

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

RIJEŠENI ZADACI

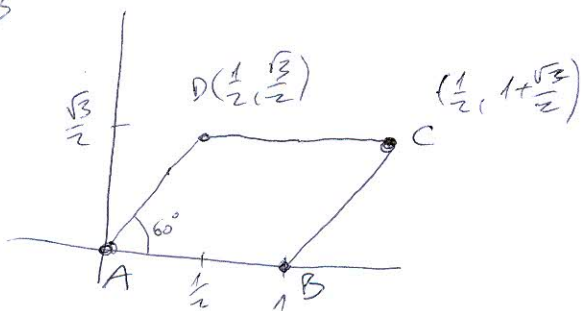
BROJ INDEKSA:

- Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y \, dx \, dy$. 20
- Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$. 20
- Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{1}{4}\mathbf{i} + (\cos(t) + 3)\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f \, ds$. 20
- Izračunati $\iint_S (x^2 + y^2) \, dS$ ako je S kružni stožac zadan jednačbom $z = \sqrt{x^2 + y^2}$ i $0 \leq z \leq 3$. 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu: 20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4, \quad f'(0) = 3, \quad f''(0) = 5.$$

Ukupno:

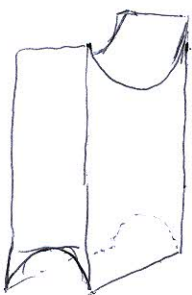
① ROMB



DAJE
OVO JE JEDNOSTAVNO
VIDI LALIC.

②

SKICA



$$V = \int_{-1}^1 \int_{-1}^1 \int_{-y^2}^{3+x^2} dz \, dy \, dx = \dots$$

$dz \, dy \, dx = \dots$

DAJE JEDNOSTAVNO
INTEGRIRANJE

③

$$\int_C f \, ds = \int_0^{4\pi} (1 + \sin t) \sqrt{\frac{17}{16}} \, dt = \dots = \pi\sqrt{17}$$

④

IZA...

④ EKSPLICITNA JEDNAČBA

PROHE $z = \sqrt{x^2 + y^2}$

NORMALA: $\frac{\partial r}{\partial x} = \begin{pmatrix} 1 \\ 0 \\ \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix}$ $\frac{\partial r}{\partial y} = \begin{pmatrix} 0 \\ 1 \\ \frac{y}{\sqrt{x^2+y^2}} \end{pmatrix}$

NORMALA
PARAMETRIZACIJA $r(x,y) = \begin{pmatrix} x \\ y \\ \sqrt{x^2+y^2} \end{pmatrix}$

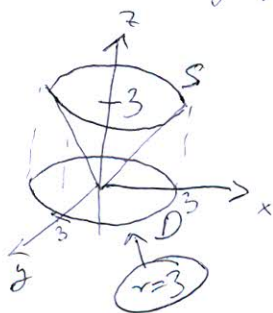
$$\vec{n} = \frac{\partial r}{\partial x} \times \frac{\partial r}{\partial y} = \begin{pmatrix} 1 & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 1 \end{pmatrix} = \begin{pmatrix} -\frac{x}{\sqrt{x^2+y^2}} \\ -\frac{y}{\sqrt{x^2+y^2}} \\ 1 \end{pmatrix} = \vec{n}$$

$$\|\vec{n}\| = \sqrt{1 + \frac{x^2+y^2}{(\sqrt{x^2+y^2})^2}} = \sqrt{2}$$

$$\iint_S (x^2+y^2) dS = \sqrt{2} \iint_D (x^2+y^2) dx dy = \left. \begin{array}{l} \text{PRIJELAZ NA} \\ \text{POLARNE} \\ \text{KOORDINATE} \end{array} \right\}$$

$$= \sqrt{2} \int_0^{2\pi} \int_0^3 [(r \cos \varphi)^2 + (r \sin \varphi)^2] \cdot r dr d\varphi = \sqrt{2} \int_0^{2\pi} \int_0^3 r^3 dr d\varphi = \sqrt{2} \cdot 2\pi \cdot \frac{3^4}{4} = \frac{81\sqrt{2}}{2}$$

D = krug ispod stišca u x-y ravnini



⑤ GLAVNI KORACI:

NAKON LAPLACEOVE TRANSFORMACIJE:

$$-9 - 3s - 4s^2 + 2F(s) + sF(s) + s^3F(s) + 2(3 - 4s + s^2F(s)) = \frac{1}{s^2}$$

ALGEBARSKO RJEŠENJE:

$$F(s) = \frac{1 + 15s^2 + 11s^3 + 4s^4}{s^2(2+s)(s^2+1)}$$

RASTAV NA PARCIJALNE RAZLOMKE:

$$F(s) = \frac{1}{2s^2} - \frac{1}{4s} + \frac{37}{20(s+2)} + \frac{31+12s}{5(s^2+1)}$$

PRIMJENA INVERZNE LAPLACEOVE TRANSFORMACIJE:

$$f(t) = -\frac{1}{4} + \frac{37e^{-2t}}{20} + \frac{t}{2} + \frac{1}{5}(12 \cos t + 31 \sin t)$$

PROVJERA ...

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: ZLATKO LALIC

BROJ INDEKSA: 57646-2009

1. Izaberi bilo koji romb R u ravnini i na njemu odredi integral $\iint_R x + y \, dx \, dy$.

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2. Izračunati volumen tijela omeđenog ravninama $x = 1$, $x = -1$, $y = 1$, $y = -1$, $z = 3 + x^2$, $z = -y^2$.

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3. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t) = \frac{t}{4}\mathbf{i} + (\cos(t) + 3)\mathbf{j} + \sin t\mathbf{k}$, $t \in [0, 4\pi]$. Zadano je skalarno polje $f(x, y, z) = 1 + z$. Izračunaj $\int_C f \, ds$.

20

4. Izračunati $\iint_S (x^2 + y^2) \, dS$ ako je S kružni stožac zadan jednačbom $z = \sqrt{x^2 + y^2}$ i $0 \leq z \leq 3$.

20

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbnu:

20

$$f'''(t) + 2f''(t) + f'(t) + 2f(t) = t, \quad f(0) = 4, \quad f'(0) = 3, \quad f''(0) = 5.$$

Ukupno:

~~115~~
20

3.

$$\mathbf{r}'(t) = \begin{pmatrix} \frac{t}{4} \\ \cos t + 3 \\ \sin t \end{pmatrix}$$

$$t \in (0, 4\pi)$$

$$f(x, y, z) = 1 + z$$

$$\mathbf{r}''(t) = \begin{pmatrix} \frac{1}{4} \\ -\sin t \\ \cos t \end{pmatrix}$$

$$\|\mathbf{r}'(t)\| = \sqrt{\left(\frac{t}{4}\right)^2 + (\cos t + 3)^2 + \sin^2 t} = \sqrt{\frac{1}{16} + \underbrace{\cos^2 t + 6\cos t + 9 + \sin^2 t}_{=1}} = \sqrt{\frac{1}{16} + 6\cos t + 10}$$

$$= \sqrt{\frac{1}{16} + 1} = \sqrt{\frac{17}{16}}$$

$$f(\mathbf{r}(t)) = 1 + z$$

$$= 1 + \sin t$$

$$\int_C f \, ds = \int_0^{4\pi} (1 + \sin t) \cdot \frac{1}{4} \, dt = \int_0^{4\pi} \frac{1}{4} \, dt + \frac{1}{4} \int_0^{4\pi} \sin t \, dt$$

$$= \frac{1}{4} t \Big|_0^{4\pi} - \frac{1}{4} \cos t \Big|_0^{4\pi} = \frac{1}{4} (4\pi - 0) - \frac{1}{4} (\cos 4\pi - \cos 0)$$

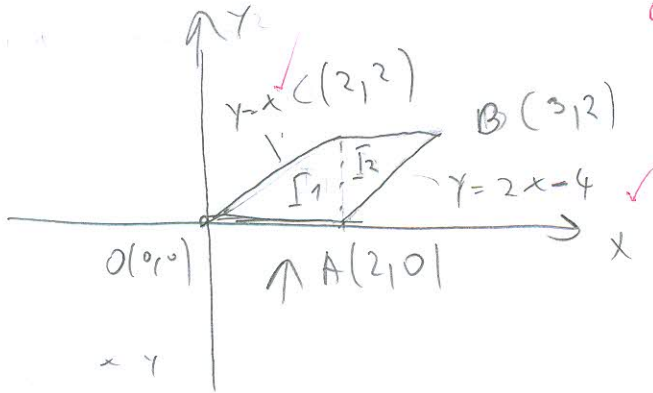
$$= \pi$$

$$\iint_R x+y \, dx \, dy$$

$$O(0,0) \mid A(2,0) \mid B(3,2) \mid C(2,2)$$

KOORDINATE

NASTAVNIK JE DOPUSTIO PARALELOGRAM
OVO NIJE NI TI ROMB NI TI PARALELOGRAM



$$\overline{AB} \quad A(2,0) \\ B(3,2)$$

$$\overline{OC} \quad O(0,0) \\ C(2,2)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0 = \frac{2 - 0}{3 - 2} (x - 2)$$

$$y - 0 = \frac{2 - 0}{2 - 0} (x - 0)$$

$$y = 2x - 4$$

$$y = *$$

$$\iint_R x+y \, dx \, dy = \underbrace{\int_0^2 dx \int_0^x (x+y) \, dy}_{I_1} + \underbrace{\int_2^3 dx \int_{2x-4}^2 (x+y) \, dy}_{I_2}$$

$$I_1 = \int_0^2 dx \int_0^x (*+y) \, dy$$

$$= \int_0^2 \left(*y + \frac{y^2}{2} \right) \Big|_0^x dx$$

$$= \int_0^2 \left(x \cdot x + \frac{x^2}{2} \right) dx$$

$$= \int_0^2 \left(x^2 + \frac{x^2}{2} \right) dx = \left(\frac{x^3}{3} + \frac{1}{2} \cdot \frac{x^3}{3} \right) \Big|_0^2 = \left(\frac{x^3}{3} + \frac{1}{6} x^3 \right) \Big|_0^2$$

$$= \left(\frac{(2)^3}{3} + \frac{1}{6} \cdot (2)^3 \right) = \frac{8}{3} + \frac{4}{3} = 4$$

$$9. \quad I_2 = \int_2^3 dx \int_{2x-4}^2 (x+y) dy$$

$$= \int_2^3 \left(xy + \frac{y^2}{2} \right) \Big|_{2x-4}^2 dx$$

$$= \int_2^3 \left(x \cdot 2 + \frac{2^2}{2} - \left(x \cdot (2x-4) + \frac{(2x-4)^2}{2} \right) \right) dx$$

$$= \int_2^3 \left(2x + 2 - 2x^2 + 4x - \frac{(4x^2 - 2 \cdot 2x \cdot 4 + 4^2)}{2} \right) dx$$

$$= \int_2^3 \left(2x + 2 - 2x^2 + 4x - \frac{4x^2 - 16x + 16}{2} \right) dx$$

$$= \int_2^3 \left(2x + 2 - 2x^2 + 4x - 2x^2 + 8x - 8 \right) dx$$

$$= \int_2^3 \left(-6 + 14x - 4x^2 \right) dx \quad \checkmark$$

$$= -6x \Big|_2^3 + 14 \frac{x^2}{2} \Big|_2^3 - 4 \frac{x^3}{3} \Big|_2^3 \quad \checkmark$$

$$= -6(3-2) + 7(3^2 - 2^2) - 4 \left(\frac{3^3}{3} - \frac{2^3}{3} \right) \quad \times$$

$$= -6 + 31 - 36 = \frac{32}{3}$$

$$= \frac{115}{3}$$

$$I = I_1 + I_2$$

$$= 4 + \frac{115}{3}$$

$$= \frac{127}{3} \quad \checkmark$$

ZADATAK JE DOBRO
POSTAVLJEN (INTEGRAL)
OSIM ŠTO NIJE ODABRAN
PARALELOGRAM.

15

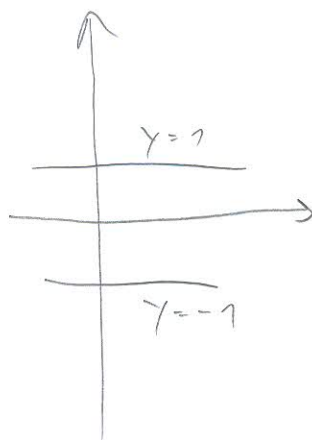
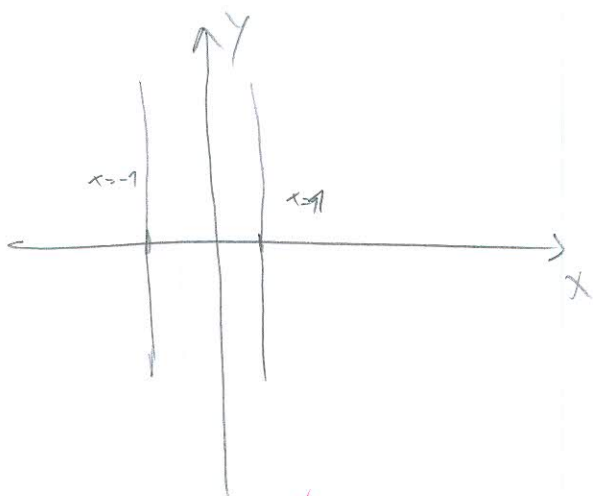
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2. Volumen

$x = 1, x = -1$

$y = 1, y = -1$

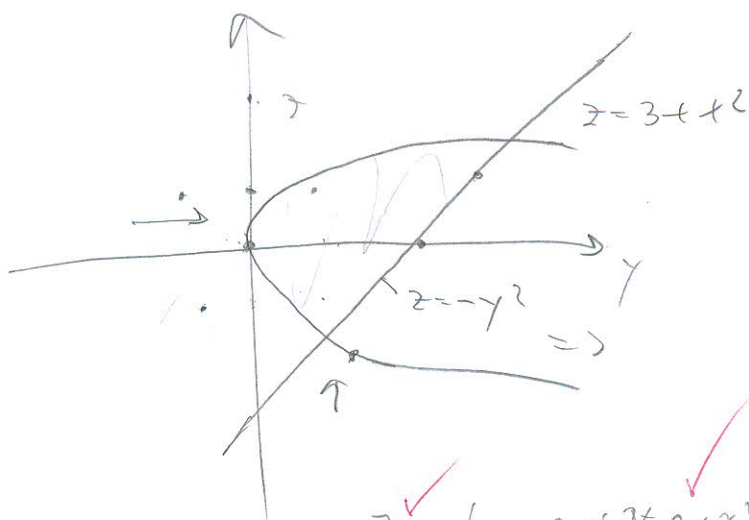
$z = 3 + x^2, z = -y^2$



$x \in [-1, 1] \checkmark \Rightarrow]r(-1, 1)$

$y \in [-1, 1] \checkmark \Rightarrow]r(-1, 1)$

$z = 3 + x^2, z = -y^2$



z	-1	-4	0
y	1	2	0

z	0	1
x	3	4

$z \in [-y^2, 3+x^2] \checkmark = [-r^2 \cos^2 t, 3+r^2 \cos^2 t] \checkmark$

$V \equiv \int_{-1}^1 \int_{-1}^1 \int_{-y^2}^{3+x^2} r \, dz \, dy \, dx \checkmark$

~~15~~
OMK

POGREŠNO POSTAVLJEN INTEGRAL
JER SU POBLIŽI POMEŠTANE
ELIPTIČNI KARTEZIJEVE KOORDINATE SA
JAKOBIJANOM KOD PRELASKA U CILINDRIČNE.

