

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa
xx00x
POPUNJAVA
NASTAVNIK
Broj ↓
bodeva

IME I PREZIME:

Tomislav Kraljic

BROJ INDEKSA:

1701-0052-2011

1. Izračunati dvostruki integral $\iint_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$.

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2. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 4$ i ravninama $z = y$ i $z = x - 2$.

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3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

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$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1, \quad x'(0) = 0.$$

4. Neka je C cilindar zadan sa $C = \{(x, y, z) : (x + 2)^2 + (y - 3)^2 \leq 1, -1 \leq z \leq 1\}$. Izračunati plošni integral

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$$\iint_{\partial C} 2x \, dy dz$$

5. Izračunati $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$

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Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x \, dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

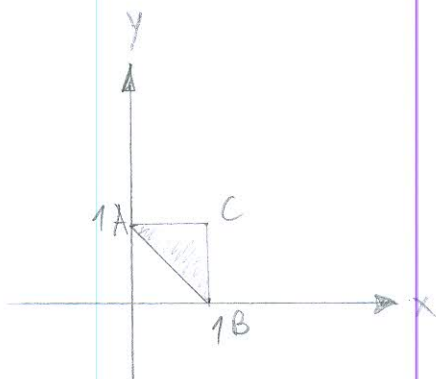
Ukupno:

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Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) \, dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) \, d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

1) Iračunati $\iint_S e^{x+y} dx dy$, gdje je trokut S vrhovima $A(0,1)$ $B(1,0)$ $C(1,1)$



$$x \in [0, 1] \quad \checkmark$$

$$y \in [-x+1, 1] \quad \checkmark$$

$$\overline{AB} \quad A(x_1, y_1) \quad B(x_2, y_2)$$

$$(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$$

$$(1 - 0)(y - 1) = (0 - 1)(x - 0)$$

$$1y - 1 \cdot 1 = -1 \cdot x + 0$$

$$y - 1 = -x$$

$$y = -x + 1$$

$$e^{x+y} = e^x \cdot e^y$$

$$\int_0^1 \int_{-x+1}^1 e^{x+y} dy dx //$$

$$\int_0^1 \int_{-x+1}^1 e^x \cdot e^y dy dx \quad \checkmark = \int_0^1 e^x \cdot e^y \Big|_{-x+1}^1 dx$$

$$= \int_0^1 e^x \cdot (e^1 - e^{-x+1}) dx = \int_0^1 e^{x+1} - e dx = e^{x+1} - ex \Big|_0^1$$

$$= e^{1+1} - e \cdot 1 - (e^{0+1} - e \cdot 0) = e^2 - e - e = e^2 - 2e \quad \checkmark //$$

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2) $V = ?$

$$x^2 + y^2 = 4$$

$$z = y \quad ; \quad z = x - 2$$

$$x^2 + y^2 = r^2$$

$$r^2 = 4$$

$$r = 2$$

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$z = z$$

$$r \in [0, 2]$$

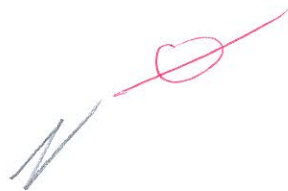
$$\varphi \in [0, 2\pi]$$

$$z \in [-2, 0] \quad \times$$

$$\iiint_V r \, dr \, d\varphi \, dz =$$

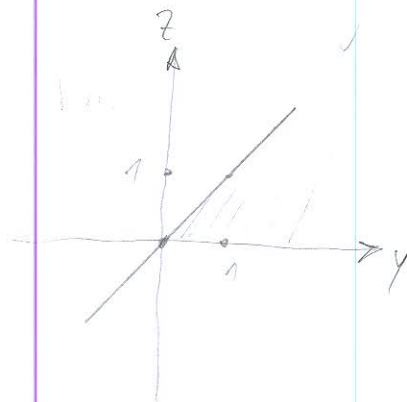
$$2\pi \cdot 2 \cdot \textcircled{0} \times$$

$$= \int_0^2 \int_0^{2\pi} \int_{-2}^0 r \, dz \, d\varphi \, dr$$



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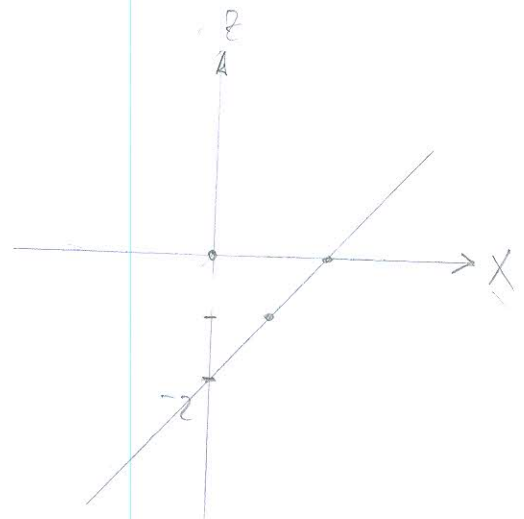
$$dx \, dy \, dz = r \, dr \, d\varphi \, dz$$



$$z = y$$

y	0	1	2
z = y	0	1	2

x	0	1	2
z = x - 2	-2	-1	0



2) \Downarrow mostovak

$$\int_0^{2\pi} \int_0^2 \int_{-2}^0 r \, dz \, dr \, d\varphi = \int_0^{2\pi} \int_0^2 r \cdot z \Big|_{-2}^0 \, dr \, d\varphi = \cancel{\emptyset}$$

$$= \int_0^{2\pi} \int_0^2 r \cdot (0 - (-2)) \, dr \, d\varphi = \int_0^{2\pi} \int_0^2 2r \, dr \, d\varphi =$$

$$= \int_0^{2\pi} \left[\frac{r^2}{2} \Big|_0^2 \right] d\varphi = \int_0^{2\pi} (2^2 - 0^2) \, d\varphi = \int_0^{2\pi} 4 \, d\varphi$$

$$= 4 \cdot \varphi \Big|_0^{2\pi} = 4 \cdot 2\pi = 8\pi$$

$$\textcircled{3} \quad x'''(t) + x'(t) = 0$$

$$x(0) = x''(0) = 1 \quad x'(0) = 0$$

$$s^3 x(s) - s^2 x(0) - s x'(0) - x''(0) + s x(s) - x'(0) = 0$$

$$s^3 x(s) - s^2 \cdot 1 - s \cdot 0 - 1 + s x(s) - 0 = 0$$

$$s^3 x(s) - s^2 - 1 + s x(s) - 0 = 0$$

$$s^3 x(s) + s x(s) = s^2 + 1 + 1$$

$$x(s) \cdot (s^3 + s) = s^2 + 2 \quad /: (s^3 + s)$$

$$x(s) = \frac{s^2 + 2}{s^3 + s} = \frac{s^2 + 2}{s(s^2 + 1)}$$

$$\frac{s^2 + 2}{s(s^2 + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1} \quad /: s \cdot (s^2 + 1)$$

$$s^2 + 2 = A(s^2 + 1) + (Bs + C) \cdot s$$

$$s^2 + 2 = As^2 + A + Bs^2 + Cs$$

$$A + B = 1$$

$$B + C = 0$$

$$A + C = 2$$

$$B + \frac{1}{2} = 0$$

$$B = -\frac{1}{2}$$

$$A - C = 1 - 0$$

$$A - C = 1$$

$$A - C = 1$$

$$-1A + C = 2$$

$$-2C = -1$$

$$C = \frac{1}{2}$$

$$A + \frac{1}{2} = 2$$

$$A = 2 - \frac{1}{2}$$

$$A = \frac{3}{2}$$

