MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTE RAŽOV **BROJ INDEKSA:** 5994

Grupa xxoxo

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - 4\,f'(t) = \cos(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y$ i $y=x-2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\widehat{ABC}} y\,dx + y\,dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: $\emptyset$

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1.

$f'''(t) - 4f'(t) = \cos(2t) \qquad f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - s^2\overset{=0}{f(0)} - s\overset{=0}{f'(0)} - \overset{=0}{f''(0)} - 4\left(sF(s) - \overset{=0}{f(0)}\right) = \frac{s}{s^2+4}$$

$$s^3F(s) - 4sF(s) = \frac{s}{s^2+4}$$

$(s-2)(s+2)$

$s^2 + \cancel{2s} - \cancel{2s} - 4$

(s^2-4)

$$F(s)(s^3-4s) = \frac{s}{s^2+4} \quad / : (s^3-4s)$$

$$F(s) = \frac{s}{(s^2+4)(s^3-4s)}$$

$$F(s) = \frac{s}{(s^2+4)\,s\,(s^2-4)} = \frac{s}{s\cdot(s-2)(s+2)(s^2+4)}$$

$$s = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+4} \quad / \cdot s(s-2)(s+2)(s^2+4)$$

$$s = A(s^2+4)(s^2-4) + Bs(s+2)(s^2+4) + Cs(s-2)(s^2+4) + (Ds^2+Es)(s^2-4)$$

$$s = (As^2+4A)(s^2-4) + (Bs^2+2Bs)(s^2+4) + (Cs^2-2Cs)(s^2+4) + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s = As^4 - \cancel{4As^2} + \cancel{4As^2} - 16A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 + 4Cs^2 - 2Cs^3 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s = As^4 - 16A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 + 4Cs^2 - 2Cs^3 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$A+B+C+D=0 \qquad 2B-2C+E=0 \qquad 4B+4C-4D=0 \qquad 8B-8C-4E=1$

$-16A=0 \qquad A=0$

ANTE RAŽOV

(1.) - NASTAVAK

$$2B-2C+E=0 \quad / \cdot(-4)$$
$$8B-8C-4E=1$$

$$-8B+8C-4E=0$$
$$8B-8C-4E=1$$
$$-8E=1$$
$$E=-\frac{1}{8}$$

$$2B-2C=\frac{1}{8}$$

$$B+C=0$$
$$B=-C$$
$$-8B-8C-4\cdot\left(-\frac{1}{8}\right)=1$$
$$-16C+\frac{1}{2}=1$$
$$-16C=1-\frac{1}{2}$$
$$-16C=\frac{1}{2}$$
$$C=-\frac{\frac{1}{2}}{16}$$
$$C=-\frac{1}{32}$$
$$B=\frac{1}{32}$$

$$B+C+D=0 \quad / \cdot(-4)$$
$$4B+4C-4D=0$$

$$-4B-4C+4D=0$$
$$4B+4C-4D=0$$
$$-8D=0$$
$$\boxed{D=0}$$

$$A=0 \quad B=\frac{1}{32} \quad C=-\frac{1}{32} \quad D=0, \quad E=-\frac{1}{8}$$

$$f(s)=0+\frac{1}{32}\cdot\frac{1}{s-2}-\frac{1}{32}\cdot\frac{1}{s+2}-\frac{1}{8}\frac{1}{s^2+4}$$

$$f(s)=\frac{1}{32}\cdot e^{2t}-\frac{1}{32}e^{-2t}-\frac{1}{8}\sin 2t$$

PROVJERA: $f(0)=\frac{1}{32}-\frac{1}{32}=0$

$$f'(t)=\frac{1}{16}e^{2t}+\frac{1}{16}e^{-2t}-\frac{1}{4}\cos 2t$$
$$f'(0)=\frac{1}{16}+\frac{1}{16}-\frac{1}{4}=\frac{1}{8}-\frac{1}{4}=-\frac{1}{8}$$

$$\mathcal{L}^{-1}\left[\frac{1}{8}\frac{1}{s^2+4}\right]=\mathcal{L}^{-1}\left[\frac{1}{16}\frac{2}{s^2+4}\right]=\frac{1}{16}\sin(2t)$$

(3)

$$x^2+z^2=1 \qquad z=y \qquad y=x-2$$

$$-x^2+1-(x-2)$$

$$r^2=1 \qquad z=-x+1 \qquad \varphi-(0,2\pi)$$
$$r=1 \qquad z=\sqrt{-x^2+1} \qquad r-(0,1)$$
$$x-2$$

$$V=\int_0^{2\pi}\int_0^1\int_{0}^{x-2} x\, x\,dr\,d\varphi=\int_0^1\left[(-x^2+1)-(x-2)\right]dx=\int_0^{2\pi}\cdot\int_0^1\left(-x^2+1-x+2\right)dx=\int_0^{2\pi}\int_0^1\left(-x^2-x+1\right)dx=$$

?

$$V=2\pi\left(-\frac{x^3}{3}-\frac{x^2}{2}+x\right)\Big|_0^1=2\pi\cdot\left(-\frac{1^3}{3}-\frac{1^2}{2}+1\right)=2\pi\cdot\left(-\frac{1}{3}-\frac{1}{2}+1\right)=2\pi\cdot\left(\frac{-2-3+6}{6}\right)$$

$$V=\frac{1}{3}\pi$$

TOČNO RJEŠ:

$$\begin{cases} x=r\cos\varphi \\ z=r\sin\varphi \\ y=y \end{cases}$$

$$V=\int_0^{2\pi}\int_0^1\int_{\cancel{r\sin\varphi}\ r\cos\varphi-2}^{r\sin\varphi} r\,dy\,dr\,d\varphi=\ldots$$

④

$$f(t) = \int_0^{2\pi} (\sin^2 t - \cos^2 2t)\, dz = \int_0^{2\pi} \big((-2\cos t + 1) - (2\sin 2t + 2)\big) dz = \int_0^{2\pi} (-2\cos t - 2\sin 2t - 1)\, dz =$$

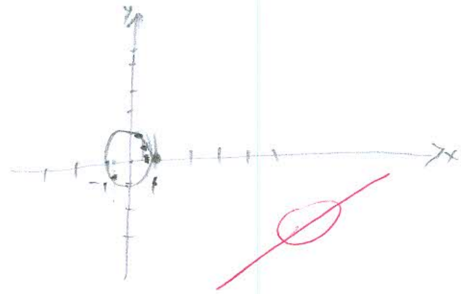
$$f(t) = (-2\cos t - 2\sin 2t - 1)\Big|_0^{2\pi} = -2\cos 6\pi - 2\sin 2\cdot 6\pi - 1 - (-2\cos 0 - 2\sin 2\cdot 0 - 1) =$$

$$f(1) = -2 - 1 + 2 + 1 = \underline{\underline{0}}$$

$x = \underline{\underline{1}}$ $\quad$ $y = \underline{\underline{0}}$

$z = 0$

180 · 6 =
1080
− 360
720
− 360



t = 10

x = 1

y =

t = 2

t = 30

100

800

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IME I PREZIME: MATEO BOBAČEK **BROJ INDEKSA:** 17-2-0113-2011

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

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$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

MATEO BOBAČEK

① $f'''(t) - 4f'(t) = \cos(2t)$ $\qquad f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - \cancel{s^2 f(0)} - \cancel{s f'(0)} - \cancel{f''(0)} - 4sF(s) + \cancel{f(0)} = \frac{s}{s^2+2^2}$$

$$s^3F(s) - 4sF(s) = \frac{s}{s^2+2^2}$$

$$F(s)\,(s^3 - 4s) = \frac{s}{s^2+4}$$

$$F(s)\, s(s^2-4) = \frac{s}{(s^2+4)} \quad / \cdot \frac{1}{s(s^2-4)}$$

$$F(s) = \frac{\cancel{s}}{\cancel{s}(s^2-4)(s^2+4)} = \frac{1}{(s^2-4)(s^2+4)} = \frac{1}{(s-2)(s+2)(s^2+4)} = \frac{A}{s-2} + \frac{B}{s+2} + \frac{Cs+D}{s^2+4} \quad / \cdot (s-2)(s+2)(s^2+4)$$

$$1 = A(s+2)(s^2+4) + B(s-2)(s^2+4) + (Cs+D)(s-2)(s+2)$$

$$1 = A(s^3+4s+2s^2+8) + B(s^3+4s-2s^2-8) + (Cs+D)(s^2-4)$$

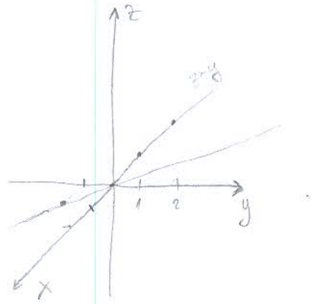
$$1 = As^3 + 4As + 2As^2 + 8A + Bs^3 + 4Bs - 2Bs^2 - 8B + Cs^3 - 4Cs + Ds^2 - 4D$$

(3.) $x^2+z^2=1$ $\quad z=y \quad y=x-2$

$x^2-1=-z^2$

$-x^2+1=z^2$

$z=\sqrt{-x^2+1}$



4. $x = \cos 2t \quad y = \sin 2t \quad , \quad z = t$

MATEJ BOBACEK

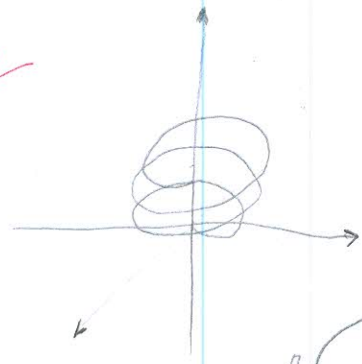
$$r(t) = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix} \qquad r'(t) = \begin{bmatrix} -\sin t \cdot 2 \\ \cos t \cdot 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2\sin t \\ 2\cos t \\ 1 \end{bmatrix}$$

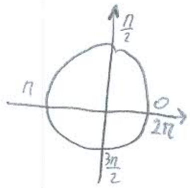
$$\| r'(t) \| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1^2} = \sqrt{4\sin^2 t + 4\cos^2 t + 1} = \sqrt{4(\sin^2 t + \cos^2 t) + 1}$$

$$= \sqrt{4+1} = \sqrt{5} \qquad t \in [0, 6\pi]$$

$$\int_0^{6\pi} \sqrt{5}\, dt = \sqrt{5}t \Big|_0^{6\pi} = \sqrt{5} \cdot 6\pi = 6\pi\sqrt{5}$$

20





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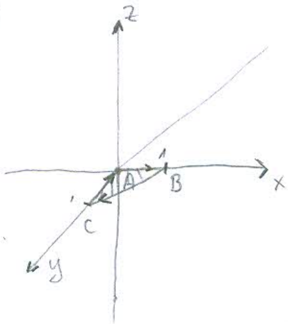


$A(0,0) \quad B(1,0)$

$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$(1-0)(y-0) = (0-0)(x-0)$

$\overline{AB} \ldots y = 0$

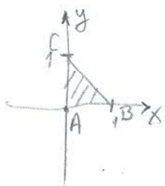
$B(1,0) \quad C(0,1)$

$(0-1)(y-0) = (1-0)(x-1)$

$-y = x - 1$

$\overline{BC} \ldots y = -x + 1$

0



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POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: BORIS KREŠIĆ **BROJ INDEKSA:** 17-1-0022-2010

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$
c	$\frac{c}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$
e^{-at}	$\frac{1}{s+a}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$

$f(t)$	$F(s) = \mathcal{L}[f](s)$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$e^{-at}\,f(t)$	$F(s+a)$
$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
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$f'(t)$	$sF(s) - f(0)$
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$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa XXOXO
IME I PREZIME: Ivan Frühwirth **BROJ INDEKSA:** 0269024744 56183-2008.	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - 4\,f'(t) = \cos(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\displaystyle\iint_{\partial K} \mathbf{F}\cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\widehat{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: ∅

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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<u>MATEMATIKA 3</u>: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: RIKARDO PEROVIĆ **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: 20

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RIKARDO PEROVIC

4.) $x = \cos 2t$ 3 namotaja

$y = \sin 2t$ $f \in [0, 6\pi]$ ✓

$z = t$

$$r(t) = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix} \qquad r'(t) = \begin{bmatrix} -2\sin t \\ 2\cos t \\ \cancel{0} \end{bmatrix} \leftarrow 1$$

∅

$$\|r'\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2} = \sqrt{4\sin^2 t + 4\cos^2 t}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t)} = \sqrt{4} = 2$$

$$I = 2\int_0^{6\pi} dt = 2 \cdot t \Big|_0^{6\pi} = 2 \cdot 6\pi = 12\pi$$

3.) $x^2 + z^2 = 1$ $z = y$ i $y = x - 2$

$x = r\cos f$ $y = r\cos f - 2$ $f \in [0, 2\pi]$

$z = r\sin f$ $y = r\sin f$ $r \in [0, 1]$

$y = y$ $(r\cos f)^2 + (r\sin f)^2 = 1$ $y \in [r\cos f - 2, r\sin f]$

$r^2\cos^2 f + r^2 \sin f^2 = 1$

$r^2(\cos^2 f + \sin^2) = 1$

$r^2 = 1$

$r = \sqrt{1} = 1$

$$V = \int_0^{2\pi}\int_0^1\int_{r\cos f - 2}^{r\sin f} r\,dy\,dr\,df = \int_0^{2\pi}\int_0^1 rf \Big|_{r\cos f - 2}^{r\sin f} dy\,dr = \int_0^{2\pi}\int_0^1 r(r\sin f - r\cos f + 2)\,dy\,dr$$

✓

$$V = \int_0^{2\pi}\int_0^1 (r^2\sin f - r^2\cos f + 2r)\,dy\,dr = \int_0^{2\pi} -\frac{r^3}{3}\cos f - \frac{r^3}{3}\sin f + 2\cdot\frac{r^2}{2}\Big|_0^1$$

$$V = \int_0^{2\pi}\left(-\frac{1}{3}\cos f - \frac{1}{3}\sin f + 1\right)df = -\frac{1}{3}\sin f + \frac{1}{3}\cos f + f \Big|_0^{2\pi}$$

$\rightarrow$ IZA

$$V = -\frac{1}{3}\sin f + \frac{1}{3}\cos f + f \Big|_0^{2\pi} \; \cancel{-\frac{1}{3}\cdot 0} + \frac{1}{3} + 2\pi - \left(0 + \frac{1}{3} + 0\right)$$

$$V = \frac{1}{3} + 2\pi - \frac{1}{3} = 2\pi \qquad \checkmark$$

20

RIKARDO PEROVIĆ

1.) $f'''(t) - 4f'(t) = \cos(2t)$

$f(0)=0$
$f'(0)=0$
$f''(0)=0$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - 4(sF(s) - f'(0)) = \frac{s}{s^2+4}$$

$$s^3F(s) - s^2\cancel{f(0)}^{0} - s\cancel{f'(0)}^{0} - \cancel{f''(0)}^{0} - 4sF(s) + 4\cancel{f'(0)}^{0} = \frac{s}{s^2+4}$$

$$s^3F(s) - 4sF(s) = \frac{s}{s^2+4}$$

$$F(s)(s^3-4s) = \frac{s}{s^2+4} =$$

$$F(s) = \frac{s}{(s^3-4s)(s^2+4)} = \frac{s}{s(s^2-4)(s^2+4)} = \frac{s}{s\underbrace{(s-2)(s+2)}_{(s^2-4)}(s^2+4)}$$

$$s^3-4s = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+4}$$

$$s^3-4s = A(s^2-4)(s^2+4) + Bs((s+2)(s^2+4)) + Cs((s-2)(s^2+4)) + (Ds+E)(s^3-4s)$$

$$s^3-4s = A(s^4+4s^2-4s^2-8) + Bs(s^3+4s+2s^2+8) + Cs(s^3-2s^2+4s-8) +$$
$$Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s^3-4s = As^4 + 4As^2 - 4As^2 - 8A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 - 2Cs^3 + 4Cs^2$$
$$- 8sC + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$0 = A + B + C + D \Rightarrow -2 = B + C + D$

$1 = 2B - 2C + E$

$0 = 4B + 4C - 4D$

$-4 = 8B - 8C - 4E$

$0 = -8A \ /:(-8)$

$\boxed{A=0}$

$-2 = B + C + D \ /\cdot(-1)$
$0 = 4B + 4C - 4D \ /:(4)$

$2 = -B - C - D$
$0 = B + C - D$

$2 = -2D \ /:(-2)$

$\boxed{D=-1}$

$-2 = B + C - 1$
$B + C = -1$
$B = -1 - C$

$1 = 2(-1-C) - 2C + 0$
$1 = -2 - 2C - 2C$
$1 = -2 - 4C$
$3 = -4C$
$\boxed{C = -\frac{3}{4}}$

$1 = 2B - 2C + E \ /\cdot(-4)$
$-4 = 8B - 8C - 4E$

$\cancel{-4} = -8B + 8C - 4E$
$\cancel{-4} = 8B - 8C - 4E$
$0 = -8E \ /:(-8)$
$\boxed{E=0}$

$-2 = B - \frac{3}{4} - 1$

$B = -2 + \frac{3}{4} + 1$

$\boxed{B = -\frac{1}{4}}$

$$f = \cancel{\frac{0}{s}} - \frac{1}{4} \cdot \frac{1}{s-2} - \frac{3}{4} \cdot \frac{1}{s+2} - \frac{1}{s^2+4} + \cancel{\frac{0}{s^2+4}}^{\,0}$$

$$f = -\frac{1}{4} \cdot e^{-2t} - \frac{3}{4} \cdot e^{2t} - 2 \cdot \sin 2t$$

PROVJERA

$f(0) = -\frac{1}{4} - \frac{3}{4} - 0 = -1$

POGREŠNO!

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**		Grupa xxoxo
IME I PREZIME: MARIN GVOZDEN	**BROJ INDEKSA:** 17-2-0137-2011.	POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 20

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$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

(1.) $f'''(t) - 4f'(t) = \cos(2t)$ $\qquad f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - s^2\underset{0}{\underset{\shortparallel}{f(0)}} - s\underset{0}{\underset{\shortparallel}{f'(0)}} - \underset{0}{\underset{\shortparallel}{f''(0)}} - 4\left(sF(s) - \underset{0}{\underset{\shortparallel}{f(0)}}\right) = \frac{s}{s^2+4}$$

$$s^3F(s) - 4sF(s) = \frac{s}{s^2+4}$$

$$F(s)(s^3 - 4s) = \frac{s}{s^2+4}$$

$$F(s)\,s(s^2-4) = \frac{s}{s^2+4}$$

$$F(s) = \frac{s}{s(s^2-4)(s^2+4)}$$

$$s = \frac{A}{s} + \frac{B}{(s^2-4)} + \frac{Cs+D}{(s^2+4)} \qquad /\cdot s(s^2-4)(s^2+4)$$

$$s = A(s^2-4)(s^2+4) + B(s^2+4)\cdot s + (Cs+D)\cdot(s^2-4)\cdot s$$

$$s = A(s^4 + \cancel{4s^2} - \cancel{4s^2} - 16) + (Bs^2 + 4B)\cdot s + (Cs^3 - 4Cs + Ds^2 - 4D)\cdot s$$

$$s = \underline{As^4} - 16A + \underline{Bs^3} + \underline{4Bs} + \underline{Cs^4} - \underline{4Cs^2} + \underline{Ds^3} - \underline{4Ds}$$

$A + C = 0$

$B + D = 0$

$-4C = 0$

$4B - 4D = 1$

$-16A = 0$

$$\begin{array}{r} 4B - 4D = 1 \\ B + D = 0 \\ \hline 5B - 3D = 1 \end{array}$$

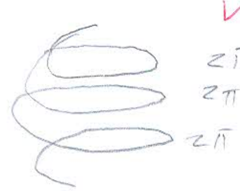
$$f(t) = \frac{0}{s} + \frac{}{(s^2-4)} + \frac{0+}{(s^2+4)}$$

MARIN GVOZDOV

(1.)

$$r = \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -2\sin 2t \\ 2\cos 2t \\ 1 \end{pmatrix}$$

SKICA:



$t \in [0, 6\pi]$

$$\|r'(t)\| = \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2 + 1^2}$$
$$= \sqrt{4\sin^2 2t + 4\cos^2 2t + 1}$$
$$= \sqrt{4\cdot(1)+1} = \sqrt{5}$$

$$\int_0^{6\pi} \sqrt{5}\,dt = \sqrt{5}\cdot t = \sqrt{5}\cdot 6\pi = 6\sqrt{5}\,\pi$$

20

*NASTAVAK 1.

$$\frac{s}{s(s^2-4)(s^2+4)} = \frac{s}{s(s-2)(s+2)(s^2+4)}$$

$$s = \frac{A}{s} + \frac{B}{(s-2)} + \frac{C}{(s+2)} + \frac{Ds+E}{(s^2+4)} \quad / \cdot s(s-4)(s+4)(s^2+4)$$

$$s = A(s-2)(s+2)(s^2+4) + Bs(s+2)(s^2+4) + Cs(s-2)(s^2+4) + (Ds+E)\cdot s\cdot(s-4)(s+4)$$

$$s = A(s^4 + \cancel{4s} - \cancel{4s} - 16) + (Bs^2 + 2Bs)(s^2+4) + (Cs^2 - 2Cs)(s^2+4) + (Ds^2+Es)(s^2-4)$$

$$s = As^4 - 16A + Bs^4 + 2Bs^3 + 4Bs^2 + 8Bs + Cs^4 - 2Cs^3 + 4Cs^2 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$A+B+C+D=0$
$2B-2C+E=0$
$4B+4C-4D=0$
$8B-8C-4E=1$

$\boxed{A=0}$

$B+C+D=0$
$4B+4C-4D=0$
$5B+5C-3D=0$

$2B-2C+E=0$
$8B-8C-4E=1$
$10B-10C-3E=1$

$2B-2C+E=0 \,/\cdot 2$
$4B-4C+2E=0$
$4B+4C-4D=0$

MARIN GVOZDEN

1.

$2B-2C+E=0 \quad /\cdot(-4)$

$-8B+8C-4E=0$

$8B-8C-4E=1$

$-8E=1$

$\boxed{E=-\frac{1}{8}}$

$B+C+D=0 \quad /\cdot(-4)$

$-4B-4C-4D=0$

$4B+4C-4D=0$

$\boxed{D=0}$

$B+C=0 \quad /\cdot 8$

$8B-\cancel{8}C=\frac{3}{8}$

$8B+\cancel{8}C=\frac{9}{8}$

$8B=\frac{3}{2}$

$\boxed{B=\frac{3}{16}}$

$A+B+C+D=0$

$0+\frac{3}{16}+C+0=0$

$\boxed{C=-\frac{3}{16}}$

$$f(t)=\frac{0}{s}+\frac{3}{16}\frac{1}{(s-2)}-\frac{3}{16}\frac{1}{(s+2)}-\frac{1}{8}\frac{1}{(s^2+4)}$$

$$f(t)=\frac{3}{16}e^{2t}-\frac{3}{16}e^{-2t}-\frac{1}{8}\sin t$$

PROVJERA:

$f'(t)=\frac{3}{8}e^{2t}+\frac{3}{8}e^{-2t}-\frac{1}{8}\cos t$

$f(0)=\frac{3}{16}-\frac{3}{16}-0=0$ ✓

$f'(0)=\frac{3}{8}+\frac{3}{8}-\frac{1}{8}=\frac{5}{8}\neq 0$ ✗ $\emptyset$

5. $$\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y\,dy$$

$$W = \begin{bmatrix} \frac{\sin y}{x} \\ \ln x \cos y \end{bmatrix} = -\mathrm{grad}\, f$$

sin
cos
sn

$$\partial f(x) = \frac{\sin y}{x}$$

$$\partial f(y) = -\ln x \cos y$$

?

MARIN VUČEDEL

(2.) $F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ $\quad F \text{ div.} = \frac{\partial(x^2+y^2)}{\partial x} + \frac{\partial(z)}{\partial y} + \frac{\partial(1)}{\partial z} = 2x$ ✓

$x = r\cos\varphi + 2$ $\quad x^2+y^2+z^2 = 1$

$y = r\sin\varphi + 1$ $\quad r^2+z^2 = 1$ ✓

$r = 1$ $\quad z^2 = 1 - r^2$ ✓

$r \in [0,1]$ ✓ $\quad z = 1 - r$ ✗

$\varphi \in [0, 2\pi]$ ✓ $\quad z \in [r, 1-r]$

$z \in [-\sqrt{1-r^2}, \sqrt{1-r^2}]$

$\emptyset$

$$\iint_{\partial K} F \cdot dS = \int_0^{2\pi}\int_0^1\int_{r}^{1-r} \left(2\cdot(r\cos\varphi+2)\right) r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^1\int_r^{1-r} (2r\cos\varphi + 4)\cdot r\,dz\,dr\,d\varphi$$

✗

$$= \int_0^{2\pi}\int_0^1\int_r^{1-r} (2r^2\cos\varphi + 4r)\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r)\cdot[1-r-r]\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r)\cdot(1-2r)\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r - 4r^3\cos\varphi - 8r^2)\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^1 (-4r^3\cancel{\cos\varphi} + 2r^2\cancel{\cos\varphi} - 8r^2 + 4r)\,dr\,d\varphi = \int_0^{2\pi} \left(-\frac{8}{3}r^3 + 2r^2\right)\Big|_0^1 d\varphi =$$

$$= \int_0^{2\pi} \left(-\frac{8}{3} + 2\right) d\varphi = \int_0^{2\pi} -\frac{2}{3}\,d\varphi = -\frac{4}{3}\pi$$

(3.)

$x^2 + z^2 = 1^2 \leftarrow$ VALJAK

$r^2 = 1$

$r = \pm 1$

$z = y$

$y = x - 2$

$z = x - 2$

$r \in [0, 1]$

$\varphi \in [0, 2\pi]$

$z \in [r\sin\varphi, r\cos\varphi - 2]$

$$V = \int_0^{2\pi}\int_0^1\int_{r\sin\varphi}^{r\cos\varphi - 2} r\, dz\, dr\, d\varphi = \int_0^{2\pi}\int_0^1 r \cdot [r\cos\varphi - 2 - r\sin\varphi]\, dr\, d\varphi =$$

$$\int_0^{2\pi}\int_0^1 (r^2\cos\varphi - r^2\sin\varphi - 2r)\, dr\, d\varphi = \int_0^{2\pi} \left(\frac{r^3}{3}\cos\varphi - \frac{r^3}{3}\sin\varphi - r^2\right)\Big|_0^1 d\varphi$$

$$\int_0^{2\pi} \left(\frac{1}{3}\cos\varphi - \frac{1}{3}\sin\varphi - 1\right) d\varphi = \left(\frac{1}{3}\sin\varphi + \frac{1}{3}\cos\varphi - \varphi\right)\Big|_0^{2\pi}$$

$$= \left(\frac{1}{3}\sin 2\pi - \frac{1}{3}\cos 2\pi - 2\pi\right) - \left(\frac{1}{3}\sin 0 + \frac{1}{3}\cos 0 - 2\pi\right)$$

$$= \left(0 - \frac{1}{3} - 2\pi\right) - \left(0 + \frac{1}{3} - 2\pi\right) = -\frac{1}{3} - \cancel{2\pi} - \frac{1}{3} + \cancel{2\pi} = -\frac{2}{3}$$

0

VOLUMEN NIKAD NIJE NEGATIVAN!!!