

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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Grupa  
XXOXO  
POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$f'''(t) - 4f'(t) = \cos(2t), \quad f(0) = f'(0) = f''(0) = 0.$$

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2. Izračunati  $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$  gdje je  $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$  i  $\partial K$  rub kugle  $K$  radijusa 1 s centrom u točki  $T(2, 1, 0)$ , a koji je orijentiran vanjskom normalom.

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3. Izračunati volumen tijela omeđenog valjkom  $x^2 + z^2 = 1$  i ravninama  $z = y$  i  $y = x - 2$ .

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4. Zadana je kružna uzvojnica (spirala) s jednadžbama  $x = \cos 2t$ ,  $y = \sin 2t$  i  $z = t$ . Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje.

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5. Izračunati  $\int_{\widehat{ABC}} y dx + y dy$  gdje je  $\widehat{ABC}$  krivulja koja ide bridovima trokuta s vrhovima  $A(0, 0, 0)$ ,  $B(1, 0, 0)$ ,  $C(0, 1, 0)$  usmjerena redom od vrha  $A$  preko  $B$  i  $C$  do ponovo vrha  $A$ . Koristiti Stokesovu formulu.

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Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln  \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln  \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln  \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln  \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x \sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Ukupno:

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Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$c$	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$t$	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
$t^n$	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$e^{-at}$	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

1.

$$f'''(t) - 4f'(t) = \cos(2t)$$

$$f(0) = f'(0) = f''(0) = 0$$

$$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0) - 4(sF(s) - f(0)) = \frac{s}{s^2 + 4}$$

$$s^3 F(s) - 4sF(s) = \frac{s}{s^2 + 4}$$

$$F(s)(s^3 - 4s) = \frac{s}{s^2 + 4} \quad \left/ \begin{matrix} (s^2 - 4) \\ (s^2 + 4) \end{matrix} \right.$$

$$\frac{(s-2)(s+2)}{s^2 + 2s - 2s - 4} = \frac{(s-2)(s+2)}{(s^2 - 4)}$$

$$F(s) = \frac{s}{(s^2 + 4)(s^3 - 4s)}$$

$$F(s) = \frac{s}{(s^2 + 4)s(s^2 - 4)} = \frac{s}{s(s-2)(s+2)(s^2 + 4)}$$

$$s = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{Ds + E}{s^2 + 4} \quad \left/ \cdot s(s-2)(s+2)(s^2 + 4) \right.$$

$$s = A(s^2 + 4)(s^2 - 4) + Bs(s+2)(s^2 + 4) + Cs(s-2)(s^2 + 4) + (Ds^2 + Es)(s^2 - 4)$$

$$s = (As^2 + 4A)(s^2 - 4) + (Bs^2 + 2Bs)(s^2 + 4) + (Cs^2 - 2Cs)(s^2 + 4) + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s = As^4 - 4As^2 + 4As^2 - 16A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 + 4Cs^2 - 2Cs^3 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s = As^4 - 16A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 + 4Cs^2 - 2Cs^3 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$A + B + C + D = 0 \quad 2B - 2C + E = 0 \quad 4B + 4C - 4D = 0 \quad 8B - 8C - 4E = 1$$

$$-16A = 0 \quad A = 0$$

1. - MASA VAK

$$2B - 2C + E = 0 \quad / \cdot (-4)$$

$$8B - 8C - 4E = 1$$

$$-8B + 8C - 4E = 0$$

$$8B - 8C - 4E = 1$$

$$-8E = 1$$

$$E = -\frac{1}{8}$$

$$B + C + D = 0 \quad / \cdot (-4)$$

$$4B + 4C - 4D = 0$$

$$-4B - 4C + 4D = 0$$

$$4B + 4C - 4D = 0$$

$$-8D = 0$$

$$D = 0$$

$$A = 0 \quad B = \frac{1}{32} \quad C = -\frac{1}{32} \quad D = 0, \quad E = -\frac{1}{8}$$

$$f(s) = 0 + \frac{1}{32} \cdot \frac{1}{s-2} - \frac{1}{32} \cdot \frac{1}{s+2} - \frac{1}{8} \cdot \frac{1}{s^2+4}$$

$$f(s) = \frac{1}{32} \cdot e^{2t} - \frac{1}{32} e^{-2t} - \frac{1}{8} \sin 2t$$

$$2B - 2C = \frac{1}{8}$$

$$B + C = 0$$

$$B = -C$$

$$-8B - 8C - 4 \cdot \left(-\frac{1}{8}\right) = 1$$

$$-16C + \frac{1}{2} = 1$$

$$-16C = 1 - \frac{1}{2}$$

$$-16C = \frac{1}{2}$$

$$C = -\frac{1}{32}$$

$$C = -\frac{1}{32}$$

$$B = \frac{1}{32}$$

PROVĚRA:  $f(0) = \frac{1}{32} - \frac{1}{32} = 0$   
 $f'(t) = \frac{1}{16} e^{2t} + \frac{1}{16} e^{-2t} - \frac{1}{4} \cos 2t$   
 $f'(0) = \frac{1}{16} + \frac{1}{16} - \frac{1}{4} = \frac{1}{8} - \frac{1}{4} = -\frac{1}{8}$

$$\mathcal{L}^{-1} \left[ \frac{1}{8} \frac{1}{s+4} \right] = \mathcal{L}^{-1} \left[ \frac{1}{16} \frac{2}{s+4} \right] = \frac{1}{16} \sin(2t)$$

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$$x^2 + z^2 = 1$$

$$z = y \quad y = x - 2$$

$$-x^2 + 1 - (x-2)$$

$$r^2 = 1$$

$$z = -x + 1$$

$$r = (0, 2\pi)$$

$$r = 1$$

$$z = \sqrt{-x^2 + 1}$$

$$r = (0, 1)$$

$$x = 2$$

$$V = \int_0^{2\pi} \int_0^1 \int_{x-2}^{\sqrt{-x^2+1}} (x) \cdot x \cdot dx \cdot dy = \int_0^{2\pi} \int_0^1 ((-x^2+1) - (x-2)) dx = \int_0^{2\pi} \int_0^1 (-x^2+1-x+2) dx = \int_0^{2\pi} \int_0^1 (x^2-x+1) dx =$$

$$V = 2\pi \left( -\frac{x^3}{3} - \frac{x^2}{2} + x \right) \Big|_0^1 = 2\pi \cdot \left( -\frac{1}{3} - \frac{1}{2} + 1 \right) = 2\pi \cdot \left( \frac{1}{3} - \frac{1}{2} + 1 \right) = 2\pi \cdot \left( \frac{2-3+6}{6} \right)$$

$$V = \frac{1}{3} \pi$$

TOČNO ŘEŠĚNÍ:  
 $\int_0^{2\pi} \int_0^1 \int_{r \cos \varphi}^{\sqrt{1-r^2}} r \, dy \, dr \, d\varphi = \dots$

$$\begin{cases} x = r \cos \varphi \\ z = r \sin \varphi \\ y = y \end{cases}$$

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$$f(t) = \int_0^{2\pi} (\sin^2 t - \cos^2 2t) dz - \int_0^{2\pi} (-2\cos t + 1) - (2\sin 2t + 2) dt = \int_0^{2\pi} (-2\cos t - 2\sin 2t - 1) dt$$

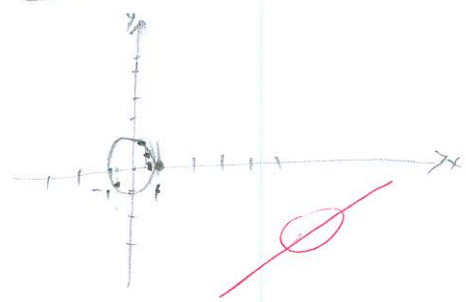
$$f(t) = (-2\cos t - 2\sin 2t - 1) \Big|_0^{2\pi} = -2\cos 6\pi - 2\sin 2 \cdot 6\pi - 1 - (-2\cos 0 - 2\sin 2 \cdot 0 - 1) =$$

$$f(t) = -2 - 1 + 2 + 1 = \underline{\underline{0}}$$

$z = 0$

$x = 1$

$y = 0$



180 - 60  
100  
- 360  
210  
- 360

$t = 10$

$x = 1$

$y =$

$t = 2$

$t = 30$

100

200

