MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xoxxo
IME I PREZIME: Rako Matijašević **BROJ INDEKSA:** 17-1-0073-2011	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti integral: 15

$$\int (1+3x^2)\,e^{-x}\,dx\,.$$

2. Riješiti integrale: 15+15

(a) $\displaystyle\int_1^3 \frac{2x}{x^2-4}\,dx$;

(b) $\displaystyle\int_0^\pi \sin^5 x\cos^3 x\,dx$.

3. Riješiti diferencijalnu jednadžbu: $y'+y+3=x$. 15

4. Odrediti ekstreme funkcije: $f(x,y)=2y^2+x^2-x+1$. Koje su ekstremne vrijednosti i gdje se postižu? 8+12

5. Pronaći ravninu koja dira kuglu $x^2+y^2+z^2=4$ u točki $(1,1,1)$. 20

Ukupno: 20

TREBA MIJENJATI GRANICE INTEGRACIJE
NE VRIJEDI

2.a) $\displaystyle\int_1^3 \frac{2x}{x^2-4}\,dx = 2\int_1^3 \frac{x}{x^2-4}\,dx = \left| \begin{array}{l} x^2-4=t\,/' \\ 2x\,dx = dt\,/2 \\ x\,dx=\frac{1}{2}dt \end{array}\right| = \boxed{2\cdot\frac{1}{2}\int_1^3 \frac{dt}{t}} = 1\cdot\int_1^3 \frac{1}{t}\,dt = \int_1^3 \frac{1}{t}\,dt$

$= \boxed{\ln|t|\Big|_1^3} = \ln|3| - \ln|1| = 1{,}099 - 0 = 1{,}0986$

b) $\displaystyle\int_0^\pi \sin^5 x\cos^3 x\,dx = \int_0^\pi \sin^t x\cos^k\,dt = \left|\begin{array}{l}\sin x = t_1\,/' \\ \cos x\,dx = dt_1\end{array}\right| = \int_0^0 t_1\,dt_1 = \int_0^0 t_1^{\,t}\,dt_1 = \frac{t_1^{\,t+1}}{t+1}\Big|_0^0$

$= \frac{\sin^{\cancel{5+1}} 0}{5+1} - \frac{\sin^{\cancel{5+1}} 0}{5+1} = 0$

$\left|\begin{array}{l} t=5\,/' \\ dt=dx\end{array}\right|$ $\left|\begin{array}{l} k=3\,/' \\ dk=dx\end{array}\right|$ Promjena granica! → $x_1=0$, $t_1=0$, $x_2=\pi$, $t_2=0$

4. $f(x,y) = 2y^2+x^2-x+1$

$\frac{\partial f}{\partial x} = 2x-1$ $\quad 2x-1=0 \Rightarrow 2x=1\,/2$, $x=\frac{1}{2}$

$\frac{\partial f}{\partial y} = 2\cdot 2y = 4y$ $\quad 4y=0\,/4$, $y=0$

$T_1\left(\frac{1}{2},0\right)$ STACIONARNA TOČKA

$\Delta T = \begin{vmatrix} A & B \\ B & C\end{vmatrix} = A\cdot C - B^2$ $\quad \begin{vmatrix} 2 & 0 \\ 0 & 4\end{vmatrix} = 2\cdot 4 - 0 = 8$

$\frac{\partial^2 f}{\partial x^2} = 2 \Rightarrow A$

$\frac{\partial^2 f}{\partial x\partial y} = 0 \Rightarrow B$

$\frac{\partial^2 f}{\partial y^2} = 4 \Rightarrow C$

$\Delta T > 0$ Ekstrem postoji

$A > 0$ Minimum funkcije

$f_{min} = 2\cdot 0^2 + \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{3}{4}$

5. $x^2+y^2+z^2=4$ $\quad T(1,1,1)$
$\qquad\qquad\qquad\qquad x_0, y_0, z_0$

Implicitni oblik!

$$\frac{\partial f}{\partial x_0}=2x \qquad R \dots \frac{x-x_0}{\frac{\partial f}{\partial x_0}}=\frac{y-y_0}{\frac{\partial f}{\partial y_0}}=\frac{z-z_0}{\frac{\partial f}{\partial z_0}}$$

$$\frac{\partial f}{\partial y_0}=2y \qquad \frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{2}=4 \quad \times$$

VIDI NAPOMENU KALCIMA

$$\frac{\partial f}{\partial z_0}=2z$$

1. $\int(1+3x^2)e^{-x}\,dx = \left| \begin{array}{l} u=1+3x^2 /' \\ du=6x\,dx \\ dv=e^{-x} /\int \\ v=e^{-x} \end{array} \right| = (1+3x^2)\cdot e^{-x} - \int e^{-x}\, 6x\,dx = \left| \begin{array}{l} 6x=u /' \\ 6dx=du /6 \\ dx=\frac{1}{6}du \\ dv=e^{-x} /\int \\ v=e^{-x} \end{array} \right|$

$= (1+3x^2)\cdot e^{-x} - \left(6x\cdot e^{-x} - 6\int dx\right)$

$= (1+3x^2)\cdot e^{-x} - (6x\cdot e^{-x} - 6x)$ $\Rightarrow$ KRIVO!

$\int(1+3x^2)e^{-x}\,dx = \left| \begin{array}{l} -x=t /' \\ -dx=dt /(-1) \\ dx=-dt \end{array} \right| = -\int(1+3x^2)e^{t}\,dt = \left| \begin{array}{l} 1+3x^2=u /' \\ 6x\,dx=du /6 \\ x\,dx=\frac{1}{6}du \\ dv=e^{t} /\int \\ v=e^{t} \end{array} \right| = 1+3x^2\cdot e^{t} - \int e^{t}\cdot 6x\,dx = \left| \begin{array}{l} 6x=u /' \\ 6dx=du /6 \\ dx=\frac{1}{6}du \\ dv=e^{t} /\int \\ v=e^{t} \end{array} \right|$

$= 1+3x^2\cdot e^{t} - \left(6x\cdot e^{t} - \int e^{t}\,dx\right)$

$= 1+3x^2\cdot e^{t} - (6x\cdot e^{t} - e^{t}) + C$

$= 1+3x^2\cdot e^{-x} - (6x\cdot e^{-x} - e^{-x}) + C$

RJEŠENJE!

PROVJERA

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xoxxo
IME I PREZIME: SABELIĆ BORIS **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: ~~0~~ 20 Koso

① $\int (1+3x^2)e^{-x}\,dx = \left| \begin{matrix} 1+3x^2 = u \\ 6x\,dx = du \\ x\,dx = \frac{1}{6}du \end{matrix} \right.$ ✓ $\begin{matrix} e^{-x}dx = dv \\ -e^{-t} = v \end{matrix}$ ✓ $\left| \begin{matrix} -x = t \\ -dx = dt \\ dx = -dt \end{matrix} \right.$

$= (1+3x^2)\cdot -e^{-t} + 6\int e^{-t}x\,dx$

$\times$

$\left| \begin{matrix} x = u & e^{-t}dv \\ dx = du & -e^{-t} = v \end{matrix} \right| = x\cdot(-e^{-t}) + \int e^{-t}dx$

$-xe^{-t} + e^{-t} = e^{-t}(-x+1)$

$= -(1+3x^2)\cdot e^{-t} + 6\left[e^{-t}(-x+1)\right] + C = -e^{-t} - 3x^2e^{-t} - 6xe^{-t} + 6e^{-t} + C$

$-xe^{-t} + e^{-t}$

$-6xe^{-t} + 6e^{-t}$

$= e^{-t}(-1-3x^2-6x+6) + C$

$e^{-t}(-3x^2-6x+5) + C$

PROVJERA: $y = e^{-t}(-3x^2-6x+5)$

$y' = e^{-t}(+3\cdot \cancel{x^2} \ldots) \stackrel{?}{=} (1+3x^2)e^{-x}$

SABOLIĆ

② $\int_1^3 \frac{2x}{x^2-4}\,dx =$

$x^2-4=(x-2)(x+2)$

$x^2+\cancel{2x}-\cancel{2x}-4$

$$\int \frac{2}{x^2-4}\,dx + \int \frac{x}{x^2-4}\,dx \overset{*}{=} 2\int \frac{dx}{x^2-4} + \int \frac{x}{x^2-4}\,dx$$

$$\frac{x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$x = Ax + 2A + Bx - 2B$

$1 = A + B \,/\cdot 2$

$0 = 2A - 2B$

$2 = 2A + \cancel{2B}$ /+

$0 = 2A - \cancel{2B}$

$2 = 4A \,/:4$

$\frac{2}{4} = \boxed{A = \frac{1}{2}} \rightarrow 1 = \frac{1}{2} + B$

$-B = \frac{1}{2} - 1$

$\boxed{B = \frac{1}{2}}$

$$\frac{1}{2}\int \frac{dx}{x-2} + \frac{1}{2}\int \frac{dx}{x+2} = \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int \frac{dt}{t} =$$

$\left|\begin{matrix} x-2=t \\ dx=dt \end{matrix}\right|$ $\left|\begin{matrix} x+2=t \\ dx=dt \end{matrix}\right|$ $\frac{1}{2}\ln|x-2| + \frac{1}{2}\ln|x+2|$

$$* \; 2\int \frac{dx}{x^2-4} + \frac{1}{2}\ln|x-2| + \frac{1}{2}\ln|x+2| =$$

$$= 2\cdot\left(\frac{1}{2\cdot 2}\ln\left|\frac{2+x}{2-x}\right|\right) + \frac{1}{2}\ln|x-2| + \frac{1}{2}\ln|x+2| \Big|_1^3 =$$

$$\left(\frac{1}{2}\ln\left|\frac{2+3}{2-3}\right| + \frac{1}{2}\ln|3-2| + \frac{1}{2}\ln|3+2|\right) - \left(\frac{1}{2}\ln\left|\frac{2+1}{2-1}\right| + \frac{1}{2}\ln|1-2| + \frac{1}{2}\ln|1+2|\right) =$$

$$(0.80 + 0 + 0.80) - (0.55 + 0 + 0.55) = \frac{8}{5} - \frac{11}{10} = \frac{1}{2} \;///$$

KRIVO

② $\int \frac{x}{x^2-4}\,dx = \frac{x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$

$x = Ax + 2A + Bx - 2B$

$1 = A + B \,/\cdot 2$ $\quad 1 = 2 + B$

$0 = 2A - 2B$ $\quad -B = 2 - 1$

$2 = 2A + \cancel{2B}$ $\quad \boxed{B = -1}$

$0 = 2A - \cancel{2B}$

$2 = 4A \,/:4$ $\quad 2\; 2\int \frac{dx}{x-2} - \int \frac{dx}{x+2}$

$\boxed{2 = A}$

$\left|\begin{matrix} x-2=t \\ dx=dt \end{matrix}\right|$ $\left|\begin{matrix} x+2=t \\ dx=dt \end{matrix}\right|$

$2\int \frac{dt}{t} - \int \frac{dt}{t} = 2\ln|x-2| - \ln|x+2|$

$4\ln|x-2| - \ln|x+2| \Big|_1^3$

$(4\ln|1| - \ln|5|) - (4\ln|-1| - \ln|3|)$

$-1.61 - (-1.09) = \underline{-0.5}$

V.A. $x=2$ unutar $[1,3]$
NE VRIJEDI N-L FORMULA.
OVO JE NEPRAVI INTEGRAL.

$\boxed{2b}$ $\int_0^{\pi} \sin^5 x \cos^3 x \, dx = \int \sin^2 x \cdot \sin^3 x \cdot \cos^2 x \cdot \cos x \, dx =$

$\int_0^{\pi} (1-\cos^2 x) \cdot \sin^3 x \cdot \cos^2 x \cdot \cos x \, dx = \int \sin^3 x \cdot \cos^2 x \cdot \cos x \, dx - \int \sin^3 x \cdot \cos^2 x \cdot \cos x \cdot \cos^2 x \, dx$

$\int (1-\cos^2 x) \cdot \sin x \cdot \cos^2 x \cdot \cos x \, dx - \int (1-\cos^2 x) \sin x \cdot \cos^2 x \cdot \cos x \cdot \cos^2 x \, dx$

$\int \sin x \cos^2 x \cos x \, dx - \int \cos^2 x \cdot \sin x \cos^2 x \cos x - \int (1-\cos^2 x) \sin x \cos^2 x \cos x \cos^2 x \, dx$

$\left| \begin{array}{l} \cos x = t \\ -\sin x = dt \\ \sin x = -dt \end{array} \right| - \int t^3 dt + \int t^5 dt - \int \sin x \cos^2 x \cos x \cos^2 x \, dx \overset{+}{-} \int \cos^2 x \sin x \cos^5 x \, dx$

2a $\int_1^3 \frac{2x}{x^2-4} dx = 2 \int \frac{x}{x^2-4} = 2 \cdot \frac{1}{2} \int \frac{dt}{t} =$

$x^2-4=t$
$2x\,dx = dt$
$x\,dx = \frac{1}{2} dt$

$\ln|x^2-4| + C \Big|_1^3$ ×

OVO JE NEPRAVI.
NE VRIJEDI
NEWTON – LE-
IBNITZOVA
FORMULA

$\ln|3^2-4| - \ln|1^2-4| =$

$1.61 - 1.1 = -0.5$

$-\int_0^{\pi} t^3 dt + \int_0^{\pi} t^5 dt + \int_0^{\pi} t^5 dt + \int_0^{\pi} t^7 dt$

$-\frac{t^4}{4}\Big|_0^{\pi} + \frac{t^6}{6}\Big|_0^{\pi} + \frac{t^6}{6}\Big|_0^{\pi} + \frac{t^8}{8}\Big|_0^{\pi} = -\frac{\cos^4 x}{4}\Big|_0^{\pi} + \frac{\cos^6 x}{6}\Big|_0^{\pi} + \frac{\cos^6}{6}\Big|_0^{\pi} + \frac{\cos^8 x}{8}\Big|_0^{\pi}$

$= \left[\left(-\frac{1}{4}\cos^4 \pi\right) - \left(\frac{1}{4}\cos^4 0\right)\right] + \left[\left(\frac{1}{6}\cos^6 \pi\right) - \left(\frac{1}{6}\cos^6 0\right)\right] + \left[\left(\frac{1}{6}\cos^6 \pi\right) - \left(\frac{1}{6}\cos^6 0\right)\right] +$

$\left[\left(\frac{1}{8}\cos^8 \pi\right) - \left(\frac{1}{8}\cos^8 0\right)\right] =$

$= -\frac{1}{2} + 0 + 0 + 0 = -\frac{1}{2}$ /// ×

SABOLIĆ

Ime i prezime: SABOLIĆ BORIS **Broj indeksa:**

④ $f(x,y) = 2y^2 + x^2 - x + 1$

$\partial_x f = 2x - 1 \longrightarrow 2x - 1 = 0$ $\quad 2x = 1 \,/ :2$

$\partial_y f = 4y$ $\quad 4y = 0$ $\quad \boxed{x = \frac{1}{2}}$ ✓

$\boxed{y = 0}$

$T\left(\frac{1}{2}, 0\right)$ ✓

$\partial_{xx} f = 2$ A

$\partial_{xy} f = 0$ B

$\partial_{yy} f = 4$ C

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

$\Delta = 2 \cdot 4 - 0$

$\Delta = 8 > 0$ ima extrem

$A > 0$ minimum

$f_{min} = 2 \cdot 0 + \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$ ✓

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xoxxo
IME I PREZIME: KREŠIMIR KALCINA **BROJ INDEKSA:** 57181/2009	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti integral: ~~15~~

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Ukupno: 8

3

$$y'+y+3=x$$

$$\frac{dy}{dx}+y=x-3 \quad /\cdot dx$$

$$dy+y=(x-3)\,dx \quad /\int$$

$$\int dy+\int y\,dy=\int x\,dx-3\int dx$$

$$y+\frac{y^2}{2}=\frac{x^2}{2}-3x+C$$ X

4

$$f(x,y)=2y^2+x^2-x+1$$

$$\frac{\partial f}{\partial x}=2x-1$$

$$\frac{\partial f}{\partial y}=4y$$

U KOJOJ TOČKI?

$$\frac{\partial^2 f}{\partial x^2}=2$$

$$\Delta=\begin{vmatrix}2&0\\0&4\end{vmatrix}=8-0=8>0 \Rightarrow \text{ ekstrem funkcije.}$$

$$\frac{\partial^2 f}{\partial y^2}=4$$

$$\frac{\partial^2 f}{\partial x^2}=2>0 \Rightarrow \text{ minimum funkcije}$$

KOLIKO IZNOSI?

$$\frac{\partial^2 f}{\partial x\,\partial y}=0$$

(2)

a) $$\boxed{\int_1^3 \frac{2x}{x^2-4}dx = \lim_{x\to 2_-}\int_1^x \frac{2x}{x^2-4} + \lim_{t\to 2_+}\int_t^3 \frac{2x}{x^2-4}}$$

$$\int \frac{2x}{x^2-4}dx = \begin{bmatrix} x^2-4=t \\ 2x\,dx = dt\end{bmatrix} = \int \frac{dt}{t} = \ln|t| + C = \ln|x^2-4| + C$$

IZVRSNO! PREPOZNALI STE NEPRAVI INTEGRAL. NISTE ZAKLJUČILI DA IZNOS NE POSTOJI.

$$\lim_{x\to 2_-}\int_1^x \ln|x^2-4| + \lim_{t\to 2_+}\int_t^3 \ln|x^2-4| = \ln|x^2-4|\Big|_1^x + \ln|x^2-4|\Big|_t^3$$

8

$$= \left(\ln|x^2-4| - \ln|1^2-4|\right) + \left(\ln|3^2-4| - \ln|t^2-4|\right)$$

$$\lim_{x\to 2_-}\frac{2x}{x^2-4} = -\infty$$

$$\lim_{x\to 2_+}\frac{2x}{x^2-4} = +\infty$$

Funkcija nije neprekidna t.d $\boxed{P=\emptyset}$

b) $$\int_0^{\pi} \sin^5 x \cos^3 x\,dx =$$

$\emptyset$

(1) $$\int (1+3x^2)\cdot e^{-x}dx =$$

$$= \begin{bmatrix} 1+3x^2 = u & e^{-x}dx = dv \\ 6x\,dx = du & -e^{-x} = v \end{bmatrix}$$

$u\cdot v - \int v\cdot du$

$$= (1+3x^2)\cdot e^{-x} - \int e^{-x}\cdot 6x\,dx$$

$$= e^{-x}\cdot 3x^2+1-6\int e^{-x}x\,dx$$

$$\begin{bmatrix} x = u & e^{-x}dx = dv \\ dx = du & e^{-x} = v \end{bmatrix}$$

$$= e^{-x}\cdot 3x^2+1-6\left(x\cdot e^{-x} - \int e^{-x}\cdot dx\right)$$

$$= e^{-x}\cdot 3x^2+1-6\left(x\cdot e^{-x} - e^{-x}\right)$$

$$= e^{-x}\cdot(3x^2+1-6x)$$

$$= \boxed{\frac{3x^2-6x+1}{e^x} + C}$$

PROVJERA: $y = e^{-x}(3x^2-6x+1)$

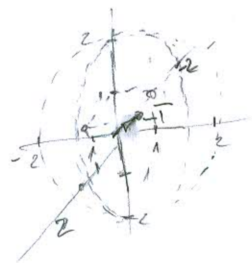
$y' = e^{-x}(-3x^2+6x-1+6x-6)$

$= e^{-x}(-3x^2+12x-7) \neq (1+3x^2)e^{-x}$

DAKLE, POGREŠNO!

IME I PREZIME: KREŠIMIR KALCINA BROJ INDEKSA: 57181/2009

(5) $x^2+y^2+z^2=4$ $\quad T\left(\overset{x}{1},\overset{y}{1},\overset{z}{1}\right)$



$\frac{\partial f}{\partial x}=2x$ $\quad T f(x)=2\cdot 1=2$

$\frac{\partial f}{\partial y}=2y$ $\quad T f(y)=2\cdot 1=2$

$\frac{\partial f}{\partial z}=2z$ $\quad T f(z)=2\cdot 1=2$

$$\frac{x-x_0}{Tf(x)}=\frac{y-y_0}{Tf(y)}=\frac{z-z_0}{Tf(z)}=\frac{1-x_0}{2}=\frac{1-y_0}{2}=\frac{1-z_0}{2}$$

X

TREBALO JE PREKO

$f(x,y)=z=\sqrt{4-x^2-y^2}$ naći $\frac{\partial f}{\partial x}(1,1)$, $\frac{\partial f}{\partial y}(1,1)$

RAVNINA $z-1=\frac{\partial f}{\partial x}(1,1)(x-1)+\frac{\partial f}{\partial y}(1,1)(y-1)$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**	Grupa xoxxo
Ime i prezime: Ivan Radović **Broj indeksa:** 57230	Popunjava nastavnik Broj ↓ bodova

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Ukupno: 0

1. $\int u\cdot v = v + \int u \cdot dv$

$$\int (1+3x^2)\,e^{-x}\,dx = \begin{vmatrix} u=(1+3x^2) & v = e^{-x} \\ u = 3\frac{x^3}{3} & dv = e \end{vmatrix} = (1+3x^2) + \int (1+3x^2)\cdot e$$

$$= (1+3x^2) + 3\,\frac{x^3}{3}\cdot e$$

4.

$f(x,y)' \; 2y^2 + x^2 - x + 1 = 4y + 2x$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MLADEN BULIĆ **BROJ INDEKSA:** 17-1-0018-2010

Grupa xoxxo | POPUNJAVA NASTAVNIK | Broj ↓ bodova

1. Riješiti integral: ~~15~~
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Ukupno: 0

$\boxed{3^2-4=9-4=5,\quad 1^2-4=1-4=-3}$

2. a) $\displaystyle\int_1^3 \frac{2x}{x^2-4}\,dx = \begin{bmatrix} x^2-4=t \\ 2x\,dx=dt \end{bmatrix} = \int_1^3 \frac{dt}{t} = \int_{-3}^{5}\frac{dt}{t} = \ln|x^2-4|\Big|_{-3}^{5}$

$= \ln|5^2-4| - \ln|(-3)^2-4| = \ln|21| - \ln|5| = 3{,}0445 - 1.6094 = 1.4351$

NE! JER JE RIJEČ O NEPRAVOM INTEGRALU. $\frac{2x}{x^2-4}$ NIJE NEPREKIDNA NA [1,3]

NE VRIJEDI NEWTON-LEIBNITZOVA FORMULA!

b) $\displaystyle\int_0^\pi \sin^5 x\cos^3 x\,dx = \begin{bmatrix} \sin x = t \\ \cos x\,dx = dt \end{bmatrix} = \int_0^\pi t^5\,dt^3 = \int_0^\pi t^5 + \int_0^\pi dt^3 = \frac{t^6}{6} + t^3\Big|_0^{0{,}054}$

$\displaystyle\Big|_0^{0{,}054} \frac{1}{6}\sin^6(0{,}054) - \frac{1}{6}\sin^6(0) + \sin^3(0{,}054) - \sin^3(0) =$

3.

$y' + y + 3 = x$

$y' + y = x - 3$

$f(x) = 3 \; , \; g(x) = x$

$f(x)\,dx = 3\int dx = 3x$

$-f(x)\,dx = -3\int dx = -3x$

$y = e^{-3x} \cdot \int e^{3x} \cdot x$

$\int e^{3x} \cdot x = \begin{bmatrix} u = x & dv = \int e^{3x} \\ du = dx & v = \frac{1}{3} e^{3x} \end{bmatrix}$

$y = e^{-3x} \cdot x \cdot \frac{1}{3} e^{3x} - \int \frac{1}{3} e^{3x} dx = e^{-3x} \cdot x \cdot \frac{1}{3} e^{3x} - \frac{1}{3}\int e^{3x} dx$

$y = e^{-3x} \cdot x \cdot \frac{1}{3} e^{3x} - \frac{1}{3} \cdot \frac{1}{3} e^{3x} = e^{-3x} \cdot x \cdot \frac{1}{3} e^{3x} - \frac{1}{9} e^{3x}$

PROVJERA: $y = \frac{1}{3}x - \frac{1}{9} e^{3x}$

$y' = \frac{1}{3} - \frac{1}{3} e^{3x}$

$y' + y = \frac{1}{3} - \frac{1}{3} e^{3x} + \frac{1}{3}x - \frac{1}{9} e^{3x}$

$= \frac{1}{3} - \frac{1}{3}x - \frac{4}{9} e^{3x} \neq x - 3$

Ime i prezime: MLADEN BULIĆ **Broj indeksa:** 17-1-0018-2010

(4.) $f(x,y) = 2y^2 + x^2 - x + 1$

$A \cdot C - B^2 = 2 \cdot 4 - 0^2 = 8$

$f_x = 2x - 1$

$f_{xx} = 2$

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ ✓

$f_y = 4y$

$f_{yy} = 4$

$4y = 0$

$\boxed{y = -4}$ ✗

$A = 2 > 0 \Rightarrow \min$

$\boxed{T(\frac{1}{2}, -4) \Rightarrow minimum}$ ✗

$f_{xx} = 2$

(A) $B = 0$

$f_{yy} = 4$

(C)

(5.) $x^2 + y^2 + z^2 = 4$ u $\begin{array}{c} x_0\,y_0\,z_0 \\ (1, 1, 1) \end{array}$

$\int (1+3x^2) e^{-x} dx = \left| \begin{array}{l} u = e^{-x} \\ du = e^{-x} dx \\ dv = \int 1 + 3x^2 \\ v = x + x^3 \end{array} \right| = e^{-x} \cdot x + x^3 - \int x + x^3 \cdot e^{-x}$

$= e^{-x} \cdot x + x^3 -$

$\times \int 1 + 3x^2 = 1 \int dx + 3 \int x^2$

$= x + 3 \frac{x^3}{3}$

$= x + x^3$

$1+3x^2 \cdot e^{-x} - \int e^{-x} 6x dx$

$1 + 3x^2 \cdot e^{-x} -$

$u = 6x dx \quad dv = e^{-x}$

$du = 6 dx \quad v = e^{-x}$

$6x \cdot e^{-x} - \int e^{-} 6 dx$

$- e^{-x} \cdot 6 \frac{x}{2}$

(1.) $\int (1+3x^2) e^{-x} dx = \left[\begin{array}{ll} u = 1+3x^2 & dv = e^{-x} \\ du = 6x\,dx & v = e^{-x} \end{array}\right] = (1+3x^2) \cdot e^{-x} -$

$\times \quad dv = -e^{-x}$

$+$

$- \int e^{-x} 6x = (1+3x^2) e^{-x} - 6x \cdot e^{-x} - e^{-x} \cdot 6x$ $\times$

$* \int e^{-x} 6x\,dx = \left[\begin{array}{ll} 6x\,dx = u & e^{-x} = dv \\ 6\,dx = du & e^{-x} = v \end{array}\right] = 6x \cdot e^{-x} - \int e^{-x} 6\,dx$

$= 6x \cdot e^{-x} - e^{-x} \cdot 6x$ $\times$

$- \left[\begin{array}{ll} \cancel{u = e^{-x}} & \cancel{dv = 1+3x^2} \\ \cancel{du = e^{-x}} & \cancel{v = x + x^3} \end{array}\right] = \cancel{e^{-x} \cdot x + x^3 \int x + x^3 \cdot e^{-x}}$

$* \left[\begin{array}{ll} \cancel{u = e^{-x}} & \cancel{dv = x + x^3} \\ \cancel{du = e^{-x}} & \cancel{v = \frac{x^2}{2} + \frac{x^4}{4}} \end{array}\right]$