MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa	xxoox
POPUNJAVA NASTAVNIK	Broj ↓ bodova

IME I PREZIME: Tomislav Kralj

BROJ INDEKSA: 1701-0052-2011

1. Izračunati dvostruki integral $\iint_S e^{x+y}\,dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20

2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. ~~20~~

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t)+x'(t)=0, \quad x(0)=x''(0)=1,\ x'(0)=0.$$

4. Neka je C cilindar zadan sa $C=\{(x,y,z) : (x+2)^2+(y-3)^2\le 1,\ -1\le z\le 1\}$. Izračunati plošni integral 20

$$\iint_{\partial C} 2x\,dy dz$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx+\ln x\cos y\,dy$ 20

Ukupno: 20

Tablica integrala

$\int dx = x+C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int \frac{dx}{x} = \ln\|x\|+C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int a^x\,dx = \frac{a^x}{\ln a}+C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

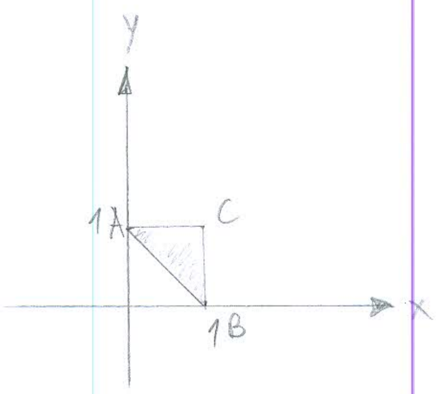
Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tomislav Kraljev 17-01-0052-2011

1) Izračunati $\iint_S e^{x+y}\,dx\,dy$, gdje je trokut S sa vrhovima

A(0,1) B(1,0) C(1,1)



$x \in [0,1]$ ✓

$y \in [-x+1, 1]$ ✓

$\overline{AB}$ $A\begin{pmatrix} x_1 & y_1 \\ 0, & 1 \end{pmatrix}$ $B\begin{pmatrix} x_2 & y_2 \\ 1, & 0 \end{pmatrix}$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-0)(y-1)=(0-1)(x-0)$

$1y-1\cdot 1=-1\cdot x+0$

$y-1=-x$

$y=-x+1$

$e^{x+y}=e^x\cdot e^y$

$$\int_0^1\int_{-x+1}^1 e^{x+y}\,dy\,dx$$

$$\int_0^1\int_{-x+1}^1 e^x\cdot e^y\,dy\,dx = \int_0^1 e^x\cdot e^y\Big|_{-x+1}^1 dx$$

$$=\int_0^1 e^x\cdot\left(e^1-e^{-x+1}\right)dx = \int_0^1 e^{x+1}-e\,dx = e^{x+1}-ex\Big|_0^1$$

$$=e^{1+1}-e\cdot 1-\left(e^{0+1}-e\cdot 0\right)=e^2-e-e=e^2-2e$$ ✓

20

2 $V=?$

$x^2+y^2=4$ $\quad z=y$ i $z=x-2$

$x^2+y^2=r^2$

$r^2=4$

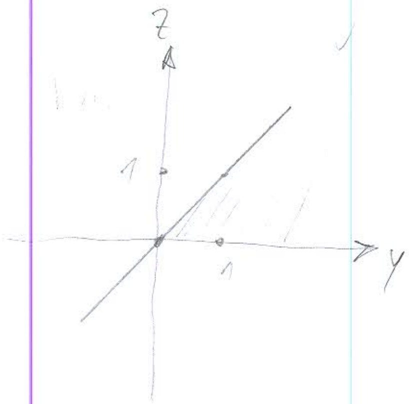
$r=2$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$



$r\in[0,2]$

$\varphi\in[0,2\pi]$

$z\in[-2,0]$

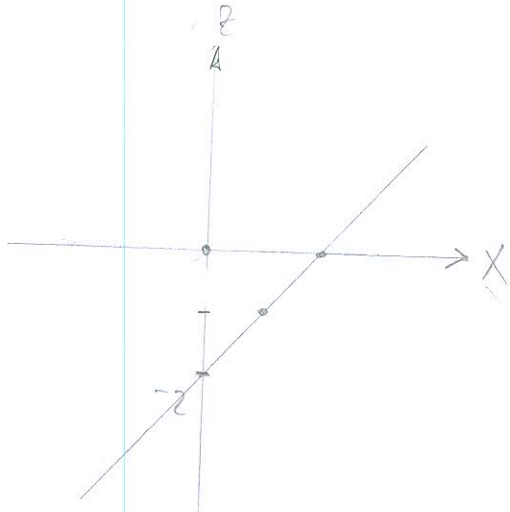
$z=y$

y	0	1
$z=y$	0	1

$\iiint_V r\,dr\,d\varphi\,dz=$

x	0	1	2
$z=x-2$	-2	-1	0

$=\int_0^{2\pi}\int_0^{2}\int_{-2}^{0} r\,dz\,dr\,d\varphi$



⇓

nastavak

Tomislav Kraljev

17-01-0052-2011

2) ⇓ nastavak

$$\int_0^{2\pi}\int_0^{2}\int_{-2}^{0} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{2} r\cdot z\Big|_{-2}^{0}\,dr\,d\varphi = \cancel{\emptyset}$$

$$= \int_0^{2\pi}\int_0^{2} r\cdot(0-(-2))\,dr\,d\varphi = \int_0^{2\pi}\int_0^{2} 2r\,dr\,d\varphi =$$

$$= \int_0^{2\pi} \cancel{2}\cdot\frac{r^2}{\cancel{2}}\Big|_0^2\,d\varphi = \int_0^{2\pi}(2^2-0^2)\,d\varphi = \int_0^{2\pi} 4\,d\varphi$$

$$= 4\cdot\varphi\Big|_0^{2\pi} = 4\cdot 2\pi = 8\pi$$

Tomislav Kraljev 17-01-0052-2011

3) $x'''(t) + x'(t) = 0$ $x(0) = x''(0) = 1$ $x'(0) = 0$

$s^3 X(s) - s^2 X(0) - s X'(0) - X''(0) + s X(s) - X(0) = 0$

$s^3 X(s) - s^2 \cdot 1 - s \cdot 0 - 1 + s X(s) - 1 = 0$

$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$

$s^3 X(s) + s X(s) = s^2 + 1 + 1$

$X(s) \cdot (s^3 + s) = s^2 + 2 \quad / : (s^3 + s)$

$X(s) = \frac{s^2+2}{s^3+s} = \frac{s^2+2}{s(s^2+1)}$

$\frac{s^2+2}{s \cdot (s^2+s)} = \frac{A}{s} + \frac{Bs+C}{(s^2+1)} \quad / \cdot s \cdot (s^2+1)$

$s^2 + 2 = A(s^2+1) + (Bs+C) \cdot s$

$s^2 + 2 = As^2 + A + Bs^2 + Cs$

$A + B = 1$

$B + C = 0$

$A + C = 2$

$B + \frac{1}{2} = 0$

$\boxed{B = -\frac{1}{2}}$

$A - C = 1 - 0$

$A - C = 1$

$A - C = 1$

$-(A + C) = 2$

$-2C = -1$

$\boxed{C = \frac{1}{2}}$

$A + \frac{1}{2} = 2$

$A = 2 - \frac{1}{2}$

$\boxed{A = \frac{3}{2}}$

$$\Rightarrow X(s) = \frac{\frac{3}{2}}{s} + \frac{\left(-\frac{1}{2}\right)\cdot s + \frac{1}{2}}{s^2+1}$$

$$X(s) = \frac{\frac{3}{2}}{s} + \left(-\frac{1}{2}\right)\cdot\frac{s}{s^2+1} + \frac{1}{2}\cdot\frac{1}{s^2+1}$$

$$X(s) = \frac{3}{2} + \left(-\frac{1}{2}\right)\cdot\cos(t) + \frac{1}{2}\cdot\sin(t)$$

$$X(s) = \frac{3}{2} - \frac{1}{2}\cdot\cos(t) + \frac{1}{2}\cdot\sin(t)$$

$X(0) = \frac{3}{2} - \frac{1}{2} = 1 \checkmark$

PROVJERA:

$x'(t) = \frac{1}{2}\sin t + \frac{1}{2}\cos t$ $x'(0) = \frac{1}{2}$ X

$\varnothing$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: SABOLIĆ BORIS

BROJ INDEKSA:

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

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$$\iint\limits_{\partial C} 2x\,\mathrm{d}y\mathrm{d}z$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,\mathrm{d}x+\ln x\cos y\,\mathrm{d}y$ 20

Ukupno: 10

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

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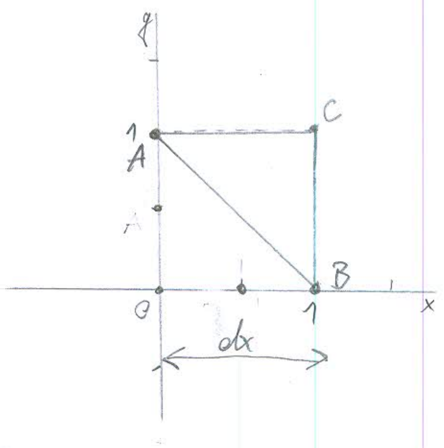
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Sabolić Boris

① $A(0,1)$, $B(1,0)$, $C(1,1)$ $\Big\}\, S$ $\qquad \iint_S e^{x+y}\,dx\,dy$



$\overline{AB}$

$y-1=\frac{0-1}{1-0}(x-0)$

$y-1=\frac{-1}{1}x$

$y=-x+1$

$x=-y+1$

$\overline{BC}$

$y-0=\dots$

$x=1$

$\overline{AC}$

$\dots$

$y=1$

$$\iint_S e^{x+y}\,dx\,dy=\int e^x dx\int e^y dy=\int_0^1 e^x dx\int_{-x+1}^{1} e^y dy=\int_0^1 e^x dx\,\left[e^y\right]^{1+x-1}=$$

10

$$\int_0^1 e^x dx\,(e^x)=e^2$$

$e^1\cdot e^1=7.389$

$e^2=7.389$

$$\iint_S (e^x+e^y)\,dx\,dy=\int e^x dx\int e^y dy=\int_0^1 e^y dy\,(e^x)^{1}_{-y+1}=\int_0^1 e^y dy\,(e^x)^{1+y-1}=\dots e^2$$

③ $x'''(t)+x'(t)=0$

$x(0)=1$

$x''(0)=1$

$x'(0)=0$

$$s^3X(s)-s^2\cancel{x(0)}^{\,1}-s\cancel{x'(0)}^{\,0}-\cancel{x''(0)}^{\,1}+sX(s)-\cancel{x(0)}^{\,1}=0$$

$$s^3X(s)-s^2-1+sX(s)-1=0$$

$(s+2)^2=s^2+4s+4$

$$X(s)(s^3+s)-s^2-2=0$$

$$X(s)\,s(s^2+1)=s^2+2$$

$$X(s)=\frac{s^2+2}{s(s^2+1)}=\cancel{\frac{s^2}{s(s^2-1)}+\frac{2}{s(s^2\ldots)}}$$

$$\frac{A}{s}+\frac{Bs+C}{s^2+1}$$

$s^2+2=$

$(s+1)(s+2)$

$s^2+2s-s+2$

s^2+s+2

$s^2-2s+s-2$

~~s^2-s-2~~

$$s^2+2=A(s^2+1)+(Bs+C)s$$

$$s^2+2=As^2+A+Bs^2+Cs$$

$1=A+B\rightarrow 1=2+B$

$\boxed{0=C}$ $\boxed{-1=B}$

$\boxed{A=2}$

$$X(s)=2\cdot\frac{1}{s}-1\cdot\frac{s+0}{s^2+1}$$

t $\cos t$

$$X(t)=2t-\cos t$$

$x'(t)=2+\sin t$

$x''(t)=\cos t$

PROVJERA

$X(0)=2\cdot 0-\cos 0=-1$ $\cancel{1}$

$x'(0)=2+\sin 0=2$ $\emptyset$

$x''(0)=1$ ✓

SABOLIĆ BORIS

$x'''(t)+x'(t)=0$ $\quad$ $x(0)=1$, $x''(0)=1$, $x'(0)=0$

$$s^3X(s)-s^2\overset{1}{\cancel{x(0)}}-s\overset{0}{\cancel{x'(0)}}-\overset{1}{\cancel{x''(0)}}+sX(s)-\overset{1}{\cancel{x(0)}}=0$$

$$\underline{s^3X(s)}-s^2-1+\underline{sX(s)}-1=0$$

$$X(s)(s^3+s)=s^2+2$$

$$X(s)=\frac{s^2+2}{s(s^2+1)} \Longrightarrow \frac{A}{s}+\frac{Bs+C}{s^2+1}$$

$$s^2+2=A(s^2+1)+(Bs+C)s$$

$$s^2+2=As^2+A+Bs^2+Cs$$

$1=A+B \rightarrow 1=2+B$

$\boxed{0=C}$ $\quad$ $\boxed{-1=B}$

$\boxed{2=A}$

$$X(s)=2\cdot\frac{1}{s}-\frac{s}{s^2+1}$$

$$X(t)=2t-\cos t$$

$x'(t)=2+\sin t$

$x''(t)=\cos t$

?

PROVJERA

$x(0)=2\cdot 0-\cos 0=-1$ –

$x'(0)=2+\sin 0=2$ –

$x''(0)=\cos 0=1$ ✓

PROVJERA:

$x(0)=-1$ X

NIJE RJEŠENJE!

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: KRISTIAN JOZIĆ

BROJ INDEKSA: 17-1-0012-2010

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20 10
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$$\iint\limits_{\widehat{\partial C}} 2x\,\mathrm{d}y\mathrm{d}z$$
5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,\mathrm{d}x+\ln x\cos y\,\mathrm{d}y$ 20

Ukupno: 0 (10)

Kos

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\vert\frac{a+x}{a-x}\right\vert+C$
$\int \frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int \sinh x\,\mathrm{d}x=\cosh x+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\vert\frac{x-a}{x+a}\right\vert+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x=\sinh x+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\vert x+\sqrt{x^2\pm a^2}\right\vert+C$
$\int \sin x\,\mathrm{d}x=-\cos x+C$	$\int \tanh x\,\mathrm{d}x=\ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \coth x\,\mathrm{d}x=\ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x}=\tanh x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x}=-\coth x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

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c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

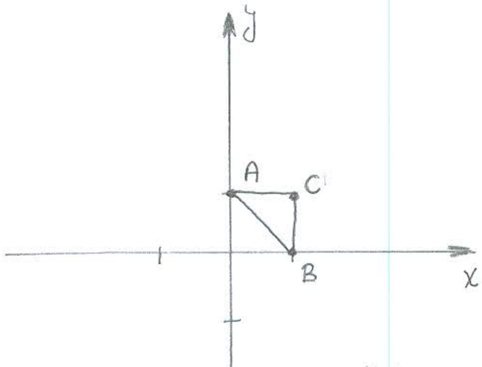
1.
$$\iint_S e^{x+y}\,dx\,dy$$

$$\int_0^1\int_{1-x}^1 e^{x+y}\,dx\,dy$$ ✓ 10

$$\int_0^1 e^{x+y}\Big|_{1-x}^{1}\,dy$$?

$$\int_0^1 e \;\ldots$$

Ø



$\overset{x_1\,y_1}{A(0,1)}$ $\overset{x_2\,y_2}{C(1,1)}$

$$\overline{AC} = y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$= y - 1 = \frac{1-1}{1-0}(0-0)$$

$$\overline{AC} = \underline{\underline{y = 1}}$$

$\overset{x_1\,y_1}{A(0,1)}$ $\overset{x_2\,y_2}{B(1,0)}$

$$\overline{AB} = y - 1 = \frac{0-1}{1-0}(x-0)$$

$$y - 1 = -1(x)$$

$$y - 1 = -x$$

$$\overline{AB} = \underline{\underline{y = 1 - x}}$$

3. $x'''(t) + x'(t) = 0 \qquad x(0) = 1 \quad x'(0) = 0$

$x''(0) = 1$

KRISTIAN JOZIĆ

$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + s Y(0) - y(0) = 0$

$s^3 Y(s) - s^2 \cdot 1 - s \cdot 0 - 1 + s Y(0) - 1 = 0$

$s^3 Y(s) - s^2 - 1 + s Y(0) - 1 = 0$

$Y(s)(s^3 + s) = s^2 + 2 \quad / : (s^3 + s)$

$Y(s) = \frac{s^2+2}{s^3+s} = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{B}{s^2+1} + \frac{C}{s+1}$

$s^2 + 2 = A(s^2+1) + B(s) + C(s^2)$

$s^2 + 2 = As^2 + A + Bs + Cs^2$

$\boxed{2 = A}$

$s^2 = As^2 + Cs^2$

$0 = A + C \Rightarrow 0 = 2 + c$

$\boxed{C = -2}$

$\boxed{B = 0}$

$\frac{2}{s} + \frac{0}{s^2+1} + \frac{-2}{s+1}$

RJEŠENJE ?

0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: Marin Smolić **BROJ INDEKSA:** 55376/2007	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 10
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t)+x'(t)=0, \quad x(0)=x''(0)=1,\ x'(0)=0.$$

4. Neka je C cilindar zadan sa $C=\{(x,y,z) : (x+2)^2+(y-3)^2 \le 1,\ -1 \le z \le 1\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} 2x \, dy dz$$

5. Izracunati $\int\limits_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ 20

Ukupno: ~~0~~ 10

Kos

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n \ne -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\, dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

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e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\, dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
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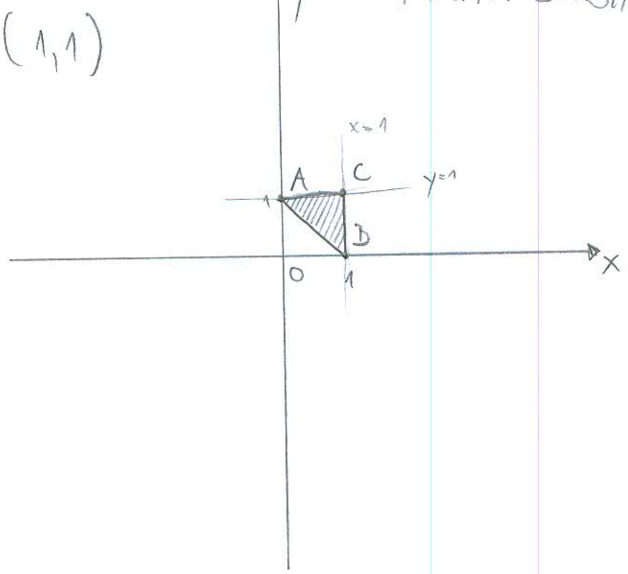
Marin Šmolić

1. $\iint_S e^{x+y}\,dx\,dy$, $A(\overset{x_1}{0},\overset{y_1}{1})$, $B(\overset{x_2}{1},\overset{y_2}{0})$, $C(1,1)$



AB...

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(1-0)(y-1)=(0-1)(x-0)$

$1(y-1)=-1(x)$

$y=-x+1$

$$\iint_S e^{x+y}\,dx\,dy=\int_0^1\int_{-x+1}^1 e^{x+y}\,dy\,dx=\int_0^1\int_{-x+1}^1 e^x\cdot e^y\,dy\,dx$$

?

10

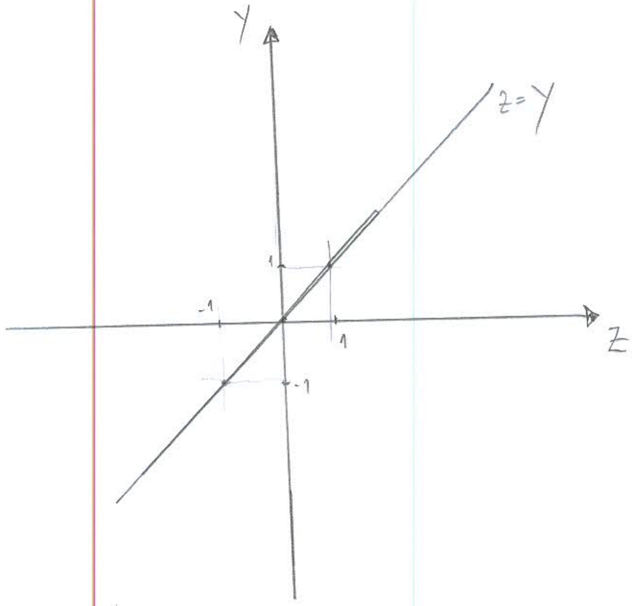
2. $V = ?$ $\quad x^2 + y^2 = 4$, $z = y$, $z = x - 2$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 2^2$

$r = \underline{2}$ //

z	0	-1	1
y	0	-1	1



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** IME I PREZIME: DARIAN RADMAN BROJ INDEKSA: 57635-2009.	Grupa xxoox POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}\,dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20

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$$\iint\limits_{\partial C} 2x\,dy dz$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx+\ln x\cos y\,dy$ 20

Ukupno: ~~0~~

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
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$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

DARIAN RADMAN

(2) $x^2+y^2=4$

$z=y$

$z=x-2 \Rightarrow x=z+2$

$(z+2)^2+z=4$

$2z^2+2\cdot z\cdot 2+2^2+z=4$

$2z^2+4z+4+z=4$

$2z^2+5z=0$

$z_{1,2}=\frac{-5\pm\sqrt{5^2+4\cdot 2\cdot 0}}{4}=\frac{-5\pm 5}{4}=\frac{0}{4}=0 \; - z_1$

$=\frac{-10}{4}=\frac{-5}{2} \; - z_2$

$\emptyset$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: Ivan Škara	**BROJ INDEKSA:** 56180-2008

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20
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Ukupno: 0

Tablica integrala

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$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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IVAN SARKA

2.) $x^2+y^2=4^2$ $z=y$

$x^2+y^2=2$ $z=x-2$

$r=2$

x	0	1
$z=x-2$	-2	-1

$x=r\cos\varphi$
$y=r\sin\varphi$
$dx\,dy\,dz=r\,dr\,d\varphi\,dz$

$r\in[0,2]$
$\varphi\in[0,2\pi]$
$z\in[0,x-2]$

$$V=\int_0^{2\pi}\int_0^2\int_0^{x-2} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_0^2 r\cdot z\Big|_0^{x-2}=r\cdot\sin\varphi+1+2)\,dr\,d\varphi$$

0

$$\int_0^{2\pi}\int_0^2(r^2\sin\varphi+3r)\,dr\,d\varphi=\int_0^{2\pi}\left(\frac{r^3}{3}\sin\varphi+3\cdot\frac{r^2}{2}\right)\Big|_0^2\,d\varphi$$

$$\int_0^{2\pi}\left(\frac{2^3}{3}\sin\varphi+3\cdot\frac{2^2}{2}\right)d\varphi=\int_0^{2\pi}\left(\frac{8}{3}\sin\varphi+3\cdot\frac{6}{2}\right)d\varphi$$

$$=-\frac{8}{3}\cos\varphi+6\varphi\Big|_0^{2\pi}=\frac{8}{3}\cos 2\pi+6\cdot 2\pi-\left(-\frac{8}{3}\cos 2\pi+6\cdot 0\right)$$

$=12\pi$

4) $C=\{(x,y,z):(x+2)^2+(y-3)^2\leq 1,\ -1\leq z\leq 1\}$

$$\iint_{\partial C} 2x\,dy\,dz$$