

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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IME I PREZIME:

Ines Valušić

BROJ INDEKSA:

17-2-0223-2012

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

- Riješiti diferencijalnu jednadžbu: $y'' - y = x^2 - xe^x$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$.
- Odrediti lokalne ekstreme funkcije: $f(x, y) = y\sqrt{x} - y^2 - x + 6y$.
- Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1, 1, z_0)$.
- $\int_0^{\pi} \cos^4 x \, dx = ?$
- $\int_0^1 x \cos(3x^2 + 4) \, dx = ?$
- Neka je $f(x) = \frac{1}{x^3}$. Odrediti $\int_1^{+\infty} f(x) \, dx$. Skicirati graf funkcije f i površinu određenu integralom.

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Ukupno:

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f	$\frac{df}{dx}$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$a^x (\alpha > 0)$	$a^x \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right] + C$	
$\int \cos x dx = \sin x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$	

3.) ~~z_0~~ $z_0 = \arctan 1 = \frac{\pi}{4}$
 $f_x = \frac{1}{1 + (\frac{y}{x})^2} \cdot \left(-\frac{y}{x^2}\right)$ ✓

~~f_x~~ $f_x(1,1) = -\frac{1}{2}$
 $f_y = \frac{1}{1 + (\frac{y}{x})^2} \cdot \frac{1}{x}$

$f_y(1,1) = \frac{1}{2}$ ✓

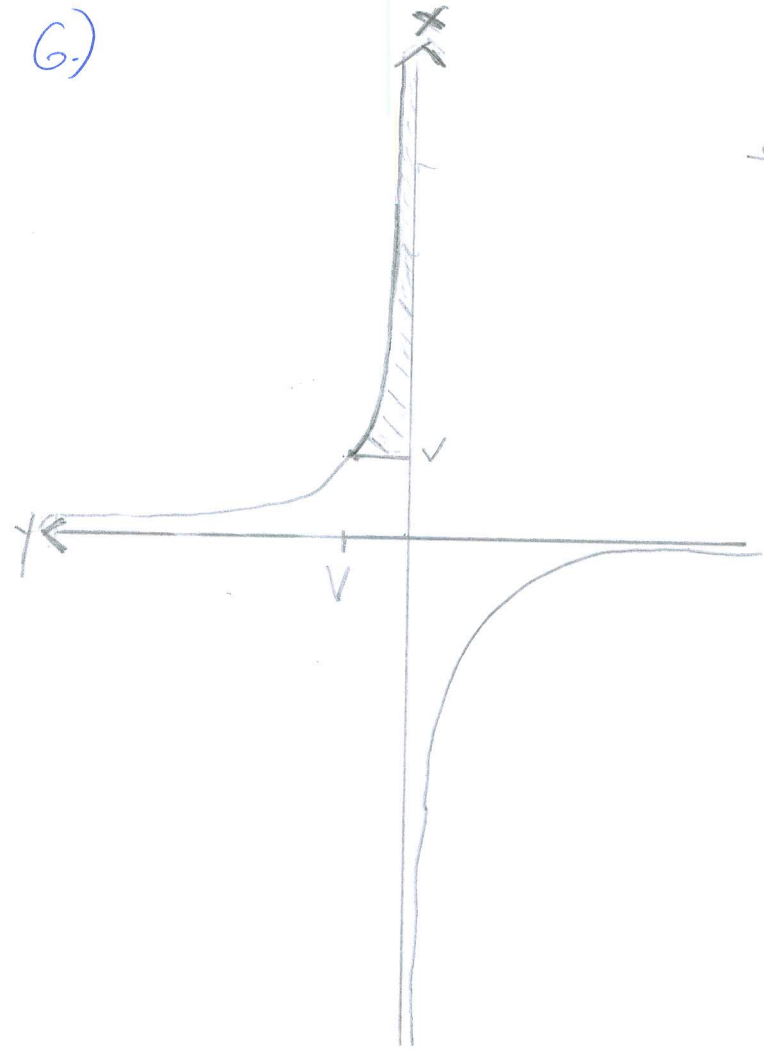
Rt. ... $z - \frac{\pi}{4} = -\frac{1}{2}(x-1) + \frac{1}{2}(y-1)$ ✓
 $z + \frac{1}{2}x - \frac{1}{2}y - \frac{\pi}{4} = 0$ ✓

$$5.) \int_0^1 x \cos(3x^2+4) dx = \left. \begin{array}{l} 3x^2+4 = t \quad x=1 \quad t=7 \\ 6x dx = dt \\ x dx = \frac{1}{6} dt \quad x=0 \quad t=4 \end{array} \right|$$

$$= \int_4^7 \cos t \cdot \frac{1}{6} dt = \left(\frac{1}{6} \sin t \right) \Big|_4^7 =$$

$$= \frac{1}{6} \sin 7 - \frac{1}{6} \sin 4 = 0,236$$

6.)



$$=$$

$$\int_1^{+\infty} \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^3} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{2x^2} \right)$$

$$= \lim_{b \rightarrow +\infty} \left(-\frac{1}{2b^2} + \frac{1}{2} \right) = \frac{1}{2}$$

$$\frac{d^2 f}{dx^2} < 0 = \text{MAX}$$

$$f_{\text{max}} = 8 - 16 - 4 + 24 = 12$$

$T(4, 4, 12)$ is Maximum

$$4.) \int_0^{\pi} \cos^4 x \, dx = \int_0^{\pi} \left(\frac{1 + \cos(2x)}{2} \right)^2 dx =$$

$$= \frac{1}{4} \int_0^{\pi} (1 + 2\cos(2x) + \cos^2(2x)) dx =$$

$$= \frac{1}{4} \int_0^{\pi} \left(1 + 2\cos(2x) + \frac{1 + \cos(4x)}{2} \right) dx$$

$$= \frac{1}{4} \left(x + 2 \cdot \frac{1}{2} \sin(2x) + \frac{1}{2} x + \frac{1}{2} \cdot \frac{1}{4} \sin(4x) \right)$$

$$= \frac{1}{4} \left(\pi + \sin 2\pi + \frac{\pi}{2} + \frac{1}{8} \sin 4\pi \right) -$$

$$- \frac{1}{4} (\sin 0 + \frac{1}{8} \sin 0) = \frac{1}{4} \cdot \frac{3\pi}{2} = \frac{3\pi}{8}$$

