

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa
XXXXX
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

SABOLIC BORIS

BROJ INDEKSA:

17-2-0010-2010

15

1. Riješiti integral:


$$\int_0^1 \frac{2x}{(x^2-4)^2} dx.$$

2. Riješiti integrale:

(a) $\int_0^\pi x \sin x dx$;

(b) $\int \frac{(2x-2) dx}{x^2-2x+4}$

$(x^2-2x+4=t \Rightarrow \int \frac{dt}{t} = \ln|x^2-2x+4| + C$ 15+15



3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x + 2y + 1$.

15

4. Odrediti ekstreme funkcije: $f(x, y) = x^3 - 3xy - y^3$. Koje su ekstremne vrijednosti i gdje se postižu?

8+12

5. Riješi diferencijalnu jednadžbu $(1 + e^x)yy' = e^x$ uz početni uvjet $y(0) = 1$.

20

① $\int_0^1 \frac{2x}{(x^2-4)^2} dx = \int \frac{dt}{t^2} = t^{-2} dt = \frac{t^{-1}}{-1} = -\frac{1}{t}$

Ukupno:

50

$2 \int \frac{x}{(x^2-4)^2} dx = 2 \cdot \frac{1}{2} \int \frac{dt}{t^2} = \dots$

$R_f = -\frac{1}{x^2-4} \Big|_0^1 = \left(-\frac{1}{1-4}\right) - \left(-\frac{1}{0-4}\right) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ ✓

② $\int_0^\pi x \sin x dx = \int \sin x dx = -\cos x = v$

$-x \cdot \cos x + \int \cos x dx =$

$-x \cos x + \sin x \Big|_0^\pi = [-\pi \cos(\pi) + \sin(\pi)] - [-0 \cdot \cos 0 + \sin 0] = (\pi + 0) - (0 + 0) = \pi$ ✓

$$(2) \int \frac{dx}{x^2 - 2x + 4}$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 16}}{-2}$$

$$x^2 - 2x + 4 = (x-2)(x+2)$$

$$x^2 + 2x - 2x - 4$$

$$(x-2)^2 = x^2 + 4x - 4$$

④ EXTREMI F-fē

$$f(x, y) = x^3 - 3xy - y^3$$

$$3x^2 - 3y = 0 \rightarrow -3y = -3x^2 / :(-3)$$

$$-3x - 3y^2 = 0 \leftarrow \boxed{y = x^2}$$

$$\partial_x f = 3x^2 - 3y$$

$$-3x - 3x^4 = 0$$

$$\partial_y f = -3x - 3y^2$$

$$x(-3 - 3x^3) = 0$$

$$\partial_{xx} f = 6x$$

A

$$\boxed{T_1 / \begin{matrix} x_1 = 0 \\ y_1 = 0 \end{matrix}} \checkmark$$

$$-3 - 3x^3 = 0$$

$$\partial_{xy} f = -3$$

B

$$-3x^3 = 3 / :(-3)$$

$$\partial_{yy} f = -6y$$

C

$$x^3 = -1$$

$$\boxed{T_2 \begin{matrix} x_2 = -1 \\ y_2 = 1 \end{matrix}} \checkmark$$

$$T_1(0, 0)$$

SEDLO

$$\left. \begin{matrix} A=0 \\ C=0 \end{matrix} \right\} \Delta = 0 \cdot 0 - 9 = -9 < 0 \leftarrow \text{nema extrema} \checkmark$$

$$T_2(-1, 1)$$

$$A = -6 \left. \begin{matrix} \Delta = -6 \cdot (-6) - 9 = 36 - 9 \\ C = -6 \end{matrix} \right\} \boxed{\Delta = 27 > 0} - \text{postoji ekstrem}$$

$$f_{\max} = -1 - 3 \cdot (-1) \cdot 1 - 1^3 =$$

$$\boxed{f_{\max} = 1} \checkmark$$

$$A < 0 - \underline{\underline{\max}} \checkmark$$

$$\textcircled{1} \int_0^1 \frac{2x}{(x^2-4)^2} dx = \left| \begin{array}{l} x^2-4=t \\ 2x dx=dt \end{array} \right| = \int_0^1 \frac{dt}{t^2} = \int_0^1 t^{-2} dt =$$

$$= \frac{t^{-1}}{-1} = -\frac{1}{t} \Big|_0^1 = \left[-\frac{1}{-3} \right] - \left[-\frac{1}{-4} \right] = \frac{1}{12} \checkmark$$

$$\textcircled{2} \int_0^{\pi} x \sin x dx = \left| \begin{array}{l} x=u \\ dx=du \\ \int \sin x dx = -\cos x = v \end{array} \right|$$

$$-x \cos x + \int \cos x dx = -x \cos x + \sin x \Big|_0^{\pi} + C$$

$$\left(-\pi \cdot \cos(\pi) + \sin(\pi) \right) - \left(-0 \cdot \cos(0) + \sin(0) \right) =$$

$$\left(\pi + 0 \right) - \left(0 + 0 \right) = \pi = 3.14 \checkmark$$

EXTREMI

$f(x,y) = x^3 - 3xy - y^3$

$\partial_x f = 3x^2 - 3y$

$\partial_y f = -3x - 3y^2$

$\partial_{xx} f = 6x$

$\partial_{xy} f = -3$

$\partial_{yy} f = -6y$

$\begin{cases} 3x^2 - 3y = 0 \\ -3x - 3y^2 = 0 \end{cases} \Rightarrow \begin{cases} 3x^2 = 3y \quad | :3 \\ x^2 = y \end{cases}$

$-3x - 3 - (x^2)^2 = 0$

$-3x - 3x^4 = 0$

$x(-3 - 3x^3) = 0$

$-3 - 3x^3 = 0$

$-3x^3 = 3 \quad | :(-3)$

$x^3 = -1$

$\begin{matrix} T_1 \\ x_1 = 0 \\ y_1 = 0 \end{matrix}$

$\begin{matrix} T_2 \\ x_2 = -1 \\ y_2 = 1 \end{matrix}$

$\begin{cases} -3y^2 = 3x \quad | :(-3) \\ y^2 = -x \\ -3x = 3y^2 \quad | :(-3) \\ x = -y^2 \quad ??? \end{cases}$

$T_1(0,0) \quad T_2(-1,1)$

$T_1(0,0) \Rightarrow A=0 \quad C=0$

$\Delta = 0 \cdot 0 - 9 = -9 < 0$ nema EXTREMA

$T_2(-1,1) \Rightarrow A = -6 \cdot (-6) - 9 = 27 > 0$

postoji ekstrem

$A < 0$ - max

$f_{max} = -1 + 3 - 1 = 1 \checkmark$

2b

$$\int \frac{dx}{x^2 - 2x + 4} =$$

$$\int \frac{dx}{(x+2)^2 - 6x} = \int \frac{dx}{(x+2)^2} + \int \frac{dx}{6x}$$

$$= \int (x^2 - 2x + 4 = t) \\ (2x - 2) dx = dt$$

$$\int \frac{dx}{(x+2)^2} + \int \frac{dx}{6x}$$

$$6x = t \\ 6dx = dt \\ dx = \frac{1}{6} dt$$

$$|x+2=t| \Rightarrow \int \frac{dt}{t^2} + \frac{1}{6} \int \frac{dt}{t} =$$

$$\int t^{-2} dt + \frac{1}{6} \ln|t| + C$$

$$-\frac{1}{x+2} + \frac{1}{6} \ln|6x| + C$$

$$x^2 - 2x + 4$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 16}}{-2} =$$

$$(x-2)(x+2)^2 = (x^2 + 4x + 4) \cdot (x-2) \\ (x+2)^2 - 6x - 2x$$

$$(x-1)(x+2) \mid (x+2)(x-1) = \\ x^2 + 2x - x - 2 \mid x^2 - x + 2x - 2 \\ x^2 - x - 2$$

$$x(x+2)^2 =$$

$$x^2 - 2x + 4$$

$$(x+2)^2 = (x^2 + 4x + 4) - 6x$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: DENI MILETIĆ

BROJ INDEKSA: 57143-2009

Grupa
XXXX
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Riješiti integral:

$$\int_0^1 \frac{2x}{(x^2-4)^2} dx.$$

15

2. Riješiti integrale:

(a) $\int_0^\pi x \sin x dx$;

(b) $\int \frac{dx}{x^2-2x+4}$

15+15

3. Odrediti domenu, kodomenu i razinske krivulje za funkciju $f(x, y) = x + 2y + 1$.

15

4. Odrediti ekstreme funkcije: $f(x, y) = x^3 - 3xy - y^3$. Koje su ekstremne vrijednosti i gdje se postižu?

8+12

5. Riješi diferencijalnu jednadžbu $(1 + e^x)yy' = e^x$ uz početni uvjet $y(0) = 1$.

20

Ukupno:

30

$$\begin{aligned}
 1. \int_0^1 \frac{2x}{(x^2-4)^2} &= \left\{ t = x^2 - 4; dt = 2x dx \right\} = \int_0^1 \frac{dt}{t^2} = \int_0^1 t^{-2} dt = \\
 &= \int_0^1 \frac{t^{-2+1}}{-2+1} dt = \int_0^1 -\frac{dt}{t} = -[\ln t]_0^1 = -[\ln |x^2-4|]_0^1 \\
 &= -[\ln |0^2-4| - \ln |1^2-4|] = 0 \quad \times
 \end{aligned}$$

