Bobaček

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MATEO BOBAČEK **Broj indeksa:** 17-2-0113-2011

Vrijeme: od ____________ **do** _________ ♣3 **Broj bodova:** 87

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)
$$\int \frac{\sin(\ln x)}{x}\,dx$$ 12

b)
$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj
$$\int \frac{2x+3}{x^2+3x-10}\,dx$$ 15

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$. 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije
$$f(x,y) = y^2 + xy + 3y + 2x^2 + 3$$ 10

b) Odredi domenu funkcije:
$$f(x,y) = \sqrt{25 - x^2 - y^2}$$ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$y' - \frac{1}{x}\cdot y = x^2$$ 15

b)
$$y'' + 4y' + 4y = \sin x$$ 10

① a) $\int \frac{\sin(\ln x)}{x}\,dx = \left| \begin{matrix} \ln x = t \\ \frac{1}{x}\,dx = dt \\ \frac{dx}{x} = dt \end{matrix} \right| = \int \sin(t)\cdot dt = -\cos(t) = -\cos(\ln x) + C$ 12

b) $\int_0^{+\infty} \frac{dx}{1+x^2} = 1 \arctan x \Big|_0^{+\infty} = \lim_{x\to 0} \arctan x = +\infty + 0 = +\infty$

konvergira ?

(2.) $\displaystyle\int \frac{2x+3}{x^2+3x-10}\,dx = \int \frac{2x+3}{(x-2)(x+5)}\,dx \ *$

$x_{1,2} = \dfrac{-3 \pm \sqrt{9+40}}{2}$

$x_1 = 2 \quad x_2 = -5$

$\dfrac{2x+3}{(x-2)(x+5)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x+5)} \quad / \cdot (x-2)(x+5)$

$2x+3 = A(x+5) + B(x-2)$

za $x_1 = 2 \Rightarrow 7 = 7A$

$A = 1$

za $x_2 = -5 \Rightarrow -7 = -7B$

$B = -1$

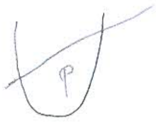
$\displaystyle * \int \frac{1}{x-2}\,dx + \int \frac{-1}{x+5}\,dx = \int \frac{1}{x-2}\,dx - \int \frac{1}{x+5}\,dx = \left| \begin{matrix} x-2=t\,/' & x+5=m\,/' \\ dx=dt & dx=dm \end{matrix} \right|$

$\displaystyle = \int \frac{dx}{t} - \int \frac{dm}{m} = \ln|t| - \ln|m| = \ln|x-2| - \ln|x+5| + C$

15

$\displaystyle = \ln\left|\frac{x-2}{x+5}\right| + C$ //

(3.) $y = x+1 \qquad y = x^2 - x - 2$



sjecišta

$x^2 - x - 2 = x + 1$

$x^2 - x - x - 2 - 1 = 0$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4+12}}{2}$

$x_1 = 3 \qquad x_2 = -1$

$\displaystyle \int_{-1}^{3} (x+1-x^2+x+2)\,dx = \int_{-1}^{3} (-x^2+2x+3)\,dx$

$\displaystyle = -\frac{x^3}{3} + \cancel{2}\frac{x^2}{\cancel{2}} + 3x \Big|_{-1}^{3}$

$= -\dfrac{3^3}{3} + 3^2 + 3\cdot 3 - \left(-\dfrac{(-1)^3}{3} + (-1)^2 + 3\cdot(-1)\right)$

$= \dfrac{32}{3}$ //

15

4. a) $f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$$\frac{\partial f}{\partial x} = y + 4x$$

$$\frac{\partial f}{\partial y} = 2y + x + 3$$

$y + 4x = 0 \Rightarrow y = -4x$

$2y + x + 3 = 0$

$2 \cdot (-4x) + x + 3 = 0$

$-8x + x + 3 = 0$

$-7x = -3$

$x = \frac{3}{7}$

$y = -4 \cdot \frac{3}{7} = -\frac{12}{7}$

$T\left(-\frac{12}{7}, \frac{3}{7}\right)$

$A = \frac{\partial^2 f}{\partial x^2} = 4$

$B = \frac{\partial^2 f}{\partial x \partial y} = 1$

$C = \frac{\partial^2 f}{\partial y^2} = 2$

$\Delta AC - B^2 = 4 \cdot 2 - 1 = 7 > 0 \quad A > 0$ minimum

$T\left(-\frac{12}{7}, \frac{3}{7}\right)$ je minimum

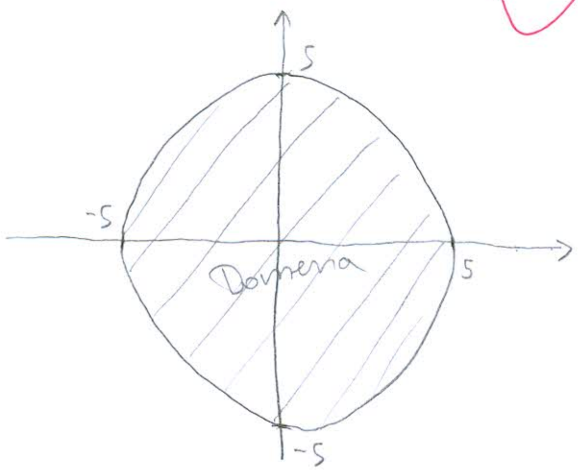
b) $f(x,y) = \sqrt{25 - x^2 - y^2}$

$25 - x^2 - y^2 \geq 0$

$-x^2 - y^2 \geq -25 \quad / \cdot (-1)$

$x^2 + y^2 \leq 25$

$S(0,0) \quad r = 5$



Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

5. a) $y' - \frac{1}{x} y = x$

$p(x) = -\frac{1}{x} \qquad g(x) = x^2$

$\int p(x)\,dx = \int -\frac{1}{x}\,dx = -\int \frac{dx}{x} = -\ln|x|$

$y = e^{\ln|x|} \cdot \left[\int e^{-\ln|x|} \cdot x^2\,dx + c\right]$

$y = x \cdot \left[\int x^{-1} \cdot x^2\,dx + c\right]$

$y = x \cdot \left[\int x\,dx + c\right]$

$y = x \cdot \left(\frac{x^2}{2} + c\right]$

$y = \frac{x^3}{2} + cx$ ✓

b) $y'' + 4y' + 4y = \sin x$

$a=0$
$b=1$
$m=0$
$\alpha \pm bi = i$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda_{1/2} = \frac{-4 \pm \sqrt{16-16}}{2}$

$\lambda_{1/2} = -2$

$y_H = e^{-2x}(C_1 + C_2 x)$

$y = A\cos x + B\sin x$

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x$

$y = -\frac{13}{16}\cos x + \frac{39}{64}\sin x$

$-A\cos x - B\sin x + 4(-A\sin x + B\cos x) + 4(A\cos x + B\sin x) = \sin x$

$-A\cos x - B\sin x - 4A\sin x + 4B\cos x + 4A\cos x + 4B\sin x = \sin x$

$(-A + 4B + 4A)\cos x + (4B - B - 4A)\sin x = \sin x$

$(4B + 3A)\cos x + (3B - 4A)\sin x = \sin x$

$3B - 4A = 1$

$-4A = (1 - 3B) / :(-4)$

$A = -\frac{1-3B}{4} \qquad A = -\frac{1 - 3\cdot\left(-\frac{3}{4}\right)}{4} = -\frac{13}{16}$

$4B + 3A = 0$

$4B = -3A / :4$

$B = -\frac{3}{4}A = -\frac{3\cdot\left(-\frac{13}{16}\right)}{4} = \frac{39}{64}$

GREŠKA U RAČUNU

$y = e^{-2x}(C_1 + C_2 x) - \frac{13}{16}\cos x + \frac{39}{64}\sin x$ ✓

10

♣3

Kraljević

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: Tomislav Kraljević **Broj indeksa:** 17-01-0052-2011

Vrijeme: od 08:15 h **do** 09:40 h ♣3 **Broj bodova:** 67

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{\sin(\ln x)}{x} dx$$

✓ 12

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj

$$\int \frac{2x+3}{x^2+3x-10} dx$$

✓ 15

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$. ✓ 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = y^2 + xy + 3y + 2x^2 + 3$$

✓ 10

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{25 - x^2 - y^2}$$

✓ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{x} \cdot y = x^2$$

5

b)

$$y'' + 4y' + 4y = \sin x$$

4 b)

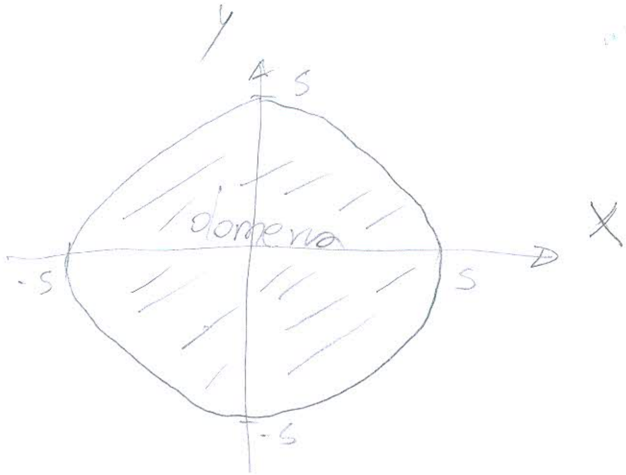
$f(x,y) = \sqrt{25-x^2-y^2}$

$25 - x^2 - y^2 \geq 0$

$-x^2 - y^2 \geq 25 \ /\cdot -1$

$x^2 + y^2 \leq 25$

$r = 5$



a) $f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$\frac{\partial f}{\partial x} = 4x + y$

$\frac{\partial^2 f}{\partial x^2} = 4$

$\frac{\partial f}{\partial y} = 2y + 3 + x$

$\frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial f}{\partial x \partial y} = (2y + x + 3) = 1$

$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$

$4x + y = 0$

$y = -4x$

$y = -4 \cdot \frac{3}{7}$

$y = \frac{31}{7}$

$2y + x + 3 = 0$

$2 \cdot (-4x) + x + 3 = 0$

$-8x + x + 3 = 0$

$-7x = -3$

$x = \frac{3}{7}$

$T = \left(\frac{3}{7}, \frac{31}{7}, 3\right)$

$\Delta = \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} = 8 - 1 = 7 > 0$ 1^0

$\frac{\partial^2 f}{\partial x^2} = 4 > 0$ T je minimum

Tomislav Kraljev

♣3

$\boxed{5}$ b) $y'' + 4y' + 4y = \sin x$

$r^2 + 4r + 4 = 0$

$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 16}}{2} = \frac{-4}{2} \Rightarrow r = -2$

$y = e^x(\cos(-2) + \sin(-2))$

$\boxed{2}$ $\int \frac{2x+3}{x^2+3x-10}\,dx$

$x_{1,2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-10)}}{2} = \frac{-3 \pm \sqrt{49}}{2} \Rightarrow x_1 = 2$

$x_2 = -5$

$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{(x-2)} + \frac{B}{(x+5)} \quad \Big| \cdot$ nazivnik

$2x + 3 = A(x+5) + B(x-2)$

$2x + 3 = Ax + 5A + Bx - 2B$

$2x + 3 = x(A+B) + (5A - 2B)$

$A + B = 2 \Rightarrow A = 2 - B$

$5A - 2B = 3 \Rightarrow -2B = 3 - 5A$

$A = 2 - 1 = 1$

$-2B = 3 - 5 \cdot (2 - B)$

$-2B = 3 - 10 + 5B$

6

$-7B = -7$

$\boxed{B = 1}$

(1)

a) $\int \frac{\sin(\ln x)}{x} dx = \begin{bmatrix} \ln x = t /' \\ \frac{1}{x} dx = dt \end{bmatrix} = \int \sin t \, dt =$

$= -\cos t = -\cos(\ln x) + C$

b) $\int_0^{+\infty} = \frac{dx}{1+x^2} = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + c \Big|_0^{\infty}$

$= \lim_{\infty \to 0} \frac{1}{2} \ln \left| \frac{1+0}{1-0} \right| + c = \frac{1}{2} \ln \left| \frac{1}{1} \right| + c$

(5)

a) $y' - \frac{1}{x} \cdot y = x^2$

$y = e^{-\int p(x)} \left(Q(x) \cdot e^{\int p(x)} dx \right)$

$\int px = \int \frac{1}{x} dx = \ln x$

$Q(x) e^{\int px} dx = \int x^2 \cdot e^{\ln(x)} = \int x^2 \cdot x = \int x^3 = \frac{x^4}{4}$

$y = e^{-\ln|x|} \left(\frac{x^4}{4} + c \right)$

$y = e^{\ln(x^{-1})} \left(\frac{x^4}{4} + c \right)$

$y = \frac{1}{x} \left(\frac{x^4}{4} + c \right)$

5

GREŠKA U RAČUNU

Tablica osnovnih derivacija

f	f'	f	f'
$x^{\alpha} (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: MARTIN SEDMAK **Broj indeksa:** 17-2-0215-2012

Vrijeme: od ____________ **do** ________ ♣3 **Broj bodova:** 62

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a) 12

$$\int \frac{\sin(\ln x)}{x}dx$$

$\int \sin(\ln x)\cdot\frac{dx}{x}$ $\left\{ \begin{array}{l} u=\ln x \\ du=\frac{1}{x}dx \end{array} \right\} =$

$= \int \sin u \, du = -\cos u = -\cos(\ln x)+C$ ✓

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj

$$\int \frac{2x+3}{x^2+3x-10}dx$$

3. *(15)* Odredi površinu koju zatvaraju pravac $y=x+1$ i parabola $y=x^2-x-2$. 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije ✓ 10

$$f(x,y)=y^2+xy+3y+2x^2+3$$

b) Odredi domenu funkcije: ✓ 10

$$f(x,y)=\sqrt{25-x^2-y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'-\frac{1}{x}\cdot y=x^2$$

b) 15

$$y''+4y'+4y=\sin x$$

$y(x) = C_1e^{-2x}+C_2x^{-2x}+\frac{3\sin x}{25}-\frac{4\cos(x)}{25}$ ✓

5 b) $y''+4y'+4y=\sin x$

$y(x)=y_H(x)+y_P(x)$

$y_H(x)=$

$\lambda^2+4\lambda+4=0$

$\lambda_{1,2}=\frac{-4\pm\sqrt{16-16}}{2}$

$\lambda_{1,2}=-2$

$y_H(x)=C_1e^{-2x}+C_2xe^{-2x}$

$y_p(x)=\frac{3\sin(x)}{25}-\frac{4\cos x}{25}$

$y_p(x)=A\sin(x)+B\cos(x)$

$y'_p(x)=A\cos(x)-B\sin(x)$

$y''_p(x)=-A\sin(x)-B\cos(x)$

$-A\sin(x)-B\cos(x)+4A\cos(x)-4B\sin(x)+4A\sin x+4B\cos(x)=\sin x$

$-A-4B+4A=1$ $\qquad -B+4A+4B=0$

$3A-4B=1$ $\qquad \cancel{3}\ 3B=-4A \quad A=-\frac{3}{4}B$

$-\frac{9}{4}B-4B=1$ $\qquad A=\frac{3}{25}$

$-\frac{25}{4}B=1 \quad B=-\frac{4}{25}$

4. b) $f(x,y) = \sqrt{25 - x^2 - y^2}$

$25 - x^2 - y^2 \geq 0$

$25 \geq x^2 + y^2 \Rightarrow$

$a^2 + b^2 = r^2$

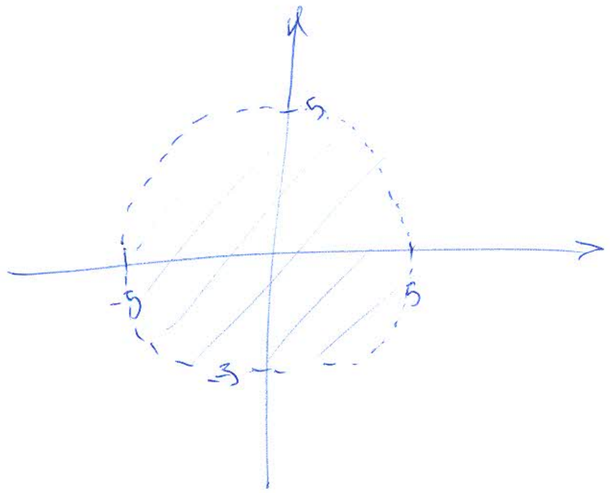
$D \in R^2 : x^2 + y^2 \leq 25$

$r^2 = 25$

$r = 5$

DOMENA JE SKUP SVIH TOČAKA UNUTAR KRUŽNICE $x^2+y^2=25$

NE UKLJUČUJUĆI I SAMU KRUŽNICU



1. b) $\int_0^{+\infty} \frac{dx}{1+x^2} = \ln\left|x + \sqrt{x^2+1}\right| \Big|_0^{+\infty}$

$\lim_{x\to 0} = \ln|\infty| - \ln|\underbrace{\sqrt{1}}_{=0}|$

$= \infty - 0$

$= \infty$

MARTIN SEDMAK

3. $y = x+1$

$y = x^2 - x - 2$

$y = y$

$x^2 - x - 2 = x + 1$

$x^2 - x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_1 = -1$

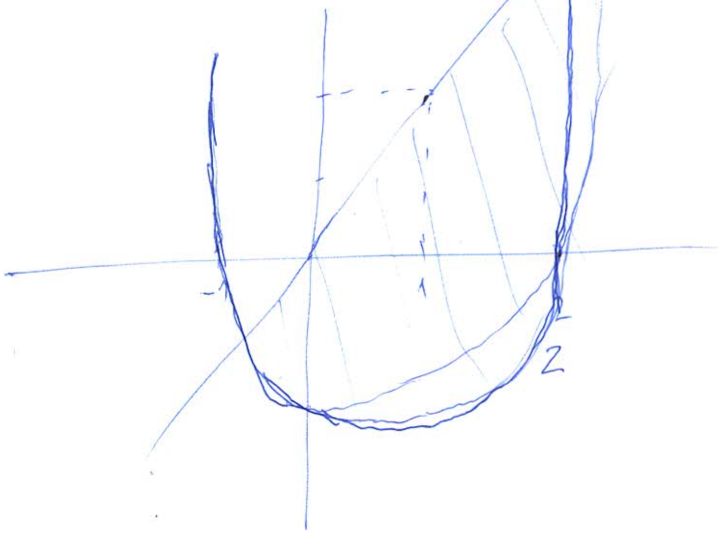
$x_2 = 3$

$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2}$

$x_1 = -1$

$x_2 = 2$

x	0	1
x-1	1	2



$$P = \int_{-1}^{3} x+1\,dx - \int_{-1}^{3} x^2 - x - 2\,dx$$

$$= \int_{-1}^{3} x\,dx + \int_{-1}^{3} dx - \int_{-1}^{3} x^2 dx + \int_{-1}^{3} x\,dx + 2\int_{-1}^{3} dx$$

$$= 2\int_{-1}^{3} x\,dx + 3\int_{-1}^{3} dx - \int_{-1}^{3} x^2 dx$$

$$= x^2\Big|_{-1}^{3} + 3x\Big|_{-1}^{3} - \frac{1}{3}x^3\Big|_{-1}^{3}$$

$$= (3)^2 - (-1)^2 + \cancel{(3\cdot(-1) - 3\cdot(3))}\ 3(3+1) - \frac{1}{3}(+1+27)$$

$$= 8 + 12 - \frac{28}{3}$$

$$= \frac{32}{3}$$

✓ 11

♣2|

4. a)

$f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$\frac{\partial}{\partial x} = 4x + y$

$\frac{\partial}{\partial y} = x + 2y + 3$

$\frac{\partial^2}{\partial x \partial y} = 1$

$4x + y = 0$

$4x = -y$

$x = \frac{-1}{4} y$

$x + 2y + 3 = 0$

$-\frac{1}{4}y + 2y = -3$

$\frac{7}{4} = -3$

$y = -\frac{12}{7}$

$x = \frac{-1}{4} \cdot \left(\frac{-12}{7}\right)$

$x = \frac{3}{7}$

$\frac{\partial}{\partial x} = 4 \cdot \frac{3}{7} + \left(-\frac{12}{7}\right)$

$= \frac{12}{7} + \left(-\frac{12}{7}\right) = 0$

$\Delta = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$\frac{\partial}{\partial y} = \frac{3}{7} + 2\left(-\frac{12}{7}\right) + 3 = -3 + 3$

$= 0$

$\Delta > 0 \wedge \frac{\partial^2}{\partial x \partial y} > 0$

TOČKA $\left(\frac{3}{7}, -\frac{12}{7}\right)$ JE MINIMUM

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MARKO FRANIĆ **Broj indeksa:** 55661

Vrijeme: od 8:16 **do** ________ ♣3 **Broj bodova:** 62

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{\sin(\ln x)}{x}dx$$

12

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj

$$\int \frac{2x+3}{x^2+3x-10}dx$$

15

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = y^2 + xy + 3y + 2x^2 + 3$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{25 - x^2 - y^2}$$

5

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{x}\cdot y = x^2$$

15

b)

$$y'' + 4y' + 4y = \sin x$$

15

a)

1) $\int \frac{\sin(\ln x)}{x}dx = \left| \begin{matrix} u = \log(x) \\ du = \frac{1}{x} \end{matrix} \right| = \int \sin(u)\,du = -\cos(u) + KONST.$

$\ln x$

$= -\cos(\log(x)) + KONST.$

12

b) $\int_0^{-\infty} \frac{dx}{1+x^2} = \int \frac{1}{1+x^2}dx = \tan^{-1}(x)$

2) $\int \frac{2x+3}{x^2+3x-10}dx = \frac{3+2x}{-10+3x+x^2} \left| \begin{matrix} u = -10+3x+x^2 \\ du = (3+2x)dx \end{matrix} \right| = \int \frac{1}{u}du = \log(u) =$

$= \log(-10+3x+x^2) + KONSTANTA$

Ln je oznaka za prirodni logaritam!!

4\. b) $f(x,y)=\sqrt{25-x^2-y^2}=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq 25\}$

5 A SKICA ??

5\. a) $y'-\frac{1}{x}y=x^2 \;/\cdot \mu(x)=(e^{\int -\frac{1}{x}dx}=\frac{1}{x})$

$$\frac{\frac{dy(x)}{dx}}{x}-\frac{y(x)}{x^2}=x$$

$$-\frac{1}{x^2}=\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\frac{\frac{dy(x)}{dx}}{x}+\frac{d}{dx}\left(\frac{1}{x}\right)y(x)=x$$

$$g\frac{df}{dx}+f\frac{dg}{dx}=\frac{d}{dx}(f\cdot g)$$

$$\frac{d}{dx}\left(\frac{y(x)}{x}\right)=x$$

$$\int\frac{d}{dx}\left(\frac{y(x)}{x}\right)dx=\int x\,dx$$

$$\frac{y(x)}{x}=\frac{x^2}{2}+c_1 \text{ - KONST.} \;/\; \mu(x)=\frac{1}{x}$$

$$y(x)=x\left(\frac{x^2}{2}+c_1\right)$$

$$y(x)=c_1x+\frac{x^3}{2}$$

b) $y''+4y'+4y=\sin x$

$$\frac{d^2(y(x))}{dx^2}+4\frac{dy(x)}{dx}+4y(x)=0 \;/\; y(x)=e^{\lambda x}$$

$$y(x)=c_1e^{-2x}+c_2e^{-2x}x+\frac{3\sin(x)}{25}-\frac{4\cos(x)}{25}$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: LUKA BORZIĆ **Broj indeksa:** 17-2-0016-2010

Vrijeme: od____________ **do**________ ♣3 **Broj bodova:** 57

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{\sin(\ln x)}{x}dx$$ 12

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj
$$\int \frac{2x+3}{x^2+3x-10}dx$$ 15

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$. 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije
$$f(x, y) = y^2 + xy + 3y + 2x^2 + 3$$

b) Odredi domenu funkcije:
$$f(x, y) = \sqrt{25 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$y' - \frac{1}{x} \cdot y = x^2$$ 15

b)
$$y'' + 4y' + 4y = \sin x$$

4. a) $f(x,y)=y^2+xy+3y+2x^2+3$

$\frac{\partial f}{\partial x} = 0+1+0+4x+0 = 1+4x$ $\quad 1+4x=0$, $4x=-1 /:4$, $x=-\frac{1}{4}$

$\frac{\partial f}{\partial y} = 2y+1+3 = 2y+4$ $\quad 2y+4=0$, $2y=-4/:2$, $y=-\frac{1}{2}$

$T(-\frac{1}{4}, -\frac{1}{2})$

$A=\frac{\partial f}{\partial x^2} = 0+1+0+4x+0 = 4$ $\quad H=AC-B^2=8-0=8$

$B=\frac{\partial f}{\partial x \partial y} = 0+1+0+4x+0=0$

$C=\frac{\partial f}{\partial y^2} = 2y+1+3+0+0=2$

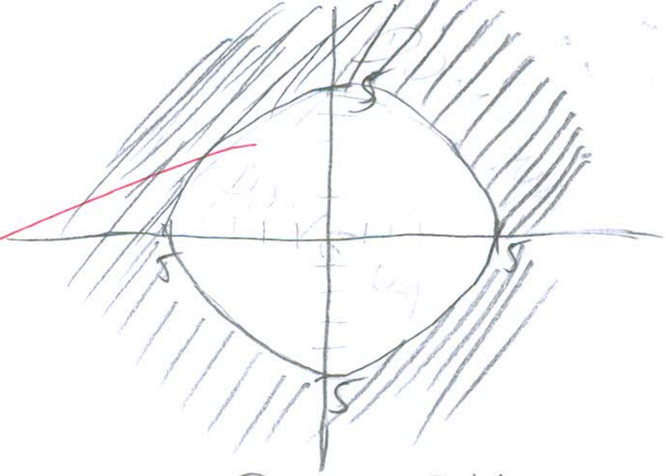
MINIMUM

b) $f(x,y) = \sqrt{\underbrace{25-x^2-y^2}_{\geq 0}} =$

$25-x^2-y^2 \geq 0$

$-x^2-y^2 \geq -25$

$x^2+y^2 = r^2$



DOMENA

1. a) $\int \frac{\sin(\ln x)}{x}\,dx = \left| \begin{matrix} u = \ln x \\ du = \frac{1}{x}dx \end{matrix} \right|$

$= \int \sin u \, du = -\cos(u) + C =$

$= -\cos(\ln(x)) + C$ ✓ 12

5. a) $y' - \frac{1}{x} \cdot y = x^2$

$y(x) = \frac{1}{e^{\int(-\frac{1}{x})dx}} \cdot \{ x^2 \cdot e^{\int(-\frac{1}{x})dx} + C_1$

$y(x) = \left[\frac{1}{e^{(\ln(\frac{1}{x}))}} \cdot \int x^2 \cdot e^{\ln(\frac{1}{x})}dx + C_1\right]$

$y(x) = x\left[\int x^2 \cdot \frac{1}{x}\,dx) + C_1\right]$

$y(x) = x \cdot \left[(\frac{x^2}{2}) + C_1\right]$

$y(x) = \frac{x^3}{2} + C_1 \cdot x$ ✓

LUKA BORZIĆ

Tablica osnovnih derivacija

f	f'
$x^{\alpha} (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

1. b) $\int_0^{+\infty} \frac{dx}{1+x^2} = \int_0^{+\infty} \frac{dx}{(1+x^2)} = \frac{1}{1} \text{arctan}\left(\frac{x}{1}\right)\Big|_0^{+\infty} = \text{arctan}(x)\Big|_0^{+\infty} =$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2} + C$

3. $y = x+1$

$y = x^2 - x - 2$

$x + 1 = 0$

$x^2 - x - 2 = 0$

$x + 1 = x^2 - x - 2$

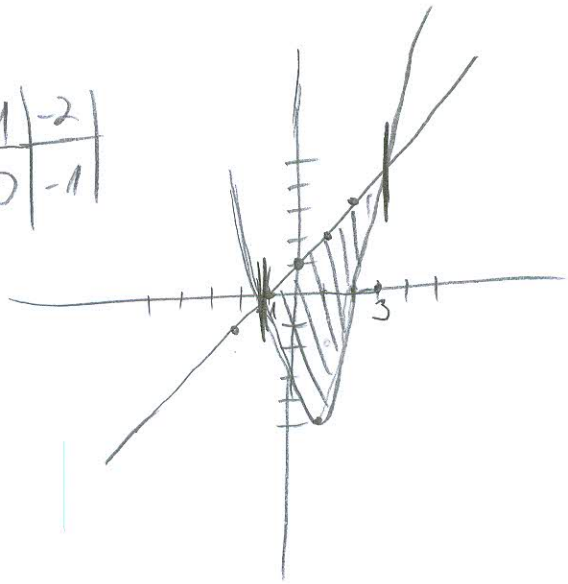
$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} =$

$x_1 = \frac{2-4}{2} = -1$

$x_2 = \frac{2+4}{2} = 3$

x	1	2	0	-1	-2
y	2	3	1	0	-1



DRUGA STRANA

$$\int_{-1}^{3}(x+1)\,dx=\int_{-1}^{3}x\,dx+\int_{-1}^{3}dx=$$

$$=\left(\frac{x^2}{2}\right)\Big|_{-1}^{3}+x\Big|_{-1}^{3}=\frac{3^2}{2}-\frac{1}{2}+3+1=8$$

$$\int_{-1}^{3}x^2-x-2\,dx=\int_{-1}^{3}(x^2)\,dx-\int_{-1}^{3}x\,dx-2\int_{-1}^{3}dx$$

$$=\frac{x^3}{3}\Big|_{-1}^{3}-\frac{x^2}{2}\Big|_{-1}^{3}-2x\Big|_{-1}^{3}=-8/3$$

$$P=8-(-8/3)=32/3$$

10

2. $$\int\frac{2x+3}{x^2+3x-10}\,dx=\left|\begin{array}{l}u=x^2+3x-10\\ du=(2x+3)\,dx\end{array}\right|=$$

$$\int\frac{du}{u}=\ln|u|=\ln|x^2+3x-10|+C$$

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: ADRIANO VIPOTNIK **Broj indeksa:** 17-2-0138-2011

Vrijeme: od 08:00 **do** ________ ♣3 **Broj bodova:** 42

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{\sin(\ln x)}{x} dx$$ 12

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj

$$\int \frac{2x+3}{x^2+3x-10} dx$$ 15

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = y^2 + xy + 3y + 2x^2 + 3$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{25 - x^2 - y^2}$$ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{x} \cdot y = x^2$$

b)

$$y'' + 4y' + 4y = \sin x$$ 5

3.) $Y = X + 1$

$Y = x^2 - x - 2$

$Y = Y$

$x + 1 = x^2 - x - 2$

$-x^2 + x + x + 2 + 1 = 0$

$-x^2 + 2x + 3 = 0 \quad / \cdot (-1)$

$x^2 - 2x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$x_1 = -1 \qquad x_2 = 3$

$y_1 = x + 1 \qquad y_2 = x + 1$

$y_1 = 0 \qquad y_2 = 4$

$S_1(-1, 0)$

$S_2(3, 4)$

a) $Y = x + 1$

X	0	1
Y=x+1	1	2

b) $Y = x^2 - x - 2$

$a = 1 > 0 \quad \cup$

$$T\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$$

$$T\left(\frac{1}{2}, \frac{-8 - 1}{4}\right)$$

$$T\left(\frac{1}{2}, \frac{-9}{4}\right)$$

$x^2 - x - 2 = 0$

$$x = \frac{1 \pm \sqrt{1 + 8}}{2}$$

$$x = \frac{1 \pm 3}{2}$$

$x_1 = -1 \qquad x_2 = 2$

$y_1 = x^2 - x - 2$

$y_1 = 1 + 1 - 2$

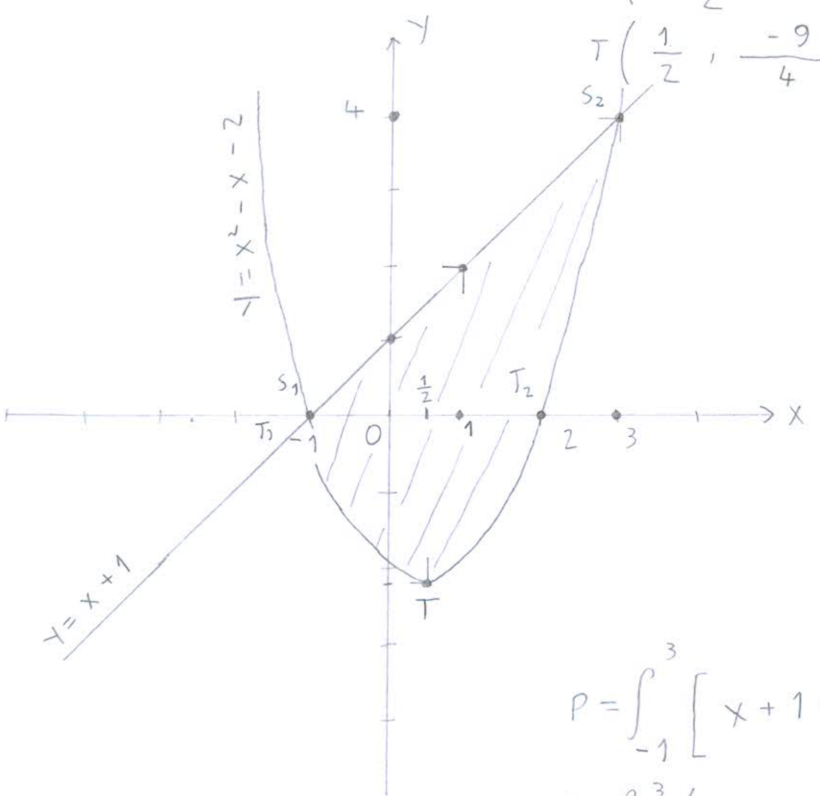
$y_1 = 0$

$T_1(-1, 0)$

$y_2 = 4 - 2 - 2$

$y_2 = 0$

$T_2(2, 0)$



$$P = \int_{-1}^{3} \left[x + 1 - \left(x^2 - x - 2\right) \right] dx$$

$$P = \int_{-1}^{3} \underbrace{\left(-x^2 + 2x + 3\right)}_{*} dx$$

$$* \int \left(-x^2 + 2x + 3\right) dx = -\int x^2 dx + 2\int x\, dx + 3\int dx = -\frac{x^3}{3} + 2\frac{x^2}{2} + x$$

$$= -\frac{x^3}{3} + x^2 + x$$

$$P = \left(-\frac{x^3}{3} + x^2 + x\right)\Bigg|_{-1}^{3}$$

$$P = -\cancel{9} + \cancel{9} + 3 - \left(+\frac{1}{3} + 1 - 1\right) = 3 - \frac{1}{3} - \cancel{1} + \cancel{1} = 3 - \frac{1}{3} = \frac{9 - 1}{3} = \frac{8}{3}$$

♣1

4.) a) $f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$f(x,y) = 2x^2 + y^2 + 3y + xy + 3$

$\partial_x f = 4x + y$ $\qquad$ $\partial_y f = 2y + x$

$\partial_{xx} f = 4$ $\qquad$ $\partial_{yy} f = 2$

$\partial_{xy} f = 1$ $\qquad$ $\partial_{yx} f = 1$

$A = \partial_{xx} f = 4 \qquad A > 0$

$$\begin{aligned} \partial_x f &= 0 \\ \partial_y f &= 0 \\ \hline 4x + y &= 0 \\ 2y + x &= 0 \end{aligned}$$

$$\begin{aligned} 4x + y &= 0 \\ x + 2y &= 0 \,/\cdot(-4) \\ \hline \cancel{4x} + y &= 0 \\ -\cancel{4x} - 8y &= 0 \\ \hline -7y &= 0 \,/ : (-7) \\ y &= 0 \end{aligned}$$

$$\begin{aligned} x + 2y &= 0 \\ x &= 0 \end{aligned}$$

$T(0,0)$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix} = 4 - 2 = 2 \qquad \Delta = 2 \qquad \Delta > 0$$

$A > 0,\ \Delta > 0 \Rightarrow$ minimum

$T(0,0)$ $f(x,y) = 2x^2 + y^2 + 3y + x \cdot y + 3$

$= 0 + 0 + 0 + 0 + 3$

$= 3$

$f(0,0)_{min} = 3$

funkcija ima minimum u točki 3

b) $f(x,y) = \sqrt{25 - x^2 - y^2}$

$25 - x^2 - y^2 \geq 0$

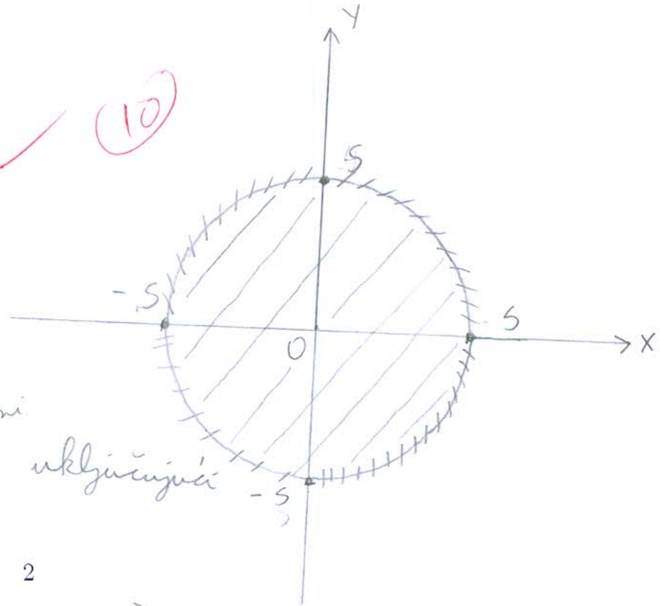
$-x^2 - y^2 = -25 \,/\cdot(-1)$

$x^2 + y^2 = 25$

funkcija za kružnicu radijusa 5.

$r = 5$

10



Domena funkcije su svi uređeni parovi (x,y) unutar kružnice i uključujući one na obodu kružnice.

$D f(x,y) = \{ (x,y) \in \mathbb{R};\ x = 5,\ y = 5 \}$

1.) a) $\int \frac{\sin(\ln x)}{x}\,dx = \begin{bmatrix} \ln x = t \\ \frac{1}{x}\,dx = dt \end{bmatrix} = \int \sin t\,dt$

$= -\cos t + C = -\cos(\ln x) + C$

b) $\int_0^{+\infty} \frac{dx}{1+x^2}$

2.) $\int \frac{2x+3}{x^2+3x-10}\,dx = \int \frac{2x}{x^2+3x-10}\,dx + \int \frac{3\,dx}{x^2+3x-10}$

$= \underbrace{2\int \frac{x}{x^2+3x-10}\,dx}_{A} + \underbrace{3\int \frac{dx}{x^2+3x-10}}_{B}$

krivo

B) $3\int \frac{dx}{x^2+3x-10} = 3\ln|x^2+3x-10| + C$

A) $2\int \frac{x}{x^2+3x-10}\,dx = \begin{bmatrix} x^2+3x-10 = t \\ (2x+3)\,dx = dt \\ x = \frac{1}{2}(x+3) - \frac{1}{2} \end{bmatrix} = 2\int \frac{\frac{1}{2}(x+3)-\frac{1}{2}}{t}\,dt$

$= \underbrace{\int \frac{x+3}{t}\,dt}_{C} - \underbrace{\int \frac{dt}{t}}_{D}$

$D = -\int \frac{dt}{t} = -\ln|t| + C = -\ln|x^2+3x-10| + C$

$C = \int \frac{x+3}{t}\,dt =$

2.) $\int \frac{2x+3}{x^2+3x-10}\,dx = \begin{bmatrix} t = x^2+3x-10 \\ dt = (2x+3)\,dx \end{bmatrix} = \int \frac{dt}{t}$

$= \ln|t| + C$

$= \ln|x^2+3x-10| + C$

točno

♣1

5.) a) $y' - \frac{1}{x} \cdot y = x^2$

$y = y_H + Y$

$y_H = C_1 \cdot e^{\frac{1}{x} \cdot x}$

$\lambda - \frac{1}{x} \cdot 1 = 0$

$\lambda - \frac{1}{x} = 0$

$\lambda_1 = \frac{1}{x}$

$r = 0 \quad n = 2 \qquad Y = r + 2$

$Y = 2$ – polinom drugog stupnja

$Y = a_2 x^2 + a_1 x + a_0$

$Y' = 2 a_2 x + a_1$

$Y'' = 2 a_2$

$2 a_2 x + a_1 - \frac{1}{x}\left(a_2 x^2 + a_1 x + a_0\right) = x^2$

$\cancel{2 a_2 x} + \cancel{a_1} - a_2 x - \cancel{a_1} - \frac{a_0}{x} = x^2$

$a_2 x - \frac{a_0}{x} = x^2$

b) $y'' + 4y' + 4y = \sin x$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda = \frac{-4 \pm \sqrt{16 - 16}}{2}$

$\lambda = \frac{-4 \pm 0}{2}$

$x = x_1 = x_2 = -2$

$y_H = e^{-2x}\left(C_1 + C_2 x\right)$

5

Tablica osnovnih derivacija

f	f'	f	f'
$x^{\alpha}(\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣3

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MLADEN BULIĆ **Broj indeksa:** 17-1-0018-2010

Vrijeme: od 8^{15} **do** ________ ♣3 **Broj bodova:** 10

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)
$$\int \frac{\sin(\ln x)}{x} dx$$

b)
$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj
$$\int \frac{2x+3}{x^2+3x-10} dx$$

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije
$$f(x, y) = y^2 + xy + 3y + 2x^2 + 3$$

~~b)~~ Odredi domenu funkcije:
$$f(x, y) = \sqrt{25 - x^2 - y^2}$$ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$y' - \frac{1}{x} \cdot y = x^2$$

b)
$$y'' + 4y' + 4y = \sin x$$

1. a) $\int \frac{\sin(\ln x)}{x} dx = \int \frac{\sin}{x} + \int \frac{\ln x}{x'} = \ln(\sin x) + \frac{1}{x} = \cot x dx + \frac{1}{x} + C$

b) $\int_0^{+\infty} \frac{dx}{1+x^2} = \int_0^{+\infty} \frac{1}{1} \arctan \frac{x}{1} + C = \arctan \frac{\infty}{1} - \arctan \frac{0}{1} = \arctan 0 - \arctan 0 = 0$

2. $\int \frac{2x+3}{x^2+3x-10}$

$x^2+3x-10=0$

$x_{1,2}=\frac{-3\pm\sqrt{9-4\cdot1\cdot(-10)}}{2\cdot1}=\frac{-3\pm\sqrt{49}}{2}=\frac{-3\pm7}{2}\Rightarrow x_1=2,\ x_2=-5$

$\frac{2x+3}{(x-2)\cdot(x+5)}=\frac{A}{(x-2)}-\frac{B}{(x+5)}\quad /\cdot(x-2)\cdot(x+5)$

$2x+3=A(x+5)-B(x-2)$

$2x+3=5A+Ax-Bx+2B$

$2x+3=x(A-B)+5A+2B$

$2=A-B\Rightarrow A=B+2$

$3=5A+2B\Rightarrow$

$5=6A+B$

$B=5-6A$

3. $y=x+1$

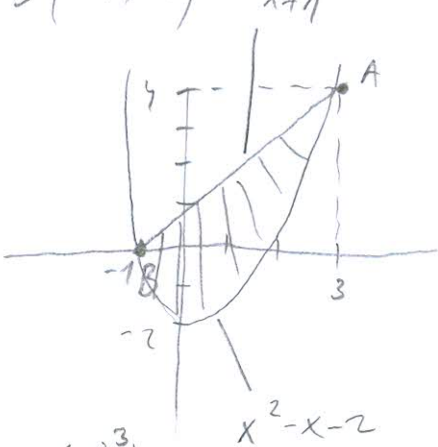
$y=x^2-x-2$

x^2-2x-3

$A=\mathrm{I}=x+1=3+1=4$

$B=\mathrm{II}=x+1=-1+1=0$

$A(3,4),\ B(-1,0)$



$x_{1,2}=\frac{+2\pm\sqrt{4-4\cdot1\cdot(-3)}}{2\cdot1}=\frac{2\pm4}{2}\Rightarrow x_1=3,\ x_2=-1$

$P=\int_{-1}^{3}2x+3-x^2=2\int_{-1}^{3}\frac{x^2}{2}+3\int_{-1}^{3}x-\int_{-1}^{3}\frac{x^3}{3}$

$=2\cdot\frac{3^2}{2}+3\cdot3-\frac{3^3}{3}-\left(2\cdot\frac{(-1)^2}{2}+3\cdot(-1)-\frac{(-1)^3}{3}\right)$

$P=9+9-9-\left(1-3+\frac{1}{3}\right)=\frac{16}{3}\approx5{,}33$

MLADEN BOŽIĆ

4. a) $f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$Zx = 4x + y \qquad Zy = 2y + 3 + x$

$Zxx = 4 \qquad Zyy = 2 \qquad Zxy = 1$

$A \cdot C - B^2 = 4 \cdot 1 - 4 = 0$

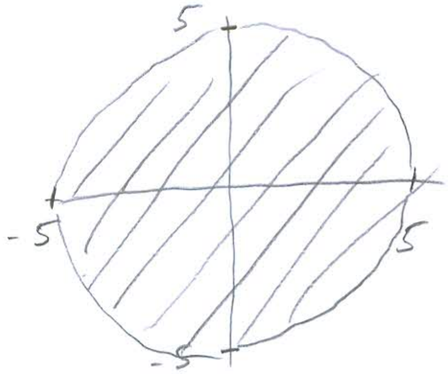
$A = 4 > 0 \rightarrow$ minimum

b) $f(x,y) = \sqrt{25 - x^2 - y^2}$

$25 - x^2 - y^2 \geq 0$

$-x^2 - y^2 \leq 25 \Rightarrow$ kružnica

$Df = [-5, 5]$



10

5. a) $y' - \frac{1}{x} \cdot y = x^2$

$f(x)dx = -\frac{1}{x}$

$g(x) = x^2$

$-\int f(x)(dx) = -\frac{1}{x} = \ln x^{-1}$

$\int fxdx = \ln x$

$e^{\ln x^{-1}} \cdot \left[\int e^{\ln x} \cdot x^2 + C\right]$

$\int e^{\ln x} \cdot x^2 = \left| \begin{matrix} u = x^2 & dv = \int e^{\ln x} \\ du = 2x & v = x \end{matrix} \right|$

$x^2 \cdot x - \int x \cdot 2x \quad \left| \begin{matrix} u = x & dv = \int 2x \\ du = dx & v = x^2 \end{matrix} \right.$

$x^2 \cdot x - x^3 - \int x^2 dx = \frac{x^3}{3}$ //

$e^{\ln x^{-1}} \cdot \left[\frac{x^3}{3} + C\right]$

b) $y'' + 4y' + 4y = \sin x$

$\lambda^2 + 4\lambda + 4$

$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = -2$

OPĆE RJEŠENJE

$C_1 e^{-2x} - C_2 x e^{-2x}$

Tablica osnovnih derivacija

f	f'
$x^{\alpha} (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: DAMIR MARINKOVSKI **Broj indeksa:** 52950

Vrijeme: od ____________ **do** _________ **♣3** **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)
$$\int \frac{\sin(\ln x)}{x} dx$$

b)
$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj
$$\int \frac{2x+3}{x^2+3x-10} dx$$

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije
$$f(x, y) = y^2 + xy + 3y + 2x^2 + 3$$

b) Odredi domenu funkcije:
$$f(x, y) = \sqrt{25 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$y' - \frac{1}{x} \cdot y = x^2$$

b)
$$y'' + 4y' + 4y = \sin x$$

1) a) $\int \frac{\sin(\ln x)}{x} dx = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \\ \sin x dv = dx \\ dv = \int \sin x dx \\ v = \cos x \end{array} \right| = I = u \cdot v - \int v \cdot du =$

$= \ln x \cdot \cos x - \int \cos x \frac{1}{x} dx$

$= \ln x \cdot \cos x - \int \cos x dx \int \frac{1}{x} dx$

$= \ln x \cdot \cos x - \sin x \cdot \ln|x| + C$

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2} = \left| \begin{array}{l} 1+x^2 = t \\ 2x\,dx = dt \\ dx = \frac{1}{2}dt \end{array} \right| \Rightarrow I = \int_0^{+\infty}$$

$(1+x)(1-x) =$

2)

a)

$$\int \frac{2x+3}{x^2+3x-10}\,dx =$$

$$\begin{array}{l} (x^2+3x-10) / \cdot 2 \\ \hline 2x^2+6x-20 \\ \cancel{x^2} \\ 2x+3 \\ \hline 4x-23 \end{array}$$

$$I = \int \frac{4x-23}{x^2+3x-10}\,dx = \int dx + \int \frac{4x-23}{x^2+3x-10}\,dx = x /$$

$$4x-23 = A(x+1) + B(x+3x-10) =$$

$$\frac{4x-23}{x^2+3x-10} = \frac{4x-23}{(x+3x-10)(x+1)} = \frac{A}{(x+3x-10)} + \frac{B}{(x+1)}$$

$$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} =$$

$$= \frac{-6 \pm \sqrt{6^2-4\cdot 2\cdot 20}}{2\cdot 2}$$

$$= \frac{-6 \pm \sqrt{36-160}}{4} = \frac{-6 \pm \sqrt{-124}}{4} = \frac{-6 \pm 11}{4}$$

$$x_1 = \frac{-6-11}{4} = \frac{-17}{4} \qquad x_2 = \frac{-6+11}{4} = \frac{5}{4}$$

3)

$y = x^2 - x - 2$; PRAVAC $y = x+1$

$$x_0 = \frac{-b}{2a} = \boxed{\frac{1}{2}}$$

$$y = \frac{4ac-b^2}{4a} = \frac{4\cdot 1\cdot(-2)-(-1)^2}{4\cdot 1} = \frac{-8+1}{4} = \boxed{-\frac{7}{4}}$$

$$\begin{array}{l} y = x^2-x-2 = 0 \\ y = x+1 \quad = 0 \\ \hline \end{array}$$

$x^2-x-2 = x+1$

$x^2-x-2-x-1 = 0$

$\underline{x^2-2x-3}$

$$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x_{12} = \frac{-2 \pm \sqrt{(-2)^2-4\cdot 1\cdot(-3)}}{2\cdot 1}$$

$$x_{12} = \frac{-2 \pm \sqrt{4+12}}{2}$$

$$x_{12} = \frac{-2 \pm \sqrt{16}}{2}$$

$$x_{12} = \frac{-2 \pm 4}{2}$$

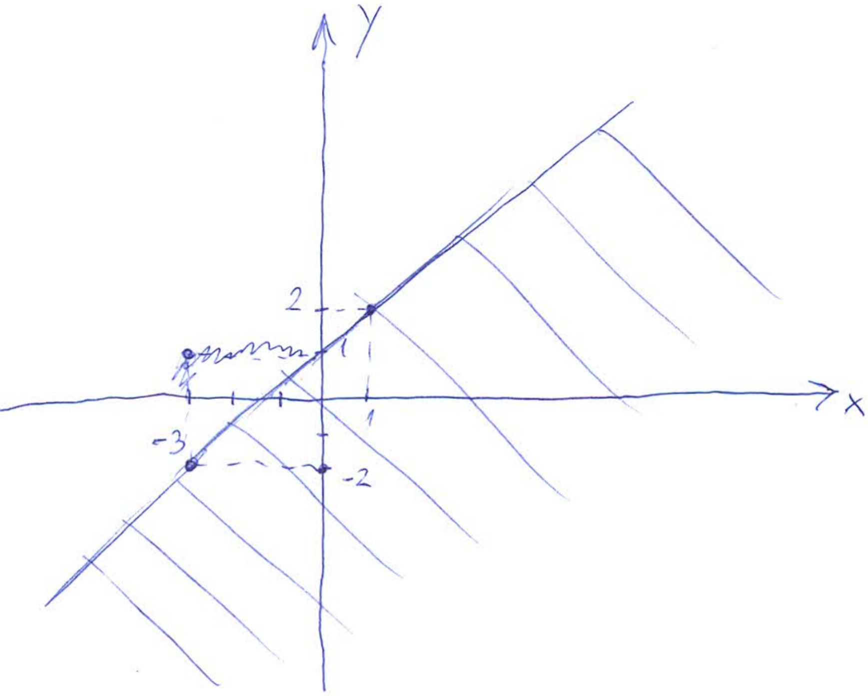
$$x_1 = \frac{-2-4}{2} = -\frac{6}{2} = \boxed{-3} \qquad x_2 = \frac{-2+4}{2} = \frac{2}{2} = \boxed{1}$$

$$y_1 = -3+1 = \boxed{-2} \qquad y_2 = 1+1 = \boxed{2} \qquad S_1(-3,-2) \quad S_2(1,2)$$

(4) b) $f(x,y)=\sqrt{25-x^2-y^2}$

♣3

DAMIR MARINKOWSKI



$$P = \int (x+1)\,dx - \int (x^2 - x - 2)\,dx$$

$=$

(4) a) $f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$\frac{\partial f}{\partial x} = 1 + 4x$ $\qquad$ $\frac{\partial f}{\partial y} = 2y + 1 + 3$

$1 + 4x = 0$
$2y + 1 + 3 = 0$

$1 + 4x = 0$
$2y + 4 = 0$

$1 + 4x - 2y - 4 = 0$

$\boxed{4x - 2y - 3}$

$\frac{\partial^2 f}{\partial x^2} = 1 + 4x = 4 \Rightarrow A$

$\frac{\partial^2 f}{\partial y^2} = 2y + 4 = 2 \Rightarrow C$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}($

$$\Delta \begin{vmatrix} A & B \\ B & C \end{vmatrix} = \begin{vmatrix} 4 & \\ & 2 \end{vmatrix} =$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: ANTE TROSKOT **Broj indeksa:** 17 1- 0002 - 2010

Vrijeme: od_____________ **do**_________ ♣**3** **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{\sin(\ln x)}{x} dx$$

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. *(15)* Integriraj

$$\int \frac{2x+3}{x^2+3x-10} dx$$

3. *(15)* Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

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b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{25 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{x} \cdot y = x^2$$

b)

$$y'' + 4y' + 4y = \sin x$$

$$\int \frac{\sin(\ln x)}{x} dx = \cancel{\int \frac{\sin(\ln x)\, dx}{x}} \quad -\cos(\ln x)$$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3