MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** ixi

IME I PREZIME: ANĐELO ŽMIRE **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{3x} + 1$. 15 8
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2$. 15
3. Izračunati tangencijalnu ravninu plohe $z = x^2 y$ u točki $(2, 1, 4)$. 15
4. $\int_0^1 (x+2)(x+1)^8 \, dx = ?$ 15
5. $\int_0^\pi \frac{dx}{\sin x\,(2\cos^2 x - 1)} = ?$ 20
6. $\int_0^2 \frac{x+2}{3x^2 - 2x - 5} dx = ?$ 20

Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x\, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

2) $f(x,y) = e^x - x + y^2$

$\frac{\partial f}{\partial x} = e^x - 1$

$\frac{\partial^2 f}{\partial x^2} = e^x$

$\frac{\partial f}{\partial y} = 2y$

$\frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial f}{\partial x \partial y} = 0$

$e^x - 1 = 0$

$e^x = 1 \ /\ln$

$x = 0$

$2y = 0$

$y = 0$

$T_1(0,0)$

$\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 > 0$ $\quad \frac{\partial f}{\partial x^2} > 0 \Rightarrow$ LOKALNI MINIMUM

(6.) $\displaystyle\int_0^2 \frac{x+2}{3x^2-2x-5}\,dx = \left[\begin{array}{l} (3x^2-2x-5) = t \,/' \\ (6x-2)\,dx = dt \end{array}\right] = \frac{1}{6}\int \frac{6x-2+14}{(3x^2-2x-5)}\,dx = \frac{1}{6}\int \frac{6x-2}{(3x^2-2x-5)}\,dx + \frac{14}{6}\int \frac{dx}{(3x^2-2x-5)}$

$\displaystyle = \frac{1}{6}\int \frac{dt}{t} + ?$

(5.) $\displaystyle\int_0^1 (x+2)(x+1)^8\,dx = \int_0^1 (x+2)(x^8+1^8)\,dx = \int_0^1 x^9 + x + 2x^8 + 2 = \int_0^1 x^9 dx + 2\int_0^1 x^8 dx + \int_0^1 x\,dx + 2\int_0^1 dx$

$\displaystyle = \frac{x^{10}}{10}\Big|_0^1 + 2\,\frac{x^9}{9}\Big|_0^1 + \frac{x^2}{2}\Big|_0^1 + 2x\Big|_0^1$

$\displaystyle = \left(\frac{1}{10} - 0\right) + \left(\frac{2}{9} - 0\right) + \left(\frac{1}{2} - 0\right) + 2$

$\displaystyle = \frac{1}{10} + \frac{2}{9} + \frac{1}{2} + 2 = \frac{9+20+45+180}{90} = \frac{254}{90}$

ANĐELO ZMIRIĆ

⑤ $\int_0^{\pi} \frac{dx}{\sin x(2\cos^2 x-1)} = \left[\begin{array}{l} 2\cos^2 x - 1 = t \\ \frac{1}{4\sin x}\,dx = dt \end{array}\right]$

$= \frac{1}{4}\int_0^{\pi} \frac{dt}{t} = \frac{1}{4}\ln|t| = \frac{1}{4}\ln|2\cos^2 x-1| + c$

Ø

①

$r^2 - 2r - 3 = 0$

$r_{1,2} = \frac{2 \pm \sqrt{4+12}}{4} = \frac{2 \pm 4}{4}$

$r_1 = \frac{3}{2} \qquad r_2 = -\frac{1}{2}$

Ø

POPUNJAVA NASTAVNIK
Broj ↓ bodova

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** ixi

IME I PREZIME: ANTON RAŠIĆ

BROJ INDEKSA: 17-2-0084-2011

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = \mathrm{e}^{3x} + 1$. 15
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4. $\int\limits_0^1 (x+2)(x+1)^8 \,\mathrm{d}x = ?$ 15
5. $\int\limits_0^{\pi} \frac{\mathrm{d}x}{\sin x\,(2\cos^2 x - 1)} = ?$ 20
6. $\int\limits_0^2 \frac{x+2}{3x^2 - 2x - 5}\,\mathrm{d}x = ?$ 20

Ukupno: 6

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

② $f(x,y) = e^x - x + y^2$

$\frac{\partial f}{\partial x} = e^x - 1$

$e^x = 1$

$x = 0$

$T(0,0)$?

$\frac{\partial f}{\partial y} = 2y \Rightarrow 2y = 0$

$y = 0$

$$\frac{\delta f}{\delta x^2} = e^x = 2{,}718$$

$$\frac{\delta^2 f}{\delta y \delta x} = 0$$

$$\frac{\delta f}{\delta y^2} = 2$$

$$\frac{\delta^2 f}{\delta x \delta y} = 0$$

$$\Delta \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$$

$$\Delta = 2 > 0$$

T(0,0) je ~~MAKSIMUM~~ MINIMUM

④ $$\int_0^1 (x+2)(x+1)^8\,dx \quad \left| \begin{array}{l} t = x+1 \\ dt = x\,dx \\ dx = \frac{dt}{x} \end{array} \right|$$

$$= \int_0^1 (x+2)(t^8) \cdot \frac{dt}{x} = \int_0^1 \left(\frac{x^2}{2} + 2x\right) \cdot \frac{t^9}{9} \cdot \frac{dt}{x} = \int_0^1 \left(\frac{x^2}{2} + 2x\right) \cdot \frac{t^9}{9} \cdot x$$

$$= \left(\frac{1}{2} + \frac{2}{1}\right) \cdot \frac{t^9}{9} \cdot 1 = \frac{5}{2} \cdot \frac{t^9}{9} \cdot 1 = \frac{5t^9}{18} = \frac{5(x+1)}{18} = \frac{5}{18}(x+1)$$

⑤ $$\int_0^{\pi} \frac{dx}{\sin x(2\cos^2 x - 1)} = \left[\begin{array}{l} t = 2\cos^2 x - 1 \\ dt = 4\cos x\,dx \\ dx = \frac{dt}{4\cos x} \end{array} \right| = \int_0^{\pi} \frac{\frac{dt}{4\cos x}}{\frac{\sin x \cdot (t)}{1}}$$

$$= \int_0^{\pi} \frac{dt}{4\sin x \cos x \cdot t} = \frac{1}{4} \cdot \frac{x}{\cos x \cdot (-\sin x) \cdot t} \Big|_0^{\pi} =$$

$$= \frac{1}{4} \cdot \frac{x}{0{,}9985 \cdot (-0{,}0548) \cdot t} = \frac{x}{-0{,}2167 \cdot 2\cos^2 x - 1}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	ixi
IME I PREZIME: Vedran Ivanković	**BROJ INDEKSA:** 17-2-0235-2012.

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{3x} + 1$. 15
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Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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~~1) $y'' - 2y' - 3y = e^{3x} + 1$~~

4.) $\displaystyle\int_0^1 (x+2)(x+1)^8 dx =$ $t = x+1$, $dt = dx$ $\quad x=0 \rightarrow t_1$, $x=1 \rightarrow t_2$ $=$

$$\int_1^2 (t+1)\cdot t^8 dt = \int_1^2 t^9 + \int_1^2 t^8 = \frac{t^{10}}{10}\Big|_1^2 + \frac{t^9}{9}\Big|_1^2 =$$

$$\frac{2^{10}}{10} - \frac{1^{10}}{10} + \frac{2^9}{9} - \frac{1^{10}}{9} = \frac{14317}{90}$$

6.) $dx = \int_0^2 \frac{x+2}{3x^2-2x-5}\, dx = \int_0^2 \frac{x+2}{3x^2+3x-5x-5}\, dx =$

$$\int_0^2 \frac{x+2}{3x(x+1)-5(x+1)}$$

$$dx = \int_0^2 \frac{x+2}{(x+1)(3x-5)} \Rightarrow \frac{A}{x+1} + \frac{B}{3x-5} = \frac{x+2}{\quad}$$

$\varnothing$?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** ixi

IME I PREZIME: ANTONIO SEKULA **BROJ INDEKSA:** 17-2-0025-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{3x} + 1$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = e^x - x + y^2$. 15
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5. $\int\limits_0^\pi \frac{dx}{\sin x \,(2\cos^2 x - 1)} = ?$ 20
6. $\int\limits_0^2 \frac{x+2}{3x^2 - 2x - 5} dx = ?$ 20

Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
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$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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2\. $f(x,y) = e^x - x + y^2$

$\partial_x f = e^x - 1$

$\partial_y f = 2y$

$\boxed{e^x - 1 = 2y}$

$y = \frac{e^x - 1}{2}$

?

x	0	1
y	0	0.85

$\boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}}$

$$\begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix}$$

$$\Delta \begin{vmatrix} e^x & 0 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1 \cdot 2 - (0) = 2$$

$\partial_{xx} f > 0$

$\Delta > 0$

MIN.

(5) $\int_0^1 (x+2)(x+8)^8\,dx = \left| \begin{array}{ll} u = (x+8)^8 & dv = (x+2) \\ du = 8(x+8)^9 & v = \frac{x^2}{2} + 2x \\ du = 8x+64 & \end{array} \right.$

$u v - \int v\, du$

$\int x+2$

$\int x\,dx + 2\int dx$

$\frac{x^2}{2} + 2x$

$(x+8)^8 \cdot \frac{x^2}{2} + 2x - \int \frac{x^2}{2} + 2x \cdot (8x+64)$

$\int \frac{x^2}{2} + 16x^2 + 128x$

$\frac{x^3}{3}$

$\frac{1}{2}\int x^2 dx + 16\int x^2 dx + 128\int x\,dx$

$\frac{1}{2}\frac{x^3}{3} + 16\frac{x^3}{3} + 128\frac{x^2}{2}$

$(x+8)^8 \cdot \frac{x^2}{2} + 2x - \frac{x^3}{6} - \frac{16x^3}{3} - 64x^2 \Big|_0^1 = 21523293 - 0$

$= 21523293$

(6) $\int_0^2 \frac{x+2}{3x^2 - 2x - 5} =$

$x(3x-2) - 5 =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** ixi

IME I PREZIME: MARKO ČULINA **BROJ INDEKSA:** 17-1-0008-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

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Tablica nekih integrala

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
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3) $z = x^2 y \qquad T(2,1,4)$

$\frac{\partial f}{\partial x} = 2x$ ✗ $\qquad \frac{\partial f}{\partial x}(T) = 4$

$\frac{\partial f}{\partial y} = 1$ ✗ $\qquad \frac{\partial f}{\partial x}(T) = 1$

$\boxed{\partial_x f = 2xy \qquad \partial_y f = x^2}$

$z - 4 = 4(x-2) + 1(y-1)$

$z - 4 = 4x - 8 + y - 1$

$z - 4 - 4x + 8 - y + 1 = 0$

$z - 4x - y + 5 = 0$

2) $f(x,y) = e^x - x + y^2$

$\frac{\partial f}{\partial x} = e^x - 1$ $\qquad e^x - 1 = 0$

$\qquad e^x = 1 \implies \underline{\underline{x=0}}$

$\frac{\partial f}{\partial y} = 2y$ $\qquad 2y = 0$

$\qquad y = 0$

$\frac{\partial f}{\partial^2 x} = e^x = A$

$\frac{\partial f}{\partial x \partial y} = 0 = B$ $\qquad \Delta = AC - B^2$

$\qquad \Delta = e^x \cdot 2 - 0^2$

$\frac{\partial f}{\partial^2 y} = 2 = C$ $\qquad \Delta = 2$ - ekstrem

?

5) $\int_{0}^{\pi} \frac{dx}{\sin x(2\cos^2 x - 1)}$ | $\cos^2 x = t$

$-2\cos x \sin x \, dx = dt$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

ČUČINA

$\int_{0}^{\pi} \frac{dx}{\sin x(2 - \sin^2 x)}$

?

$\left(x + \frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + 9$

$$4\ \int_0^1 (x+2)(x+1)^8 dx = \left|\begin{matrix} x+1=t \\ dx=dt \end{matrix}\right| = \int_0^1 (x+2)(t^8) dx = \int_0^1 (x+2)\left(\frac{t^9}{9}\right) = \int_0^1 \left(\frac{x^2}{2}+2x\right)\left(\frac{t^9}{9}\right)$$

$$\int_0^1 \left(\frac{x^2}{2}+2x\right)\left(\frac{(x+1)^9}{9}\right)$$

= ?

$$6.\ \int_0^2 \frac{x+2}{3x^2-2x-5} dx = \int_0^2 \frac{x+2}{x+(-1)^2-(-1)^2-5} = \int_0^2 \frac{x+2}{x-5} dx =$$

$$q=-5$$

$$\frac{p}{2}=-\frac{2}{2}=-1$$

$$\Big|_0^2 \left(\frac{2+2}{2-5}\right) - \left(\frac{0+2}{0-5}\right) = \left(\frac{4}{-3}\right) - \left(\frac{2}{-5}\right) = -\frac{14}{15}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** ixi	POPUNJAVA NASTAVNIK
IME I PREZIME: FILIP MEŠTROVIĆ **BROJ INDEKSA:**	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{3x} + 1$.	15
2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2$.	15
3. Izračunati tangencijalnu ravninu plohe $z = x^2y$ u točki $(2, 1, 4)$.	15
4. $\int_0^1 (x+2)(x+1)^8 \, dx = ?$	15
5. $\int_0^\pi \frac{dx}{\sin x (2\cos^2 x - 1)} = ?$	20
6. $\int_0^2 \frac{x+2}{3x^2 - 2x - 5} dx = ?$	20
	Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha - 1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \|\cos x\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
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$y'' - 2y' - 3y = e^{3x} + 1 \qquad r^2 - 2r - 3 = 0 \qquad r_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} \qquad \boxed{r_1 = 3 \quad r_2 = -1}$

$y_H = C_1 e^{3x} + C_2 e^{-x}$ ✓

$y = y_H + y_{p_1} + y_{p_2}$

$e^{3x} = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$

$\alpha = 3 \quad \beta = 0 \quad 3 + 0i = 3 = r_1 \quad \boxed{k=1}$

$P_m = 1 \quad Q_n = 1 \Rightarrow m, n = 0$ $\boxed{N = 0}$

$y_{p_1} = Axe^{3x}$

$y'_{p_1} = Ae^{3x} + A3xe^{3x}$

$y''_{p_1} = 3Ae^{3x} + 3Ae^{3x} + 9Axe^{3x}$

$1 = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$

$\alpha = 0 \quad \beta = 0i \quad \alpha + \beta i = 0 \quad k = 0$

$P_m(x) = 0 \quad m = 0$
$Q_n(x) = 0 \quad n = 0$ $\boxed{N = 0}$

$y_{p_2} = A \ B$

$y'_{p_2} = 0 \quad y''_{p_2} = 0$

$\Rightarrow B = -\frac{1}{3}$

$3e^{3x} + 3e^{3x} + 9\cancel{xe^{3x}} - 2e^{3x} - 6\cancel{xe^{3x}} - 3\cancel{xe^{3x}} = e^{3x}$

$\underline{4e^{3x} = e^{3x}}$

DOBILI BI $4Ae^{3x} = e^{3x}$

$\boxed{A = \frac{1}{4}}$

$\Downarrow$

$y = \frac{1}{4}xe^{3x} - \frac{1}{3} + C_1e^{3x} + C_2e^{-x}$

$\int_0^2 \frac{x+2}{3x^2-2x-5}dx =$

$\frac{x+2}{3x^2-2x-5}$

$(x-2)(x+\frac{2}{3})$

$3x^2-2x-5$

$x_{12} = \frac{4 \pm \sqrt{4+60}}{6}$ $\frac{-4}{6}$ $-\frac{2}{3}$

x^2

$x_1 = 2$ $x_2 =$

?

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** ixi	POPUNJAVA NASTAVNIK
IME I PREZIME: TOMISLAV GLAVAH **BROJ INDEKSA:** 17-0115-2011	Broj ↓ bodova

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Ukupno: $\emptyset$

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