MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii
IME I PREZIME: Ines Valušić **BROJ INDEKSA:** 17-2-0223-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = y\sqrt{x} - y^2 - x + 6y$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1, 1, z_0)$. 15
4. $\int_0^{\pi} \cos^4 x \, dx = ?$ 20
5. $\int_0^1 x \cos(3x^2 + 4) \, dx = ?$ 15
6. Neka je $f(x) = \frac{1}{x^3}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno: 85

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3.) $z_0 = \arctan 1 = \frac{\pi}{4}$

$f_x = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right)$ ✓

$f_x(1,1) = -\frac{1}{2}$

$f_y = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}$

$f_y(1,1) = \frac{1}{2}$ ✓

$R_t \ldots \; z - \frac{\pi}{4} = -\frac{1}{2}(x-1) + \frac{1}{2}(y-1)$ ✓

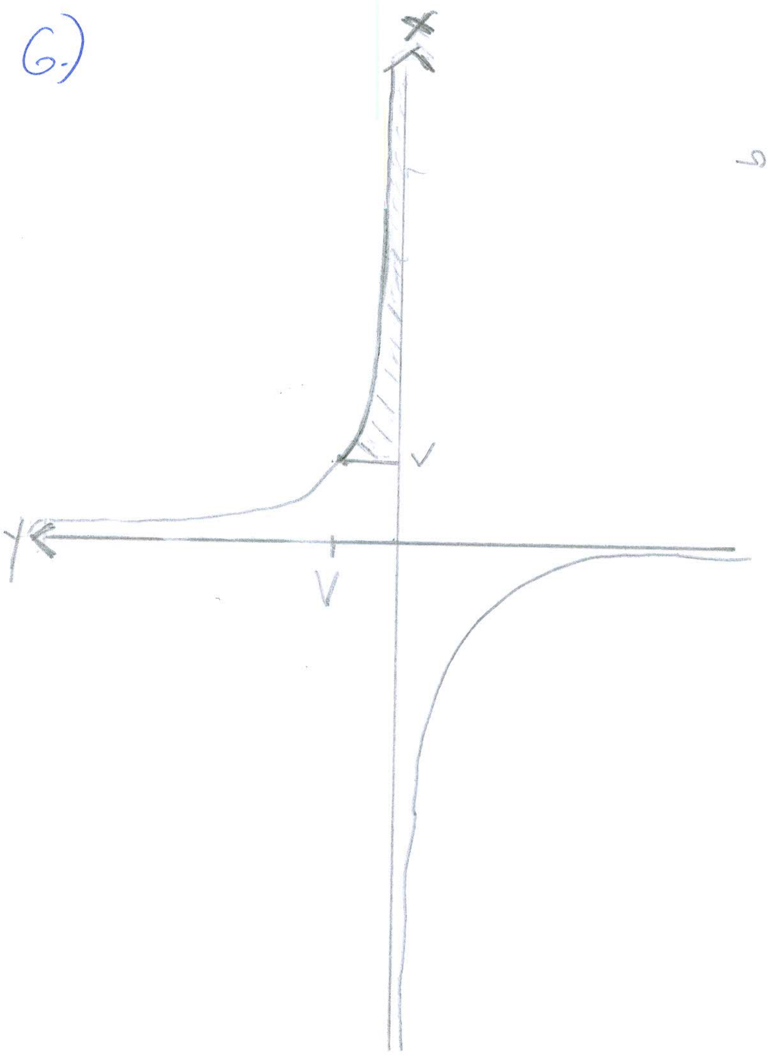
$z + \frac{1}{2}x - \frac{1}{2}y - \frac{\pi}{4} = 0$

5.) $\int_0^1 x\cos(3x^2+4)\,dx = \left| \begin{array}{lll} 3x^2+4 & x=1 & t=7 \\ 6x\,dx = dt & & \\ x\,dx = \frac{1}{6}dt & x=0 & t=4 \end{array} \right|$

$= \int_4^7 \cos t \cdot \frac{1}{6}\,dt = \left(\frac{1}{6}\sin t\right)\Big|_4^7 =$

$= \frac{1}{6}\sin 7 - \frac{1}{6}\sin 4 = 0{,}236$

6.)



$\int_1^{+\infty} \frac{dx}{x^3} = \lim_{b\to+\infty} \int_1^b \frac{dx}{x^3} = \lim_{b\to+\infty} \left(-\frac{1}{2x^2}\right)\Big|_1^b =$

$= \lim_{b\to+\infty} \left(-\frac{1}{2b^2} + \frac{1}{2}\right) = \frac{1}{2}$

$\frac{\partial^2 f}{\partial x^2} < 0 = \text{MAX}$

$f_{max} = 8 - 16 - 4 + 24 = 12$

$T(4{,}4, 12)$ je Maximum

4.) $\int_0^{\pi} \cos^4 x \, \partial x = \int_0^{\pi} \left(\frac{1+\cos(2x)}{2}\right)^2 \partial x =$

$= \frac{1}{4} \int_0^{\pi} \left(1 + 2\cos(2x) + \cos^2(2x)\right) \partial x =$

$= \frac{1}{4} \int_0^{\pi} \left(1 + 2\cos(2x) + \frac{1+\cos(4x)}{2}\right) \partial x$

$= \frac{1}{4} \left(x + 2 \cdot \frac{1}{2} \sin(2x) + \frac{1}{2} x + \frac{1}{2} \cdot \frac{1}{4} \sin 4x\right) \Big|_0^{\pi}$

$= \frac{1}{4} \left(\pi + \sin 2\pi + \frac{\pi}{2} + \frac{1}{8} \sin 4\pi\right) -$

$- \frac{1}{4} \left(\sin 0 + \frac{1}{8} \sin 0\right) = \frac{1}{4} \cdot \frac{3\pi}{2} = \frac{3\pi}{8}$

Ines Vukušić

2) $\frac{\partial f}{\partial x} = \frac{y}{2\sqrt{x}} - 1 = 0 \Rightarrow y = 2\sqrt{x}$

$\frac{\partial f}{\partial y} = \sqrt{x} - 2y + 6 = 0$

$\sqrt{x} - 4\sqrt{x} + 6 = 0$

$3\sqrt{x} = 6$

$\sqrt{x} = 2$

$x = 4$

$\cancel{x = -4}$

$T_1 = (4,4)$

$\cancel{T_2 = (4,}$

$\frac{\partial^2 f}{\partial x^2} = \frac{y}{2} \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = -\frac{y}{4\sqrt{x^3}}$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2\sqrt{x}}$

$\frac{\partial^2 f}{\partial y^2} = -2$

$\sqrt{x} = 2$

$x = 4 \qquad y = 4$

$\frac{\partial^2 f}{\partial x^2} = \frac{y}{2} \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = -\frac{y}{4\sqrt{x^3}}$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{2\sqrt{x}}$

$\frac{\partial^2 f}{\partial y^2} = -2$

$\Delta = -\frac{1}{8} \cdot (-2) - \left(\frac{1}{4}\right)^2 = \frac{1}{4} - \frac{1}{16} = \frac{3}{16} > 0$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii

IME I PREZIME: NENO VULIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
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Ukupno: 35

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$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
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6 $\int_0^1$

$y'' - y = x^2 - xe^x \qquad y(0) = 0 \text{ i } y'(0) = 1$ (1)

$f(x,y) = y\sqrt{x} - y^2 - x + 6y$

$z = \arctan\left(\frac{y}{x}\right)$ u točki $M(1,1,20)$

NENO VULIĆ

1. $y'' - y = x^2 - xe^x$ uz uvjet $y(0) = 0$ i $y'(0) = 1$

LAPLACEOVA TRANSFORMACIJA

$$f^2 Y(f) - f y(0) - y'(0) - Y(f) = \frac{2}{f^3} - \frac{1}{(f-1)^2}$$

$$(f^2-1)Y(f) = \frac{2}{f^3} - \frac{1}{(f-1)^2} + 1 = \frac{2(f-1)^2 - f^3 + f^3(f-1)^2}{f^3 \cdot (f-1)^2}$$

$$Y(f) = \frac{2f^2 - 4f + 2 - f^3 + f^5 - 2f^4 + f^3}{f^3(f-1)^3 \cdot (f+1)}$$

$$Y(f) = \frac{f^5 - 2f^4 + 2f^2 - 4f + 2}{f^3(f-1)^3 \cdot (f+1)} = \frac{Af^2 + Bf + C}{f^3} + \frac{Df^2 + Ef + F}{(f-1)^3}$$

$$+ \frac{G}{f+1} \quad / \cdot \text{NAZIVNIK}$$

$f^5 - 2f^4 + 2f^2 - 4f + 2 = (Af^2 + Bf + C) \cdot (f-1)^3 \cdot (f+1) + (Df^2 + Ef + F) \cdot (f+1) f^3 + G(f-1)^3 \cdot f^3 = (Af^3 + Bf^2 + Cf + Af^2 + Bf + C) \cdot (f^3 - 3f^2 + 3f - 1) + (Df^3 + Ef^2 + Ff + Df^2 + Ef + F) \cdot f^3 + Gf^6 - 3Gf^5 + 3Gf^4 - Gf^3 = Af^6 + Bf^5 + Cf^4 - 3Af^5 - 3Bf^4 - 3Cf^3 - 3Af^4 - 3Bf^3 - 3Cf^2 + Af^5 + Bf^4 + Cf^3 - 3Af^5 - 3Bf^4 - 3Cf^3 + 3Af^4 + 3Bf^3 + 3Cf^2 + 3Af^3 + 3Bf^2 + 3Cf - Af^3 - Bf^2 - Cf - Af^2 - Bf - C + Df^6 + Ef^5 + Ff^4 + Df^5 + Ef^4 + Ef^3 + Gf^6 - 3Gf^5 + 3Gf^4 - Gf^3 = (A + B + G)f^6 + (B + A + E + D - 3G)f^5 + (C + B - 3B - 3A + 3A + F + E + 3G)f^4 + (C - 3A - 3 - 3B + 3B + 3A - A + F - 6)f^3 + (3C - 3C + 3B - B - A)f^2 + (3C - C - B)f - C$

$C = -2$

$2C - B = -4$

$-4 - B = -4 \Rightarrow B = 0$

$-A = 2$

$A = -2$

$2 - 3 + 2 + f - 6 = 0$

$f = 9$

③

$$\frac{y}{\sqrt{x}} = 2$$

$$y = 2\sqrt{x}$$

$$\sqrt{x} - 4\sqrt{x} = -6$$

$$-3\sqrt{x} = -6$$

$$\sqrt{x} = 2$$

$$x = 4$$

$$y = 4$$ STACIONARNA TOČKA JE $(4,4)$

$$\frac{Df}{Dx^2} = -\frac{1}{4}y \cdot \frac{1}{\sqrt{x^3}} = -\frac{1}{4 \cdot 2} = -\frac{1}{8} < 0$$

$$\frac{Df}{Dy^2} = -2$$

$$\frac{Df}{DyDx} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{4}$$

$$\begin{vmatrix} -\frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & -2 \end{vmatrix} = \frac{1}{4} - \frac{1}{16} > 0$$

$(4,4)$ JE LOKALNI MAKSIMUM

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④ $$\int_0^{\pi} \cos^4 x\,dx = \int_0^{\pi} \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int_0^{\pi} (1 + 2\cos 2x + \cos^2 2x)\,dx$$

$$= \frac{1}{4}\left[x\Big|_0^{\pi} + \sin 2x\Big|_0^{\pi} + \int_0^{\pi} \frac{1+\cos 4x}{2}\,dx\right]$$

$$= \frac{1}{4}\left[\pi + \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)\Big|_0^{\pi}\right]$$

$$= \frac{1}{4}\left[\pi + \frac{1}{2}(\pi)\right]$$

$$= \frac{1}{4} \cdot \frac{3\pi}{2} = \frac{3\pi}{8}$$

VULIĆ NENO (2)

$2-3+2+f-6=0$

$f=9$

$-2+9+E+3G=-2$

$E+3G=-9$

$-2+E+D-3G=1$

$2E-2+D=-8$

$2E+D=-6$

$D+G=2$

$E+3(2-D)=-9$

$-3B=-15$

$2(3D-15)+D=-6$

$7D=-6+30$

$D=\frac{24}{7}$

$G=2-\frac{24}{7}$

$G=-\frac{10}{7}$

$E=-9+\frac{30}{7}$

$E=-\frac{33}{7}$

$$Y(f)=\frac{-2}{f}-\frac{2}{f^3}+\frac{\frac{24}{7}f^2-\frac{33}{7}f+9}{(f-1)^3}+\frac{-\frac{10}{7}}{f+1}$$

$Y(x)=-2+\frac{x^2}{2}\cdot 2-\frac{10}{7}\cdot E^{-x}+$ NEŠTO ŠTO? 0

LAKŠE BI BILO DA STE KORISTILI PROCEDURU IZ VJEŽBENICE.

2) $f(x,y)=y\sqrt{x}-y^2-x+6y$

$\frac{Df}{Dx}=\frac{1}{2}\cdot y\cdot\frac{1}{\sqrt{x}}-1=0$ ✓

$\frac{Df}{Dy}=\sqrt{x}-2y+6=0$ ✓

~~$\frac{y}{2\sqrt{x}}-1=\sqrt{x}-2y+6$~~

~~$\left(\frac{1}{2\sqrt{x}}+2\right)y=\sqrt{x}+7$~~

4
STRAN

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii

IME I PREZIME: TOMISLAV TUTP **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. — 15
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Ukupno: 15

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$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
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TU7A

5.) $\int_0^1 x\cos(3x^2+4)\,dx$

$3x^2+4=t$

$6x\,dx=dt$

$dx=\frac{dt}{6x}$

$\int \cancel{x}\cos(t)\frac{dt}{6\cancel{x}}$

$\frac{1}{6}\int\cos(t)\,dt$

$=\frac{1}{6}\sin(t)$

$=\frac{1}{6}\sin(3x^2+4)$

$\frac{1}{6}\sin(3+4)=0.11$

$\frac{1}{6}\sin(4)=-0.13$

$0.11-(-0.13)$

$0.11+0.13\approx 0.24$

4.) $\int_0^{\pi}\cos^4 x\,dx = \int_0^{\pi}$

?

2. $f(x,y) = y\sqrt{x} - y^2 - x + 6y$

$y^2x - y^4 - x^2 + 6y^2$

$yx^{\frac{1}{2}} - y^2 - x + 6y$

$1 - 4y^3 + 12y$

$2y - 2x$

$\frac{\partial f}{\partial x} = y \quad -1$

$\frac{\partial f}{\partial y} = x^{\frac{1}{2}} - 2y + 6$

$\frac{\partial f}{\partial x^2} = 0$

$\frac{\partial f}{\partial y^2} = -2$

$\frac{\partial f}{\partial xy} = 1$

$\Delta = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix}$

$\Delta = 0 - 1 = -1$

NEMA EKSTREMA

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii

IME I PREZIME: ŠIME-BORNA MAGAŠ **BROJ INDEKSA:** 17-2-0108-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

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$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(4) $\displaystyle\int_0^{\pi} \cos^4 x\,dx = \left[\begin{array}{l} \cos^4 = t \\ x\,dx = dt \end{array}\right] =$

⑤ $\int_0^1 x\cos(3x^2+4)\,dx$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii

IME I PREZIME: ROMANA NOVOSEL **BROJ INDEKSA:** 17-2-0097-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = y\sqrt{x} - y^2 - x + 6y$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1, 1, z_0)$. 15
4. $\int_0^\pi \cos^4 x \, dx = ?$ 20
5. $\int_0^1 x \cos(3x^2 + 4) \, dx = ?$ 15 — 15
6. Neka je $f(x) = \frac{1}{x^3}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno: 15

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

(2) $\cancel{f(x,y) = y\sqrt{x} - y^2 - x + 6y}$

$\cancel{2xy =}$

0

(4.) $\int_0^\pi \cos^4 x \, dx = \left[\begin{array}{l} \cos^4 x = t \\ -\sin x^4 dx = dt /\cdot \\ \sin^4 dx = -dt \end{array}\right.$

0

(5) $\int_0^1 x\cos(3x^2+4)\,dx = \left[\begin{array}{l} 3x^2+4 = t \\ 6x\,dx = dt /:6 \\ x\,dx = \frac{1}{6}dt \end{array}\right] = \int_0^1 \cos t \cdot \frac{1}{6}dt = \frac{1}{6}\int_0^1 \cos t\,dt = \frac{1}{6}\sin(3x^2+4)\Big|_0^1$

$\underbrace{\frac{1}{6}\sin(3\cdot 1^2+4)}_{0.102} - \frac{1}{6}\sin(3\cdot 0^2+4) =$

0.66   -0.76

$- \ -0.115 = 0.217$ ✓

② $f(x,y) = 4\sqrt{x} - y^2 - x + 6y$

$2xf = 4\sqrt{x} - 1$

$2xxf = 4$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii

IME I PREZIME: MATEA ČOLINA **BROJ INDEKSA:** 17-2-0206-2012

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = y\sqrt{x} - y^2 - x + 6y$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1, 1, z_0)$. 15
4. $\int_0^{\pi} \cos^4 x \, dx = ?$ 20
5. $\int_0^{1} x \cos\left(3x^2 + 4\right) dx = ?$ 15
6. Neka je $f(x) = \frac{1}{x^3}$. Odrediti $\int_1^{+\infty} f(x) dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iii

IME I PREZIME: MARIN GALOŠIĆ **BROJ INDEKSA:** 17-2-0001-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. 15
2. Odrediti lokalne ekstreme funkcije: $f(x, y) = y\sqrt{x} - y^2 - x + 6y$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1, 1, z_0)$. 15
4. $\displaystyle\int_0^{\pi} \cos^4 x \, dx = ?$ 20
5. $\displaystyle\int_0^{1} x \cos\left(3x^2 + 4\right) dx = ?$ 15
6. Neka je $f(x) = \dfrac{1}{x^3}$. Odrediti $\int_1^{+\infty} f(x)dx$. Skicirati graf funkcije f i površinu određenu integralom. 20

Ukupno:

f	$\frac{df}{dx}$
x^α $(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x$ $(\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
α^x $(\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	