

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: Ivan Kovačević Broj indeksa: 17-2-0125-2012

Vrijeme: od \_\_\_\_\_ do \_\_\_\_\_ ♣4

Broj bodova: 70

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (12+8) Integriraj

a)

$$\int e^{\sin^2 x} \sin(2x) dx$$

✓ (12)

b)

$$\int_e^{+\infty} \frac{dx}{x \ln^3 x}$$

✓ (8)

2. (15) Integriraj

$$\int \frac{x^3 - x + 2}{x^2 - 1} dx$$

✓ (15)

3. (15) Odredi površinu koju zatvaraju parabola  $y = 2 - 2x - x^2$  i pravac  $y = 3x - 3$ .

✓ (15)

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 - y^3 + 3xy$$

✓ (10)

b) Odredi domenu funkcije:

$$f(x, y) = \ln(x - y) + \frac{1}{y - 1}$$

✓ (10)

5. (15+15) Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{(x+1)}y = 1 - x^2$$

b)

$$y'' + 8y' + 16y = \cos x$$

$$1.) \quad a) \quad \int e^{\sin^2 x} \sin(2x) dx \quad \left| \begin{array}{l} \sin^2 x = t \\ 2 \sin x \cos x dx = dt \\ \sin(2x) dx = dt \end{array} \right.$$

$$\int e^t dt = e^t + c = e^{\sin^2 x} + c \quad \checkmark$$

$$b) \quad \int_e^{+\infty} \frac{dx}{x \ln^2 x} = \lim_{b \rightarrow +\infty} \int_e^b \frac{dx}{x \ln^2 x} = \left. \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \\ x = b, t = \ln b \\ x = e, t = 1 \end{array} \right| = \lim_{b \rightarrow +\infty} \int_1^{\ln b} \frac{1}{t^3} dt$$

$$= \lim_{b \rightarrow +\infty} \int_1^{\ln b} t^{-3} dt = \lim_{b \rightarrow +\infty} \left| \frac{t^{-2}}{-2} \right|_1^{\ln b} = \lim_{b \rightarrow +\infty} \left[ -\frac{1}{2 \ln^2 b} + \frac{1}{2 \cdot 1^2} \right]$$

$$= \lim_{b \rightarrow +\infty} \left[ -\frac{1}{2 \cdot \ln^2 b} + \frac{1}{2} \right] = -0 + \frac{1}{2} = \frac{1}{2} \quad \checkmark$$

$$2.) \quad \int \frac{x^3 - x + 2}{x^2 - 1} dx = \int x dx + \int \frac{2 dx}{x^2 - 1} = \int x dx + 2 \int \frac{dx}{x^2 - 1}$$

$$(x^3 - x + 2) : (x^2 - 1) = x$$

$$\frac{-(x^3 - x)}{2}$$

$$= \int x dx - 2 \int \frac{dx}{x^2 - 1}$$

$$= \frac{x^2}{2} - 2 \cdot \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + c$$

$$= \frac{x^2}{2} - \ln \left| \frac{x+1}{x-1} \right| + c \quad \checkmark$$

$$3.) \quad y = 2 - 2x - x^2$$

$$y = 3x - 3$$

x	-1	0	1	-2
y	3	2	-1	2

$$2 - 2x - x^2 = 3x - 3$$

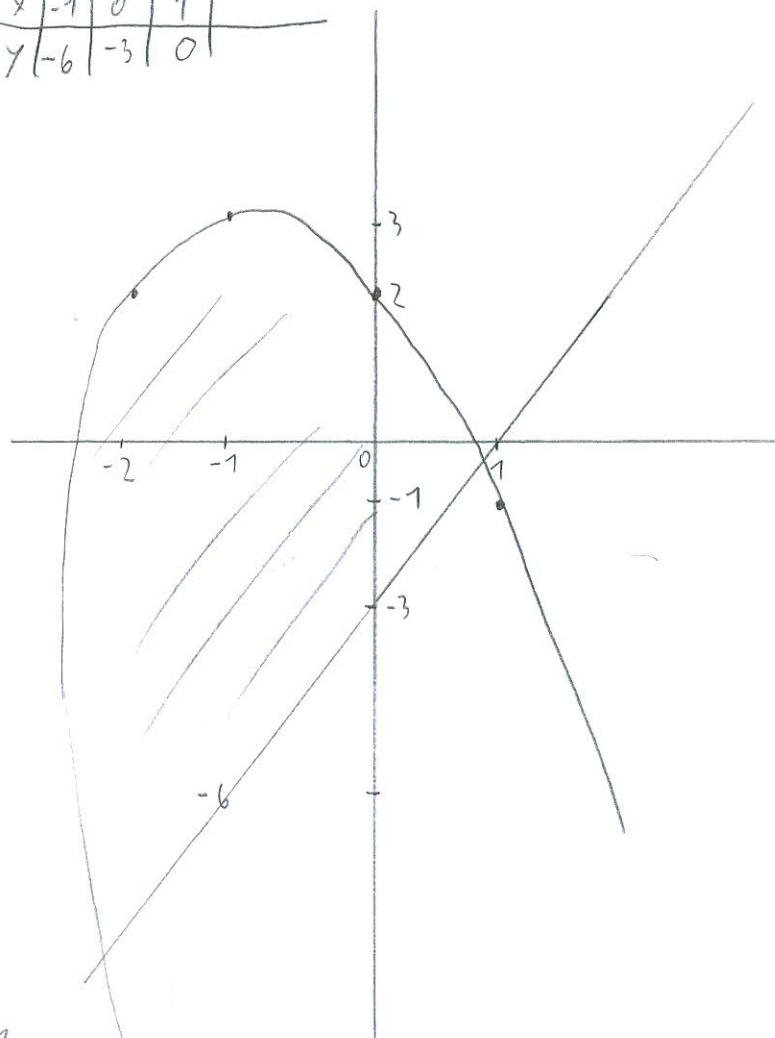
$$2 + 3 - 2x - 3x - x^2 = 0$$

$$-x^2 - 5x + 5 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

$$x_1 = -5,85 \quad x_2 = 0,25$$

x	-1	0	1
y	-6	-3	0



$$P = \int_{-5,85}^{0,25} [2 - 2x - x^2] - [3x - 3] dx =$$

$$= \int_{-5,85}^{0,25} (-x^2 - 3x + 5) dx = \left( -\frac{x^3}{3} - \frac{3x^2}{2} + 5x \right) \Big|_{-5,85}^{0,25}$$

$$P = 50,31$$

$$4.) \quad b) \quad f(x, y) = \ln(x - y) + \frac{1}{y - 1}$$

$$I \quad x - y > 0 \Rightarrow y < x$$

$$II \quad y - 1 \neq 0 \Rightarrow y \neq 1$$

$$Df = \{(x, y) : y < x, y \neq 1\}$$

✓ (10)



Tablica osnovnih derivacija

$f$	$f'$	$f$	$f'$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
$e^x$	$e^x$	$\coth x$	$\frac{1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left  \frac{x+a}{x-a} \right  + C$
$\int \frac{1}{x} dx = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln  x + \sqrt{x^2+a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

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♣3

$$4.) \quad f(x, y) = x^2 - y^3 + 3xy$$

$$x^2 - y^3 + 3xy$$

$$\frac{\partial f}{\partial x} = 2x + 3y$$

$$\frac{\partial f}{\partial y} = -3y^2 + 3x$$

$$\frac{\partial f}{\partial x \partial y} = +3$$

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = -6y$$

stacionarne točke

$$2x + 3y = 0 \quad / \cdot (-3)$$

$$-3y^2 + 3x = 0 \quad / (2)$$

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$$-6y^2 + 9x = 0$$

$$-6y^2 + 6x = 0$$


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$$-6y^2 - 9y = 0$$

$$y_1 = 0 \quad y_2 = -\frac{3}{2}$$

$$x_1 = 0 \quad x_2 = \frac{9}{4}$$

$$T_1(0, 0, ) \quad T_2(0, -\frac{2}{3}, ) \quad T_3(\frac{9}{4}, 0, ) \quad T_4(\frac{9}{4}, -\frac{3}{2}, )$$

Extremi

$$\Delta = \begin{vmatrix} 2 & +3 \\ +3 & -6y \end{vmatrix} = (2 \cdot (-6y)) - (3 \cdot 3) =$$

$$T_1) \quad 2 \cdot (-6 \cdot 0) - 3 \cdot 3 = -9 \quad T_1 \rightarrow \text{sedlasta točka}$$

$$T_2) \quad 2 \cdot (-6 \cdot -\frac{2}{3}) - 3 \cdot 3 = -17 \quad T_2 \rightarrow \text{sedlasta točka}$$

$$T_3) \quad 2 \cdot (-6 \cdot 0) - 3 \cdot 3 = -9 \quad T_3 \rightarrow \text{sedlasta točka}$$

$$T_4) \quad 2 \cdot (-6 \cdot -\frac{3}{2}) - 3 \cdot 3 = 9 \quad T_4 \rightarrow \text{minimum}$$



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29. lipnja 2013.

Ime i prezime: Luka Peros

Broj indeksa: 02184

Vrijeme: od \_\_\_\_\_ do \_\_\_\_\_ ♣4

Broj bodova:

57

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