

1. Riješiti integral:

$$\int (1 + 3x^2) e^{-x} dx.$$

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2. Riješiti integrale:

(a) $\int_1^3 \frac{2x}{x^2-4} dx;$

(b) $\int_0^\pi \sin^5 x \cos^3 x dx.$

15+15

3. Riješiti diferencijalnu jednadžbu: $y' + y + 3 = x.$

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4. Odrediti ekstreme funkcije: $f(x, y) = 2y^2 + x^2 - x + 1.$ Koje su ekstremne vrijednosti i gdje se postižu?

8+12

5. Pronaći ravninu koja dira kuglu $x^2 + y^2 + z^2 = 4$ u točki $(1, 1, 1).$

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TREBA MIJENJATI GRANICE INTEGRALOKA NE VRIJEDI

Ukupno:

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2. a) $\int_1^3 \frac{2x}{x^2-4} dx = 2 \int_1^3 \frac{x}{x^2-4} dx$

$\left| \begin{array}{l} x^2-4 = t \\ 2x dx = dt/2 \\ x dx = dt/2 \end{array} \right| = 2 \cdot \frac{1}{2} \int_1^3 \frac{dt}{t} = 1 \cdot \int_1^3 \frac{1}{t} dt = \int_1^3 \frac{1}{t} dt$

$= \ln|t| \Big|_1^3 = \ln|3| - \ln|1| = 1,099 - 0 = 1,0986$

b) $\int_0^\pi \sin^5 x \cos^3 x dx = \int_0^\pi \sin^4 x \cos^2 x dx = \int_0^\pi (\sin^2 x)^2 \cos^2 x dx$

$\left| \begin{array}{l} t = \sin x \\ dt = \cos x dx \\ k = 3 \\ dk = -\sin x dx \end{array} \right|$

Rangiraj granice!

$x_1 = 0, t_1 = 0, x_2 = \pi, t_2 = 0$

$\int_0^\pi \sin^4 x \cos^2 x dx = \int_0^\pi (1 - \sin^2 x)^2 \sin x dx = \int_0^\pi (1 - 2\sin^2 x + \sin^4 x) \sin x dx$

$= \int_0^\pi (\sin x - 2\sin^3 x + \sin^5 x) dx = \left[-\cos x + \frac{2}{4} \cos^4 x - \frac{1}{6} \cos^6 x \right]_0^\pi$

$= \left[-\cos \pi + \frac{1}{2} \cos^4 \pi - \frac{1}{6} \cos^6 \pi \right] - \left[-\cos 0 + \frac{1}{2} \cos^4 0 - \frac{1}{6} \cos^6 0 \right]$

$= \left[1 + \frac{1}{2} - \frac{1}{6} \right] - \left[-1 + \frac{1}{2} - \frac{1}{6} \right] = \left[\frac{5}{6} \right] - \left[-\frac{5}{6} \right] = \frac{5}{6} + \frac{5}{6} = \frac{10}{6} = \frac{5}{3}$

4. $f(x, y) = 2y^2 + x^2 - x + 1$

$\frac{\partial f}{\partial x} = 2x - 1$

$2x - 1 = 0 \Rightarrow 2x = 1/2$

$\frac{\partial f}{\partial y} = 2 \cdot 2y = 4y$

$4y = 0/4$

$y = 0$

$\frac{\partial^2 f}{\partial x^2} = 2 \Rightarrow A$

$\frac{\partial^2 f}{\partial x \partial y} = 0 \Rightarrow B$

$\frac{\partial^2 f}{\partial y^2} = 4 \Rightarrow C$

$\Delta T = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = A \cdot C - B^2 = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 2 \cdot 4 - 0 = 8$

$\Delta T > 0$ Ekstrem postoji

$A > 0$ Minimum funkcije

$f_{min} = 2 \cdot 0^2 + \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 1 = \frac{3}{4}$

$T_1\left(\frac{1}{2}, 0\right)$ STACIONARNA TOČKA ✓

$$5. x^2 + y^2 + z^2 = 4$$

implicitni derivata!

$$T(1,1,1)$$

$$x_0, y_0, z_0$$

$$\frac{\partial f}{\partial x_0} = 2x$$

$$\frac{\partial f}{\partial y_0} = 2y$$

$$\frac{\partial f}{\partial z_0} = 2z$$

$$R \dots \frac{x-x_0}{\frac{\partial f}{\partial x_0}} = \frac{y-y_0}{\frac{\partial f}{\partial y_0}} = \frac{z-z_0}{\frac{\partial f}{\partial z_0}} = 0$$

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-1}{2} = 0 \quad \times$$

JIDI NAROVNENU KALCINA

~~1. $\int (1+3x^2)e^{-x} dx = \left| \begin{array}{l} u = 1+3x^2 / ' \\ du = 6x dx \\ dv = e^{-x} / \int \\ v = e^{-x} \end{array} \right| = (1+3x^2) \cdot e^{-x} - \int e^{-x} 6x dx =$~~

~~$\left. \begin{array}{l} ux = u / ' \\ u dx = du / u' \\ dx = \frac{1}{u'} du \\ v = e^{-x} / \int \\ v = e^{-x} \end{array} \right| = (1+3x^2) \cdot e^{-x} - (6x \cdot e^{-x} - 6 \int dx)$~~

~~$= (1+3x^2) \cdot e^{-x} - (6x \cdot e^{-x} - 6x) + C$~~

~~\Rightarrow KRIVU!~~

$\int (1+3x^2)e^{-x} dx = \left| \begin{array}{l} -x = t / ' \\ -dx = dt / (-1) \\ dx = -dt \end{array} \right| = -\int (1+3x^2)e^t dt = \left| \begin{array}{l} 1+3x^2 = u / ' \\ u dx = du / 6 \\ x dx = \frac{1}{6} du \\ dv = e^t / \int \\ v = e^t \end{array} \right| = 1+3x^2 \cdot e^t - \int e^t \cdot 6x dx =$

$= 1+3x^2 \cdot e^t - (6x \cdot e^t - \int e^t dx)$

$= 1+3x^2 \cdot e^t - (6x \cdot e^t - e^t) + C$

$= (1+3x^2) \cdot e^{-x} - (6x \cdot e^{-x} - e^{-x}) + C$

\Rightarrow RJEŠENJE!

PROJEKTA

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: SABOLIĆ BORIS

BROJ INDEKSA:

Grupa
XXXX
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

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Ukupno:

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Kosov

① $\int (1 + 3x^2) e^{-x} dx = \begin{cases} 1 + 3x^2 = u \\ 6x dx = du \\ x dx = \frac{1}{6} du \end{cases} \quad \begin{cases} e^{-x} dx = dv \\ -e^{-t} = v \end{cases} \quad \begin{cases} -x = t \\ -dx = dt \\ dx = -dt \end{cases}$

* $(1 + 3x^2) \cdot e^{-t} + 6 \int e^{-t} x dx$

$\begin{cases} x = u \\ dx = du \end{cases} \quad \begin{cases} e^{-t} dv \\ -e^{-t} = v \end{cases} = x \cdot e^{-t} + \int e^{-t} dx$

$-x e^{-t} + e^{-t} = e^{-t}(-x + 1)$

* $(1 + 3x^2) \cdot e^{-t} + 6 \left[e^{-t}(-x + 1) \right] + C = -e^{-t} - 3x^2 e^{-t} - 6x e^{-t} + 6e^{-t} + C$

$\begin{matrix} -x e^{-t} + e^{-t} \\ -6x e^{-t} + 6e^{-t} \end{matrix}$

$= e^{-t}(-1 - 3x^2 - 6x + 6) + C =$

$e^{-t}(-3x^2 - 6x + 5) + C$

PROVJERA:

$y = e^{-t}(-3x^2 - 6x + 5)$

$y' = e^{-t}(-3 \cdot 2x - 6) = (1 + 3x^2) e^{-x}$

SABOLIĆ

$$\textcircled{2} \int_1^3 \frac{2x}{x^2-4} dx =$$

$$x^2-4 = (x-2)(x+2)$$

$$x^2+2x-2x-4$$

$$\int \frac{2}{x^2-4} dx + \int \frac{x}{x^2-4} dx = 2 \int \frac{dx}{x^2-4} + \int \frac{x}{x^2-4} dx$$

$$\textcircled{2} \frac{x}{x^2-4} dx = \frac{x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$x = Ax + 2A + Bx - 2B$$

$$1 = A + B \quad | \cdot 2 \quad 1 = 2 + B$$

$$0 = 2A - 2B \quad | -B = 2 - 1$$

$$2 = 2A + 2B \quad | \boxed{B = -1}$$

$$0 = 2A - 2B$$

$$2 = 4A \quad | :4 \quad 2 \int \frac{dx}{x-2} + \int \frac{dx}{x+2}$$

$$\boxed{2 = A}$$

$$\left| \begin{array}{l} x-2=t \\ dx=dt \end{array} \right| \quad \left| \begin{array}{l} x+2=t \\ dx=dt \end{array} \right|$$

$$2 \int \frac{dt}{t} - \int \frac{dt}{t} = 2 \ln|x-2| - \ln|x+2|$$

$$4 \ln|x-2| - \ln|x+2| \Big|_1^3$$

$$(4 \ln|1| - \ln|5|) - (4 \ln|-1| - \ln|3|)$$

$$-1.61 - (-1.09) = \underline{\underline{-0.5}}$$

$$\frac{x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$x = Ax + 2A + Bx - 2B$$

$$1 = A + B \quad | \cdot 2$$

$$0 = 2A - 2B$$

$$2 = 2A + 2B$$

$$0 = 2A - 2B$$

$$2 = 4A \quad | :4$$

$$\frac{2}{4} = A = \frac{1}{2} \rightarrow 1 = \frac{1}{2} + B$$

$$-B = \frac{1}{2} - 1$$

$$\boxed{B = \frac{1}{2}}$$

$$\frac{1}{2} \int \frac{dx}{x-2} + \frac{1}{2} \int \frac{dx}{x+2} = \frac{1}{2} \int \frac{dt}{t} + \frac{1}{2} \int \frac{dt}{t} =$$

$$\left| \begin{array}{l} x-2=t \\ dx=dt \end{array} \right| \quad \left| \begin{array}{l} x+2=t \\ dx=dt \end{array} \right| \quad \frac{1}{2} \ln|x-2| + \frac{1}{2} \ln|x+2|$$

$$* 2 \int \frac{dx}{x^2-4} + \frac{1}{2} \ln|x-2| + \frac{1}{2} \ln|x+2| =$$

$$= 2 \cdot \left(\frac{1}{2 \cdot 2} \ln \left| \frac{2+x}{2-x} \right| \right) + \frac{1}{2} \ln|x-2| + \frac{1}{2} \ln|x+2| \Big|_1^3 =$$

$$\left(\frac{1}{2} \ln \left| \frac{2+3}{2-3} \right| + \frac{1}{2} \ln|3-2| + \frac{1}{2} \ln|3+2| \right) - \left(\frac{1}{2} \ln \left| \frac{2+1}{2-1} \right| + \frac{1}{2} \ln|1-2| + \frac{1}{2} \ln|1+2| \right) =$$

$$(0.80 + 0 + 0.80) - (0.55 + 0 + 0.55) = \frac{8}{5} - \frac{11}{10} = \frac{1}{2} //$$

KRIVO

V.A. $x=2$ IZVAN
 [1,3]
 NE VRIJEDI N-L
 FORMULA.
 OVO JE NEPRAVI
 INTEGRAL.

