

1. Riješiti diferencijalnu jednadžbu: $y'' - 2y' - 3y = e^{3x} + 1$.

~~15~~

2. Odrediti lokalne ekstreme funkcije $f(x, y) = e^x - x + y^2$.

~~15~~

3. Izračunati tangencijalnu ravninu plohe $z = x^2y$ u točki (2, 1, 4).

15

4. $\int_0^1 (x+2)(x+1)^8 dx = ?$

15

5. $\int_0^\pi \frac{dx}{\sin x (2 \cos^2 x - 1)} = ?$

~~20~~

6. $\int_0^2 \frac{x+2}{3x^2 - 2x - 5} dx = ?$

20

Ukupno:

15

f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

② $f(x,y) = e^x - x + y^2$

$\frac{\partial f}{\partial x} = e^x - 1$

$e^x - 1 = 0$

$2y = 0$

$e^x = 1 \quad / \ln$

$y = 0$

$\frac{\partial^2 f}{\partial x^2} = e^x$

$x = 0$

$T_1(0,0)$

$\frac{\partial^2 f}{\partial y^2} = 2$

$\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 > 0$

$\frac{\partial^2 f}{\partial x^2} > 0$

\Rightarrow LOKALNI MINIMUM

$\frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial^2 f}{\partial x \partial y} = 0$

$$\textcircled{6} \int_0^2 \frac{x+2}{3x^2-2x-5} dx = \left[\begin{array}{l} (3x^2-2x-5) = t \\ (6x-2) dx = dt \end{array} \right] = \frac{1}{6} \int \frac{6x-2+14}{(3x^2-2x-5)} dx = \frac{1}{6} \int \frac{6x-2}{(3x^2-2x-5)} dx + \frac{14}{6} \int \frac{dx}{(3x^2-2x-5)}$$

$$= \frac{1}{6} \int \frac{dt}{t} + \text{?}$$

$$\textcircled{7} \int_0^1 (x+2)(x+1)^2 dx = \int_0^1 (x+2)(x^2+1^2) dx = \int_0^1 x^2 + x + 2x^2 + 2 = \int_0^1 x^2 dx + 2 \int_0^1 x dx + \int_0^1 x dx + 2 \int_0^1 dx$$

$$= \frac{x^3}{3} \Big|_0^1 + 2 \frac{x^2}{2} \Big|_0^1 + \frac{x^2}{2} \Big|_0^1 + 2x \Big|_0^1$$

$$= \left(\frac{1}{3} - 0 \right) + \left(\frac{2}{2} - 0 \right) + \left(\frac{1}{2} - 0 \right) + 2$$

$$= \frac{1}{3} + \frac{2}{2} + \frac{1}{2} + 2 = \frac{9+20+45+120}{90} = \frac{254}{90}$$

5) $\int_0^{\pi} \frac{dx}{\sin x (2 \cos^2 x - 1)}$ $\left[\begin{array}{l} 2 \cos^2 x - 1 = t \\ \frac{1}{4 \sin x} dx = dt \end{array} \right]$ AUXILIAR ZMIRI

$$= \frac{1}{4} \int_0^{\pi} \frac{dt}{t} = \frac{1}{4} \ln|t| = \frac{1}{4} \ln|2 \cos^2 x - 1| + C$$

1) $r^2 - 2r - 3 = 0$

$$r_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm 4}{2}$$

$$r_1 = \frac{3}{2} \quad r_2 = -\frac{1}{2}$$

