

odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: ZLATKO LALIC

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1. Koristeći Laplaceovu transformaciju nađi realnu funkciju  $f$  koja zadovoljava sljedeće uvjete:

$$f'''(t) + f'(t) = 1, \quad f''(0) = 1, \quad f'(0) = 1, \quad f(0) = 1.$$

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2. Poznato je da zemljina sila teža oko razine površine dobro odgovara izrazu  $f = -mg\vec{k}$ , gdje su konstante  $m$  masa tijela i  $g$  konstanta ubrzanja oko 9.81, vektor  $\vec{k}$  pokazuje prema gore (od zemlje). Pretpostavimo da je zemlja ravna ploča, barem u dijelu prostora od interesa i da je koordinatni vektori pokazuju  $\vec{i}$  prema sjeveru,  $\vec{j}$  prema zapadu i  $\vec{k}$  prema gore. Efektivno stoga promatramo vektorsko polje sile zadano sa

$$f(x\vec{i} + y\vec{j} + z\vec{k}) = -mg\vec{k}.$$

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Pokaži da je ovako zadana sila teže potencijalno polje?

3. Izračunati dvostruki integral:

$$\iint_S xy \, dx \, dy,$$

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gdje je  $S$  jedno od dva područja omeđenih kružnicom  $x^2 + y^2 = 4$  i pravcem koji prolazi točkama  $A(0, 2)$  i  $B(2, 0)$ .

4. Izračunati volumen paraboloida omeđenog plohama:  $z = x^2 + y^2$ ,  $z = 5$ .

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5. Izračunati:  $\int_{\hat{\Gamma}} \mathbf{w} \cdot d\mathbf{r}$ , ako je  $\mathbf{w}(x, y, z) = (y, z, x)$  i krivulja zadana parametrizacijom

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$$x = \frac{1}{2} \cos t, \quad y = \frac{1}{2} \sin t, \quad z = \frac{\sqrt{3}}{2}, \quad t \in [0, \pi].$$

Ukupno:

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5.

$$\mathbf{r}(t) = \begin{pmatrix} \frac{1}{2} \cos t \\ \frac{1}{2} \sin t \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} \frac{1}{2} \sin t \\ \frac{\sqrt{3}}{4} \\ \frac{1}{2} \cos t \end{pmatrix}$$

$$\mathbf{r}'(t) = \begin{pmatrix} -\frac{1}{2} \sin t \\ \frac{1}{2} \cos t \\ 0 \end{pmatrix}$$

$$\int_{\hat{\Gamma}} \mathbf{w} \cdot d\mathbf{r} = \int_0^\pi \mathbf{w}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt$$

$$\mathbf{w} = \begin{pmatrix} \frac{1}{2} \sin t \\ \frac{\sqrt{3}}{4} \\ \frac{1}{2} \cos t \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{2} \sin t \\ \frac{1}{2} \cos t \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} \sin^2 t \\ \frac{\sqrt{3}}{4} \cos t \\ 0 \end{pmatrix} = \int_0^\pi \left( -\frac{1}{4} \sin^2 t + \frac{\sqrt{3}}{4} \cos t \right) dt = \frac{15}{4}$$

$$\int_0^{\pi} \frac{1}{4} \cdot \frac{1 + \cos 2t}{2} dt = \int_0^{\pi} \frac{\sqrt{3}}{4} \cos t dt$$

$$= -\frac{1}{2} \cdot \frac{1}{4} \cdot \int_0^{\pi} dt + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \int_0^{\pi} \cos 2t dt + \frac{1}{2} \cdot \frac{\sqrt{3}}{4} \sin t \Big|_0^{\pi}$$

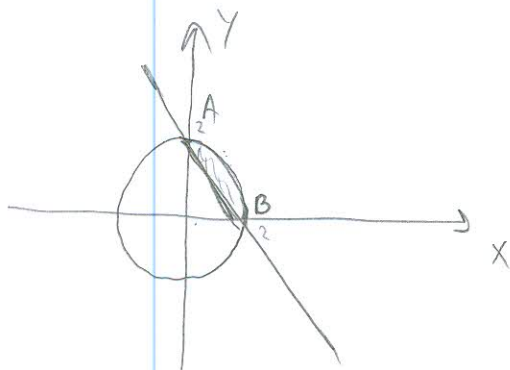
$$= -\frac{1}{8} t \Big|_0^{\pi} + \frac{1}{8} \cdot \frac{1}{2} (\sin 2\pi - \sin 0)$$

$$= -\frac{\pi}{8} \quad \checkmark \quad \underline{\underline{E}}$$

$$\int \cos 2t = \int_{2t=2}^{2t=2} \cos 2t dt = \frac{1}{2} \int \cos z dz$$

$$= \frac{1}{2} \sin z = \frac{1}{2} \sin 2t$$

3.



kurve

$$x^2 + y^2 = 4$$

$$r^2 = 4/r$$

$$r = 2$$

$$t \in (0, 2\pi)$$

$$r \in (0, 2)$$

$$x = r \cos t$$

$$y = r \sin t$$

$$\int_1 = \int \cos t \sin t dt$$

$$= \int_{\sin t = z} \cos t dt = dz$$

$$= \int z dz = \frac{z^2}{2}$$

$$\int_1 = \frac{\sin^2 t}{2}$$

$$2\pi \quad 2$$

$$\int_0^{2\pi} \int_0^2 (r \cos t \cdot r \sin t) r dr dt$$

$$= \int_0^{2\pi} r^3 \int_0^2 \cos t \sin t dt dr$$

$$\int_1 \quad 2\pi$$

$$= \int_0^2 r^3 \left( \frac{\sin^2 t}{2} \Big|_0^{2\pi} \right) dr = r^3 \frac{\sin^2 2\pi - \sin^2 0}{2}$$

$$= 0$$

1. 1

$$F(s) = 1 \cdot \frac{1}{s^2} + \frac{2}{s} + 1 \cdot \frac{s}{s^2+1} + \frac{0}{s^2+1}$$

$$f(s) = t + 2 - \cos t$$

PROVERBA

$$f(0) = 1$$

$$f'(0) = 0 + 2 - \underbrace{\cos 0}_{=1} = 1 \quad f''(0) = 0 + 0 + 1 = 1$$

$$f'''(0) = 1 + 0 + 0 = 1$$

2.

$$\text{rot } \vec{g} = \nabla \times \vec{g} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

VEKTORSKO POLJE JE POTENCIJALNO ZA

$$f(x\vec{i} + y\vec{j} + z\vec{k})$$

OVO ZNAČI DA NIJE ROTACIONO ✓  
ONDA JE JEST POTENCIJALNO ✓

POGREŠNO POLJE JE PROVERAVANO:

TREBALO JE

$$P(x, y, z) = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix}$$





$$1. f'''(1) + f'(1) = 1 \quad f''(0) = 1, \quad f'(0) = 1, \quad f(0) = 1$$

$$\left[ \underbrace{s^3 F(s)}_{=1} - \underbrace{s^2 f(0)}_{=1} - \underbrace{s f'(0)}_{=1} - \underbrace{f''(0)}_{=1} \right] + \left[ \underbrace{s F(s)}_{=1} - \underbrace{f(0)}_{=1} \right] = \frac{1}{s}$$

$$s^3 F(s) - s^2 - s - 1 + s F(s) - 1 = \frac{1}{s}$$

$$F(s) (s^3 + s) = \frac{1}{s} + s^2 + s + 1 + 1$$

$$F(s) (s^3 + s) = \frac{1}{s} + s^2 + s + 2$$

$$F(s) (s^3 + s) = \frac{1 + s^3 + s^2 + 2s}{s} \quad | : (s^3 + s)$$

$$F(s) = \frac{1 + s^3 + s^2 + 2s}{(s^4 + s^2)} \quad (s^4 + s^2) = s^2(s^2 + 1)$$

$$\frac{1 + s^3 + s^2 + 2s}{s^2(s^2 + 1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs + D}{s^2 + 1} \quad | \cdot s^2(s^2 + 1)$$

$$1 + s^3 + s^2 + 2s = A(s^2 + 1) + Bs(s^2 + 1) + (Cs + D)(s^2)$$

$$1 + s^3 + s^2 + 2s = As^2 + A + Bs^3 + Bs + Cs^3 + Ds^2$$

$$s^3: B + C = 1 \quad \checkmark \quad \Rightarrow 2 + C = 1$$

$$s^2: A + D = 1 \quad \checkmark \quad \boxed{C = -1}$$

$$s^1: \boxed{B = 2} \quad \checkmark \quad \rightarrow 1 + D = 1$$

$$s^0: \boxed{A = 1} \quad \checkmark \quad \boxed{D = 0}$$

$$4. \quad z = x^2 + y^2, \quad z = 5$$

ZLATKO LACIĆ

$$x^2 + y^2 = z$$

$$r^2 = z \quad |r \quad \Rightarrow \quad z = r^2$$

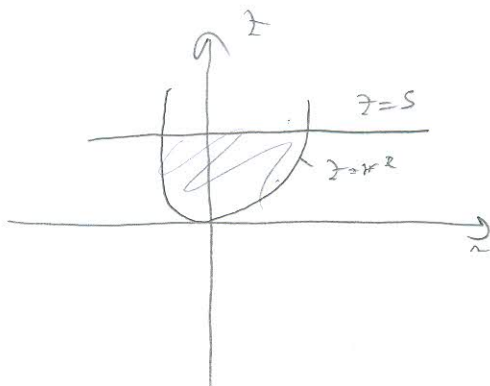
$$r = \sqrt{z} \quad z = 5$$

$$r = \sqrt{5}$$

$$r \in (0, \sqrt{5})$$

$$z \in (r^2, 5]$$

$$\theta \in (0, 2\pi)$$



$$V = \int_0^{2\pi} \int_0^{\sqrt{5}} \int_{r^2}^5 r \, dz \, dr \, d\theta = 2\pi \int_0^{\sqrt{5}} \int_{r^2}^5 r \, dz \, dr \quad \checkmark$$

$$= 2\pi \int_0^{\sqrt{5}} r z \Big|_{r^2}^5 \, dr = 2\pi \int_0^{\sqrt{5}} r (5 - r^2) \, dr$$

$$= 2\pi \int_0^{\sqrt{5}} (5r - r^3) \, dr = 2\pi \left( 5 \frac{r^2}{2} - \frac{r^4}{4} \right) \Big|_0^{\sqrt{5}}$$

$$= 2\pi \left[ \frac{5}{2} (\sqrt{5})^2 - \frac{(\sqrt{5})^4}{4} \right] = 2\pi \left[ \frac{25}{2} - \frac{25}{4} \right] = 2\pi \left( \frac{25}{4} \right) = \frac{25}{2} \pi \quad \checkmark$$

