MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** JOSIP ŠIMIČEV **BROJ INDEKSA:**	Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. ~~20~~

3. Zadan je trokut s vrhovima $A(-2, 4)$ $B(10, 5)$ i $C(0, -1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. ~~20~~ 10

4. Izračunati integral funkcije $f(x, y) = \dfrac{2}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x, y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ ~~20~~

Ukupno: 30

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \; n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1 - at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$

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1)
$$f'''(t) + f''(t) = \sin(2t) \qquad f'(0)=0 \text{ i } f(0) = f''(0) = 1$$

$$s^3 f(s) - s^2 f(0) - s f'(0) - f''(0) + s^2 f(s) - s f(0) - f'(0) = \frac{2}{s^2+2^2}$$

$$s^3 f(s) - s^2 - 0 - 1 + s^2 f(s) - s - 0 = \frac{2}{s^2+4}$$

$$f(s)(s^3 + s^2) = \frac{2}{s^2+4} + s^2 + 1 + s$$

$$f(s)(s^3+s^2) = \frac{2+s^4+4s^2+s^2+4+s^3+4s}{s^2+4} = \frac{s^4+s^3+5s^2+4s+6}{s^2+4} \quad / : (s^3+s^2)$$

$$f(s) = \frac{s^4+s^3+5s^2+4s+6}{s^2(s+1)(s^2+4)}$$

$$\frac{s^4+s^3+5s^2+4s+6}{s^2(s+1)(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+1)} + \frac{Ds+E}{(s^2+4)} \quad / \cdot s^2(s+1)(s^2+4)$$

$$s^4+s^3+5s^2+4s+6$$
$$= As^4 + 4As^2 + 4As^3 + 4As + Bs^3 + Bs^2 + 4Bs + 4B + Cs^4 + 4Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$$

$A + C + D = 1$

$4A + B + D + E = 1$

$4A + B + 4C + E = 5$

$4A + 4B = 4$

$4B = 6$

$\boxed{B = \frac{6}{4} = \frac{3}{2}}$

$4A + 4B = 4 / : 4$

$A + B = 1$

$A = 1 - \frac{3}{2} = \boxed{-\frac{1}{2} = A}$

$A + C + D = 1$

$C + D = \frac{3}{2}$

$C = \frac{3}{2} - D$

$4A + B + D + E = 1$

$- \; 4A + B + 4C + E = 5$

$D - 4C = -4$

$-4C = -4 - D / : (-1)$

$4C = 4 + D$

$\frac{3}{2} - D = 4 + D$

$-D - D = 4 - \frac{3}{2} =$

$-2D = \frac{5}{2} / : (-2)$

$D = -\frac{5}{4} \qquad C = \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$

$4A + B + D + E = 1$

$E = -2 - \frac{3}{2} + \frac{5}{4} - \frac{11}{4} = -5$

$A = -\frac{1}{2}$ $\quad B = \frac{3}{2}$ $\quad C = \frac{11}{4}$ $\quad D = -\frac{5}{4}$ $\quad E = -5$

$$= -\frac{1}{2} s + \frac{3}{2}\frac{1}{s^2} + \frac{11}{4}\frac{1}{(s+1)} + \frac{5}{4}\frac{s}{(s+2^2)} - 5\frac{1}{(s+2^2)}$$

$$f(t) = -\frac{1}{2} + \frac{3}{2}t + \frac{11}{4}e^{-t} - \frac{5}{4}\cos(2t) - 5\sin(2t)$$

$$f'(t) = \frac{3}{2} - \frac{11}{4}e^{-t} + \frac{10}{4}\sin(2t) - 10\cos(2t)$$

0

PROVJERA:

$f(0) = -\frac{1}{2} + \frac{11}{4} - \frac{5}{4} = \frac{-2+11-5}{4} = \frac{4}{4} = 1$ ✓

$f'(0) = \frac{3}{2} - \frac{11}{4} - 10 = \frac{6-11-40}{4} = \frac{-45}{4} \neq 0$ ✗

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5. $f(x,y) = -y \qquad x \dots \begin{cases} x = \sin y \\ y = \frac{\pi}{2}x \end{cases}$

$x = \sin y \qquad y = \frac{\pi}{2}x = \frac{\pi}{2}\sin y$

$x = \sin\frac{\pi}{2}x$

$$\int_0^{\frac{\pi}{2}}\int_0^{\sin\frac{\pi}{2}} -y\,dy\,dx = \int_0^{\frac{\pi}{2}} -\frac{y^2}{2}\Big|_0^{\sin\frac{\pi}{2}} dx = \int_0^{\frac{\pi}{2}} -\sin^2\frac{\pi}{2}\cdot\frac{1}{2}\,dx = \cos^2\frac{\pi}{2}x\cdot\frac{1}{2}x\Big|_0^{\frac{\pi}{2}} = \cos^2\frac{\pi}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\cdot\frac{\pi}{2}$$

$$= \cos^2\frac{2\pi}{4} - \frac{1}{2}\frac{\pi}{2}$$

$$= -\frac{1}{4}\pi$$

✗

0

4. $f(x,y) = \frac{2}{\sqrt{x^2+y^2}} \qquad \varphi \in [0, \frac{3\pi}{2}] \quad r = 2 \quad r \in [0,2] \qquad x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\varphi$

$$\iint \frac{2}{\sqrt{x^2+y^2}}\,dx\,dy = \int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{\sqrt{r^2}}\cdot r\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2r}{r}\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}} 2r\Big|_0^2 d\varphi$$

$$= \int_0^{\frac{3\pi}{2}} 2\cdot 2\,d\varphi = 4\cdot\frac{3\pi}{2} = \frac{12\pi}{2} = 6\pi$$ ✓ 20

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2) $x^2+y^2=5z \quad z\leq 1$

$5z=r^2$

$z=\frac{r^2}{5} \quad z\in\left[\frac{r^2}{5},1\right]$

$r^2=5z$

$r^2=5$

$r=\sqrt{5} \quad r\in[0,\sqrt{5}]$

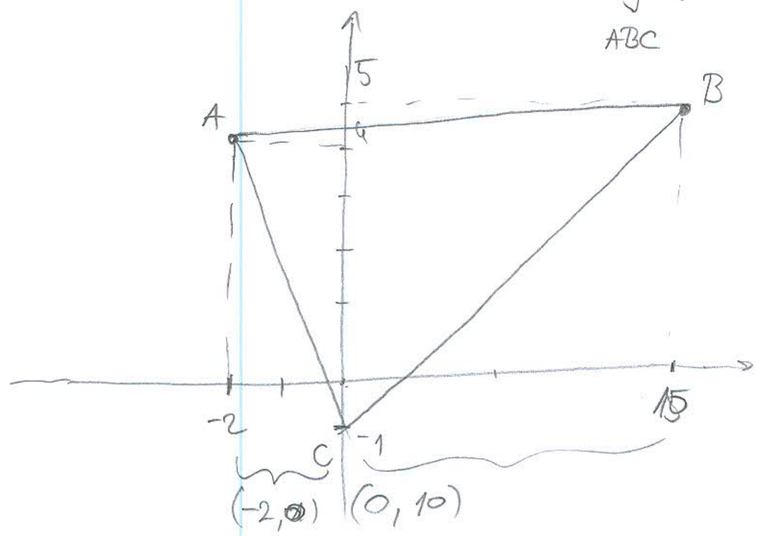
$\varphi\in[0,2\pi]$

$$P=\int_0^{2\pi}\int_0^{\sqrt{5}}\int_{\frac{r^2}{5}}^{1} r\, dz\, dr\, d\varphi=\int_0^{2\pi}\int_0^{\sqrt{5}} r\cdot z\Big|_{\frac{r^2}{5}}^{1} dr\, d\varphi=\int_0^{2\pi}\int_0^{\sqrt{5}} r\cdot\left(1-\frac{r^2}{5}\right) dr\, d\varphi$$

$$=\int_0^{2\pi}\int_0^{\sqrt{5}} r-\frac{1}{5}r^3\, dr\, d\varphi=\int_0^{2\pi}\frac{r^2}{2}-\frac{1}{5}\frac{r^3}{3}\Big|_0^{\sqrt{5}} d\varphi=\int_0^{2\pi}\frac{5}{2}-\frac{1}{5}\frac{(\sqrt{5})^3}{3}\, d\varphi$$

$$=\int_0^{2\pi}\frac{5}{2}-\frac{\sqrt{5}}{3}\, d\varphi=\cancel{2}\pi\cdot\frac{5}{\cancel{2}}-\frac{2\sqrt{5}}{3}\pi=5\pi-\frac{2\sqrt{5}\pi}{3}\approx 11.024$$

3. $A(-2,4)$, $B(10,5)$, $C(0,-1)$ $\oint\limits_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$



$\overline{AB}$ ... $A(-2,4)$ $B(10,5)$

$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(10+2)(y-4) = (5-4)(x+2)$

$12y - 48 = x+2$

$12y = x + 50 \;/:12$

$y = \frac{1}{12}x + \frac{50}{12}$

$\overline{BC}$ ... $B(10,5)\,C(0,-1)$

$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(0-10)(y-5) = (-1-5)(x-10)$

$-10y + 50 = -6x + 60$

$-10y = -6x + 10 \;/:10$

$y = \frac{6}{10}x + 1$

$\overline{AC}$ ... $A(-2,4)\,C(0,-1)$

$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(0+2)(y-4) = (-1-4)(x+2)$

$2y - 8 = -5x - 10$

$2y = -5x - 2 \;/:2$

$y = -\frac{5}{2}x - 1$

GREENOVA F.

$P(x,y) = (x^2-y)$

$Q(x,y) = \sin(y^3)$

$$\frac{Q(\sin(y^3))}{\partial x} - \frac{P(x^2-y)}{\partial y} = 0 + 1 = 1$$

10

$$= \int_{-2}^{0}\int_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{50}{12}} 1\,dy\,dx + \int_{0}^{10}\int_{\frac{6}{10}x+1}^{\frac{1}{12}x+\frac{50}{12}} 1\,dy\,dx = \int_{-2}^{0} y\Big|_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{50}{12}} dx + \int_{0}^{10} y\Big|_{\frac{6}{10}x+1}^{\frac{1}{12}x+\frac{50}{12}} dx$$

10

$$= \int_{-2}^{0} \left(\frac{1}{12}x + \frac{50}{12}\right) + \frac{5}{2}x + 1 \,dx + \int_{0}^{10} \frac{1}{12}x + \frac{50}{12} - \frac{6}{10}x + 1\,dx$$

$$= \int_{-2}^{0} \frac{31}{12}x + \frac{31}{6}\,dx + \int_{0}^{10} -\frac{31}{60}x + \frac{31}{6}\,dx = \left(\frac{31}{12}\cdot\frac{x^2}{2} + \frac{31}{6}x\right)\Big|_{-2}^{0} + \left(-\frac{31}{60}\frac{x^2}{2} + \frac{31}{6}x\right)\Big|_{0}^{10}$$

$$= \left(\frac{31}{12}\cdot\frac{(-2)^2}{2} + \frac{31}{6}(-2)\right) + \left(-\frac{31}{60}\frac{(10)^2}{2} + \frac{31}{6}\cdot 10\right)$$

$$= \frac{-31}{6} + \frac{155}{6} = \frac{62}{3}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: JASMIN NEKIĆ **BROJ INDEKSA:** 17-1-0058-2011	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$.   ~~20~~
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$.   20
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$.   20
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu.   20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

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1. $f'''(t) + f''(t) = \sin(2t)$, $f'(0)=0$ $f(0)=f''(0)=1$

$$\left(s^3F(s) - s^2f(0) - sf'(0) - f''(0)\right) + \left(s^2F(s) - sf(0) - f'(0)\right) = \frac{2}{s^2+2^2}$$

$$s^3F(s) - s^2 - 0 - 1 + s^2F(s) - 1 - 0 = \frac{2}{s^2+2^2}$$

$$s^3F(s) + s^2F(s) - s^2 - 2 = \frac{2}{s^2+2^2}$$

$$s^3F(s) + s^2F(s) = \frac{2}{s^2+4} + s^2 + 2 \;/ : s^3$$

$$F(s) = \frac{2}{s^3(s^2+4)} + \frac{s^2}{s^3} + \frac{2}{s^3} = \frac{2 + s^2(s^2+4) + 2(s^2+4)}{s^3(s^2+4)} = \frac{2+s^4+4s^2+2s^2+8}{s^3(s^2+4)} =$$

$$= \frac{s^4+6s^2+10}{s^3(s^2+4)} = \frac{As^2}{s^3} + \frac{Bs}{s^2} + \frac{C}{s} + \frac{Ds}{s^2+4} =$$

$$s^4+6s^2+10 = As^4 + As^2 + 4A + Bs^4 + 4Bs^2 + Cs^4 + 4Cs^2 + Ds^4$$

$$= s^4(A+B+C+D) + s^2(A+4B+4C) + 4A$$

$A+B+C+D = 1$

$A+4B+4C = 6 \Rightarrow 4C = 6 - \frac{5}{2} + 4B$

$4A = 10$

$\underline{A = \frac{5}{2}}$

$4C = \frac{7}{2} + 4B \;/ \cdot \frac{1}{4}$

$C = \frac{7}{8} + B$

$C = \frac{7}{8} + \frac{21}{40}$

$C = \frac{35+21}{40}$

$\underline{C = \frac{56}{40} = \frac{7}{5}}$

$\frac{5}{2} + 4B + \frac{7}{8} + B = 6$

$5B = 6 - \frac{5}{2} - \frac{7}{8}$

$5B = \frac{48}{8} - \frac{20}{8} - \frac{7}{8}$

$5B = \frac{21}{8} \;/ \cdot \frac{1}{5}$

$\underline{B = \frac{21}{40}}$

$D = 1 - \frac{5}{2} - \frac{7}{5} - \frac{21}{40}$

$D = \frac{40-100-56-21}{40}$

$\underline{D = -\frac{137}{40}}$

$$F(s) = \frac{5}{2}\cdot\frac{s^2}{s^3} + \frac{21}{40}\cdot\frac{s}{s^2} + \frac{7}{5}\cdot\frac{1}{s} - \frac{137}{40}\cdot\frac{s}{s^2+4}$$

$$= \frac{5}{2}\cdot\frac{1}{s} + \frac{21}{40}\cdot\frac{1}{s} + \frac{7}{5}\cdot\frac{1}{s} - \frac{137}{40}\cdot\frac{s}{s^2+2^2}$$

$$F(s) = \frac{5}{2} + \frac{21}{40} + \frac{7}{5} - \frac{137}{40}\cos(2t)$$

$$F(s) = \frac{100+21+56}{40} - \frac{137}{40}\cos(2t)$$

$$F(s) = \frac{177}{40} - \frac{137}{40}\cos(2t)$$

PROVJERA:

$$f(0) = \frac{177}{40} - \frac{137}{40} = \frac{40}{40} = 1 \checkmark$$

$$f'(t) = +2\cdot\frac{137}{40}\sin(2t)$$

$$f'(0) = 0 \checkmark$$

$$f''(t) = \frac{177}{10}\cos(2t)$$

$$f''(0) = 17.7 \neq 1$$

✗ $\emptyset$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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14.05.2014.

Grupa xxoxx
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. 20
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ ~~20~~

Ukupno: 80

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

$$\ldots) + f''(t) = \sin(2t) \;/\; f(0)=0$$

$$\ldots - s^2f(0) - s\underbrace{f'(0)}_{0} - f''(0) + s^2F(s) - sf(0) - \underbrace{f'(0)}_{0} = \frac{2}{s^2+2^2}$$

$$\ldots - s^2 - 1 + s^2F(s) - s = \frac{2}{s^2+4}$$

$$\ldots + s^2F(s) = \frac{2}{s^2+4} + s^2+s+1$$

$$\ldots(s^3+s^2) = \frac{2+s^2(s^2+4)+s(s^2+4)+(s^2+4)}{s^2+4}$$

$$\ldots = \frac{\dfrac{2+s^4+4s^2+s^3+4s+s^2+4}{s^2+4}}{s^3+s^2}$$

$$\ldots) = \frac{s^4+s^3+5s^2+4s+6}{(s^2+4)(s^3+s^2)} = \frac{s^4+s^3+5s^2+4s+6}{(s^2+4)s^2(s+1)}$$

$$F(s) = \frac{s^4+s^3+5s^2+4s+6}{s^2(s+1)(s^2+4)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{Ds+E}{s^2+4} \;/\cdot s^2(s+1)(s^2+4)$$

$$\ldots s^3+5s^2+4s+6 = As(s+1)(s^2+4)+B(s+1)(s^2+4)+Cs^2(s^2+4)+(Ds+E)s^2(s+1)$$

$$\ldots s^3+5s^2+4s+6 = As(s^3+4s+s^2+4)+B(s^3+4s+s^2+4)+Cs^4+4Cs^2+(Ds+E)(s^3+s^2)$$

$$\ldots s^3+5s^2+4s+6 = As^4+4As^2+As^3+4As+Bs^3+4Bs+Bs^2+4B+Cs^4+4Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$$\ldots s^3+5s^2+4s+6 = (A+C+D)s^4+(A+B+D+E)s^3+(4A+B+4C+E)s^2+(4A+4B)s+4B$$

$A+C+D=1$

$A+B+D+E=1$

$4A+B+4C+E=5$

$4A+4B=4 \Rightarrow 4A=4-4B$

$4B=6 \Rightarrow B=\frac{6}{4}$

$\boxed{B=\frac{3}{2}}$

$4A=4-6$

$4A=-2$

$A=-\frac{2}{4}$

$\boxed{A=-\frac{1}{2}}$

$C+D=\frac{3}{2}$

$D+E=0$

$4C+E=\frac{11}{2}$

$C+D=\frac{3}{2} \;/\cdot(-4)$

$4C+E=\frac{11}{2}$

$-4C-4D=-6$

$4C+E=\frac{11}{2}$

$E-4D=-\frac{1}{2}$

$E-4D=-\frac{1}{2}$

$D+E=0 \;/\cdot(-1)$

$-E-4D=-\frac{1}{2}$

$-E-D=0$

$-5D=-\frac{1}{2} \;/\cdot(-1)$

$5D=\frac{1}{2}$

$D=\frac{\frac{1}{2}}{5}$

$\boxed{D=\frac{1}{10}}$

$C+D=\frac{3}{2}$

$C=\frac{3}{2}-\frac{1}{10}$

$C=\frac{14}{10}$

$D+E=0$

$\boxed{E=-\frac{1}{10}}$

$\boxed{C=\frac{7}{5}}$

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17-2-0155-2011
14.05.2014.

⇘ NASTAVAK 1. ZAD.

$$F(s)=\frac{-\frac{1}{2}}{s}+\frac{\frac{3}{2}}{s^2}+\frac{\frac{7}{5}}{s+1}+\frac{\frac{1}{10}s-\frac{1}{10}}{s^2+4}$$

$$F(s)=-\frac{1}{2}\cdot\frac{1}{s}+\frac{3}{2}\cdot\frac{1}{s^2}+\frac{7}{5}\cdot\frac{1}{s+1}+\frac{1}{10}\cdot\frac{s}{s^2+2^2}-\frac{1}{10}\cdot\frac{1}{s^2+2^2}$$

$$F(s)=-\frac{1}{2}\cdot\frac{1}{s}+\frac{3}{2}\cdot\frac{1}{s^2}+\frac{7}{5}\cdot\frac{1}{s+1}+\frac{1}{10}\cdot\frac{s}{s^2+2^2}-\frac{1}{20}\cdot\frac{2}{s^2+2^2}$$

$$\boxed{f(t)=-\frac{1}{2}+\frac{3}{2}t+\frac{7}{5}e^{-t}+\frac{1}{10}\cos(2t)-\frac{1}{20}\cdot\sin(2t)}$$

PROVJERA:

$$f(0)=-\frac{1}{2}+\frac{7}{5}+\frac{1}{10}=\frac{-1+14+1}{10}=1 ✓$$

$$f'(0)=\frac{3}{2}-\frac{7}{5}-\frac{1}{10}=\frac{15-14-1}{10}=0 ✓$$

$$f''(0)=\frac{7}{5}-\frac{2}{5}=1 ✓$$

PROVERA:

$$f'(t)=\frac{3}{2}-\frac{7}{5}e^{-t}-\frac{1}{5}\sin(2t)-\frac{1}{10}\cos(2t)$$

$$f''(t)=\frac{7}{5}e^{-t}-\frac{2}{5}\cos(2t)+\frac{1}{5}\sin(2t)$$

$$f'''(t)=-\frac{7}{5}e^{-t}+\frac{4}{5}\sin(2t)+\frac{2}{5}\cos(2t)$$

$$\cancel{-\frac{7}{5}e^{-t}}+\frac{4}{5}\sin(2t)+\cancel{\frac{2}{5}\cos(2t)}+\cancel{\frac{7}{5}e^{-t}}-\cancel{\frac{2}{5}\cos(2t)}+\frac{1}{5}\sin(2t)=\sin(2t)$$

$$\frac{4}{5}\sin(2t)+\frac{1}{5}\sin(2t)=\sin(2t)$$

$$\frac{5}{5}\sin(2t)=\sin(2t)$$

$$\boxed{\sin(2t)=\sin(2t)}$$ ✓

20

2. $x^2+y^2=5z, \quad z\leq 1.$

$$P(S)=\iint_D\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$$

$x^2+y^2=az$ – PARABOLOID

$5z=x^2+y^2 \qquad 5z=x^2+y^2$

$5\partial z=2x\partial x \qquad 5\partial z=2y\partial y$

$\frac{\partial z}{\partial x}=\frac{2x}{5}$ ✓ $\qquad \frac{\partial z}{\partial y}=\frac{2y}{5}$ ✓ $\qquad x^2+y^2=r^2$

$$\sqrt{1+\left(\frac{2x}{5}\right)^2+\left(\frac{2y}{5}\right)^2}=\sqrt{1+\frac{4x^2}{25}+\frac{4y^2}{25}}=\sqrt{1+\frac{4(x^2+y^2)}{25}}=\boxed{\sqrt{1+\frac{4}{25}r^2}}$$ ✓ ⇒

NASTAVAK

2.) NASTAVAK..

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{5}]$

$x^2+y^2=5z$

$x^2+y^2=r^2$

$r^2=5z$

$r^2=5\cdot 1$

$r^2=5$

$r=\sqrt{5}$

$x=r\cos\varphi$

$y=r\sin\varphi$

$dx\,dy=r\,dr\,d\varphi$

$$P(S)=\int_0^{2\pi}\int_0^{\sqrt{5}}\sqrt{1+\frac{4}{25}r^2}\,r\,dr\,d\varphi=\int_0^{2\pi}\frac{25}{12}\sqrt{\left(1+\frac{4}{25}r^2\right)^3}\Bigg|_0^{\sqrt{5}}d\varphi=$$

$$=\int_0^{2\pi}\left[\frac{25}{12}\sqrt{\left(1+\frac{4}{5}\right)^3}-\frac{25}{12}\sqrt{1^3}\right]d\varphi$$

$$=\int_0^{2\pi}\left(\frac{25}{12}\sqrt{\left(\frac{9}{5}\right)^3}-\frac{25}{12}\right)d\varphi$$

$$\int\sqrt{1+\frac{4}{25}r^2}\,r\,dr$$

$1+\frac{4}{25}r^2=t\,/\,'$

$\frac{8}{25}r\,dr=dt$

$r\,dr=\frac{dt}{\frac{8}{25}}$

$r\,dr=\frac{25}{8}dt$

$$\int t^{\frac{1}{2}}\cdot\frac{25}{8}dt=\frac{25}{8}\int t^{\frac{1}{2}}dt=\frac{25}{8}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}=\frac{25}{\cancel{8}_4}\cdot\frac{\cancel{2}}{3}t^{\frac{3}{2}}$$

$$=\frac{25}{12}\sqrt{t^3}=\frac{25}{12}\sqrt{\left(1+\frac{4}{25}r^2\right)^3}$$

$\sqrt{125}=\sqrt{25\cdot 5}=$

$=5\sqrt{5}$

$$=\int_0^{2\pi}\left(\frac{25}{12}\sqrt{\frac{729}{125}}-\frac{25}{12}\right)d\varphi=\int_0^{2\pi}\left(\frac{\cancel{25}}{12}\cdot\frac{27}{5\sqrt{5}}-\frac{25}{12}\right)d\varphi$$

$\frac{45}{4\sqrt{5}}-\frac{25}{12}=2{,}947\approx 3$

$$=\int_0^{2\pi}\left(\frac{5}{12}\cdot\frac{27}{\sqrt{5}}-\frac{25}{12}\right)d\varphi=\int_0^{2\pi}\left(\frac{5}{4}\cdot\frac{9}{\sqrt{5}}-\frac{25}{12}\right)d\varphi=\int_0^{2\pi}\left(\frac{45}{4\sqrt{5}}-\frac{25}{12}\right)d\varphi$$

$$\approx\int_0^{2\pi}3\,d\varphi=3\varphi\Big|_0^{2\pi}=3\cdot(2\pi-0)=3\cdot 2\pi=\boxed{6\pi}$$ – POVRŠINA PLAŠTA $P(S)$

20

NASTAVAK

⇒ NASTAVAK ③

$$= \int_{-2}^{0} y\Big|_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}} dx + \int_{0}^{10} y\Big|_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{25}{6}} dx =$$

$$\frac{1}{12}+\frac{5}{2}=\frac{1+30}{12}$$

$$\frac{25}{6}+\frac{1}{1}=\frac{25+6}{6}=\frac{31}{6}$$

$$\frac{1}{12}-\frac{3}{5}=\frac{5-36}{60}=-\frac{31}{60}$$

$$\frac{25}{6}+\frac{1}{1}=\frac{31}{6}$$

$$=\int_{-2}^{0}\left[\frac{1}{12}x+\frac{25}{6}-\left(-\frac{5}{2}x-1\right)\right]dx+\int_{0}^{10}\left[\frac{1}{12}x+\frac{25}{6}-\left(\frac{3}{5}x-1\right)\right]dx$$

$$=\int_{-2}^{0}\left(\frac{1}{12}x+\frac{25}{6}+\frac{5}{2}x+1\right)dx+\int_{0}^{10}\left(\frac{1}{12}x+\frac{25}{6}-\frac{3}{5}x+1\right)dx$$

$$=\int_{-2}^{0}\left(\frac{31}{12}x+\frac{31}{6}\right)dx+\int_{0}^{10}\left(-\frac{31}{60}x+\frac{31}{6}\right)dx$$

$$=\left(\frac{31}{24}x^2+\frac{31}{6}x\right)\Big|_{-2}^{0}-\frac{31}{120}x^2\Big|_{0}^{10}+\frac{31}{6}x\Big|_{0}^{10}$$

$$=0-\left(\frac{31}{24}\cdot 4-\frac{31}{3}\right)-\frac{31}{120}\left(10^2-0^2\right)+\frac{31}{6}(10-0)$$

$$=-\frac{31}{6}+\frac{31}{3}-\frac{31}{120}\cdot 100+\frac{310}{6}=-\frac{31}{6}+\frac{31}{3}-\frac{310}{12}+\frac{310}{6}$$

$$=-\frac{31}{6}+\frac{31}{3}-\frac{155}{6}+\frac{155}{3}=\frac{-31+62-155+310}{6}$$

$$=\frac{186}{6}=\frac{93}{3}=\underline{31}$$

✓ 20

④ $$f(x,y)=\frac{2}{\sqrt{x^2+y^2}}$$

$\varphi\in\left[0,\frac{3\pi}{2}\right]$

$r=2$

$r\in[0,2]$

$T(x_0,y_0)=(0,0)$

$x=r\cos\varphi$

$y=r\sin\varphi$

$dxdy=r\,dr\,d\varphi$

$$\iint_K \frac{2}{\sqrt{x^2+y^2}}dxdy=\int_0^{\frac{3\pi}{2}}\int_0^2\frac{2}{\sqrt{(r\cos\varphi)^2+(r\sin\varphi)^2}}\,r\,dr\,d\varphi$$

$$=\int_0^{\frac{3\pi}{2}}\int_0^2\frac{2}{\sqrt{r^2\cos^2\varphi+r^2\sin^2\varphi}}\,r\,dr\,d\varphi=\int_0^{\frac{3\pi}{2}}\int_0^2\frac{2}{\sqrt{r^2\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}}}\,r\,dr\,d\varphi$$

$$=\int_0^{\frac{3\pi}{2}}\int_0^2\frac{2}{\sqrt{r^2}}\,r\,dr\,d\varphi=\int_0^{\frac{3\pi}{2}}\int_0^2\frac{2}{r}\,r\,dr\,d\varphi=$$

⇒ NASTAVAK

↓ NASTAVAK ...

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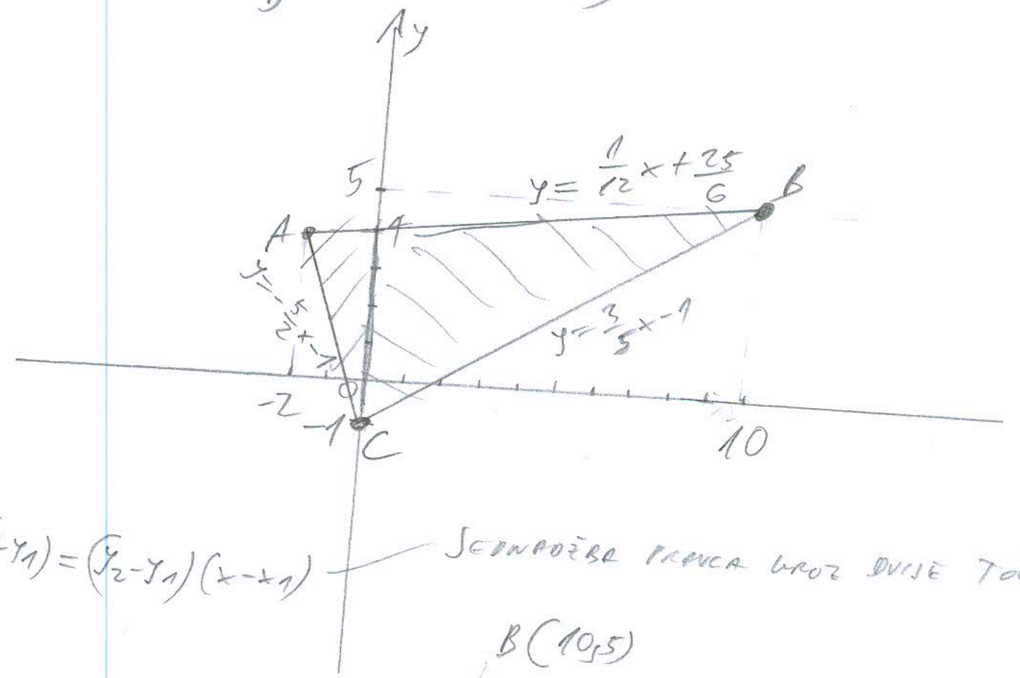
17-2-0155-2011

14.05.2014.

(3.) $A(-2,4)$
$B(10,5)$
$C(0,-1)$

$$\oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$$

$$\oint_C P_{(x,y)}\,dx + Q_{(x,y)}\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$ — GREENOVA FORMULA



$(x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$ — JEDNADŽBA PRAVCA KROZ DVIJE TOČKE

$A(-2,4)$
$B(10,5)$

$12(y-4) = 1(x+2)$

$12y-48 = x+2$

$12y = x+50 \;/:12$

$y = \frac{1}{12}x + \frac{50}{12}$

$\overline{AB} \ldots \; \underline{y = \frac{1}{12}x + \frac{25}{6}}$

$B(10,5)$
$C(0,-1)$

$-10(y-5) = -6(x-10)$

$-10y+50 = -6x+60$

$-10y = -6x+10 \;/\cdot(-1)$

$10y = 6x-10 \;/:10$

$y = \frac{6}{10}x-1$

$\overline{BC} \ldots \; \underline{y = \frac{3}{5}x-1}$

$A(-2,4)$
$C(0,-1)$

$2(y-4) = -5(x+2)$

$2y-8 = -5x-10$

$2y = -5x-2 \;/:2$

$\overline{AC} \ldots \; \underline{y = -\frac{5}{2}x-1}$

$P_{(x,y)} = x^2-y$

$Q_{(x,y)} = \sin(y^3)$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial(\sin(y^3))}{\partial x} - \frac{\partial(x^2-y)}{\partial y} = 0-0+1 = \underline{1}$$

✓

$$\oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy = \int_{-2}^{0}\int_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}} 1\cdot dy\,dx + \int_{0}^{10}\int_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{25}{6}} 1\cdot dy\,dx =$$

=> NASTAVAK

⇒ NASTAVAK... (4.) ZAD

JURE MILKOVIĆ

17-2-0155-2011

14.05.2014.

$$= \int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{\cancel{r}}\cancel{r}\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}}\int_0^2 2\,dr\,d\varphi$$

$$= \int_0^{\frac{3\pi}{2}} 2r\Big|_0^2 d\varphi = \int_0^{\frac{3\pi}{2}} 2(2-0)\,d\varphi = \int_0^{\frac{3\pi}{2}} 4\,d\varphi = 4\varphi\Big|_0^{\frac{3\pi}{2}}$$

$$= 4\cdot\left(\frac{3\pi}{2}-0\right) = \frac{12\pi}{2} = 6\pi$$ ✓ 20

(5.) $f(x,y) = -y$

$X \ldots \begin{cases} x = \sin y \\ y = \frac{\pi}{2}x \end{cases}$

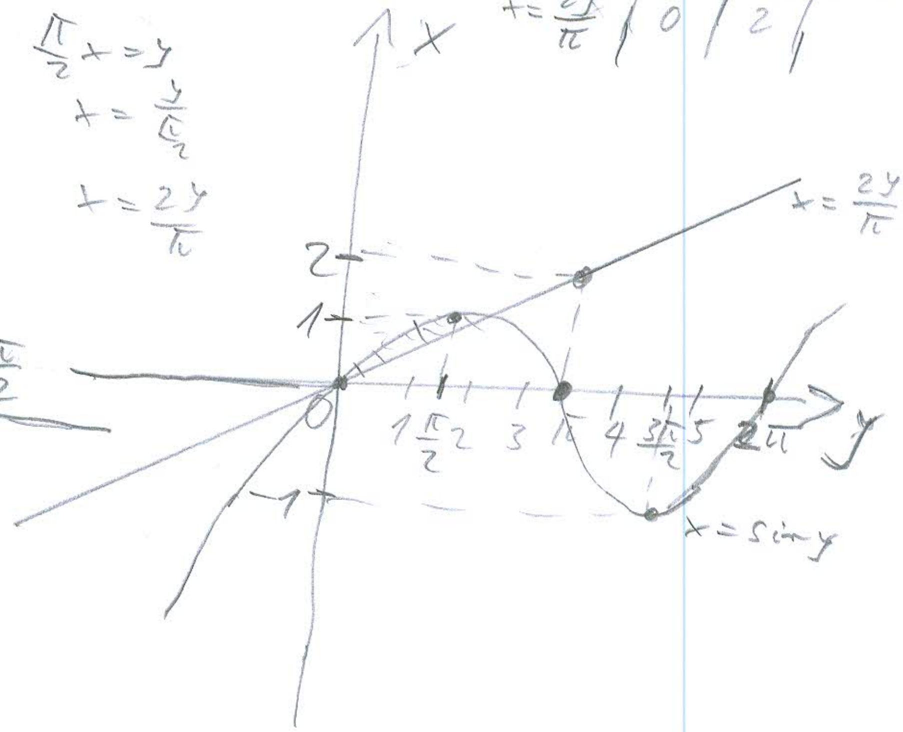
$\frac{\pi}{2}x = y$

$x = \frac{y}{\frac{\pi}{2}}$

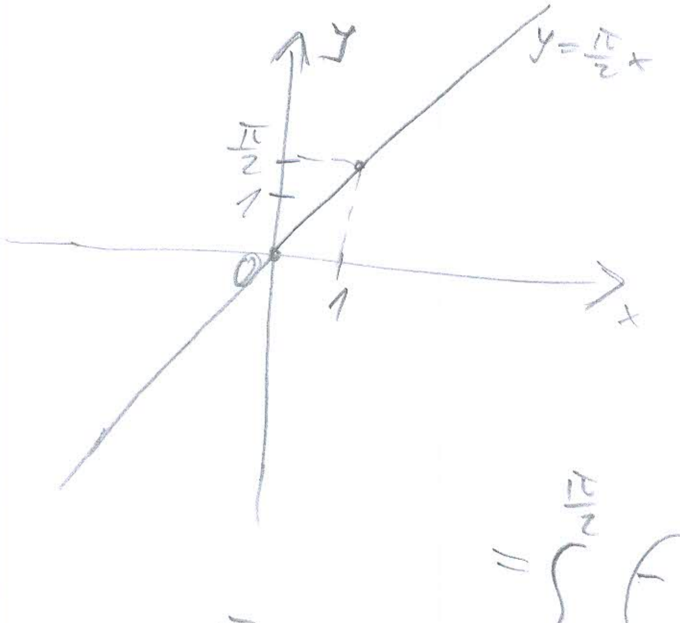
$x = \frac{2y}{\pi}$

y	0	π
$x=\frac{2y}{\pi}$	0	2

y	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$x=\sin y$	0	1	0	-1



x	0	1
$y=\frac{\pi}{2}x$	0	$\frac{\pi}{2}$



$$\iint_X -y\,dx\,dy = \int_0^{\frac{\pi}{2}}\int_{\frac{2y}{\pi}}^{\sin y} -y\,dy\,dx$$

$$= \int_0^{\frac{\pi}{2}} -\frac{y^2}{2}\Big|_{\frac{2y}{\pi}}^{\sin y} dx =$$ X

$$= \int_0^{\frac{\pi}{2}}\left(-\frac{\sin^2 y}{2} + \frac{4y^2}{\frac{\pi}{2}}\right)dx = \int_0^{\frac{\pi}{2}}\left(-\frac{\sin^2 y}{2} + \frac{4y^2}{2\pi}\right)dx$$

$$= \int_0^{\frac{\pi}{2}}\left(-\frac{\sin^2 y}{2} + \frac{2y^2}{\pi}\right)dx = \left(-\frac{\sin^2 y}{2}x + \frac{2y^2}{\pi}x\right)\Big|_0^{\frac{\pi}{2}} = -\frac{\sin^2 y}{2}\cdot\frac{\pi}{2} + \frac{2y^2}{\pi}\cdot\frac{\pi}{2} \Rightarrow$$

nastavak..

$$\frac{-\sin 2y\pi}{4} + y^2$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: Ivan Colić BROJ INDEKSA: 17-2-0152-2011	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. — 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. — 20
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. — 20
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. — 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ — 20

Ukupno: $\emptyset$

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

11. $f'''(t) + f''(t) = \sin(2t)$ $\qquad f'(0) = 0 \;, \; f(0) = f''(0) = 1$ $\qquad \sin 2t = \frac{2}{s^2+4}$

$a=2$

$$s^3F(s) - s^2 \underbrace{f(0)}_{1} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{1} + s^2F(s) - s\underbrace{f(0)}_{1} - \underbrace{f'(0)}_{0} = \frac{2}{s^2+4}$$

$$s^3F(s) - s^2 - 1 + s^2F(s) - s = \frac{2}{s^2+4}$$

$$F(s)\left(s^3+s^2\right) = \frac{2}{s^2+4} + s^2 + 1 + s$$

$$F(s)\, s^2(s+1) = \frac{2 + s^2(s^2+4) + s^2 + 4 + s(s^2+4)}{s^2+4}$$

$$F(s)\, s^2(s+1) = \frac{2 + s^4 + \cancel{4s^2} + \cancel{s^2} + 4 + \cancel{s^3} + \cancel{4s}}{s^2+4}$$

$$F(s) = \frac{s^4 + 5s^2 + s^3 + 4s + 6}{s^2(s+1)(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+4} \quad / \cdot s^2(s+1)(s^2+4)$$

$$s^4 + s^3 + 5s^2 + 4s + 6 = A(s^4 + s^3 + 4s^2 + 4s) + B(s^3 + 4s + s^2 + 4) + C(s^4 + 4s^2) + (Ds+E)(s^3+s^2)$$

$$s^4 + s^3 + 5s^2 + 4s + 6 = As^4 + As^3 + 4As^2 + 4As + Bs^3 + 4Bs + Bs^2 + 4B + Cs^4 + 4Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$$

s^4: $A + C + D = 1$

$A + B + D + E = 1$

$4A + B + 4C + E = 5$

$4A + 4B = 4 \quad /:4$

$A + B = 1 \qquad A = 1 - \frac{6}{4}$

$4B = 6 \qquad A = -\frac{1}{2}$

$\boxed{B = \frac{6}{4}}$

$C + D = \frac{3}{2}$

$-\frac{1}{2} + 4C + E = 5$

$4C + E = \frac{11}{2}$

$4\cdot\left(\frac{3}{2} + E\right) + E = \frac{11}{2}$

$6 + 4E + E = \frac{11}{2}$

$5E = \frac{11}{2} - 6$

$5E = -\frac{1}{2}$

$E = -\frac{\frac{1}{2}}{5}$

$E = -\frac{1}{10}$

$1 + D + E = 1$

$D + E = 0$

$D = -E$

$\boxed{D = \frac{1}{10}}$

$C - E = \frac{3}{2}$

$C = \frac{3}{2} + E$

$C = \frac{3}{2} + \frac{1}{10}$

$\boxed{C = \frac{8}{5}}$

$$F_s = -\frac{1}{2} + \frac{6}{4}t + \frac{8}{5}\,\frac{1}{s+1} + \frac{1}{10}\,\frac{s}{s^2+4} - \frac{1}{10}\,\frac{1}{s^2+4}$$

$$f(s) = -\frac{1}{2} + \frac{6}{4}t + \frac{8}{5}e^{-t} + \frac{1}{10}\cos 2t - \frac{1}{20}\cdot\frac{2}{s^2+2^2}$$

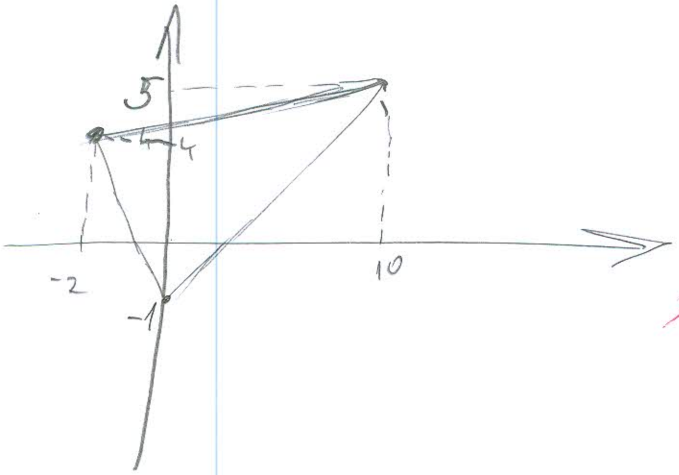
$$f(s) = -\frac{1}{2} + \frac{6}{4}t + \frac{8}{5}e^{-t} + \frac{1}{10}\cos 2t - \frac{1}{20}\sin 2t$$

PROVJERA:

$f(0) = -\frac{1}{2} + \frac{8}{5} + \frac{1}{10} = \frac{-5+16+1}{10} = \frac{12}{10}$

POGREŠNO!

3.



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4.

Ivan Colić

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Adriano Vipotnik **BROJ INDEKSA:** 17-2-0138-2011

Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. ~~20~~
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C} (x^2 - y)\,dx + \sin(y^3)\,dy$. 20
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ 20

Ukupno: 40

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Adriano Vipotnik

2.) $x^2+y^2=5z$, $z \leq 1$ $\qquad \varphi \in [0, 2\pi] \qquad r \in [0, \sqrt{5}]$

$x^2+y^2=5z$ - ~~stožac~~ PARABOLOID

$x^2+y^2=r^2$

$\partial z(5z) = \partial x(x^2+y^2)$

$r^2 = 5z \qquad z=1$

$\frac{\partial z}{\partial x} = \frac{2x}{5}$

$r^2 = 5 \,/\sqrt{\ }$

$\frac{\partial z}{\partial y} = \frac{2y}{5}$

$r = \pm\sqrt{5}$

$$P(s) = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$$

$$\sqrt{1+\left(\frac{2x}{5}\right)^2+\left(\frac{2y}{5}\right)^2} = \sqrt{1+\frac{4x^2}{25}+\frac{4y^2}{25}} = \sqrt{1+\frac{4x^2+4y^2}{25}} = \sqrt{1+\frac{4(x^2+y^2)}{25}}$$

$$= \sqrt{1+\frac{4r^2}{25}} = 1+\frac{2r}{5}$$

$$P(s) = \iint_D \left(\frac{2r}{5}+1\right) r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{5}} \left(\frac{2}{5}r^2+r\right) dr\,d\varphi = \int_0^{2\pi} \left(\frac{2}{5}\frac{r^3}{3}+\frac{r^2}{2}\right)\Big|_0^{\sqrt{5}} d\varphi$$

$$= \int_0^{2\pi} \left(\frac{2}{3}\sqrt{5}+\frac{5}{2}\right) d\varphi = \left(\frac{2}{3}\sqrt{5}\,\varphi+\frac{5}{2}\varphi\right)\Big|_0^{2\pi} = \frac{4}{3}\sqrt{5}\,\pi + 5\pi$$

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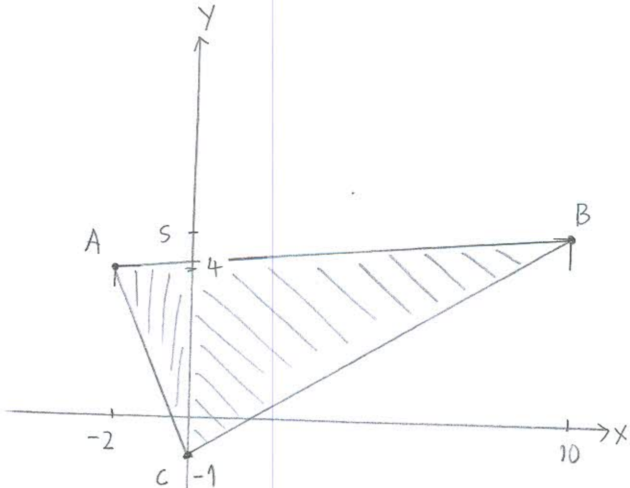
Adriano Vipotnik

3.) $A(-2,4)$, $B(10,5)$ i $C(0,-1)$

$$\oint_{\overleftarrow{ABC}} (x^2-y)\,dx + \sin(y^3)\,dy$$



$x\in[-2,0]$ | $x\in[0,10]$

$A(-2,4)$, $B(10,5)$

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$
$$(10+2)(y-4)=(5-4)(x+2)$$
$$12y-48 = x+2$$
$$12y = x+50 \,/:12$$
$$y=\frac{1}{12}x+\frac{25}{6}$$

$A(-2,4)$, $C(0,-1)$

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$
$$(0+2)(y-4)=(-1-4)(x+2)$$
$$2y-8=-5x-10$$
$$2y=-5x-2 \,/:2$$
$$y=-\frac{5}{2}x-1$$

$C(0,-1)$, $B(10,5)$

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$
$$(10-0)(y+1)=(5+1)(x-0)$$
$$10y+10=6x$$
$$10y=6x-10 \,/:10$$
$$y=\frac{3}{5}x-1$$

$$\oint_{\overleftarrow{ABC}} P\,dx+Q\,dy=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy$$

$$P\,dx = x^2-y$$
$$Q\,dy=\sin(y^3)$$

$$\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=\frac{\partial(\sin(y^3))}{\partial x}-\frac{\partial(x^2-y)}{\partial y}=0-(-1)=1$$

$$\iint_D 1\,dx\,dy$$

$$\int_{-2}^{0}\int_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}}1\,dy\,dx+\int_{0}^{10}\int_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{25}{6}}1\,dy\,dx=\int_{-2}^{0}y\Big|_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}}dx+\int_{0}^{10}y\Big|_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{25}{6}}dx$$

$$=\int_{-2}^{0}\left(\frac{1}{12}x+\frac{25}{6}+\frac{5}{2}x+1\right)dx+\int_{0}^{10}\left(\frac{1}{12}x+\frac{25}{6}-\frac{3}{5}x+1\right)dx=\int_{-2}^{0}\left(\frac{31}{12}x+\frac{31}{6}\right)dx+\int_{0}^{10}\left(-\frac{31}{60}x+\frac{31}{6}\right)dx$$

$$=\left(\frac{31}{12}\frac{x^2}{2}+\frac{31}{6}x\right)\Big|_{-2}^{0}+\left(-\frac{31}{60}\frac{x^2}{2}+\frac{31}{6}x\right)\Big|_{0}^{10}=\frac{31}{6}-\frac{31}{3}-\frac{155}{6}+\frac{155}{3}=\boxed{\frac{62}{3}}$$

Adriana Vipotnik

1.) $f'''(t) + f''(t) = \sin(2t)$ $\qquad f'(0) = 0 \qquad f(0) = f''(0) = 1$

$$s^3 F(t) - s^2 - 1 + s^2 F(t) - s = \frac{2}{s^2+2^2}$$

$$s^3 F(t) + s^2 F(t) = \frac{2}{s^2+4} + s^2 + 1 + s$$

$$s^2(s+1) F(t) = \frac{2 + s^4 + 4s^2 + s^2 + 4 + s^3 + 4s}{s^2+4}$$

$$s^2(s+1) F(t) = \frac{s^4 + s^3 + 5s^2 + 4s + 6}{s^2+4} \Big/ : s^2(s-1)$$

$$F(t) = \frac{\frac{s^4+s^3+5s^2+4s+6}{s^2+4}}{s^2(s-1)} = \frac{s^4+s^3+5s^2+4s+6}{s^2(s-1)(s^2+4)}$$

DALJE...

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Adriano Vipotnik

4.) $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$ $\quad r = 2 \quad T(0,0)$

$\varphi \in \left[0, \frac{3\pi}{2}\right]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r \in [0, 2]$

$x^2 + y^2 = r^2$

$f(x,y) = \frac{2}{\sqrt{r^2}} = \frac{2}{r}$

$$\int_0^{\frac{3}{2}\pi}\int_0^2 \left(\frac{2}{r}\right) r \, dr \, d\varphi = \int_0^{\frac{3}{2}\pi}\int_0^2 \left(\frac{2r}{r}\right) dr \, d\varphi = \int_0^{\frac{3}{2}\pi}\int_0^2 2 \, dr \, d\varphi$$

$$= \int_0^{\frac{3}{2}\pi} 2r \Big|_0^2 d\varphi = \int_0^{\frac{3}{2}\pi} 4 \, d\varphi = 4\varphi \Big|_0^{\frac{3}{2}\pi} = 6\pi$$

20