MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MATIJA JAKOBAC **BROJ INDEKSA:** 57921 - 2009

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati $\iint_S \cos(x+y)\,dxdy$, gdje je S područje omeđeno pravcima $x=0$, $y=4\pi$, $y=x$. 20 15
2. Izračunati volumen tijela omeđenog valjkom $x^2+y^2=5^2$ i ravninama $z=y+5$ i $z=-1$. 20
3. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t)=(\cos(t)+1)\,\mathbf{i}+\frac{t}{2}\mathbf{j}+\sin t\mathbf{k}$, $t\in[0,5\pi]$. Zadano je skalarno polje $f(x,y,z)=x^2+y^2+z^2$. Izračunaj $\int_C f\,ds$. 20
4. Izračunati $\iint_S (x+y)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z=\sqrt{x^2+y^2}$ i $0\le z\le 4$. 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t)+2f''(t)=0,\quad f(0)=f''(0)=2,\ f'(0)=0.$$

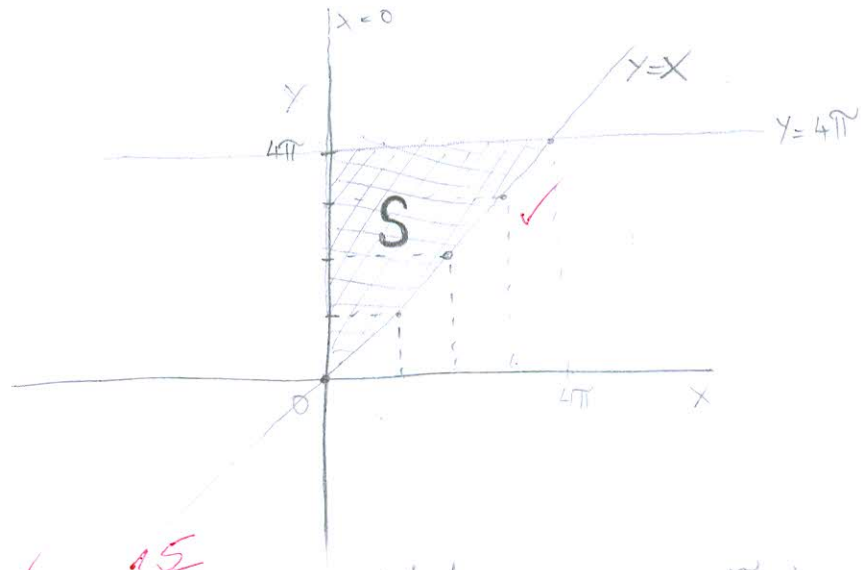
Ukupno: 15

1. $\iint_S \cos(x+y)\,dxdy$

$x=0$

$y=4\pi$

$y=x$



15

$$\iint_S \cos(x+y)\,dxdy = \int_0^{4\pi}\int_x^{4\pi} \cos(x+y)\,dy\,dx = \int_0^{4\pi}\int_x^{4\pi} \cos x\,dy\,dx + \int_0^{4\pi}\int_x^{4\pi} \cos y\,dy\,dx$$

$$= \int_0^{4\pi} \cos x + y\Big|_x^{4\pi}\,dx + \int_0^{4\pi} \sin y\Big|_x^{4\pi}\,dx = \int_0^{4\pi} \left((\cos x + 4\pi) - (\cos x + x)\right)dx$$

$$+ \int_0^{4\pi} \left[(\sin 4\pi - \sin x)\right]dx = \int_0^{4\pi} (\cos x + 4\pi - \cos x - x)\,dx + \int_0^{4\pi} -\sin x\,dx$$

$$= \int_0^{4\pi} (4\pi - x)\,dx + \int_0^{4\pi} -\sin x\,dx = 4\pi x - \frac{x^2}{2}\Big|_0^{4\pi} + \cos x\Big|_0^{4\pi} =$$

$$= (16\pi^2 - 8\pi^2) + (1-1) = \boxed{8\pi^2}$$

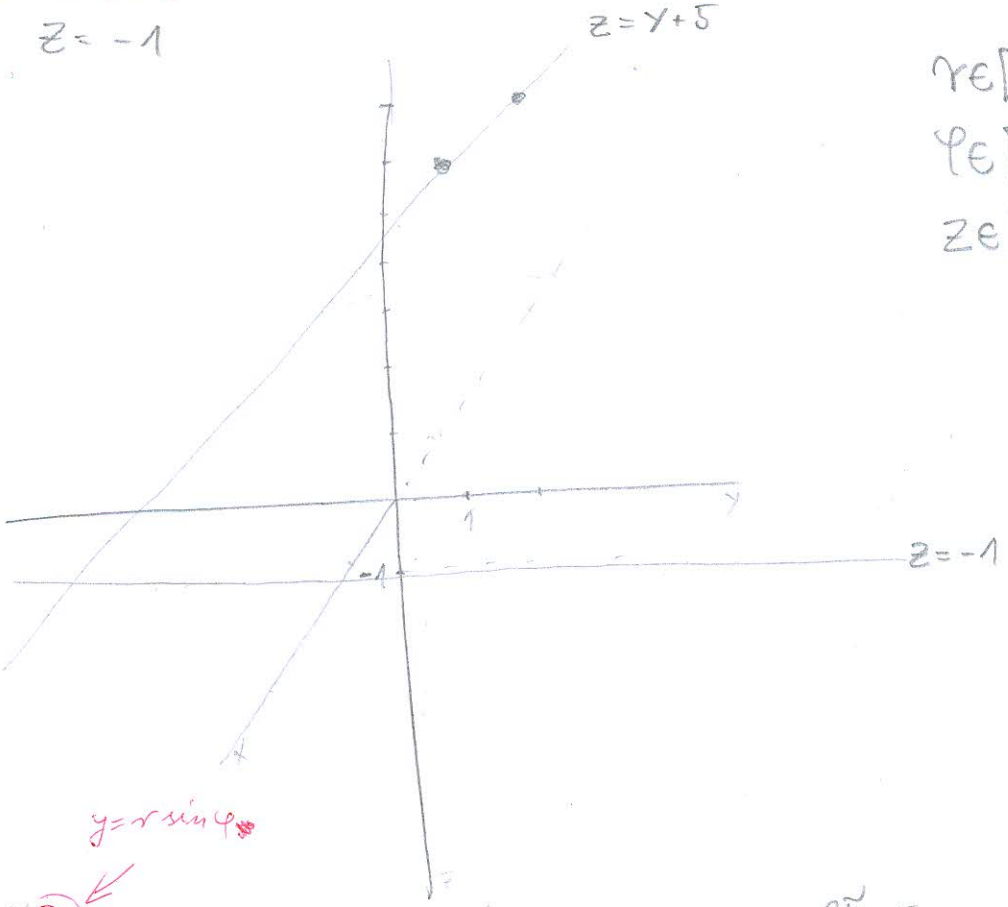
2.)

VALJAK: $x^2+y^2=5^2$

RAVNINE: $z=y+5$

$z=-1$

$r=5$



$r\in[0,5]$

$\varphi\in[0,2\pi]$

$z\in[-1,y+5]$

$y=r\sin\varphi$

$$V=\int_0^{2\pi}\int_{-1}^{y+5}\int_0^5 r\,dr\,d\varphi\,dz = \int_0^{2\pi}\int_{-1}^{y+5}\frac{r^2}{2}\Big|_0^5 dz\,d\varphi = \int_0^{2\pi}\int_{-1}^{y+5}\frac{25}{2}\,dz\,d\varphi$$

$$= \int_0^{2\pi}\frac{25}{2}z\Big|_{-1}^{y+5} = \int_0^{2\pi}\left[\frac{25}{2}(y+5)-\frac{25}{2}(-1)\right]d\varphi = \int_0^{2\pi}\frac{25}{2}y+\frac{125}{2}+\frac{25}{2}\,d\varphi$$

$=$

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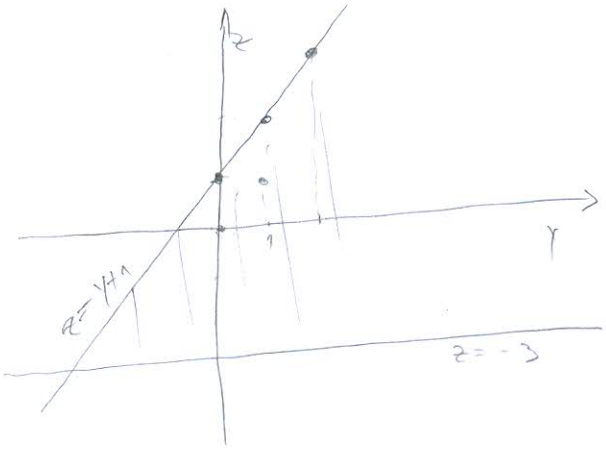
IME I PREZIME: MATE LADIĆ **BROJ INDEKSA:** 17-1-0006-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati $\iint_S \cos(x+y)\,dxdy$, gdje je S područje omeđeno pravcima $x=0$, $y=3\pi$, $y=x$. 20
2. Izračunati volumen tijela omeđenog valjkom $x^2+y^2=3^2$ i ravninama $z=y+1$ i $z=-3$. 20
3. Neka je C krivulja sa parametrizacijom $\mathbf{r}(t)=(\cos(t)+1)\,\mathbf{i}+\frac{t}{2}\mathbf{j}+\sin t\,\mathbf{k}$, $t\in[0,3\pi]$. Zadano je skalarno polje $f(x,y,z)=x^2+y^2+z^2$. Izračunaj $\int_C f ds$. 20
4. Izračunati $\iint_S (x+y)\mathrm{d}S$ ako je S kružni stožac zadan jednadžbom $z=\sqrt{x^2+y^2}$ i $0\le z\le 3$. 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t)-y(t)=e^{-t},\quad y(0)=y''(0)=1,\ y'(0)=2\,.$$

Ukupno:



y	0	1	2
z=y+1	1	2	3

$$V=\int_0^{3\pi}\int_0^{\sqrt{3}}\int_{-3}^{r\sin\varphi+1} r\,dz\,dr\,d\varphi=\int_0^{3\pi}\int_0^{\sqrt{3}} r\cdot z\,\Big|_{-3}^{r\sin\varphi+1}\,dr\,d\varphi=\int_0^{3\pi}\int_0^{\sqrt{3}} r\cdot(r\sin\varphi+1)+3\,dr\,d\varphi$$

$$=\int_0^{3\pi}\int_0^{\sqrt{3}}(r^2\sin\varphi+r+3)\,dr\,d\varphi=\int_0^{3\pi}\frac{r^3}{3}\sin\varphi+\frac{r^2}{2}+3r\,\Big|_0^{\sqrt{3}}\,d\varphi$$

$$\int_0^{3\pi}\left(\frac{(\sqrt{3})^3}{3}\sin\varphi+\frac{(\sqrt{3})^2}{2}+3\cdot\sqrt{3}\right)d\varphi=\int_0^{3\pi}\left(\frac{(\sqrt{3})^3}{3}\sin\varphi+\frac{3}{2}+3\sqrt{3}\right)d\varphi$$

$$=\int_0^{3\pi}\left(\sqrt{3}\sin\varphi+\frac{3}{2}+3\sqrt{3}\right)d\varphi=\int_0^{3\pi}\left(\frac{2\cdot\sqrt{3}\sin\varphi+3+6\sqrt{3}}{2}\right)d\varphi$$

③ $r(t) = (\cos(t)+1)\,i + \frac{t}{2}\,j + \sin t\,k \qquad t \in [0, 3\pi]$

$f(x,y,z) = x^2+y^2+z^2 \qquad \int_C f\,ds$

$r(t) = \begin{pmatrix} \cos(t)+1 \\ \frac{1}{2}t \\ \sin t \end{pmatrix} \Rightarrow r'(t) = \begin{pmatrix} -\sin t \\ \frac{1}{2} \\ \cos t \end{pmatrix} \Rightarrow \|r'\| = \sqrt{(-\sin t)^2 + \left(\frac{1}{2}\right)^2 + (\cos t)^2}$ ✓

$= \sqrt{\sin^2 t + \cos^2 t + \frac{1}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ ✗

$= \int_0^{3\pi} \frac{1}{2}\cdot\left(\cos(t)+1+\frac{1}{2}t+\sin t\right)dt$ ✗

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④ $\iint_S (x+y)\,ds \qquad z = \sqrt{x^2+y^2} \;;\; 0 \le z \le 3$

$\varphi \in [0, 2\pi] \qquad r \in [0, 3]$

$3 = \sqrt{r^2}$

$r = 3$

$z = \sqrt{x^2+y^2} \;/^2$

$z^2 = x^2+y^2$

$2z\,\partial z = 2x\,\partial x$

$\frac{\partial z}{\partial x} = \frac{2x}{2z} = x$

$z = \sqrt{x^2+y^2} \;/^2$

$z^2 = x^2+y^2$

$2z\,\partial z = 2y\,\partial y$

$\frac{\partial z}{\partial y} = \frac{2y}{2z} = y$

$= \sqrt{1+x^2+y^2} = \sqrt{1+r^2}$

$= \int_0^{2\pi}\int_0^3 \sqrt{1+r^2}\; r\,dr\,d\varphi$ ← OVO BI BILO O.K. ZA POVRŠINU

$\iint_S (x+y)\,ds = \left\{\begin{matrix} y = r\sin\varphi \\ x = r\cos\varphi \end{matrix}\right\} = \int_0^{2\pi}\int_0^3 r(\sin\varphi+\cos\varphi)\sqrt{1+r^2}\;dr\,d\varphi$

MATE LADIĆ

(5) $y''(t) - y(t) = e^{-t}$ $\quad y(0) = y''(0) = 1$

$y'(0) = 2$

$$s^3 Y(t) - s^2 y(0) - s y'(0) - y''(0) - s^2 Y(t) - s y(0) - y'(0) = \frac{1}{s-a}$$

$$s^3 Y(t) - s^2 - 2s - 1 - s^2 Y(t) - s - 2 = \frac{1}{s-a}$$

$$s^3 Y(t) - s^2 Y(t) - s^2 - 3s - 3 = \frac{1}{s-a}$$

$$s^3 Y(t) - s^2 Y(t) = \frac{1}{s-a} + s^2 + 3s + 3$$

$$Y(t)(s^3 - s^2) = \frac{(s-a)(s^2+3s+3)}{s-a} = \frac{s^2+3s+3}{(s^3-s^2)} = \frac{s^2+3s+3}{s^2(s-1)} =$$

$$\frac{s^2+3s+3}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} \quad / \cdot s^2(s-1)$$

$$s^2+3s+3 = As(s-1) + B(s-1) + Cs^2$$

$$s^2+3s+3 = As^2 - As + Bs - B + Cs^2$$

$$s^2+3s+3 = (A+C)s^2 - (A+B)s - B$$

$A + C = 1$

$-(A+B) = 3$

$-B = 3$

$B = -3$

$-(A-3) = 3$

$-A + 3 = 3$

$-A = 3 - 3$

$-A = 0$

$A = 0$

$0 + C = 1$

$C = 1$

$A = 0$
$B = -3$
$C = 1$

$$\frac{0}{s} + \left(\frac{-3}{s^2}\right) + \frac{1}{s-1}$$

$y(t) = C - 3t + e^t$

PROVJERA ?

~~VRIJEDI~~

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