

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA
NASTAVNIK
Broj ↓
Bodova

IME I PREZIME: *Antonio-Niordž Galešić* BROJ INDEKSA: *17-1-0018-2018*

1. Izračunati volumen tijela omeđenog ravninama $x = 0, x = 1, y = 0, z = 0, z = 1 - y$. 20

2. Neka je C krivulja sa parametrizacijom $r(t) = \frac{t}{5}i + (\cos(t) + 3)j + \sin tk, t \in [0, 5\pi]$. Zadano je skalarno polje $f(x, y, z) = x + z$. Izračunaj $\int_C f ds$. 20

3. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \dots x^2 + y^2 \leq 2$. Nije potrebno računati površinu baze. 20

4. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednažbu: 20

$$y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

5. Izračunati diferencijal od $f(x, y) = \left[\frac{\cos(xy)}{\frac{x}{y}} \right]$ u točki $T(1, 2)$. 20

Ukupno:

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4) $y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 5$

$$s^2 Y(s) - s y(0) - y'(0) + 2(s Y(s) - y(0)) + 2 Y(s) = 0$$

$$s^2 Y(s) - 5 + 2s Y(s) + 2 Y(s) = 0$$

$$s^2 Y(s) + 2s Y(s) + 2 Y(s) = 5$$

$$Y(s) (s^2 + 2s + 2) = 5 \quad / : s^2 + 2s + 2$$

$$Y(s) = \frac{5}{s^2 + 2s + 2} = 5 \cdot \frac{1}{(s - (-1))^2 + 1^2}$$

$$y(t) = 5 (1 \cdot e^{-t} \sin t)$$

$$\underline{\underline{y(t) = 5 e^{-t} \sin t \quad \checkmark}}$$

PROVERA

$$y(0) = 5 e^0 \cdot \sin 0$$

$$y(0) = 5 \cdot 0 = 0$$

$$y'(t) = 5 e^{-t} \cos t = 5 e^0 \cos 0 = 5 \cdot 1 = 5 //$$

$$2) \quad r(t) = \begin{pmatrix} x = \frac{t}{5} \\ y = \cos t + 3 \\ z = \sin t \end{pmatrix} \quad r'(t) = \begin{pmatrix} x = \frac{1}{5} \\ y = -\sin t \\ z = \cos t \end{pmatrix} \quad t \in [0, 5\pi]$$

$$\|r'(t)\| = \sqrt{\left(\frac{1}{5}\right)^2 + (-\sin t)^2 + (\cos t)^2} = \sqrt{\frac{1}{25} + \underbrace{\sin^2 t + \cos^2 t}_1}$$

$$\|r'(t)\| = \sqrt{\frac{1}{25} + 1} = \sqrt{\frac{26}{25}} \quad \checkmark$$

$$\int_0^{5\pi} \left(\frac{t}{5} + \sin t\right) \cdot \sqrt{\frac{26}{25}} dt = \sqrt{\frac{26}{25}} \int_0^{5\pi} \frac{t}{5} dt + \sqrt{\frac{26}{25}} \int_0^{5\pi} \sin t dt$$

$$\frac{1}{5} \cdot \sqrt{\frac{26}{25}} \cdot \frac{t^2}{2} \Big|_0^{5\pi} - \sqrt{\frac{26}{25}} \cdot \cos t \Big|_0^{5\pi} = \frac{1}{5} \cdot \sqrt{\frac{26}{25}} \cdot \frac{25\pi^2}{2} - \sqrt{\frac{26}{25}} \cdot (\cos 5\pi - \cos 0)$$

$$= \frac{1}{5} \cdot \sqrt{\frac{26}{25}} \cdot \frac{25\pi^2}{2} = \frac{25}{10} \pi^2 \sqrt{\frac{26}{25}} = \frac{1}{2} \pi^2 \sqrt{26} \quad \checkmark$$

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$$5) \quad f(x, y) = \begin{bmatrix} \cos(xy) \\ \frac{x}{y} \end{bmatrix} \quad T(1, 2)$$

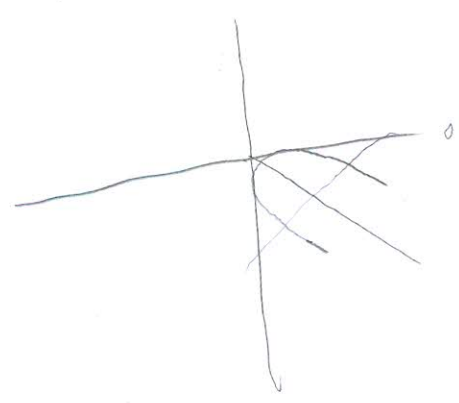
$$\frac{\partial}{\partial x} \cos(xy) = -\sin x \quad \text{diff} = -\sin x + \frac{1}{y}$$

$$\left(\frac{x}{y}\right)' = \frac{1}{y} \quad \text{diff} = -\sin 1 + \frac{1}{2} \quad \times$$

$$\text{diff} = 0,4825475936$$

$$\frac{y^2}{3} \cos^2 \theta + \sin^2 \theta$$

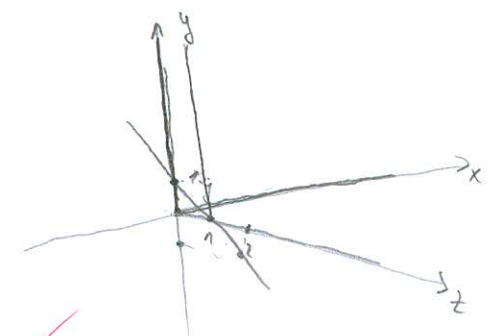
$$y = r \sin \theta$$



$$\int r dr d\theta dz$$

z	0	1	2
y	1	0	-1

$x=0$
 $x=1$
 $y=0$
 $z=0$
 $z=1-y$



$$\int_0^1 \int_{1-y}^0 \int_0^1 dy dz dx = \int_0^1 dy \int_{1-y}^0 dz \cdot x = \int_0^1 dy \left[\frac{z^2}{2} \right]_{1-y}^0 + z \Big|_{1-y}^0$$

$$\int_0^1 dy \int_{1-y}^0 dz \cdot (-z+1) = \int_0^1 dy \int_{1-y}^0 (-z dz + dz) = \int_0^1 dy \left[-\frac{z^2}{2} + z \right]_{1-y}^0$$

$$\int_0^1 dy \left[\frac{(1+y)^2}{2} - 1+y \right] = \int_0^1 dy \left[\frac{y^2 + 2y + 1}{2} - 1 + y \right] = \int_0^1 dy \left[\frac{y^2}{2} + y - \frac{1}{2} \right]$$

$$\left[\frac{y^3}{6} + \frac{y^2}{2} - \frac{y}{2} \right]_0^1 = \frac{1}{6} + \frac{1}{2} - \frac{1}{2} = \frac{1}{6}$$

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POPUNJAVA
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bodova

IME I PREZIME: MATEO MAVAR

BROJ INDEKSA: 17-2-0087-2011

1. Izračunati volumen tijela omeđenog ravninama $x = 0, x = 1, y = 0, z = 0, z = 1 - y$. 20
2. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{5}\mathbf{i} + (\cos(t) + 3)\mathbf{j} + \sin t\mathbf{k}, t \in [0, 5\pi]$. Zadano je skalarno polje $f(x, y, z) = x + z$. Izračunaj $\int_C f ds$. 20
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Ukupno:

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1. $x=0, x=1, y=0, z=0, z=1-y$

$x \in [0, 1]$

$z \in [0, 1-y]$

$y \in [0, 1]$

$z = 1 - y$
 $0 = 1 - y$
 $y = 1$

$$\int_0^1 \int_0^{1-y} \int_0^1 dz dy dx \quad \checkmark$$

$$\int_0^1 \int_0^1 z \Big|_0^{1-y} dy dx$$

$$\int_0^1 \int_0^1 (1-y) dy dx$$

$$\int_0^1 \left(y - \frac{y^2}{2} \Big|_0^1 \right) dx = \int_0^1 \left(1 - \frac{1}{2} \right) dx = \int_0^1 \frac{1}{2} dx$$

$$\frac{1}{2} x \Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2} \quad \checkmark$$

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MATEO MAVAR

2. $r(t) = \frac{t}{5}i + (\cos t + 3)j + \sin t k$
 $t \in [0, 5\pi]$

$f(x, y, z) = x + z$ Izračunaj $\int_C f d\alpha$

$r(t) = \begin{pmatrix} \frac{t}{5} \\ \cos t + 3 \\ \sin t \end{pmatrix}, r'(t) = \begin{pmatrix} \frac{1}{5} \\ -\sin t \\ \cos t \end{pmatrix}$

$\|r'(t)\| = \sqrt{\left(\frac{1}{5}\right)^2 + (-\sin t)^2 + (\cos t)^2} = \sqrt{\frac{1}{25} + \sin^2 t + \cos^2 t} = \sqrt{\frac{1}{25} + 1}$
 $= \sqrt{\frac{26}{25}} = \frac{\sqrt{26}}{5} \checkmark$

$\int_C f d\alpha = \int_0^{5\pi} \frac{\sqrt{26}}{5} (x+z) dt = \int_0^{5\pi} \frac{\sqrt{26}}{5} \left(\frac{t}{5} + \sin t\right) dt \checkmark$

$= \frac{\sqrt{26}}{5} \cdot \left(\frac{1}{5} \frac{t^2}{2} - \cos t\right) \Big|_0^{5\pi}$

$= \frac{\sqrt{26}}{5} \cdot 5\pi \left(\frac{1}{5} \cdot \frac{25\pi^2}{2} + 1 - \left(\frac{1}{5} \cdot \frac{0}{2} - 1\right)\right)$

$= \pi \sqrt{26} \cdot \left(\frac{25}{2}\pi^2 + 1 + 1\right) = \pi \sqrt{26} \cdot \left(\frac{5}{2}\pi^2 + 2\right) = 427,2921031 \checkmark$
20

3. $P(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$

$3z = x^2 + y^2$

$r^2 = 3z$

$z = 3$

$r^2 = 9$

$r = 3$

$z = \frac{x^2}{3} + \frac{y^2}{3} / 3$

$3z = x^2 + y^2$

$3 \frac{\partial z}{\partial x} = 2x$

$\frac{\partial z}{\partial x} = \frac{2x}{3}$

$3z = x^2 + y^2$

$3 \frac{\partial z}{\partial y} = 2y$

$\frac{\partial z}{\partial y} = \frac{2y}{3}$

$\theta \in [0, 2\pi]$

$r \in [0, 3]$ X



3.

MATEO NAVAR

$$\sqrt{1 + \left(\frac{2x}{3}\right)^2 + \left(\frac{2y}{3}\right)^2}$$

$$= \sqrt{1 + \left(\frac{4x^2}{9} + \frac{4y^2}{9}\right)}$$

$$= \sqrt{1 + \frac{4x^2 + 4y^2}{9}}$$

$$= \sqrt{1 + \frac{4(x^2 + y^2)}{9}}$$

$$= \sqrt{1 + \frac{4r^2}{9}}$$

$$\int \sqrt{1 + \frac{4r^2}{9}}$$

$$1 + \frac{4r^2}{9} = t$$

$$\frac{8}{3} r dr = dt \cdot \frac{8}{3}$$

$$r dr = \frac{dt}{8}$$

$$r dr = \frac{3}{8} dt$$

$$\int \sqrt{t} \cdot \frac{3}{8} dt$$

$$\int \frac{3}{8} t^{\frac{1}{2}} dt = \frac{3}{8} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{3}{8} \cdot \frac{2}{3} \sqrt{t^3} = \frac{1}{4} \sqrt{\left(1 + \frac{4r^2}{9}\right)^3}$$

$$P(s) = \int_0^{2\pi} \int_0^3 \frac{1}{4} \sqrt{\left(1 + \frac{4r^2}{9}\right)^3} r dr d\varphi$$

$$\frac{6855}{27}$$

$$= \int_0^{2\pi} \int_0^3 \frac{1}{4} r \sqrt{1 + \frac{4r^2}{27}} dr d\varphi = \int_0^{2\pi} \frac{1}{4} \cdot \frac{r^2}{2} \sqrt{1 + \frac{4}{27} \cdot \frac{r^6}{6}} d\varphi$$

$$= \int_0^{2\pi} \frac{1}{4} \cdot \frac{4}{2} \sqrt{1 + \frac{4}{27} \cdot \frac{64}{6}} d\varphi = \int_0^{2\pi} \frac{1}{2} \sqrt{1 + \frac{4^2 \cdot 2^3}{27 \cdot 2^1}} d\varphi$$

$$= \int_0^{2\pi} \frac{1}{2} \sqrt{19} d\varphi = \int_0^{2\pi} \frac{\sqrt{19}}{2} d\varphi$$

$$= \frac{\sqrt{19}}{2} \varphi \Big|_0^{2\pi} = \frac{\sqrt{19}}{2} \cdot 2\pi - \frac{\sqrt{19}}{2} \cdot 0 = \frac{\sqrt{19}}{2} \cdot 2\pi = \sqrt{19} \pi$$

