrt{4}} - 1 = -\frac{3}{4}$$

$$\text{Pt... } z - z_0 = f_x(t)(x - x_0) + f_y(t)(y - y_0)$$

$$\text{Pt... } z - 3 = -\frac{3}{4}(x - 4) + 6(y - 1)$$

$$\frac{\partial f}{\partial y} = \sqrt{x} - 2y + 6$$

$$f_y(t) = \sqrt{4} - 2 \cdot 1 + 6$$

$$= 2 - 2 + 6$$

$$= 6$$

$$\text{Pt... } z - 3 = -\frac{3}{4}x + 3 + 6y - 6$$

$$z - 3 = -\frac{3}{4}x + 6y - 3$$

$$z = -\frac{3}{4}x + 6y - 3 + 3$$

$$z = -\frac{3}{4}x + 6y$$

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