

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: NIKOLA KUCIĆEVIĆ

BROJ INDEKSA: 17-1-0002-2010

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

1. Neka je  $K$  kugla radijusa  $r = 2$  sa centrom u ishodištu. Izračunati  $\iiint_K (6x + 5y) dx dy dz$ . 20

2. Izračunati dvostruki integral:  $\iint_S x(y + 5) dx dy$ , gdje je  $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ i } x \geq y\}$ . 20

3. Izračunati  $\int_{(-1,2)}^{(2,3)} (x + y) (dx + dy)$ . 15

4.  $X$  je zadan kao četverokut s vrhovima  $O(0,0)$ ,  $A(\frac{6}{2}, 0)$ ,  $B(6, \frac{5}{2})$  i  $C(\frac{6}{2}, \frac{5}{2})$ . Izračunati dvostruki integral 15

$$\iint_X y dx dy.$$

5. Neka je  $K$  kugla radijusa  $r = 1$  sa centrom u ishodištu. Kako preko definicije izračunati  $\iint_{\partial K} 2dS$ ? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1, \quad y'(0) = -1, \quad y''(0) = 1.$$

Ukupno:

12

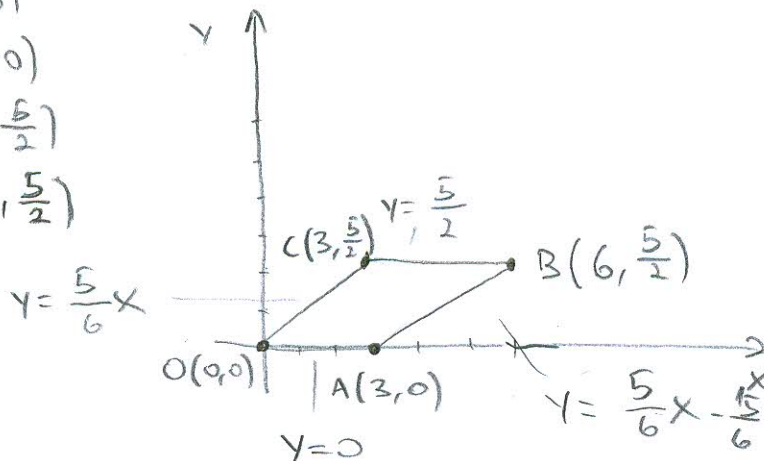
4.

$O(0,0)$

$A(3,0)$

$B(6, \frac{5}{2})$

$C(3, \frac{5}{2})$



$$(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$$

$$\overline{OC}: \quad O(0,0) \\ C(3, \frac{5}{2})$$

$$\overline{AB}: \quad A(3,0) \\ B(6, \frac{5}{2})$$

$$(y - 0)(3) = (x - 0) \frac{5}{2}$$

$$3y = \frac{5}{2}x$$

$$y = \frac{5}{6}x \quad \checkmark$$

$$(y)(6 - 3) = (x - 3) \frac{5}{2}$$

$$3y = \frac{5}{2}x - \frac{15}{2}$$

$$y = \frac{5}{6}x - \frac{15}{6} \quad \checkmark$$



IME I PREZIME: NIKOLA KREŠEVIĆ

BROJ INDEKSA:

(4.) 
$$\iint_x y dx dy = \int_0^3 \int_0^{\frac{5x}{6}} y dy dx + \int_3^6 \int_{\frac{5x}{6} - \frac{15}{6}}^{\frac{5}{2}} y dy dx \quad \checkmark \quad 12$$

$$= \int_0^3 \frac{y^2}{2} \Big|_0^{\frac{5x}{6}} dx + \int_3^6 \frac{y^2}{2} \Big|_{\frac{5x}{6} - \frac{15}{6}}^{\frac{5}{2}} dx$$

$$\left(\frac{5x-15}{6}\right)^2 = \frac{1}{2} \left(\frac{25x^2 - 75x + 225}{36}\right)$$

$$= \int_0^3 \frac{25x^2}{72} dx + \int_3^6 \frac{25}{72} - \left(\frac{25x^2}{72} - \frac{225}{72}\right) dx$$

$$= \int_0^3 \frac{25x^2}{72} dx + \int_3^6 \frac{25}{72} - \left(\frac{25x^2}{72} - \frac{225}{72}\right) dx = \frac{25}{72} \frac{x^3}{3} \Big|_0^3 + \int_3^6 \frac{450}{72} - \frac{25x^2}{72} dx$$

$$= \frac{25}{72} \cdot 9 + \frac{25}{72} \cdot 6 - \frac{25}{72} \left(\frac{x^3}{3}\right) \Big|_3^6 = \frac{225}{72} + \frac{25}{72} \cdot 6 - \frac{25}{72} \left(\frac{6^3}{3} - \frac{3^3}{3}\right)$$

$$= \frac{225}{72} + \frac{150}{72} - \frac{25}{72} \cdot 63 = \frac{375}{72} - \frac{1575}{72} = -\frac{1200}{72} = -12.5$$

6.  $Y'''(t) - 2Y''(t) + Y'(t) = 0$      $Y(0) = 1$   
 $Y'(0) = -1$   
 $Y''(0) = 1$

$$s^3 Y(s) - s^2 \cancel{Y(0)} - s \cancel{Y'(0)} - \cancel{Y''(0)} - 2(s^2 Y(s) - s \cancel{Y(0)} - \cancel{Y'(0)}) + s Y(s) - \cancel{Y(0)} = 0$$

$$s^3 Y(s) - s^2 + s - 1 - 2(s^2 Y(s) - s + 1) + s Y(s) - 1 = 0$$

$$s^3 Y(s) - s^2 + s - 1 - 2s^2 Y(s) + 2s - 2 + s Y(s) - 1 = 0$$

$$s^3 Y(s) - 2s^2 Y(s) + s Y(s) = s^2 - s - 2s + 1 + 2 + 1$$

$$Y(s) \cdot (s^3 - 2s^2 + s) = s^2 - 3s + 4 \quad | : (s^3 - 2s^2 + s)$$

$$Y(s) = \frac{s^2 - 3s + 4}{s^3 - 2s^2 + s} = \frac{s^2 - 3s + 4}{s(s^2 - s + 1)}$$

$$\frac{s^2 - 3s + 4}{s(s^2 - s + 1)} = \frac{A}{s} + \frac{Cs + D}{s^2 - s + 1} \quad | \cdot s(s^2 - s + 1)$$

$$s^2 - 3s + 4 = A(s^2 + s + 1) + (Cs + D)(s)$$

$$s^2 - 3s + 4 = As^2 + As + A + Cs^2 + Ds$$

$$s^2 - 3s + 4 = As^2 + Cs^2 + As + Ds + A$$

$$s^2 - 3s + 4 = s^2(A + C) + s(A + D) + A$$

$$A + C = 1 \quad A + D = -3 \quad (A = 4)$$

$$C = 1 - 4 \quad D = -3 - 4$$

$$(C = -3) \quad (D = -7)$$

$$Y(s) = \frac{4}{s} + \frac{-3s - 7}{s^2 - s + 1} \quad \text{INVERZNA TRANSFORMACJA ?}$$



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POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: MATE LADIĆ

BROJ INDEKSA: 17-1-0066-7010

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Ukupno:

5

Ⓟ

$$r = 2$$

$$\iiint_K (6x + 5y) dx dy dz$$

$$T(x_0, y_0, z_0)$$

$$r \in [0, 2]$$

$$\varphi \in [0, 2\pi]$$

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$z = z$$

$$dx dy dz = r dr d\varphi dz$$

$$\iiint_K (6x + 5y) dx dy dz$$

$$= \int_0^2 \int_0^{2\pi} \int_0^{\sqrt{4-z^2}} (6r \cos \varphi + 5r \sin \varphi) r dz d\varphi dr$$

$$x^2 + y^2 + z^2 = r^2$$

$$x^2 + y^2 = r^2$$

$$r^2 + z^2 = 2^2$$

$$r^2 + z^2 = 4$$

$$z^2 = 4 - r^2$$

$$z = \sqrt{4 - r^2}$$

$$z = \pm \sqrt{4 - r^2}$$

$$= \int_0^2 \int_0^{2\pi} (6r^2 \cos \varphi + 5r^2 \sin \varphi) dz d\varphi dr = \int_0^2 \int_0^{2\pi} (6r^2 \cos \varphi + 5r^2 \sin \varphi) d\varphi dr$$

$$= \int_0^2 (6r^2 \cos \varphi + 5r^2 \sin \varphi) \cdot (2\sqrt{4 - r^2}) d\varphi dr$$

$$= \int_0^2 (12r^2 \cos \varphi \sqrt{4 - r^2} + 10r^2 \sin \varphi \sqrt{4 - r^2}) d\varphi dr$$

$$= 0$$

KAD SU U ZAD. ZADANE GRANICE OD  $[0, 2\pi]$  I ZADANI SU SINUS I COSINUS INTEGRAL JE 0 ✓

5

$$\textcircled{5} \quad r = 1 \quad T(0,0) \quad \int_{\partial K} z ds \quad r =$$

VEKTORSKO  
PARAMETRIZACIJA JE POČJE

IMA 3 KOORDINATE

$$r(t) = \begin{pmatrix} 1 \cos t \\ 1 \sin t \end{pmatrix} = \begin{pmatrix} 1 \cdot \cos t \\ 1 \cdot \sin t \end{pmatrix} = r'(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

← OVDJE SAMO 2 KOORP.

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2} = \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1$$

$$\int_{\partial K} z ds = \int_0^{2\pi} (1 \cdot 2) dt = 2t \Big|_0^{2\pi} = 2 \cdot 2\pi - 2 \cdot 0 = 4\pi$$

