MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix

IME I PREZIME: PETAR JELAVIĆ MITROVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$. 15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = 5x^3y^2 - 9$ u točki $T(3,1,z_0)$. 15
4. $\int_0^1 x e^x \, dx = ?$ 20
5. $\int_0^2 \frac{2x}{x^2-1} = ?$ ~~15~~
6. $\int_{-1}^{+\infty} \frac{dx}{1+x^2} = ?$ 20

Ukupno: 55

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $I = \int_0^1 x e^x \, dx$

$u = x \Rightarrow du = dx$

$dv = e^x dx \Rightarrow v = \int e^x dx \Rightarrow v = e^x$

$I = x \cdot e^x \Big|_0^1 - \int_0^1 e^x \cdot dx$

$I = xe^x \Big|_0^1 - e^x \Big|_0^1 = e^x(x-1) \Big|_0^1$

$= e^1 \cdot (1-1) - e^0(0-1) = e \cdot 0 - 1 \cdot (-1) = 1$ ✓

6. $I = \int_{-1}^{+\infty} \frac{dx}{1+x^2} = \lim_{b \to \infty} \int_{-1}^{b} \frac{dx}{1+x^2}$

$= \lim_{b \to \infty} \operatorname{arctg} x \Big|_{-1}^{b} =$

$= \lim_{b \to \infty} (\operatorname{arctg} b - \operatorname{arctg}(-1))$

$= \lim_{b \to \infty} \left(\frac{\pi}{2} - \left(-\frac{\pi}{4}\right)\right)$

$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$ ✓

5.

$$\int_0^2 \frac{2x}{x^2-1}\,dx = \left[\begin{array}{l} x^2-1=t \\ 2x\,dx=dt \end{array} \quad \begin{array}{c|c} x & t \\ \hline 2 & 3 \\ 0 & -1 \end{array}\right]$$

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$$= \int_{-1}^{3} \frac{dt}{t} = \ln|t|\Big|_{-1}^{3} = \ln|3| - \ln(-1)$$
$$= \ln 3$$

2.

$$z = f(x,y) = \frac{1}{1+x^2+y^2}$$

$$z_x = \frac{-1}{(1+x^2+y^2)^2}\cdot 2x = \frac{-2x}{1+x^2+y^2}$$

$$z_y = \frac{-1}{(1+x^2+y^2)}\cdot 2x = \frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{-2x}{(1+x^2+y^2)^2} = 0 \quad \Big/\cdot(1+x^2+y^2)^2 \Rightarrow -2x=0 \quad \underline{\underline{x=0}}$$

$$\frac{-2y}{(1+x^2+y^2)^2} = 0 \quad \Big/\cdot(1+x^2+y^2)^2 \Rightarrow -2y=0 \quad \underline{\underline{y=0}}$$

$$z_{xx} = \frac{-2\cdot(1+x^2+y^2)^2+2x\cdot 2(1+x^2+y^2)\cdot 2x}{(1+x^2+y^2)^4} \qquad T(0,0)$$

$$= \frac{1+\cancel{x^2+y^2}\cdot\left[-2(1+x^2+y^2)+8x^2\right]}{(1+x^2+y^2)^{\cancel{4}\,3}}$$

$$= \frac{-2-2x^2-2y^2+8x^2}{(1+x^2-y^2)^3} = \frac{6x^2-2y^2-2}{(1+x^2+y^2)^3}$$

$$z_{xy} = \frac{2x}{(1+x^2+y^2)^{\cancel{4}\,3}} = 2(1+\cancel{x^2+}y^2)\cdot 2y = \frac{8xy}{(1+x^2+y^2)^3}$$

$$z_{xy} = \frac{6y^2-2x^2-2}{(1+x^2+y^2)^3}$$

$$A = \frac{6\cdot 0^2-2\cdot 0^2-2}{(1+0+0)^3} = -2$$

$$B = \frac{8\cdot 0\cdot 0}{(1+0+0)^3} = 0$$

$$C = \frac{6\cdot 0^2-2\cdot 0^2}{(1+0+0)^3} = -2$$

$$D = (-2)\cdot(-2)-0^2 = 4 > 0$$ POSTOJI EKSTREM

$$A = -2 < 0 \quad \max \ u \ T(0,0)$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix

IME I PREZIME: ALEN BURA **BROJ INDEKSA:** 17-2-0095-2011

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$. 15
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Ukupno: 20

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
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Tablica nekih integrala		
$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x \, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
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$$6. \int_{-1}^{+\infty} \frac{dx}{1+x^2} = \int_{-1}^{+\infty} \frac{dx}{(\sqrt{1})^2 + x^2} = \lim_{x \to +\infty} = \frac{1}{\sqrt{1}} \operatorname{arctg} \frac{x}{\sqrt{1}} \Big|_{-1}^{+\infty} = \left[\left(\frac{1}{\sqrt{1}} \operatorname{arctg} \frac{+\infty}{\sqrt{1}} - \frac{1}{\sqrt{1}} \operatorname{arctg} \frac{-1}{\sqrt{1}}\right)\right]$$

0

$$= -\frac{\pi}{4}$$

$$4. \int_0^1 xe^x dx = \begin{Bmatrix} u = x \\ du = dx \\ dv = e^x dx \\ v = e^x \end{Bmatrix} = u \cdot v - \int v \, du = \left(xe^x - \int e^x dx\right)\Big|_0^1 =$$

$$= \left(xe^x - e^x\right)\Big|_0^1 = \left(1 \cdot e^1 - e^1 - (0e^0 - e^0)\right) = 1$$

$$5.\ \int_0^2 \frac{2x}{x^2-1}\,dx = \left\{\begin{array}{l} t = x^2-1 \\ dt = 2x\,dx \end{array}\right\} = \int \frac{dt}{t} = \ln|t| \Big|_0^2 =$$

$$x^2-1 \neq 0$$
$$x^2 = 1 \quad /\sqrt{\ }$$
$$x \neq 1$$

$$\ln|x^2-1| \Big|_0^2 = \left(\ln(2^2-1) - \ln(0^2-1)\right)\Big|_0^2 = 1{,}098612289$$

$\cancel{0}$

FUNKCIJA JE NEOGRANIČENA S PREKIDOM U $x = 1$, INTEGRAL JE NEPRAVI, A NEWTON-LEIBNITZOVA FORMULA NE VRIJEDI.

$$2.\ f(x,y) = \frac{1}{1+x^2+y^2}$$

$$\frac{df}{dx} = \frac{1'\,(1+\cancel{x^2}-y^2) - 1\cdot(1+x^2+y^2)'}{1+x^4+y^4}$$

$$\frac{df}{dx} = \frac{-2\cancel{x}}{1+x^{\cancel{4}}+y^4} = \frac{-2}{1+x^3+y^4} = 0 \Rightarrow \frac{1}{1+x^3+y^4} = 2 \quad /\cdot(1+x^3+y^4)$$

$\times$

$$1 = 2 + 2x^3 + 2y^4$$
$$-2x^3 = 2y^4 - 1 + 2$$
$$-2x^3 = 2y^4 + 1 \quad /:(-2)$$
$$x^3 = -y^4 - \frac{1}{2} \quad /\sqrt[3]{x}$$
$$x = \frac{-y^4 - \frac{1}{2}}{\sqrt[3]{x}}$$

$$\frac{df}{dy} = \frac{1'\,(1+x^2+y^2) - 1\cdot(1+x^2+y^2)'}{1+x^4+y^4}$$

$$= \frac{-2y}{1+x^4+y^{\cancel{4}}} = \frac{-2}{1+x^4+y^3} = 0$$

$$\frac{1}{1+x^4+y^3} = 2 \quad /\cdot(1+x^4+y^3)$$
$$1 = 2 + 2x^4 + 2y^3$$
$$\cancel{-2y^3 = 2x^4 + 1 \quad /:(-2)}$$
$$\cancel{y^3 = -x^4 - \frac{1}{2} \quad /\sqrt[3]{\ }}$$

$$1.\ y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$$

DALJE...

$\cancel{0}$

$$k^2 + 2k + 5 = 0$$
$$k_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$k = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot1\cdot5}}{2}$$
$$k_1 = \frac{-2+\sqrt{-16}}{2} = \frac{-2+4i}{2} \qquad k_2 = \frac{-2-4i}{2}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix	POPUNJAVA NASTAVNIK
IME I PREZIME: Ivan Vukašinu BROJ INDEKSA: 1720182-12	Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$. — 15
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Ukupno: 15

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

(2)

$$f(x,y) = \frac{1}{1+x^2+y^2}$$

$$\frac{df}{dx} = \frac{1'\cdot(1+x^2+y^2) - 1\cdot(1+x^2+y^2)'}{(1+x^2+y^2)^2}$$

$$= \frac{-1\cdot\left((1)' + (x^2)' + (y^2)'\right)}{(1+x^2+y^2)^2} = \frac{-2x}{(1+x^2+y^2)^2}$$

$$\frac{-2x}{(1+x^2+y^2)^2} = 0 \ / \cdot (1+x^2+y^2)^2$$

$$-2x = 0 \ / : -2$$

$$x = 0$$

$$\frac{df}{dy} = \frac{1'\cdot(1+x^2+y^2) - 1\cdot(1+x^2+y^2)'}{(1+x^2+y^2)^2}$$

$$= \frac{-2y}{(1+x^2+y^2)^2}$$

$$\frac{-2y}{(1+x^2+y^2)^2} = 0 \ / \cdot 1+x^2+y^2$$

$$-2y = 0 \ / : -2$$

$$y = 0$$

$$T_0(0,0)$$

(5) $\int_0^2 \frac{dx}{x^2-1}$ $\left[\begin{matrix} x=t \\ dx=dt \end{matrix}\right]$

$\int_0^2 \frac{dt}{t^2-1^2} = \left| \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \right| \Rightarrow \left| \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| \right|_0^2$

$\left| \frac{1}{2} \ln \left| \frac{x^2-1}{x^2+1} \right| \right|_0^2$

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$= \left[\frac{1}{2} \ln \left| \frac{4-1}{4+1} \right| - \frac{1}{2} \ln \left| \frac{0-1}{0+1} \right| \right]$

$= \left[\frac{1}{2} \ln \left| \frac{3}{5} \right| - \frac{1}{2} \ln \left| \overset{N/R}{-1} \right| \right]$

$\varnothing$

Ivan Vukašina

2.) Nastavak

$$\frac{d^2f}{dx^2} = \left(\frac{-2x}{(1+x^2+y^2)^2}\right)' = \frac{(-2x)'\cdot(1+x^2+y^2)^2 - \left[(-2x)\cdot\left((1+x^2+y^2)^2\right)'\right]}{(1+x^2+y^2)^4}$$

$$= \frac{-2(1+x^2+y^2)^2 - \left[(-2x)\cdot 2(1+x^2+y^2)\cdot(2x)\right.}{(1+x^2+y^2)^4}$$

$$= \frac{-2-2x^2-2y^2 - \left[-4x^2(2+2x^2+2y^2)\right]}{(1+x^2+y^2)^4}$$

$$= \frac{-2-2x^2-2y^2 - \left[-8x^2 - 8x^4 - 8x^2y^2\right]}{(1+x^2+y^2)^4}$$

$$= \frac{-2-2x^2-2y^2+8x^2-8x^4-8x^2y^2}{(1+x^2+y^2)^2}$$

$$= \frac{-2-6x^2-2y^2-8x^4-8x^2y^2}{(1+x^2+y^2)^4} \quad \begin{pmatrix} x=0 \\ y=0 \end{pmatrix}$$

$$\frac{d^2f}{dx^2} = \frac{-2}{1} = -2$$

$$\frac{d^2f}{dy^2} = \frac{-2y}{(1+x^2+y^2)^2} = \frac{(-2y)'\cdot(1+x^2+y^2)^2 - \left[(-2y)\cdot\left((1+x^2+y^2)^2\right)'\right]}{(1+x^2+y^2)^4}$$

$\cancel{1+x+y \cdot 1+x+y}$

$1+x+y \quad x+x^2+y \ldots$

$$= \frac{-2(1+x^2+y^2)^2 - \left[-2y\cdot 2(1+x^2+y^2)\cdot 2y\right]}{(1+x^2+y^2)^4}$$

$$= \frac{-2+8y(1+x^2+y^2)}{(1+x^2+y^2)^4} = -2 = \frac{d^2f}{dy^2}$$

$\longrightarrow$

$$\frac{d^2 f}{dxy} = \frac{-2x}{(1+x^2+y^2)^2} = \frac{(-2x)'\cdot(1+x^2+y^2) - [(-2x)\cdot(1+x^2+y^2)]'}{(1+x^2+y^2)^4}$$

$$= \frac{-[-2x \cdot 2(1+x^2+y) \cdot 2y]}{(1+x^2+y^2)^4} \quad \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$$= 0 \;//$$

$$\Delta \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \rightarrow$$ pozitivno znači ekstrem. ✓

$A = -2 < 0$ imamo maksimum u tački $T_0(0,0)$ ✓

③ $f(x,y) = 5x^3 + y^2 - 9$ $\quad T(3,1,\cancel{z}20)$

$T(3,1,127)$

$z - z_0 = f_x(x - x_0) \cdot f_y(y - y_0)$

$z_0 = 5(3)^3 + 1 - 9$

$z_0 = 127 \;//$

$f_x = \frac{df}{dx} = (5x^3 - y^2 - 9)$

$f_x = 15x^2 - 0 - 0 \;//$

$f_y = \frac{df}{dy} = (5x^3 - y^2 - 9)$

$f_y = 0 - 2y - 0 \;//$

$z - 127 = 15x(x-3) - 2y(y-1)$

$z - 127 = 15x^2 - 45x - 2y^2 + 2y$

$15x^2 - 45x - 2y^2 + 2y - z + 127 = 0 \;//$

Rt

✗

$$\frac{(x-3)}{15x} = \frac{(x-1)}{-2y} = \frac{(z-127)}{-1} \;//$$

✗ ✗

0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANĐELA ŠTUC **BROJ INDEKSA:** 17-1-0173-2013	

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Ukupno: 0

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x\ln\alpha}$
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$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$	

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: NIKOLA TOMASOV **BROJ INDEKSA:** 17-2-0161-2012	

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$. 15

2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15

3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = 5x^3y^2 - 9$ u točki $T(3,1,z_0)$. 15

4. $\displaystyle\int_0^1 xe^x \,\mathrm{d}x = ?$ 20

5. $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$ 15

6. $\displaystyle\int_{-1}^{+\infty} \frac{\mathrm{d}x}{1+x^2} = ?$ 20

Ukupno: 0

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x\ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x\ (\alpha > 0)$	$\alpha^x \ln \alpha$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

4. $$\int_0^1 x e^x dx = \int_0^1 x\, dx + \int_0^1 e^x dx = \frac{x^2}{2}\Big|_0^1 + e^x\Big|_0^1 + C$$

$$= \left(\frac{1}{2} - \frac{0}{2}\right) + \left(e^1 - e^0\right) = 2{,}2$$

5. $$\int_0^2 \frac{2x}{x^2-1} =$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix

IME I PREZIME: MATE RADAŠ **BROJ INDEKSA:** 17-2-0183-2012

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$.	15
2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$.	15
3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = 5x^3y^2 - 9$ u točki $T(3,1,z_0)$.	15
4. $\int_0^1 xe^x \, dx = ?$	20
5. $\int_0^2 \frac{2x}{x^2-1} = ?$	15
6. $\int_{-1}^{+\infty} \frac{dx}{1+x^2} = ?$	20
Ukupno:	0

f	$\frac{df}{dx}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
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$\sin x$	$\cos x$
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$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
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$\arctan x$	$\frac{1}{1+x^2}$

Tablica nekih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$	
$\int \cos x \, dx = \sin x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	

3. $f(x,y) = 5x^3y^2 - 9 \quad T(3,1,z_0)$

5. $\int_0^2 \frac{2x}{x^2-1} = ?$

$x = 2 \Rightarrow \frac{2\cdot 2}{2^2-1} = \frac{4}{3}$

$x = 0 \Rightarrow \frac{2\cdot 0}{0^2-1} = \frac{0}{0}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** iix

IME I PREZIME: IVAN DONAT GRŽAN **BROJ INDEKSA:** 57 648 - 2009

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti diferencijalnu jednadžbu: $y'' + 2y' + 5y = x^2 e^{3x} + \sin(2x)$. 15

2. Odrediti lokalne ekstreme funkcije $f(x,y) = \frac{1}{1+x^2+y^2}$. 15

3. Izračunati tangencijalnu ravninu na graf funkcije $f(x,y) = 5x^3y^2 - 9$ u točki $T(3,1,z_0)$. 15

4. $\displaystyle\int_0^1 xe^x \, \mathrm{d}x = ?$ 20

5. $\displaystyle\int_0^2 \frac{2x}{x^2-1} = ?$ 15

6. $\displaystyle\int_{-1}^{+\infty} \frac{\mathrm{d}x}{1+x^2} = ?$ 20

Ukupno:

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
$\log_\alpha x \ (\alpha > 0)$	$\frac{1}{x \ln \alpha}$
e^x	e^x
$\alpha^x \ (\alpha > 0)$	$\alpha^x \ln \alpha$
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Tablica nekih integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1}, \ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \sqrt{a^2-x^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$	