MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: NIKOLA KNEŽEVIĆ **BROJ INDEKSA:** 17-1-0002-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x + 5y)\,dx dy dz$. 20

2. Izračunati dvostruki integral: $\iint\limits_S x(y+5)\,dx dy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 \,:\, x^2 + y^2 \leq 1 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(dx + dy)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2},0)$, $B(6,\frac{5}{2})$ i $C(\frac{6}{2},\frac{5}{2})$. Izračunati dvostruki integral 15 12

$$\iint\limits_X y\,dx dy.$$

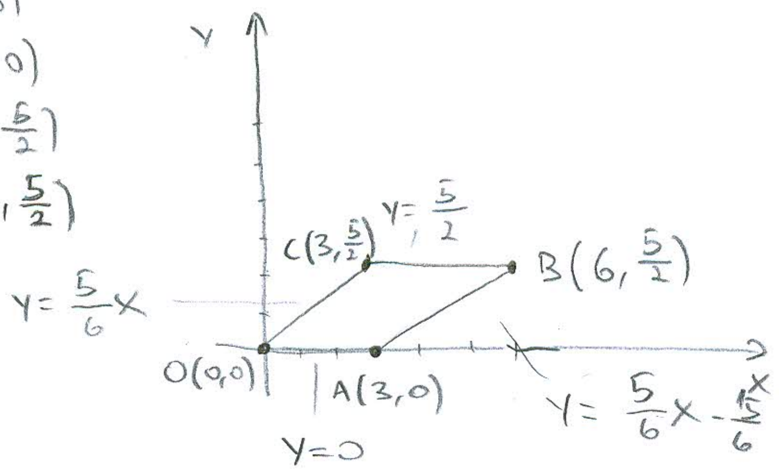
5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2dS$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 12

4.
$O(0,0)$
$A(3,0)$
$B(6,\frac{5}{2})$
$C(3,\frac{5}{2})$



$(Y-Y_1)(X_2-X_1)=(X-X_1)(Y_2-Y_1)$

$\overline{OC}$ $O(0,0)$ $C(3,\frac{5}{2})$

$(Y-0)(3)=(X-0)\frac{5}{2}$

$3Y=\frac{5}{2}X$

$Y=\frac{5}{6}X$

$\overline{AB}$ $A(3,0)$ $B(6,\frac{5}{2})$

$(Y)(6-3)=(X-3)\frac{5}{2}$

$3Y=\frac{5}{2}X-\frac{15}{2}$

$Y=\frac{5}{6}X-\frac{15}{6}$

IME I PREZIME: NIKOLA KNEŽEVIĆ **BROJ INDEKSA:**

4.

$$\iint_X y\,dx\,dy = \int_0^3 \int_0^{\frac{5x}{6}} y\,dy\,dx + \int_3^6 \int_{\frac{5x}{6}-\frac{15}{6}}^{\frac{5}{2}} y\,dy\,dx$$

✓ 12

$$= \int_0^3 \frac{y^2}{2}\Big|_0^{\frac{5x}{6}} dx + \int_3^6 \frac{y^2}{2}\Big|_{\frac{5x}{6}-\frac{15}{6}}^{\frac{5}{2}} dx$$

$$\frac{\left(\frac{5x}{6}-\frac{15}{6}\right)^2}{2} = \frac{1}{2}\left(\frac{25}{36}x^2 - \frac{275}{36}x + \frac{225}{36}\right)$$

$$= \int_0^3 \frac{\frac{25x^2}{36}}{2}\,dx + \int_3^6 \frac{\frac{25}{4}}{2} - \underbrace{\left(\frac{\frac{25x^2}{36}}{2} - \frac{\frac{225}{36}}{2}\right)}_{\times} dx$$

25

$\cancel{75}$

$\cancel{225}$

$$= \int_0^3 \frac{25x^2}{72}\,dx + \int_3^6 \frac{25}{8} - \left(\frac{25x^2}{72} - \frac{225}{72}\right) dx = \frac{25}{72}\,\frac{x^3}{3}\Big|_0^3 + \int_3^6 \frac{\cancel{450}}{\cancel{72}} - \frac{25x^2}{72}\,dx$$

$\cancel{36}$

$$= \frac{25}{72}\cdot 9 + \frac{25}{4} - \frac{25}{72}\left(\frac{x^3}{3}\right)\Big|_3^6 = \frac{225}{72} + \frac{25}{4} - \frac{25}{72}\left(\frac{6^3}{3} - \frac{3^3}{3}\right)^{72}{}_{4}$$

$$= \frac{225}{72} + \frac{450}{72} - \frac{25}{72}\cdot 63 = \frac{675}{72} - \frac{1575}{72} = -\frac{900}{72} = 12.5$$

×

(6.) $y'''(t) - 2y''(t) + y'(t) = 0$ $\quad y(0) = 1$

$y'(0) = -1$

$y''(0) = 1$

$$s^3Y(s) - s^2\cancelto{1}{y(0)} - s\cancelto{-1}{y'(0)} - \cancelto{1}{y''(0)} - 2\left(s^2Y(s) - s\cancelto{1}{y(0)} - \cancelto{-1}{y'(0)}\right) + sY(s) - \cancelto{1}{y(0)} = 0$$

$$s^3Y(s) - s^2 + s - 1 - 2\left(s^2Y(s) - s + 1\right) + sY(s) - 1 = 0$$

$$s^3Y(s) - s^2 + s - 1 - 2s^2Y(s) + 2s - 2 + sY(s) - 1 = 0$$

$$s^3Y(s) - 2s^2Y(s) + sY(s) = s^2 - s - 2s + 1 + 2 + 1$$

$$Y(s)\left(s^3 - s^2 + s\right) = s^2 - 3s + 4 \quad /:\left(s^3 - s^2 + s\right)$$

$$Y(s) = \frac{s^2 - 3s + 4}{s^3 - s^2 + s} = \frac{s^2 - 3s + 4}{s(s^2 - s + 1)}$$

$$\frac{s^2 - 3s + 4}{s(s^2 - s + 1)} = \frac{A}{s} + \frac{Cs + D}{s^2 - s + 1} \quad / \cdot s(s^2 - s + 1)$$

$$s^2 - 3s + 4 = A(s^2 + s + 1) + (Cs + D)(s)$$

$$s^2 - 3s + 4 = As^2 + As + A + Cs^2 + Ds$$

$$s^2 - 3s + 4 = As^2 + Cs^2 + As + Ds + A$$

$$s^2 - 3s + 4 = s^2(A + C) + s(A + D) + A$$

$A + C = 1 \qquad A + D = -3 \qquad A = 4$

$C = 1 - 4 \qquad D = -3 - 4$

$C = -3 \qquad D = -7$

$$Y(s) = \frac{4}{s} + \frac{-3s - 7}{s^2 - s + 1}$$

INVERZNA TRANSFORMACIJA?

Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MATE LADIĆ BROJ INDEKSA: 17-1-0006-2010	

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x + 5y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20 5

2. Izračunati dvostruki integral: $\iint\limits_S x(y + 5)\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x + y)\,(\mathrm{d}x + \mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2}, 0)$, $B(6, \frac{5}{2})$ i $C(\frac{6}{2}, \frac{5}{2})$. Izračunati dvostruki integral 15

$$\iint\limits_X y\,\mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 5

1) $r = 2$

$$\iiint\limits_K (6x+5y)\,dx\,dy\,dz$$

$T(x_0, y_0, z_0)$

$r \in [0, 2]$

$\varphi \in [0, 2\pi]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$$\iiint\limits_K (6x+5y)\,dx\,dy\,dz = \int_0^{2\pi}\int_0^{2}\int_{\sqrt{-4-z^2}}^{\sqrt{4-z^2}} (6\cdot r\cos\varphi + 5\cdot r\sin\varphi)\, r\,dz\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2}\int_{\sqrt{-4-z^2}}^{\sqrt{4-z^2}} (6r\cos\varphi + 5r\sin\varphi)\, r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{2}\int_{\sqrt{-4-z^2}}^{\sqrt{4-z^2}} (6r^2\cos\varphi + 5r^2\sin\varphi)\,dz\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} (6r^2\cos\varphi + 5r^2\sin\varphi)\cdot z \Big|_{\sqrt{-4-z^2}}^{\sqrt{4-z^2}} dr\,d\varphi = \int_0^{2\pi}\int_0^{2} (6r^2\cos\varphi + 5r^2\sin\varphi)\left(\sqrt{4-z^2} + \sqrt{4-z^2}\right) = \int_0^{2\pi}\int_0^{2} (6r^2\cos\varphi + 5r^2\sin\varphi)\cdot\left(2\sqrt{4-z^2}\right)dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} \left(12r^2\cos\varphi\sqrt{4-z^2} + 10r^2\sin\varphi\sqrt{4-z^2}\right)dr\,d\varphi$$

$$= 0$$

$x^2+y^2+z^2 = r^2$

$x^2+y^2 = r^2$

$r^2+z^2 = 2^2$

$r^2+z^2 = 4$

$z^2 = 4 - z^2$

$z = \sqrt{4-z^2}$

$z = \pm\sqrt{4-z^2}$

$z \in \left[\sqrt{-4-z^2}, \sqrt{4-z^2}\right]$

KAD SU U ZAD. ZADANE GRANICE OD $[0, 2\pi]$ I ZADANI SU SINUS I COSINUS INTEGRAL JE 0 ✓. 5

(5) $r = 1$ $\quad T(0,0)$ $\quad \int\limits_{\partial K}\int z\,ds$ $\quad r =$

PARAMETRIZACIJA JE VEKTORSKO POLJE

IMA 3 KOORDINATE

$$r(t) = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix} = \begin{pmatrix} 1\cdot\cos t \\ 1\cdot\sin t \end{pmatrix} = r'(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

← OVDJE SAMO 2 KOORD.

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2} = \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1$$

$$\int\limits_{\partial K}\int z\,ds = \int_0^{2\pi} (1\cdot 2)\,dt = 2t\Big|_0^{2\pi} = 2\cdot 2\pi - 2\cdot 0 = 4\pi$$

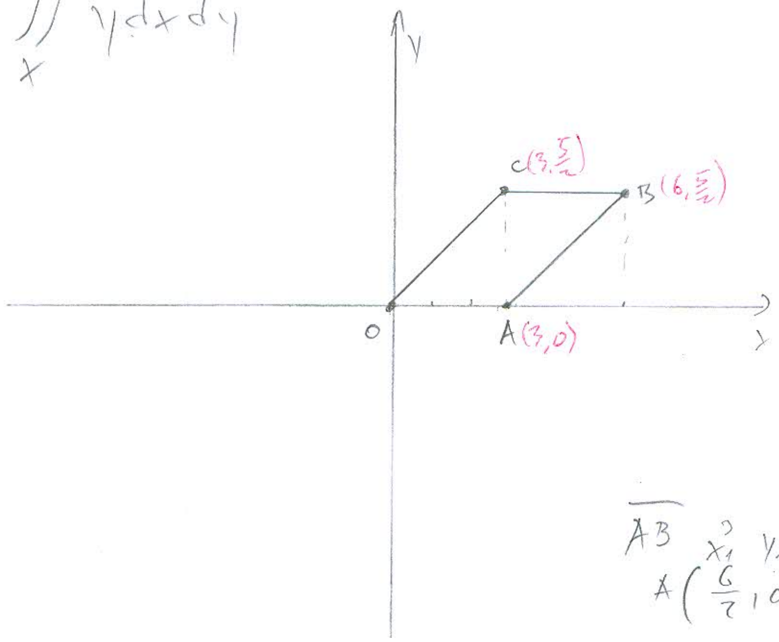
X

Ime i prezime: MATE LADIĆ **Broj indeksa:**

$O(0,0)$ $A\left(\frac{6}{2},0\right)$ $B\left(6,\frac{5}{2}\right)$, $C\left(\frac{6}{2},\frac{5}{2}\right)$

$O,$

$\iint_X y\,dx\,dy$



$\overline{CB}$:

$C\left(\frac{6}{2},\frac{5}{2}\right)$, $B\left(6,\frac{5}{2}\right)$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(6-3)\left(y-\frac{5}{2}\right)=\left(\frac{5}{2}-\frac{5}{2}\right)(x-3)$

$3y-\frac{15}{2}=x-3$

$3y=x-3+\frac{15}{2}$ $/:3$

$\overline{AB}$

$A\left(\frac{6}{2},0\right)$ $B\left(6,\frac{5}{2}\right)$ $\overline{CB}\dots y=\frac{1}{3}x+\frac{3}{2}$

$(x_2-y_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(6-3)(y-6)=\left(\frac{5}{2}-0\right)(x-3)$

$3y=\frac{5}{2}x-\frac{15}{2}$ $/:3$

$\overline{AB}\dots y=\frac{5}{6}x-\frac{5}{2}$

$\overline{OA}\dots y=x$

$\overline{OC}$

$O(0,0)$ $C\left(\frac{6}{2},\frac{5}{2}\right)$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(3-0)(y-0)=\left(\frac{5}{2}-0\right)(x-0)$

$3y=\frac{5}{2}x$ $/:3$

$\overline{OC}\dots y=\frac{5}{6}x$

$$=\iint_X y\,dx\,dy=\int_0^3\int_x^{\frac{5}{6}x} y\,dy\,dx+\int_3^6\int_{\frac{5}{6}x-\frac{5}{2}}^{\frac{1}{3}x+\frac{3}{2}} y\,dy\,dx=$$

$$\int_0^3 \frac{y^2}{2}\Big|_x^{\frac{5}{6}x}dx+\int_3^6\frac{y^2}{2}\Big|_{\frac{5}{6}x-\frac{5}{2}}^{\frac{1}{3}x+\frac{3}{2}}dx=\int_0^3\left(\frac{\left(\frac{5}{6}x\right)^2}{2}-\frac{x^2}{2}\right)dx+\int_3^6\left(\frac{\left(\frac{1}{3}x+\frac{3}{2}\right)^2}{2}-\frac{\left(\frac{5}{6}x-\frac{5}{2}\right)^2}{2}\right)dx$$

$$=\int_0^3\left(\frac{\frac{25}{36}x^2}{2}-\frac{x^2}{2}\right)dx+\int_3^6\left(\frac{\frac{1}{9}x^2+x+\frac{9}{4}}{2}\right)dx\left(\frac{\frac{25}{30}x^2-\frac{25}{6}x-\frac{25}{4}}{2}\right)dx$$

③ $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy)$ $\qquad (x+y)\,dx$

$(x+y)\,dy$

$$\begin{pmatrix} x+y \\ x+y \end{pmatrix} = -\operatorname{grad} \cdot f \begin{pmatrix} \partial_x f \\ \partial_y f \end{pmatrix}$$

$= \partial_x f = -(x+y) = -x-y \Rightarrow f = \int(-x-y)\,dx = \underline{-\frac{x^2}{2} - yx + C(y)}$

$= \partial_y f = -(x+y) = -x-y = \cancel{-x} + C'(y) = \cancel{-x} - y \Rightarrow C'(y) = -y$

$\Rightarrow C(y) = \int -y\,dy$

$= \underline{-\frac{y^2}{2} + C}$

$= f(x,y) - f(x,y)$

$= f(-1,2) - f(2,3)$

$\underline{-\frac{x^2}{2} - yx + \frac{y^2}{2}} = -\frac{(-1)^2}{2} - (2\cdot(-1)) + \underline{\frac{2^2}{2}} - \left(-\frac{2^2}{2} - 3\cdot 2 + \underline{\frac{3^2}{2}}\right)$

$= -\frac{1}{2} + 2 + 2 - 2 + 6 - \frac{9}{2} = 3$

Ime i prezime: MATE LADIĆ **Broj indeksa:**

$y'''(t) - 2y''(t) + y'(t) = 0 \qquad y(0) = 1 \quad y'(0) = -1 \quad y''(0) = 1$

$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - 2(s^2Y(s) - sy(0) - y'(0)) + sY(s) - y(0) = 0$

$s^3Y(s) - s^2\cdot 1 - s\cdot(-1) - 1 - 2s^2Y(s) - 2 - 2 + sY(s) - 1 = 0$

$s^3Y(s) - s^2 + s - 1 - 2s^2Y(s) + 2 - 2 + sY(s) - 1 = 0$

$s^3Y(s) - 2s^2Y(s) + sY(s) = s^2 - s + 1 - 2 + 2 + 1$

$s^3Y(s) - 2s^2Y(s) + sY(s) = s^2 - s + 2$

$Y(s)(s^3 - 2s^2 + s) = \frac{s^2 - s + 2}{s^3 - 2s^2 + s} = \frac{s^2 - s + 2}{s(s-1)^2} =$

$\frac{A}{s} + \frac{B}{s-1} + \frac{C}{(s-1)^2} \Big/$ ✓

$A(s-1)\cdot(s-1)^2 + Bs(s-1)^2 + Cs(s-1)$

$A(s-1)(s^2 - 2s + 1) + Bs(s - 2s + 1) + Cs^2 - Cs$

$(s^3 - 2s^2 + s - s^2 + 2s - 1)A + Bs^2 - 2Bs^2 + Bs + Cs^2 - Cs$

$(s^3 - 3s^2 + 3s - 1)A + Bs^2 - 2Bs^2 + Bs + Cs^2 - Cs$

$As^3 - 3As^2 + 3As - A + Bs^2 - 2Bs^2 + Bs + Cs^2 - Cs$

$As^3 - (4A - B + C)s^2 + (2A + B - C)s$

$s^3 - 2s^2 + s = As^3 - (4A - B + C)s^2 + (2A + B - C)s \qquad 2 + 2 + C - C = 1$

$A = 1$

$4A + B - C = -2$

$2A + B - C = 1$

$4A + B - C = -2$

$4 + B - C = -2$

$-B - C = 2 \,|\, C$

$- B$

DALJE...?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

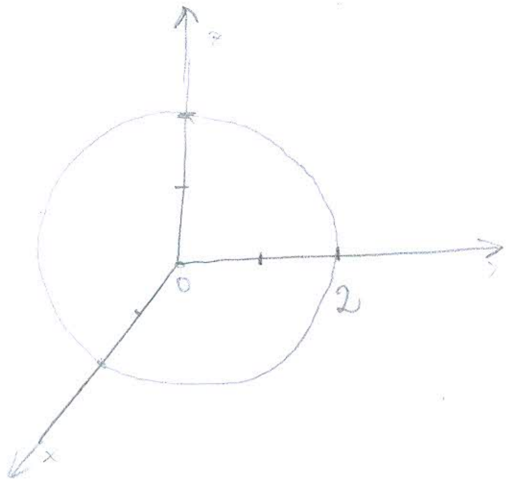
IME I PREZIME: MATIJA JAKOBAC **BROJ INDEKSA:** 57921

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x + 5y)\, dxdydz$. 20
2. Izračunati dvostruki integral: $\iint\limits_S x(y+5)\, dxdy$, gdje je $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ i } x \geq y\}$. 20
3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x + y)\, (\mathrm{d}x + \mathrm{d}y)$. 15
4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2}, 0)$, $B(6, \frac{5}{2})$ i $C(\frac{6}{2}, \frac{5}{2})$. Izračunati dvostruki integral 15
$$\iint\limits_X y\, dxdy.$$
5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15
$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 0

1.



$x^2 + y^2 + z^2 = 2^2$

$x^2 + y^2 = r^2$

$r^2 + z^2 = 4$

$r^2 = 4 - z^2$

$r = \sqrt{4 - z^2}$

$x \in [0, \sqrt{4-z^2}]$

$r \in [0, 2]$

$\varphi \in [0, 2\pi]$

$\iiint\limits_K (6x+5y)\, dxdydz$

$$\boxed{V = \int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-z^2}} (6x+5y)\, dxdydz}$$

$x = r\cos\varphi$

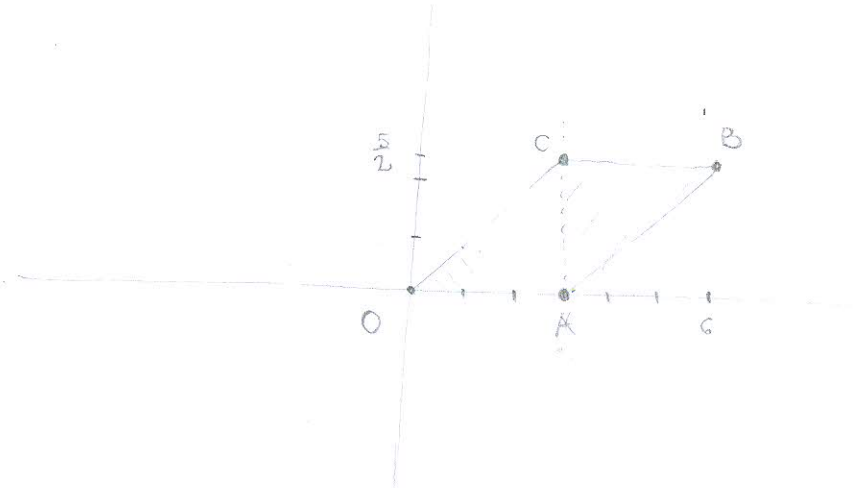
$y = r\sin\varphi$

$$\boxed{V = \int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-z^2}} (6r\cos\varphi + 5\sin\varphi)\, r\, dr\, dz\, d\varphi}$$

$$= \iiint (6r^2\cos\varphi + 5r^2\sin\varphi)\, dr\, dz\, d\varphi$$

...

4. $O(0,0)$, $A\left(\frac{6}{2},0\right)$, $B\left(6,\frac{5}{2}\right)$, $C\left(\frac{6}{2},\frac{5}{2}\right)$ $\qquad \iint\limits_{X} y\,dx\,dy$



$$\int_0^3\int_0^{\frac{15}{6}x} y\,dx\,dy + \int_3^6\int_{\frac{1}{3}x+\frac{9}{2}}^{\frac{5}{2}} y\,dx\,dy$$

$O(\overset{x_1}{0},\overset{y_1}{0})$ $\quad C(\overset{x_2}{3},\overset{y_2}{\frac{5}{2}})$

$$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$$

$$(3-0)(y-0)=\left(\frac{5}{2}-0\right)(x-0)$$

$$3y=\frac{5}{2}x \quad /:3$$

$$y=\frac{\frac{5}{3}}{\frac{2}{3}}x$$

$\overline{OC}$ .... $\quad y=\frac{15}{6}x$

$A(\overset{x_1}{3},\overset{x_2}{2})$ $\quad B(\overset{x_2}{6},\overset{x_2}{\frac{5}{2}})$

$$(6-3)(y-2)=\left(\frac{5}{2}-2\right)(x-3)$$

$$3y-6=\frac{1}{2}x-\frac{3}{2}$$

$$3y=\frac{1}{2}x-\frac{9}{2}+6$$

$$3y=\frac{1}{2}x+\frac{9}{2} \quad /:3$$

$$y=\frac{\frac{1}{3}}{\frac{2}{3}}x+\frac{\frac{9}{3}}{\frac{2}{3}}$$

$\overline{AB}$ .... $\quad y=\frac{2}{6}x+\frac{27}{6}$

$$\varnothing \quad P=\int_0^3 \frac{y^2}{2}\Bigg|_0^{\frac{5}{2}x}dx+\int_3^6\frac{y^2}{2}\Bigg|_{\frac{1}{3}x+\frac{9}{2}}^{\frac{5}{2}}dx$$

$$=\int_0^3\frac{\left(\frac{5}{2}x\right)^2}{2}dx+\int_3^6\left[\frac{\left(\frac{5}{2}\right)^2}{2}-\frac{\left(\frac{1}{3}x+\frac{9}{2}\right)^2}{2}\right]dx$$

$$=\int_0^3\frac{\frac{25}{4}x^2}{\frac{2}{1}}dx+\int_3^6\left[\frac{\frac{25}{4}}{\frac{2}{1}}-\frac{\left(\frac{1}{9}x^2-\frac{2}{3}x\cdot\frac{9}{2}+\frac{81}{4}\right)}{2}\right]dx$$

$$=\int_0^3\frac{25x^2}{8}dx+\int_3^6\left[\frac{25}{8}-\frac{\left(\frac{1}{9}x^2-3x+\frac{81}{4}\right)}{2}\right]dx$$

$$=\frac{25}{8}\cdot\frac{x^3}{3}\Bigg|_0^3+\int_3^6\left[\frac{25}{8}-\frac{\frac{1}{9}x^2}{2}+\frac{3x}{2}-\frac{81}{4}\right]dx$$

$$=\frac{25}{8}\cdot\frac{3^3}{3}-0+\int_3^6\left(-\frac{1}{18}x^2+\frac{3}{2}x-\frac{137}{8}\right)dx$$

$$=\frac{225}{24}+\left(-\frac{1}{18}\cdot\frac{x^3}{3}+\frac{3}{2}\cdot\frac{x^2}{2}-\frac{137}{8}x\right)\Bigg|_3^6$$

$$=\frac{225}{24}+\left(-\frac{x^3}{54}+\frac{3x^2}{4}-\frac{137}{8}x\right)\Bigg|_3^6$$

$$=\frac{225}{24}+\left[\left(-\frac{6^3}{54}+\frac{3\cdot 36}{4}-\frac{137}{8}\cdot 6\right)-\left(-\frac{3^3}{54}+\frac{3\cdot 3^2}{4}-\frac{137}{8}\cdot 3\right)\right]$$

Ime i prezime: MATIJA JAKOBAC **Broj indeksa:** 57921

NASTAVAK 4. ZADATKA ...

$$= \frac{225}{24} + \left[\left(-\frac{216}{54} + \frac{108}{4} - \frac{1376}{8}\right) - \left(-\frac{27}{54} + \frac{27}{4} - \frac{411}{8}\right)\right]$$

$$= \frac{225}{24} + \left[(-4 + 27 - 172) - \left(-\frac{1}{2} + \frac{27}{4} - \frac{411}{8}\right)\right]$$

$$= \frac{225}{24} + \left(-149 - \left(\frac{-4+54-411}{8}\right)\right] = \frac{225}{24} + \left(-149 + \frac{361}{8}\right)$$

$$= \frac{225}{24} + \left(\frac{-1192+361}{8}\right) = \frac{225}{24} - \frac{831}{8} = \frac{225-2493}{24} = \frac{2268}{24} = \frac{189}{2} = 94\frac{1}{2}$$

NASTAVAK 1. ZADATKA ...

$$\begin{array}{r} 16 \cdot 12 \\ \hline 32 \\ 160 \\ \hline 192 \\ 40 \\ \hline 232 \end{array}$$

$$= \int_0^{2\pi}\int_0^{2} \left. \cancel{6}^{2}\, \frac{r^3}{\cancel{3}_1}\cos\varphi + 5\frac{r^3}{3}\sin\varphi \right|_0^{\sqrt{4-z^2}} dz\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} \left[2\left(\sqrt{4-z^2}\right)^3\cos\varphi + \frac{5}{3}\cdot\left(\sqrt{4-z^2}\right)^3\sin\varphi\right] dz\,d\varphi.$$

$$= \int_0^{2\pi}\int_0^{2} \left[2\left(4-z^2\right)\cos\varphi + \frac{5}{3}\left(4-z^2\right)\sin\varphi\right] dz\,d\varphi$$

$$\overline{12}$$

$$= \int_0^{2\pi}\int_0^{2} \left(8 - 2z^2\cos\varphi + \frac{20}{12} - \frac{5}{3}z^2\sin\varphi\right) dz\,d\varphi$$

$$= \int_0^{2\pi} \left. 8z - 2\frac{z^3}{3}\cos\varphi + \frac{20}{12}z - \frac{5}{3}\cdot\frac{z^3}{3}\sin\varphi \right|_0^2 d\varphi$$

$$= \int_0^{2\pi}\left(8\cdot 2 - 2\cdot\frac{(2)^3}{3}\cos\varphi + \frac{20}{12}\cdot 2 - \frac{5}{3}\cdot\frac{(2)^3}{3}\sin\varphi\right) d\varphi$$

$$= \int_0^{2\pi}\left(16 - \frac{16}{3}\cos\varphi + \frac{40}{12} - \frac{40}{9}\sin\varphi\right) d\varphi = \int_0^{2\pi}\left(-\frac{16}{3}\cos\varphi - \frac{40}{9}\sin\varphi + 232\right) d\varphi$$

$$= -\frac{16}{3}\sin\varphi + \frac{40}{3}\cos\varphi + 232\varphi \Big|_{0}^{2\pi}$$

$$= \left[\overset{0}{-\frac{16}{3}\sin 2\pi} + \frac{40}{3}\cdot 1 + 232\cdot 2\pi\right] - \left[0 + \frac{40}{3}\right]$$

$$= \cancel{\frac{40}{3}} + 464\pi - \cancel{\frac{40}{3}} = 464\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: Igor Brajica **BROJ INDEKSA:** 52803-2005	Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x + 5y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

2. Izračunati dvostruki integral: $\iint\limits_S x(y+5)\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1 \text{ i } x \ge y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x + \mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2},0)$, $B(6,\frac{5}{2})$ i $C(\frac{6}{2},\frac{5}{2})$. Izračunati dvostruki integral 15

$$\iint\limits_X y\,\mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~15~~ $\cancel{0}$

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno:

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - 2\left(s^2F(s) - s f(0) - s f'(0) - f'(0)\right) + sF(s) - f(0) = 0$

$s^3F(s) - s^2 + 1 - 1 - 2\left(s^2F(s) - s^2 + s - 1\right) + sF(s) - 1 = 0$

$s^3F(s) - s^2 + 1 - 1 - 2sF(s) + 2s^2 - 2s + 2 + sF(s) - 1 = 0$

$F(s)(s^3 - 2s + s) = s^2 - \cancel{1} + \cancel{1} - \cancel{2s^2} + \cancel{2s} - 2 + 1 = s - 2s^2 - 2 + 1 / s - 2s^2 - 1$

$-2s^2 + -1$

$F(s)(s^3 - 2s + s) = s^2 - 2 + 1 \quad /: (s^3 - 2s + s)$

$F(s) = \dfrac{s^2 - 2 + 1}{\cancel{s^3 - 2s + 2}} = \dfrac{s^2 - 2 + 1}{s^3 - 2s + 2} = \dfrac{s^2 - 2 + 1}{s(s^2 - 2 + 1)} = \dfrac{A}{s} + \dfrac{Bs + C + E}{s^2 - 2 + 1}$

$-2s^2 + s - 1$

$s^2 - 2 + 1 = As^2 - 2A + A + Bs^2 + Cs + Es$

$1 = A + B \qquad 1 = -1 + B \qquad \boxed{B = 0}$

$0 = C + E$

$-1 = A \qquad \boxed{A = -1}$

$(2,3)$

$\int$

3

DALJE ?

Ime i prezime: Igor Bragica **Broj indeksa:** 52803-2005

$$s^3F(s) - \underbrace{s^2 f(0)}_{-s^2} - \underbrace{s f'(0)}_{+s} - \underbrace{f''(0)}_{-1} - 2\left(s^2F(s) - sf(0) - f'(0)\right) + sF(s) - f(0) =$$

$$-2s^2F(s) + 2s - 2 + sF(s) - 1 = 0 \ldots$$

$$s^3F(s) - s^2 + s - 1 - 2s^2F(s) + 2s - 2 + sF(s) - 1 = 0$$

$$F(s)\left(s^3 - 2s^2 + s\right) = s^2 - s + 1 - 2s + 2 + 1$$

$$F(s)\left(s^3 - 2s^2 + s\right) = s^2 - 3s + 4$$

DALJE ?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: JURE PAVIĆ BROJ INDEKSA: 51894-2005	

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x+5y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. — ~~20~~

2. Izračunati dvostruki integral: $\iint\limits_S x(y+5)\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1 \text{ i } x \geq y\}$. — 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x+\mathrm{d}y)$. — ~~15~~

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2},0)$, $B(6,\frac{5}{2})$ i $C(\frac{6}{2},\frac{5}{2})$. Izračunati dvostruki integral — 15

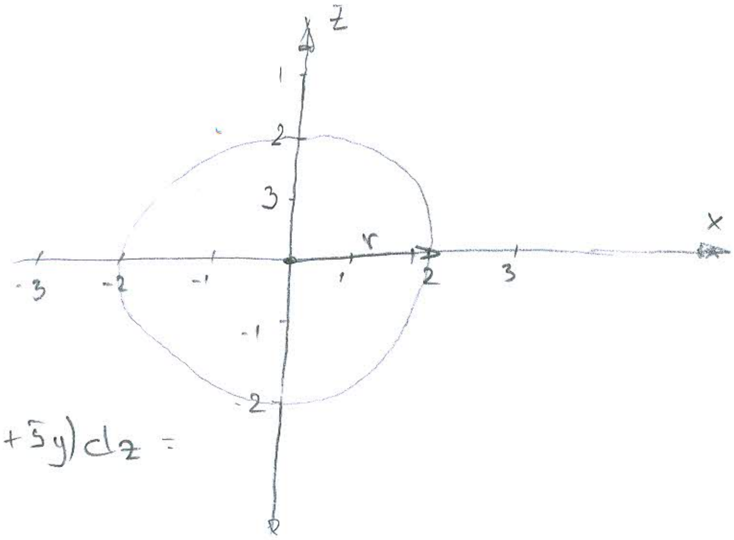
$$\iint\limits_X y\,\mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? — 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 0



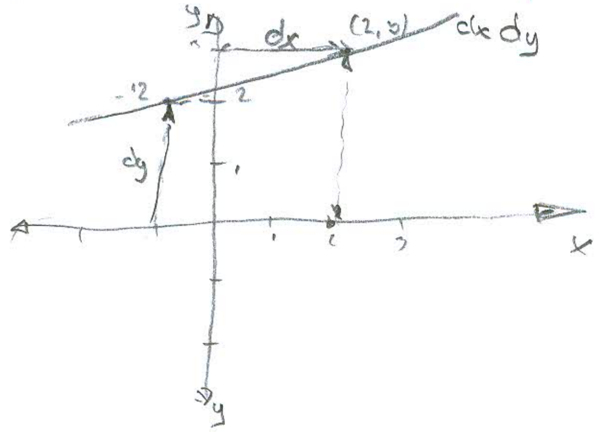
1.) $r = 2$

$\iiint\limits_K (6x+5y)\,dx\,dy\,dz$

$\int_k (6x+5y)\,dx \int_k (6x+5y)\,dy \int_k (6x+5y)\,dz =$

$\int 6x\,dx + 5\,dx\,y \int 6x\,dy + 5y^2 \int 6x\,dz + 5y\,dz$

3.) $$\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy)$$



$$\int_{(-1,2)}^{(2,3)} (2+3)dx \cdot (-1+2)\, dy$$

$$\int_{(-1,2)}^{(2,3)} 5\, dx \cdot dy$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: Tony Car **BROJ INDEKSA:** 17-2-0045-2010	Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x + 5y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

2. Izračunati dvostruki integral: $\iint\limits_S x(y + 5)\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x + y)\,(\mathrm{d}x + \mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0, 0)$, $A(\frac{6}{2}, 0)$, $B(6, \frac{5}{2})$ i $C(\frac{6}{2}, \frac{5}{2})$. Izračunati dvostruki integral 15

$$\iint\limits_X y\,\mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: BUTERIN ŠIME

BROJ INDEKSA: 17-2-0049-2010

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (6x + 5y)\,dxdydz$. 20

2. Izračunati dvostruki integral: $\iint\limits_S x(y + 5)\,dxdy$, gdje je $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x + y)\,(\mathrm{d}x + \mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0, 0)$, $A(\frac{6}{2}, 0)$, $B(6, \frac{5}{2})$ i $C(\frac{6}{2}, \frac{5}{2})$. Izračunati dvostruki integral 15

$$\iint\limits_X y\,dxdy.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2dS$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno:

0

6. $s^3F(s) - s^2f(0) - sf'(0) - f''(0) - 2s^2F(s) - sf(0) - f'(0) + sF(s) - f(0) = 0$

$s^3F(s) -$

3. $\int_{-1,2}^{2,3} (x+y)(dx+dy$

$\int_{-1}^{2}\int_{2}^{3} x + y \,/\, dx\,dy = \int_{-1}^{2} = \frac{x^3}{3} +$

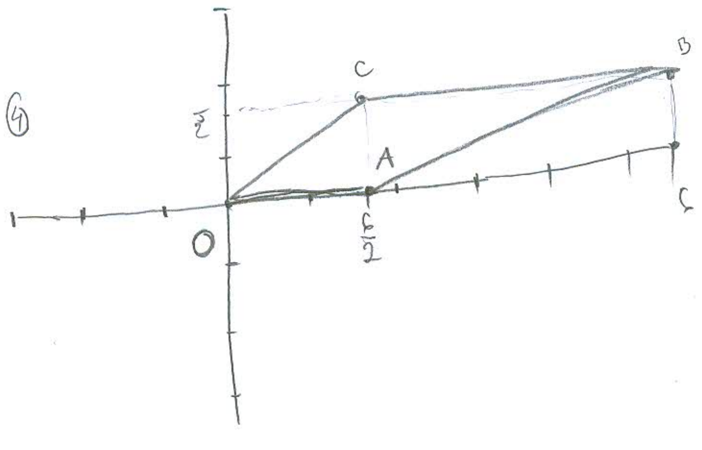
$\overline{AB}$.. =

$\overline{BC}$

$\overline{OA}$

$\overline{OC}$

4.



0