MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: Ivan Kovačević **Broj indeksa:** 17-2-0125-2012

Vrijeme: od ____________ **do** ________ ♣4 **Broj bodova:** 70

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$ ✓ 12

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$ ✓ 8

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$ ✓ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2-2x-x^2$ i pravac $y = 3x-3$. ✓ 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - y^3 + 3xy$$ ✓ 10

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x-y) + \frac{1}{y-1}$$ ✓ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{(x+1)}y = 1-x^2$$

b)

$$y'' + 8y' + 16y = \cos x$$

1.) a) $\int e^{\sin^2 x} \sin(2x)\,dx$

$$\left| \begin{array}{l} \sin^2 x = t \quad /' \\ 2\sin x \cos x\,dx = dt \\ \sin(2x)\,dx = dt \end{array} \right|$$

$$\int e^t\,dt = e^t + C = e^{\sin^2 x} + C$$

b) $$\int_e^{+\infty} \frac{dx}{x \ln^2 x} = \lim_{b \to +\infty} \int_e^b \frac{dx}{x \cdot \ln^2 x} = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x}\,dx = dt \\ x = b\,,\ t = \ln b \\ x = e \quad t = 1 \end{array} \right| = \lim_{b \to +\infty} \int_1^{\ln b} \frac{1}{t^3}\,dt$$

$$= \lim_{b \to +\infty} \int_1^{\ln b} t^{-3}\,dt = \lim_{b \to +\infty} \left| \frac{t^{-2}}{-2} \right| \Big|_1^{\ln b} = \lim_{b \to +\infty} \left[-\frac{1}{2\ln^2 b} + \frac{1}{2 \cdot 1^2} \right]$$

$$= \lim_{b \to +\infty} \left[-\frac{1}{2 \cdot \ln^2 b} + \frac{1}{2} \right] = -0 + \frac{1}{2} = \frac{1}{2}$$

2.) $$\int \frac{x^3 - x + 2}{x^2 - 1}\,dx = \int x\,dx + \int \frac{2\,dx}{x^2 - 1} = \int x\,dx + 2\int \frac{dx}{x^2 - 1}$$

$$(x^3 - x + 2) : (x^2 - 1) = x$$
$$\underline{-(x^3 - x)}$$
$$2$$

$$= \int x\,dx - 2\int \frac{dx}{(1)^2 - x^2}$$

$$= \frac{x^2}{2} - 2 \cdot \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C$$

$$= \frac{x^2}{2} - \ln \left| \frac{x+1}{x-1} \right| + C$$

3.) $y = 2 - 2x - x^2$

$y = 3x - 3$

x	-1	0	1	-2
y	3	2	-1	2

$2 - 2x - x^2 = 3x - 3$

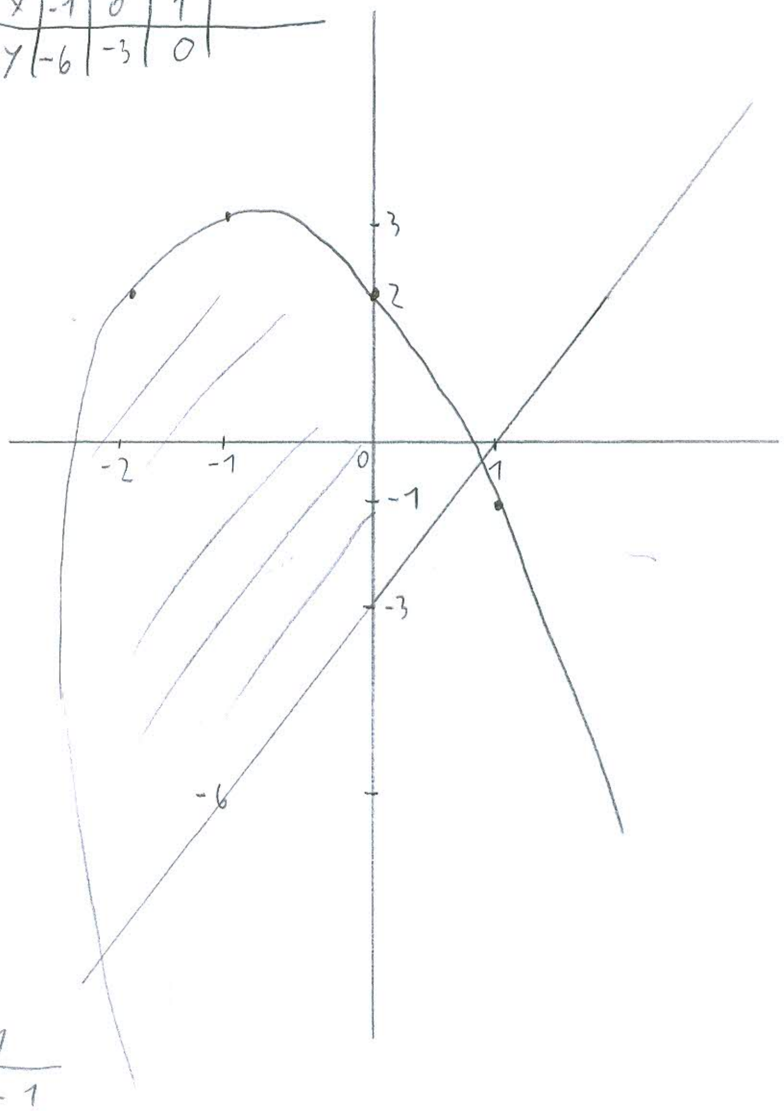
x	-1	0	1
y	-6	-3	0

$2 + 3 - 2x - 3x - x^2 = 0$

$-x^2 - 5x + 5 = 0$

$x_{12} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$

$x_1 = -5{,}85 \qquad x_2 = 0{,}25$



$P = \int_{-5,85}^{0,25} [2 - 2x - x^2] - [3x - 3] \, dx =$

$= \int_{-5,85}^{0,25} (-x^2 - 3x + 5) \, dx - \left(-\frac{x^3}{3} - \frac{5x^3}{2} + 5x\right) \Big|_{-5,85}^{0,25}$

$P = 50{,}31$

4.) b) $f(x, y) = \ln(x - y) + \dfrac{1}{y - 1}$

I $\quad x - y > 0 \Rightarrow y < x$

II $\quad y - 1 \neq 0 \Rightarrow y \neq 1$

$Df = \{(x, y) : y < x, \; y \neq 1\}$

10

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

Tablica osnovnih derivacija

f	f'	f	f'
$x^{\alpha} (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

4.) a $f(x,y) = x^2 - y^3 + 3xy$ $x^2 - y^3 + 3xy$

$\frac{\partial f}{\partial x} = 2x + 3y$ $\frac{\partial f}{\partial y} = -3y^2 + 3x$ $\frac{\partial f}{\partial x \partial y} = +3$

$\frac{\partial^2 f}{\partial x^2} = 2$ $\frac{\partial^2 f}{\partial y^2} = -6y$

stacijonarne tocke

$2x+3y=0 \quad /\cdot(-3)$

$-3y^2 + 3x = 0 \quad /(2)$

$-6y^2 + 9x = 0$

$-6y^2 + 6x = 0$

$-6y^2 - 9y = 0$

$y_1 = 0 \quad y_2 = -\frac{3}{2}$

$x_1 = 0 \quad x_2 = \frac{9}{4}$

$T_1(0, 0, \ )$ $T_2(0, -\frac{2}{3}, \ )$ $T_3(\frac{9}{4}, 0, \ )$ $T_4(\frac{9}{4}, -\frac{3}{2}, \ )$

Extremi

$$\Delta = \begin{vmatrix} 2 & +3 \\ +3 & -6y \end{vmatrix} = (2 \cdot (-6y)) - (3 \cdot 3) =$$

$T_1)$ $2 \cdot (6 \cdot 0) - 3 \cdot 3 = -9$ $T_1 \rightarrow$ sedlasta tocka

$T_2)$ $2 \cdot (-6 \cdot -\frac{2}{3}) - 3 \cdot 3 = -17$ $T_2 \rightarrow$ sedlasta tocka

$T_3)$ $2 \cdot (-6 \cdot 0) - 3 \cdot 3 = -9$ $T_3 \rightarrow$ sedlasta tocka

$T_4)$ $2 \cdot (-6 \cdot \frac{3}{2}) - 3 \cdot 3 = 9$ $T_4 \rightarrow$ minimum

10

Lukaš

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: LUKA PEROŠ **Broj indeksa:** 02184

Vrijeme: od ________ **do** ________ ♣4 **Broj bodova:** 57

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$ 12

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2-2x-x^2$ i pravac $y = 3x-3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - y^3 + 3xy$$

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x-y) + \frac{1}{y-1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{(x+1)}y = 1-x^2$$ 15

b)

$$y'' + 8y' + 16y = \cos x$$ 15

2. $\int \frac{x^3-x+2}{x^2-1}\,dx = \dots$

$(\cancel{x^3}-x+2):(x^2-1) = x + \frac{2}{x^2-1}$

$\underline{\cancel{-x^3}+x}$

2

$I = \int x\,dx + 2\int \frac{1}{x^2-1}\,dx = \frac{x^2}{2} + \cancel{2}\frac{1}{\cancel{2}}\ln\left|\frac{x-1}{x+1}\right| = \frac{x^2}{2} + \ln\left|\frac{x-1}{x+1}\right| + C$

3. $y = -x^2-2x+2 \quad , \quad y = 3x-3$

$x_0 = -\frac{b}{2a} = -\frac{-2}{-2} = \boxed{-1}$ $\quad y_0 = \frac{4ac-b^2}{4ac} = \frac{-8+4}{-8} = \frac{-4}{-8} = \boxed{\frac{1}{4}}$

$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{2\pm\sqrt{4+8}}{-2} = \frac{2\pm\sqrt{12}}{-2} = \frac{2\pm\sqrt{4\cdot 3}}{-2} = \frac{2\pm\sqrt{6}}{-2}$

$x_1 = \frac{2+\sqrt{6}}{-2} = -2{,}22 \; , \quad x_2 = \frac{2-\sqrt{6}}{-2} = 0{,}22$

$y = 3x-3$

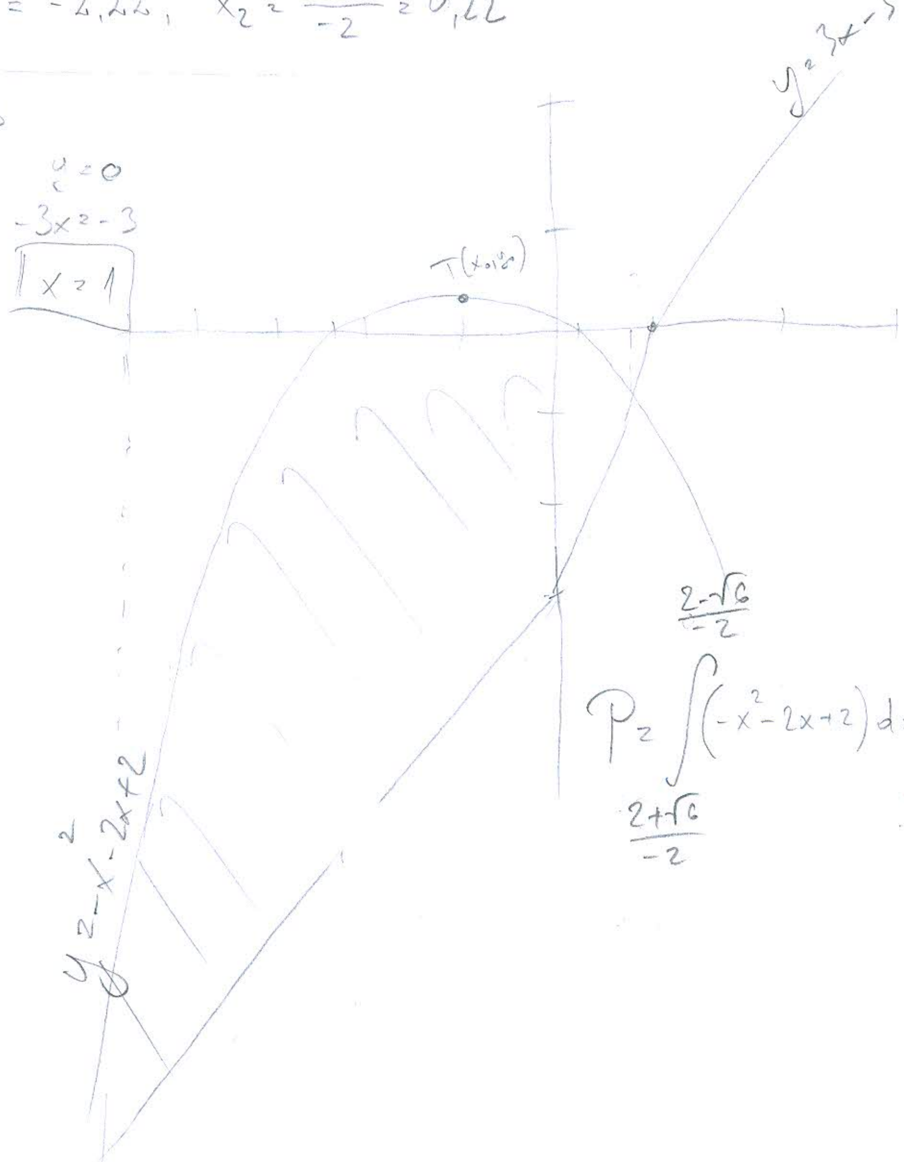
$x = 0$

$\boxed{y = -3}$

$y = 0$

$-3x = -3$

$\boxed{x = 1}$



$-x^2-2x+2 = 0$

$\underline{3x-3 = 0}$

$y = y$

$-x^2-2x+2-3x+3 = 0$

$-x^2-5x+5 = 0$

$x_{1,2} = \frac{5\pm\sqrt{25+20}}{-2}$

$x_1 = \frac{5+\sqrt{45}}{-2} = \frac{5+\sqrt{15}}{-2} = -4{,}43$

$x_2 = \frac{5-\sqrt{45}}{-2} = \frac{5-\sqrt{15}}{-2} = 0{,}56$

$P = \int_{\frac{2+\sqrt{6}}{-2}}^{\frac{2-\sqrt{6}}{-2}} (-x^2-2x+2)\,dx - \int_{\frac{5+\sqrt{15}}{-2}}^{0} (-x^2-2x+2)\,dx - \int_{\frac{5+\sqrt{15}}{-2}}^{\frac{5-\sqrt{15}}{-2}} (3x-3)\,dx$

5. a) $y' - \frac{1}{(x+1)} y = 1 - x^2$

$f(x)$ $\quad$ $g(x)$

$-\int \frac{x^2+1}{x+1} dx =$

$(x^2+1) : (x+1) = x+1$

$\cancel{-x^2} + x$

$x + \cancel{1}$

$\cancel{x} + \cancel{1}$

$\int x\,dx + \int dx = \frac{x^2}{2} + x$

$y = e^{-\int(-\frac{1}{x+1})dx} \left[\int e^{-\int \frac{1}{x+1} dx} \cdot (1-x^2) dx + C \right]$

$y = \cancel{e}^{\cancel{\ln}(x+1)} \left[\int \cancel{e}^{-\cancel{\ln}(x+1)} \cdot (1-x^2) dx + C \right]$

$y = (x+1) \left[\int \frac{1}{x+1} \cdot (1-x^2) dx + C \right]$

$y = (x+1) \left[-\int \frac{x^2+1}{x+1} dx + C \right]$

$y = (x+1) \left[-\frac{x^2}{2} + x + C \right]$

b) $y'' + 8y' + 16 = \cos x$

$y = y_0 + \eta$

$r^2 + 8r + 16 = 0$

$r_{1,2} = \frac{-8 \pm \sqrt{64-64}}{2}$

$r_{1,2} = \frac{-8 \pm 0}{2}, \quad r_1 = -4, \quad r_2 = -4$

$y_0 = C_1 e^{-4x} + x C_2 e^{-4x}$

$y = C_1 e^{-4x} + x C_2 e^{-4x} + \frac{225}{4335} \cos x + \frac{8}{289} \sin x$

$\eta = A \cos x + B \sin x$

$\eta' = -A \sin x + B \cos x$

$\eta'' = -A \cos x - B \sin x$

$-A\cos x - B\sin x + 8(-A \sin x + B \cos x) + 16(A \cos x + B \sin x) = \cos x$

$-A\cos x - B\sin x - 8A\sin x + 8B\cos x + 16A\cos x + 16B\sin x = \cos x$

$15A + 8B = 1 \quad / \cdot 8$

$-8A + 15B = 0 \quad / \cdot 15$

$120A + 64B = 8$

$-120A + 225B = 0 \quad / +$

$289B = 8$

$\boxed{B = \frac{8}{289}}$

$15A + 8 \cdot \frac{8}{289} = 1$

$15A + \frac{64}{289} = 1$

$15A = 1 - \frac{64}{289}$

$15A = \frac{225}{289} \quad / : 15$

$A = \frac{\frac{225}{289}}{\frac{15}{1}} = \boxed{\frac{225}{4335}}$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

1\. a) $f(x,y) = x^2 - y^3 + 3xy$

$z_x = 2x + 3y$

$z_y = -3y^2 + 3x$

$z_{xx} = 2$ $\Rightarrow A = 2 > 0 =$ MINIMUM

$z_{xy} = 3$ $\Rightarrow B = 3$

$z_{yy} = -6y = -6 \cdot 0 = 0$ $\Rightarrow C = 0$

$2x + 3y = 0 \Rightarrow 3y = -2x$

$3x - 3y^2 = 0 \qquad y = -\frac{2}{3}x$

$3x - \cancel{3}(-\frac{2}{3}x) = 0$

$3x + 2x = 0$

$\boxed{x = 0} \qquad \boxed{y = 0}$

$T(0,0)$

$\Delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & 0 \end{vmatrix} = 0 + 9 = 9$ ima ekstrem u $T(0,0)$, minimum (2).

b) $f(x,y) = \ln(x-y) + \frac{1}{y-1}$

$x - y > 0$

$-y > -x \quad / \cdot (-1)$

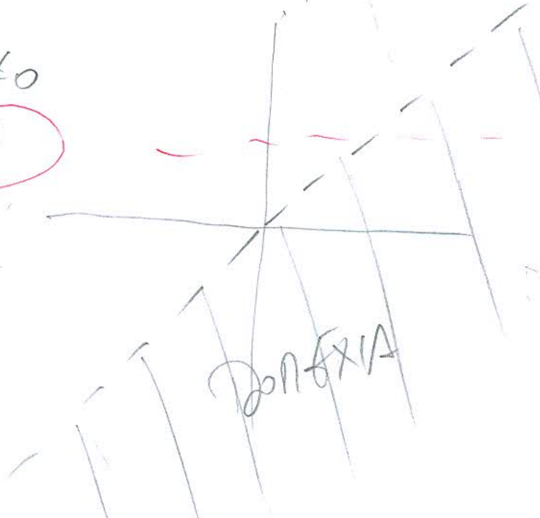
$y < x$

$y = x$

$y - 1 \neq 0$

$y \neq 1$

x	0	1	2
y	0	1	2



1.) a) $\int e^{\sin^2 x} \sin(2x)\,dx = \int e^{\sin^2 x} 2\sin x \cos x \, dx = \left| \begin{matrix} \sin^2 x = t \\ 2\sin x \cos x \, dx = dt \end{matrix} \right|$

$= \int e^t \, dt = e^t = e^{\sin^2 x} + C$

b) $\int\limits_{e}^{+\infty} \frac{dx}{x \ln^3 x} = \lim\limits_{\varepsilon \to 0} \int\limits_{\varepsilon}^{+\infty} \frac{dx}{x \ln^3 x}$

$\int \frac{1}{x \ln^3 x} \, dx = \left| \begin{matrix} \ln x = t \\ \frac{1}{x} dx = dt \end{matrix} \right| = \int \frac{dt}{t^3} = \int t^{-3} dt = \frac{t^{-2}}{-2} = -\frac{1}{t^2} = -\frac{1}{\ln^2 x}$

$\lim\limits_{\varepsilon \to 0} \left[-\frac{1}{\ln^2 x} \right] \Bigg|_{\varepsilon}^{-\infty} = \lim\limits_{\varepsilon \to 0} \left[-\frac{1}{(\ln \infty)^2} - \left(-\frac{1}{\ln \varepsilon} \right) \right] = \lim\limits_{\varepsilon \to 0} \left[0 + \frac{1}{\ln \varepsilon} \right] =$

$= \frac{1}{\ln 0} = \frac{1}{-\infty}$

♣3

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: FRANE JORDAN **Broj indeksa:** 55161-2007

Vrijeme: od 8 **do** 11 ♣4 **Broj bodova:** 42

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x} \sin(2x) dx$$ ✓ 12

b)

$$\int\limits_{e}^{+\infty} \frac{dx}{x \ln^3 x}$$

2. *(15)* Integriraj

$$\int \frac{x^3 - x + 2}{x^2 - 1} dx$$ ✓ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2 - 2x - x^2$ i pravac $y = 3x - 3$. ✓ 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 - y^3 + 3xy$$

b) Odredi domenu funkcije:

$$f(x, y) = \ln(x - y) + \frac{1}{y - 1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{(x+1)} y = 1 - x^2$$

b)

$$y'' + 8y' + 16y = \cos x$$

② $\int \frac{x^3-x+2}{x^2-1}\,dx = \int \frac{x(x^2-1)+2}{x^2-1}\,dx = \int \frac{x\cancel{(x^2-1)}}{\cancel{x^2-1}}\,dx + 2\int \frac{dx}{x^2-1} =$

$= \int x\,dx + 2\int \frac{dx}{x^2-1} = \frac{x^2}{2} + \cancel{2}\cdot\frac{1}{\cancel{2}} \ln\left|\frac{x-1}{x+1}\right| + C$

$= \frac{x^2}{2} + \ln\left|\frac{x-1}{x+1}\right| + C$ ✓

① a) $\int e^{\sin^2 x} \sin(2x)\,dx = \left| \begin{array}{l} u = \sin^2 x \;/' \\ du = 2\sin x\cos x\,dx \\ dx = \frac{du}{2\sin x\cos x} \end{array} \right| =$

$= \int e^u \cdot 2\,\cancel{\sin x\cos x} \cdot \frac{du}{\cancel{2\sin x\cos x}} = \int e^u\,du =$ ✓

$= e^u + C = e^{\sin^2 x}\,dx$

b) $\int_e^{+\infty} \frac{dx}{x\ln^3 x} = \left| \begin{array}{l} t = \ln \;/' \\ \frac{1}{x}\,dx = dt \\ dx = x\,dt \end{array} \right| = \int_1^{+\infty} \frac{\cancel{x}\,dt}{\cancel{x}\,t^3} = \int_1^{\infty} t^{-3}\,dt = \frac{t^{-2}}{-2}\Bigg|_1^{\infty} =$

$= -\frac{1}{2t^2}\Bigg|_1^{\infty} = -\frac{1}{2}\left(\frac{1}{\infty} - \frac{1}{1}\right) = -\frac{1}{2}(0-1) = \frac{1}{2}$

③ $y = 2 - 2x - x^2 \Longrightarrow x_{1,2} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

$y = 3x - 3$

$x_1 = 1 + \sqrt{3}$ $\approx 2{,}73$ $\qquad x_2 = 1 - \sqrt{3}$ $\approx -0{,}73$

$3x - 3 = 2 - 2x - x^2$

$x^2 + 5x - 5 = 0$

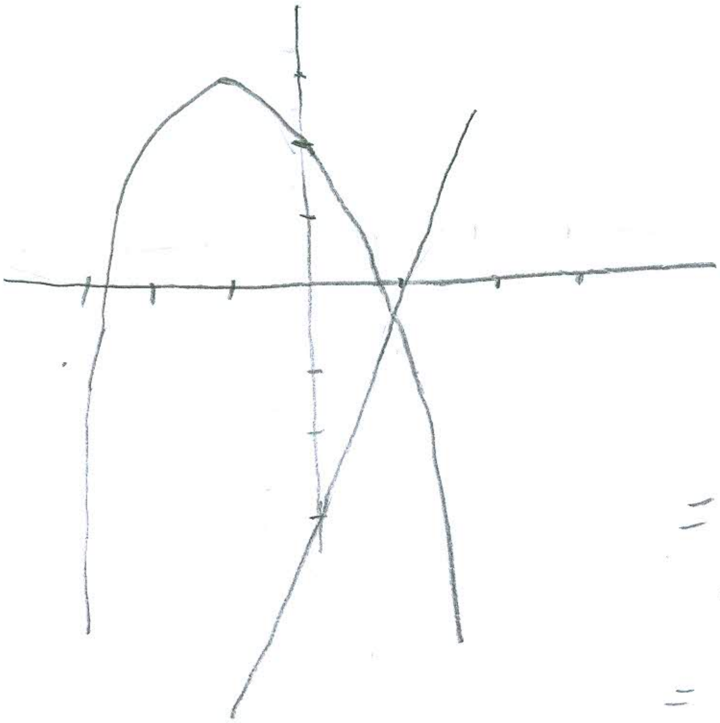
$x_{1,2} = \frac{-5 \pm \sqrt{25 + 20}}{2} = \frac{-5 \pm 3\sqrt{5}}{2}$

$x_1 = \frac{-5 + 3\sqrt{5}}{2}$ $\qquad x_2 = \frac{-5 - 3\sqrt{5}}{2}$

$\approx 0{,}85$ $\qquad \approx -5{,}85$

$y_1 = 3\left(\frac{-5 + 3\sqrt{5}}{2}\right) - 3$ $\qquad y_2 = 3\left(\frac{-5 - 3\sqrt{5}}{2}\right) - 3$

$y_1 \approx -0{,}43$ $\qquad y_2 \approx 20{,}56$



$$P = \int_{\frac{-5-\sqrt{5}}{2}}^{\frac{-5+3\sqrt{5}}{2}} \left(\left(2 - 2x - x^2\right) - \left(3x - 3\right)\right) dx =$$

$$= \int \left(2 - 2x - x^2 - 3x + 3\right) dx =$$

$$= \int \left(-x^2 - 5x + 5\right) dx =$$

$$= \left(-\frac{x^3}{3} - \frac{5x^2}{2} + 5x\right)\Bigg|_{\frac{-5-3\sqrt{3}}{2}}^{\frac{-5+3\sqrt{5}}{2}} =$$

$$P=\left[\frac{\left(\frac{-5+3\sqrt{5}}{2}\right)^3}{3}-\frac{\left(\frac{-5-3\sqrt{5}}{2}\right)^3}{3}\right]-\left[\frac{5\cdot\left(\frac{-5+3\sqrt{5}}{2}\right)^2}{2}-\frac{5\cdot\left(\frac{-5-3\sqrt{5}}{2}\right)^2}{2}\right]$$

♣2|

$$+\left[5\,\frac{-5+3\sqrt{5}}{2}-5\,\frac{-5-3\sqrt{5}}{2}\right]=-67{,}08-(-83{,}85)+33{,}54$$

$$\approx 50{,}3$$ ✓ 15

4 b) $f(x,y)=\ln(x-y)+\frac{1}{y-1}$

1

$x-y>0$

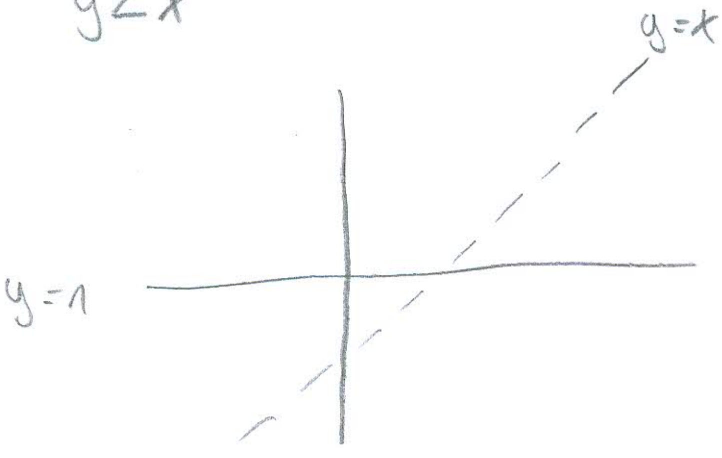
$x>y$

$y<x$

2

$y-1\neq 0$

$y\neq 1$



DOMENA SU SVE TOČKE ISPOD PRAVCA $y=x$ NE UKLJUČUJUĆI TAJ PRAVAC I PRAVAC $y=1$.

a) $f(x,y)=x^2-y^2+3xy$

$f'_x=2x+3y$

$2x+3y=0$

$f'_y=-3y^2+3x$

$-3y^2+3x=0$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x\rvert + C$	$\int \cot x dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1-\frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣4

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: DONATO PREDOVAN **Broj indeksa:**

Vrijeme: od ____________ **do** ________ ♣4 **Broj bodova:** 40

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$ ✓ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y=2-2x-x^2$ i pravac $y=3x-3$. ✓ 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^2-y^3+3xy$$ ✓ 10

b) Odredi domenu funkcije:

$$f(x,y)=\ln(x-y)+\frac{1}{y-1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'-\frac{1}{(x+1)}y=1-x^2$$

b)

$$y''+8y'+16y=\cos x$$

2.

$$x^3-x+2 : x^2-1 = x$$
$$-x^3+x$$
$$2$$

$$\frac{P}{Q}=Q+\frac{r}{Q}$$

$$=\int x+\frac{2}{x^2-1}dx=\int x\,dx+2\int\frac{1}{x^2-1}dx=$$

$$=\frac{x^2}{2}+2\cdot\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|$$ ✓

$$=\frac{x^2}{2}+\ln\left|\frac{x+1}{x-1}\right|+C$$

3. $y=2-2x-x^2$ (1)
$y=3x-3$ (2)

$$2-2x-x^2=3x-3$$
$$2-2x-x^2-3x+3=0$$
$$-x^2-5x+5=0 \,/:(-1)$$
$$x^2+5x-5=0$$
$$x_{1/2}=\frac{-5\pm\sqrt{25+4\cdot5}}{2}=\frac{-5\pm\sqrt{9\cdot5}}{2}=\frac{-5\pm3\sqrt{5}}{2}$$

$$x_1=\frac{-5+3\sqrt{5}}{2}=0.85 \qquad x_2=\frac{-5-3\sqrt{5}}{2}=-5.85$$

$y(0)=2$ → gornja
$y_1(0)=-3$ → donja

$$P=\int_{-5.85}^{0.85}\left(2-2x-x^2-(3x-3)\right)dx$$
$$=\int_{-5.85}^{0.85}(2-2x-x^2-3x+3)\,dx=\int_{-5.85}^{0.85}-x^2-5x+5$$
$$=-\int_{-5.85}^{0.85}x^2\,dx-5\int_{-5.85}^{0.85}x\,dx+\int 5\,dx$$
$$=5x\Big|_{-5.85}^{0.85}-\frac{5}{2}x^2\Big|_{-5.85}^{0.85}-\frac{1}{3}x^3\Big|_{-5.85}^{0.85}$$
$$=5(0.85+5.85)-\frac{5}{2}(0.7225-34.22)-\frac{1}{3}(0.614+200.2)$$
$$=33.5+83.74-66.938=50.3$$

15

DONATO PREDOVAN

4. a) $f(x,y) = x^2 - y^3 + 3xy$

$$\frac{df}{dx} = 2x + 3y \qquad \frac{d^2f}{dx^2} = 2$$

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & -6y \end{vmatrix} = -12y - 9$$

$$\frac{df}{dy} = -3y^2 + 3x \qquad \frac{d^2f}{dy^2} = -6y$$

$$\frac{d^2f}{dxdy} = \frac{d^2f}{dydx} = 3$$

$2x + 3y = 0 \Rightarrow 3y = -2x \Rightarrow y = -\frac{2}{3}x$

$3x - 3y^2 = 0$

$3x - 3\left(\frac{4}{9}x^2\right) = 0$

$3x - \frac{4}{3}x^2 = 0$

$x\left(3 - \frac{4}{3}x\right) = 0$

$x_1 = 0 \qquad 3 - \frac{4}{3}x = 0$

$-\frac{4}{3}x = -3 \quad / \cdot \left(-\frac{4}{3}\right)$

$y_1 = 0 \qquad x_2 = \frac{9}{4}$

$y_2 = -\frac{3}{2}$

$-\frac{2}{3} \cdot \frac{9}{4}$

$T_1(0,0)$

$T_2\left(\frac{9}{4}, -\frac{3}{2}\right)$

1. $T_1(0,0)$

$\Delta = -12 \cdot 0 - 9$

$= -9 < 0 \rightarrow$ nema ekstrema

$T_1 \rightarrow$ SEDLASTA TOČKA

2. $T_2\left(\frac{9}{4}, -\frac{3}{2}\right)$

$\Delta = -12 \cdot \left(-\frac{3}{2}\right) - 9$

$= 18 - 9 = 9 > 0 \rightarrow$ ima ekstrema

$A = 2 > 0$

Točka $T_2 \rightarrow$ lokalni minimum

10

b) $f(x,y) = \ln(x-y) + \frac{1}{y-1}$

$x - y > 0 \qquad y - 1 \neq 0$

$x > y \qquad y \neq 1$

$D = \mathbb{R} \setminus \{1\}$

$x = y$

x	0	2
y	0	2

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$\boxed{1.}$

a) $$\int e^{\sin^2 x} \sin(2x)\,dx = \left| \begin{matrix} u = e^{\sin^2 x} & dv = 2\sin x\,dx \\ du = e^{\sin^2 x}\,dx & v = -2\cos x \end{matrix} \right.$$

$$= e^{\sin^2 x} \cdot (-2\cos x) + \int 2\cos x \cdot e^{\sin^2 x}\,dx \Rightarrow \int 2\cos x \cdot e^{\sin^2 x}\,dx = \left| \begin{matrix} u = e^{\sin^2 x} & dv = 2\cos x\,dx \\ du = e^{\sin^2 x}\,dx & v = 2\sin x \end{matrix} \right.$$

$$= e^{\sin^2 x} \cdot (-2\cos x) + e^{\sin^2 x} \cdot 2\sin x - \int 2\sin x \cdot e^{\sin^2 x}\,dx$$

a) $$\int e^{\sin^2 x} \sin(2x)\,dx = \left| \sin^2 x = t \,/' \right| =$$

$$= \int e^t \ldots\,dx = e^t$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x+a}{x-a} \right\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x \sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x \sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: MATE ĆADIĆ **Broj indeksa:** 17-1-0006-2010

Vrijeme: od 8^{15} **do** 10^{00} ♣**4** **Broj bodova:** 27

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$

12

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$

15

3. *(15)* Odredi površinu koju zatvaraju parabola $y=2-2x-x^2$ i pravac $y=3x-3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^2-y^3+3xy$$

b) Odredi domenu funkcije:

$$f(x,y)=\ln(x-y)+\frac{1}{y-1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'-\frac{1}{(x+1)}y=1-x^2$$

b)

$$y''+8y'+16y=\cos x$$

1) a) $\int e^{\sin^2 x}\sin(2x)dx=\left[\begin{matrix}\sin^2 x=t\\ 2\sin x dx=dt\end{matrix}\right]=\int e^t\cdot dt=e^t=e^{\sin^2 x}+C$ ✓

b) $\int_e^{+\infty}\frac{dx}{x\ln^3 x}=f(x)=\frac{1}{x\ln^3 x}$ $\int\frac{dx}{x\ln^3 x}=\int\frac{dx}{x}+\int\frac{dx}{\ln^3 x}=\ln|x|\perp$?

$x\ln x\neq 0$

$x\neq 0$

$D(f)=\mathbb{R}\setminus\{0\}$

MATE LADIC

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

1\. a) $f(x,y) = x^2 - y^3 + 3xy$

$\partial_x f = 2x + 3y$

$\partial_{xx} f = 2$

♣2|

$\partial_{xy} f = 3$

$\partial_y f = -3y^2 + 3x$

$\partial_{yy} f = -6y$

$\partial_{yx} f = 3$ $\quad\quad T(1,0)$

$\partial_x f = 0$

$\partial_y f = 0$

$2x + 3y = 0$

$2x = -3y \; / : 2$

$x = -\frac{3}{2} y$

$2 \cdot \left(-\frac{3}{2} y\right) + 3y = 0$

$-3y + 3y = 0$

$0y = 0$

$-3y^2 + 3x = 0$

$-3y^2 = -3x \; / : -3$

$y^2 = -x$

$y = -x$

$-3 \cdot (-x)^2 + 3x = 0$

$-3x^2 + 3x = 0$

$x(-3x + 3) = 0$

$x_1 = 0$

$-3x = -3$

$x = 1$

$A = \partial_{xx} f = 2$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -6y \end{vmatrix} = -12y - 9$$

DOMENA

$f(x,y) = \ln(x - y) + \frac{1}{y-1}$

$x - y > 0$

$y - 1 \neq 0$

$y = 1$

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MARINO ŽUBČIĆ **Broj indeksa:** 17-2-0216-2012

Vrijeme: od 08 : 15 **do** 10 : 15 ♣4 **Broj bodova:** 15

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$ √ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y=2-2x-x^2$ i pravac $y=3x-3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^2-y^3+3xy$$

b) Odredi domenu funkcije:

$$f(x,y)=\ln(x-y)+\frac{1}{y-1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'-\frac{1}{(x+1)}y=1-x^2$$

b)

$$y''+8y'+16y=\cos x$$

0.85
-0.43

3 $y=2-2x-x^2$ $y=3x-3$

$2-2x-x^2=3x-3$

$2-2x-x^2-3x+3=0$

$-x^2-5x+5=0 \;/\cdot(-1)$

$x^2+5x-5=0$

$x_{1,2}=\frac{-5\pm\sqrt{25+20}}{2}$

$x_{1,2}=\frac{-5\pm\sqrt{45}}{2}$

$x_1=\frac{-5+3\sqrt5}{2}$

$x_2=\frac{-5-3\sqrt5}{2}$ -5.35

$y_1=\frac{-21+9\sqrt5}{2}$

$y_2=-\frac{21+9\sqrt5}{2}$

-0.43

MARINO ZUBČIĆ

(4) $f(x, y) = x^2 - y^3 + 3xy$

$\partial_x f = 2x + 3y$ $\qquad$ $\partial_y f = -3y^2 + 3x$

$\partial_{xx} f = 2$ $\qquad$ $\partial_{yy} f = -6y$

$\partial_{xy} f = 3$ $\qquad$ $\partial_{xy} f = 3$

$A = \partial_{xx} f$

$A = 2 > 0$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -6 \end{vmatrix} =$$

$$= 2 \cdot (-6) - 3 \cdot 3 = -12 - 9 = -21$$

FUNKCIJA NEMA EKSTREMA

(2) $$\int \frac{x^3 - x + 2}{x^2 - 1} dx = \int x\,dx + \int \frac{2}{x^2-1} dx = \int x\,dx + 2\int \frac{dx}{x^2 - 1^2} = \frac{x^2}{2} + 2\left(\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|\right) + C$$

$$= \frac{x^2}{2} + \ln\left|\frac{x+1}{x-1}\right| + C$$

15

$$x^3 - x + 2 : (x^2 - 1) = x + \frac{2}{x^2-1}$$
$$\underline{-x^3 \pm x}$$
$$2$$

(1) a) $$\int e^{\sin^2 x} \sin(2x)\,dx = \begin{bmatrix} \sin 2x = t \\ \cos 2x \cdot 2x\,dx = dt \\ \cos^2 x \cdot 2dx = dt \end{bmatrix} =$$

$2\cos^2 x\,dx = dt$

$= \int e^t$

♣3

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x\rvert + C$	$\int \cot x dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣4

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: STIPE DUŠEVIĆ **Broj indeksa:** 17-2-0051-2010

Vrijeme: od ____________ **do** _________ ♣4 **Broj bodova:** 5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=2-2x-x^2$ i pravac $y=3x-3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^2-y^3+3xy$$ 5

b) Odredi domenu funkcije:

$$f(x,y)=\ln(x-y)+\frac{1}{y-1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'-\frac{1}{(x+1)}y=1-x^2$$

b)

$$y''+8y'+16y=\cos x$$

1. a) $\int e^{\sin^2 x}\sin(2x)\,dx$

4. a) $f(x,y) = x^2 - y^3 + 3xy$

$f'(x,y)_x = 2x + 3y$

$f'(x,y)_y = -3y^2 + 3x$

$2x + 3y = 0 \Rightarrow 2x = -3y$

$-3y^2 + 3x = 0 \qquad x = -\frac{3}{2}y$

$-3y^2 - \frac{9}{2}y = 0$

$y\left(-3y - \frac{9}{2}\right) = 0$

$y_1 = 0 \qquad 2x + 3 \cdot 0 = 0 \qquad 2x + 3 \cdot \left(-\frac{3}{2}\right) = 0$

$-3y = \frac{9}{2} \quad | \cdot \left(-\frac{1}{3}\right) \qquad 2x = 0 \qquad 2x - \frac{9}{2} = 0$

$x_1 = 0$

$y_2 = -\frac{3}{2} \qquad 2x = \frac{9}{2} \qquad A(0,0)$

$x_2 = \frac{9}{4} \qquad B\left(\frac{9}{4}, -\frac{3}{2}\right)$

$f''(x,y)_{xx} = 2$

$f''(x,y)_{xy} = 3$

$f''(x,y)_{yy} = -6y$

$f''(x,y)_{yx} = 3$

J

$\Delta = \begin{vmatrix} 2 & 3 \\ \underset{\downarrow 0}{-6y} & 3 \end{vmatrix} = 5 > 0$ min

$\Delta = \begin{vmatrix} 2 & 3 \\ \underset{\downarrow 9}{-6y} & 3 \end{vmatrix} = 5 - 27 = -22 < 0$ max

MIN

- maksimum u točki $B\left(\frac{9}{4}, -\frac{3}{2}\right)$
- minimum u točki $A(0,0)$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

3. $y = 2 - 2x - x^2 \Rightarrow y = -x^2 - 2x + 2$

$y = 3x - 3$

$3x - 3 = -x^2 - 2x + 2$

$3x - 3 + x^2 + 2x - 2 = 0$

$x^2 + 5x - 5 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-5 \pm \sqrt{25 + 20}}{2}$

$x_{1,2} = \frac{-5 \pm \sqrt{45}}{2}$

$x_{1,2} = \frac{-5 \pm 3\sqrt{5}}{2}$

?

x	0	1	-1	2	3
y	2	1	5	2	5

x	0	1	2
y	-3	0	3

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: IVAN SKOBLAR **Broj indeksa:** 56203 - 2008

Vrijeme: od ____________ **do** _________ ♣4 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$

b)

$$\int_e^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int \frac{x^3 - x + 2}{x^2 - 1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2 - 2x - x^2$ i pravac $y = 3x - 3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 - y^3 + 3xy$$

b) Odredi domenu funkcije:

$$f(x, y) = \ln(x - y) + \frac{1}{y - 1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{(x + 1)}y = 1 - x^2$$

b)

$$y'' + 8y' + 16y = \cos x$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
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$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
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♣4

1. (12 + 8)

a) $$\int e^{\sin^2 x} \sin(2x)\, dx$$

b) $$\int_e^{+\infty} \frac{dx}{x \ln^3 x}$$

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: MIRO LUKIN **Broj indeksa:** 54493 - 2007

Vrijeme: od____________ **do**_________ ♣4 **Broj bodova:** Ø

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x} \sin(2x)dx$$

b)

$$\int_e^{+\infty} \frac{dx}{x \ln^3 x}$$

2. *(15)* Integriraj

$$\int \frac{x^3 - x + 2}{x^2 - 1} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2 - 2x - x^2$ i pravac $y = 3x - 3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 - y^3 + 3xy$$

b) Odredi domenu funkcije:

$$f(x, y) = \ln(x - y) + \frac{1}{y - 1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

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$$y' - \frac{1}{(x+1)} y = 1 - x^2$$

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$$y'' + 8y' + 16y = \cos x$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
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$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \cot x dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣4

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: ANTONIO SEKULA **Broj indeksa:** 17-2-0025-2010

Vrijeme: od 08:20 **do** ________ ♣4 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int e^{\sin^2 x}\sin(2x)dx$$

b)

$$\int_{e}^{+\infty}\frac{dx}{x\ln^3 x}$$

2. *(15)* Integriraj

$$\int\frac{x^3-x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2 - 2x - x^2$ i pravac $y = 3x - 3$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - y^3 + 3xy$$

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x-y) + \frac{1}{y-1}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{(x+1)}y = 1 - x^2$$

b)

$$y'' + 8y' + 16y = \cos x$$

② $\int \frac{x^3-x+2}{x^2-1}\,dx = \int x + \frac{2}{(x^2-1)}\,dx = \int x\,dx + \int \frac{2}{x^2-1}\,dx =$

$(x^3-x+2):(x^2-1)=x$ $\qquad = \frac{x^2}{2} + 2\arctan x + C$

$\ominus\, x^3 \oplus x$

$\quad +2$

$\int x\,dx = \frac{x^2}{2} + C$

$\int \frac{2}{x^2-1}\,dx = 2\int \frac{dx}{x^2-1} = 2\cdot 1 \arctan x + C$

③

$y = 2-2x-x^2$

$y = 3x-3$

$2-2x-x^2 = 3x-3$

$-x^2-2x+2-3x+3=0$

$-x^2-5x+5=0 \;/\cdot(-1)$

$x^2+5x-5=0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-5 \pm \sqrt{5^2-4\cdot 1\cdot(-5)}}{2\cdot 1}$

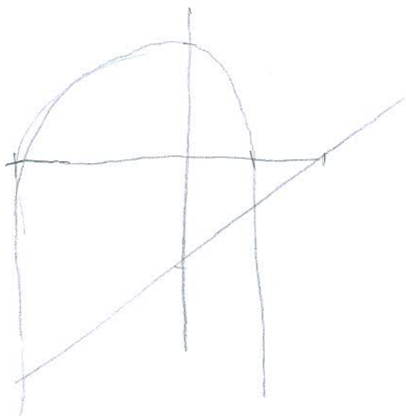
$x_{1,2} = \frac{-5 \pm \sqrt{25-20}}{2}$

$x_{1,2} = \frac{-5 \pm \sqrt{5}}{2}$

?

$x_1 = \frac{-5+\sqrt5}{2}$

$x_2 = \frac{-5-\sqrt5}{2}$



$\int 2-2x-x^2-(3x-3)\,dx = \int 2-2x-x^2-3x+3\,dx =$

$2\int_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} dx - 2\int_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} x\,dx - \int_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} x^2\,dx - 3\int_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} x\,dx + 3\int_{\frac{5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} dx =$

$x\Big|_{\frac{-5+\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} - x^2\Big|_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} - \frac{x^3}{3}\Big|_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} - 3\frac{x^2}{2}\Big|_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}} + 3x\Big|_{\frac{-5-\sqrt5}{2}}^{\frac{-5+\sqrt5}{2}}$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
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$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
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$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x\rvert + C$	$\int \cot x dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

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