

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: **JOSIP ŠIMIČEV**

BROJ INDEKSA:

Grupa  
XXOXX  
POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:  $f'''(t) + f''(t) = \sin(2t)$ ,  $f'(0) = 0$  i  $f(0) = f''(0) = 1$ . 20

2. Izračunajte površinu oplošja paraboloida  $x^2 + y^2 = 5z$ ,  $z \leq 1$ . 20

3. Zadan je trokut s vrhovima  $A(-2, 4)$ ,  $B(10, 5)$  i  $C(0, -1)$ . Izračunati  $\oint_{\overline{ABC}} (x^2 - y) dx + \sin(y^3) dy$ . 20

4. Izračunati integral funkcije  $f(x, y) = \frac{2}{\sqrt{x^2 + y^2}}$  na prve tri četvrtine kruga ( $\varphi \in [0, \frac{3\pi}{2}]$ ) radijusa  $r = 2$  sa središtem u ishodištu. 20

5. Odrediti integral funkcije  $f(x, y) = -y$  na području  $X$  koje je ograničeno krivuljama  $X \dots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$  20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln  \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln  \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln  \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln  \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Ukupno:

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Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$c$	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$t$	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
$t^n$	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$e^{-at}$	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



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$$1) f'''(t) + f''(t) = \sin(2t) \quad f'(0) = 0 \quad f(0) = f''(0) = 1$$

$$s^3 f(s) - s^2 f(0) - s f'(0) - f''(0) + s^2 f(s) - s f(0) - f'(0) = \frac{2}{s^2 + 2^2}$$

$$s^3 f(s) - s^2 - 0 - 1 + s^2 f(s) - s - 0 = \frac{2}{s^2 + 4}$$

$$f(s)(s^3 + s^2) = \frac{2}{s^2 + 4} + s^2 + 1 + s$$

$$f(s)(s^3 + s^2) = \frac{2 + s^4 + 4s^2 + s^2 + 4 + s^3 + 4s}{s^2 + 4} = \frac{s^4 + s^3 + 5s^2 + 4s + 6}{s^2 + 4} \quad | : (s^3 + s^2)$$

$$f(s) = \frac{s^4 + s^3 + 5s^2 + 4s + 6}{s^2(s+1)(s^2+4)}$$

$$\frac{s^4 + s^3 + 5s^2 + 4s + 6}{s^2(s+1)(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+4} \quad | \cdot s^2(s+1)(s^2+4)$$

$$\begin{aligned} & s^4 + s^3 + 5s^2 + 4s + 6 \\ &= A s^4 + 4A s^2 + 4A s^3 + 4A s + B s^3 + B s^2 + 4B s + 4B + C s^4 + 4C s^2 + D s^4 + D s^3 + E s^3 + E s^2 \end{aligned}$$

$$A + C + D = 1$$

$$4A + B + D + E = 1$$

$$4A + B + 4C + E = 5$$

$$4A + 4B = 4$$

$$4B = 6$$

$$\boxed{B = \frac{6}{4} = \frac{3}{2}}$$

$$4A + 4B = 4 \quad | : 4$$

$$A + B = 1$$

$$A = 1 - \frac{3}{2} = \boxed{\frac{-1}{2} = A}$$

$$A + C + D = 1$$

$$C + D = \frac{3}{2}$$

$$D = \frac{3}{2} - D$$

$$4A + B + D + E = 1$$

$$-4A + B + 4C + E = 5$$

$$D - 4C = -4$$

$$-4C = -4 - D \quad | : (-4)$$

$$4C = 4 + D$$

$$\frac{3}{2} - D = 4 + D$$

$$-D - D = 4 - \frac{3}{2} =$$

$$-2D = \frac{5}{2} \quad | : (-2)$$

$$D = -\frac{5}{4} \quad C = \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$$

$$4A + B + D + E = 1$$

$$E = -2 - \frac{3}{2} + \frac{5}{4} - \frac{11}{4} = -5$$

$$A = -\frac{1}{2} \quad B = \frac{3}{2} \quad C = \frac{11}{4} \quad D = -\frac{5}{4} \quad E = -5$$

$$= -\frac{1}{2} s + \frac{3}{2} \frac{1}{s^2} + \frac{11}{4} \frac{1}{(s+1)} + \frac{5}{4} \frac{s}{(s+2)} - 5 \frac{1}{(s+2)^2}$$

$$f(t) = -\frac{1}{2} + \frac{3}{2}t + \frac{11}{4}e^{-t} - \frac{5}{4}\cos(2t) - 5\sin(2t)$$

$$f'(t) = \frac{3}{2} - \frac{11}{4}e^{-t} + \frac{10}{4}\sin(2t) - 10\cos(2t)$$

PROJEKTA:

$$f(0) = -\frac{1}{2} + \frac{11}{4} - \frac{5}{4} = \frac{-2+11-5}{4} = \frac{4}{4} = 1 \checkmark$$

$$f'(0) = \frac{3}{2} - \frac{11}{4} - 10 = \frac{6-11-40}{4} = \frac{-45}{4} \neq 0 \times$$

5.  $f(x,y) = -y$

$$x \dots \begin{cases} x = \sin y \\ y = \frac{\pi}{2} x \end{cases}$$



$$x = \sin y$$

$$y = \frac{\pi}{2} x = \frac{\pi}{2} \sin y$$

$$x = \sin \frac{\pi}{2} x$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\sin \frac{\pi}{2}} -y \, dy \, dx$$

$$= \int_0^{\frac{\pi}{2}} \left[ -\frac{y^2}{2} \right]_0^{\sin \frac{\pi}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} -\frac{\sin^2 \frac{\pi}{2}}{2} dx$$

$$= \int_0^{\frac{\pi}{2}} -\sin^2 \frac{\pi}{2} \cdot \frac{1}{2} dx$$

$$= \cos^2 \frac{\pi}{2} x \cdot \frac{1}{2} x \Big|_0^{\frac{\pi}{2}}$$

$$= \cos^2 \frac{\pi}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \cos^2 \frac{2\pi}{4} - \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= -\frac{1}{4} \pi$$

4.  $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$

$$\theta \in [0, \frac{3\pi}{2}] \quad r = 2 \quad r \in [0, 2]$$

$$x^2 + y^2 = r^2$$

$$dx dy = r dr d\theta$$

$$\iint \frac{2}{\sqrt{x^2+y^2}} dx dy = \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{\sqrt{r^2}} \cdot r dr d\theta = \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2r}{r} dr d\theta = \int_0^{\frac{3\pi}{2}} 2r \Big|_0^2 d\theta$$

$$= \int_0^{\frac{3\pi}{2}} 2 \cdot 2 d\theta = 4 \cdot \frac{3\pi}{2} = \frac{12\pi}{2} = 6\pi \quad \checkmark \quad \underline{20}$$

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$$2) \quad x^2 + y^2 = 5z \quad z \leq 1$$

$$5z = r^2$$

$$z = \frac{r^2}{5} \quad z \in \left[ \frac{r^2}{5}, 1 \right]$$

$$r^2 = 5z$$

$$r^2 = 5$$

$$r = \sqrt{5} \quad r \in [0, \sqrt{5}]$$

$$\varphi \in [0, 2\pi]$$

$$P = \int_0^{2\pi} \int_0^{\sqrt{5}} \int_{\frac{r^2}{5}}^1 r \, dz \, dr \, d\varphi = \int_0^{2\pi} \int_0^{\sqrt{5}} r \cdot z \Big|_{\frac{r^2}{5}}^1 \, dr \, d\varphi = \int_0^{2\pi} \int_0^{\sqrt{5}} r \cdot \left(1 - \frac{r^2}{5}\right) \, dr \, d\varphi$$

$$= \int_0^{2\pi} \int_0^{\sqrt{5}} r - \frac{1}{5} r^3 \, dr \, d\varphi = \int_0^{2\pi} \left. \frac{r^2}{2} - \frac{1}{5} \frac{r^4}{4} \right|_0^{\sqrt{5}} \, d\varphi = \int_0^{2\pi} \left( \frac{5}{2} - \frac{1}{5} \frac{(\sqrt{5})^4}{4} \right) \, d\varphi$$

$$= \int_0^{2\pi} \left( \frac{5}{2} - \frac{\sqrt{5}}{3} \right) \, d\varphi = 2\pi \cdot \frac{5}{2} - \frac{2\sqrt{5}}{3} \pi = 5\pi - \frac{2\sqrt{5}\pi}{3} \approx 11.024$$

