

Bobaček

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MATEO BOBAČEK Broj indeksa: 17-2-0113-2011

Vrijeme: od _____ do _____ ♣3

Broj bodova: 87

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (12+8) Integriraj

a)

$$\int \frac{\sin(\ln x)}{x} dx$$

12

b)

$$\int_0^{+\infty} \frac{dx}{1+x^2}$$

2. (15) Integriraj

$$\int \frac{2x+3}{x^2+3x-10} dx$$

15

3. (15) Odredi površinu koju zatvaraju pravac $y = x + 1$ i parabola $y = x^2 - x - 2$.

15

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = y^2 + xy + 3y + 2x^2 + 3$$

10

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{25 - x^2 - y^2}$$

10

5. (15+15) Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' - \frac{1}{x} \cdot y = x^2$$

15

b)

$$y'' + 4y' + 4y = \sin x$$

10

① a) $\int \frac{\sin(\ln x)}{x} dx = \left| \begin{matrix} \ln x = t \\ \frac{1}{x} dx = dt \\ \frac{dx}{x} = dt \end{matrix} \right| = \int \sin(t) \cdot dt = -\cos(t) = -\cos(\ln x) + C =$ 12

b) $\int_0^{+\infty} \frac{dx}{1+x^2} = 1 \arctan x \Big|_0^{+\infty} = \lim_{x \rightarrow \infty} \arctan x = +\infty + 0 = +\infty$

konvergira ?

② $\int \frac{2x+3}{x^2+3x-10} dx = \int \frac{2x+3}{(x-2)(x+5)} dx$ *

$x_{1,2} = \frac{-3 \pm \sqrt{9+40}}{2}$

$x_1 = 2 \quad x_2 = -5$

$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} \quad | \cdot (x-2)(x+5)$

$2x+3 = A(x+5) + B(x-2)$

za $x_1 = 2 \Rightarrow 7 = 7A$

$A = 1$

za $x_2 = -5 \Rightarrow -7 = -7B$

$B = -1$

* $\int \frac{1}{x-2} dx + \int \frac{-1}{x+5} dx = \int \frac{1}{x-2} dx - \int \frac{1}{x+5} dx = \left| \begin{matrix} x-2=t & x+5=m \\ dx=dt & dx=dm \end{matrix} \right|$

$= \int \frac{dx}{t} - \int \frac{dm}{m} = \ln|t| - \ln|m| = \ln|x-2| - \ln|x+5| + C$

$= \ln \left| \frac{x-2}{x+5} \right| + C$

✓ 15

③ $y = x+1 \quad y = x^2 - x - 2$

sjecišta

$x^2 - x - 2 = x + 1$

$x^2 - x - x - 2 - 1 = 0$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_1 = 3 \quad x_2 = -1$

✓ P

$\int_{-1}^3 (x+1 - x^2 + x + 2) dx = \int_{-1}^3 (-x^2 + 2x + 3) dx$

$= -\frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 3x \Big|_{-1}^3$

$= -\frac{3^3}{3} + 3^2 + 3 \cdot 3 - \left(-\frac{(-1)^3}{3} + (-1)^2 + 3 \cdot (-1) \right)$

$= \frac{32}{3}$

✓ 15

$$(4) a) f(x, y) = y^2 + xy + 3y + 2x + 3$$

$$\frac{\partial f}{\partial x} = y + 4x$$

$$\frac{\partial f}{\partial y} = 2y + x + 3$$

$$y + 4x = 0 \Rightarrow y = -4x$$

$$2y + x + 3 = 0$$

$$2 \cdot (-4x) + x + 3 = 0$$

$$-8x + x + 3 = 0$$

$$-7x = -3$$

$$x = \frac{3}{7}$$

$$T\left(-\frac{12}{7}, \frac{3}{7}\right)$$

$$A = \frac{\partial^2 f}{\partial x^2} = 4$$

$$B = \frac{\partial^2 f}{\partial x \partial y} = 1$$

$$C = \frac{\partial^2 f}{\partial y^2} = 2$$

$$\Delta AC - B^2 = 4 \cdot 2 - 1 = 7 > 0 \quad A > 0$$

minimum

$$T\left(-\frac{12}{7}, \frac{3}{7}\right) \text{ je minimum}$$

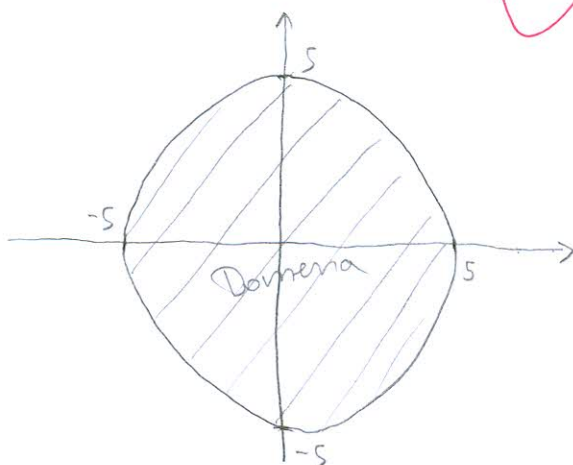
$$b) f(x, y) = \sqrt{25 - x^2 - y^2}$$

$$25 - x^2 - y^2 \geq 0$$

$$-x^2 - y^2 \geq -25 \quad | \cdot (-1)$$

$$x^2 + y^2 \leq 25$$

$$S(0, 0) \quad r = 5$$



Tablica osnovnih derivacija

| f | f' | f | f' |
|----------------------------|-----------------------|----------------|--------------------------|
| $x^\alpha (\alpha \neq 0)$ | $\alpha x^{\alpha-1}$ | $\cosh x$ | $\sinh x$ |
| $\ln x$ | $\frac{1}{x}$ | $\tanh x$ | $\frac{1}{\cosh^2 x}$ |
| e^x | e^x | $\coth x$ | $\frac{-1}{\sinh^2 x}$ |
| $\sin x$ | $\cos x$ | $\arcsin x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\cos x$ | $-\sin x$ | $\arctan x$ | $\frac{1}{1+x^2}$ |
| $\tan x$ | $\frac{1}{\cos^2 x}$ | $\sinh^{-1} x$ | $\frac{1}{\sqrt{1+x^2}}$ |
| $\cot x$ | $\frac{-1}{\sin^2 x}$ | $\tanh^{-1} x$ | $\frac{1}{1-x^2}$ |
| $\sinh x$ | $\cosh x$ | $\coth^{-1} x$ | $\frac{1}{\sqrt{x^2-1}}$ |

Tablica osnovnih integrala

| | | |
|--|---|---|
| $\int dx = x + C$ | $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$ | $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$ |
| $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$ | $\int \tan x dx = -\ln \cos x + C$ | $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left \frac{x+a}{x-a} \right + C$ |
| $\int \frac{1}{x} dx = \ln x + C$ | $\int \cot x dx = \ln \sin x + C$ | $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$ |
| $\int e^x dx = e^x + C$ | $\int \frac{dx}{\cos^2 x} = \tan x + C$ | $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$ |
| $\int a^x dx = \frac{a^x}{\ln a} + C$ | $\int \frac{dx}{\sin^2 x} = -\cot x + C$ | $\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$ |
| $\int \sin x dx = -\cos x + C$ | $\int \sinh x dx = \cosh x + C$ | $\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$ |
| $\int \cos x dx = \sin x + C$ | $\int \cosh x dx = \sinh x + C$ | $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$ |

♣3

5. a) $y' - \frac{1}{x}y = x$

$p(x) = -\frac{1}{x}$ $q(x) = x^2$

$\int p(x) dx = \int -\frac{1}{x} dx = -\int \frac{dx}{x} = -\ln|x|$

$y = e^{\ln|x|} \cdot \left[\int e^{-\ln|x|} \cdot x^2 dx + c \right]$

$y = x \cdot \left[\int x^{-1} \cdot x^2 dx + c \right]$

$y = x \cdot \left[\int x dx + c \right]$

$y = x \cdot \left(\frac{x^2}{2} + c \right)$

$y = \frac{x^3}{2} + cx$ ✓

$a=0$
 $b=1$ $m=i$
 $m=0$

b) $y'' + 4y' + 4y = \sin x$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda_{1/2} = \frac{-4 \pm \sqrt{16-16}}{2}$

$\lambda_{1/2} = -2$

$y_H = e^{-2x} (C_1 + C_2 x)$

$y = A \cos x + B \sin x$ $y = -\frac{13}{16} \cos x + \frac{39}{64} \sin x$

$y' = -A \sin x + B \cos x$

$y'' = -A \cos x - B \sin x$

$-A \cos x - B \sin x + 4(-A \sin x + B \cos x) + 4(A \cos x + B \sin x) = \sin x$

$-A \cos x - B \sin x - 4A \sin x + 4B \cos x + 4A \cos x + 4B \sin x = \sin x$

$(-A + 4B + 4A) \cos x + (4B - B - 4A) \sin x = \sin x$

$(4B + 3A) \cos x + (3B - 4A) \sin x = \sin x$

$3B - 4A = 1$

$-4A = (1 - 3B) / (-4)$

$A = -\frac{1-3B}{4}$ $A = -\frac{1-3 \cdot (-\frac{3}{4})}{4} = -\frac{13}{16}$

$4B + 3A = 0$

$4B = -3A / 4$

$B = -\frac{3}{4}A = -\frac{3 \cdot (-\frac{13}{16})}{4} = \frac{39}{64}$

GRSEJKA
u HESOV

$y = e^{-2x} (C_1 + C_2 x) - \frac{13}{16} \cos x + \frac{39}{64} \sin x$ ✓

10

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Krajević

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: Tomislav Krajević Broj indeksa: 17-01-0052-2011

Vrijeme: od 08:15 h do 09:40 h ♣3

Broj bodova:

07

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4) 5)

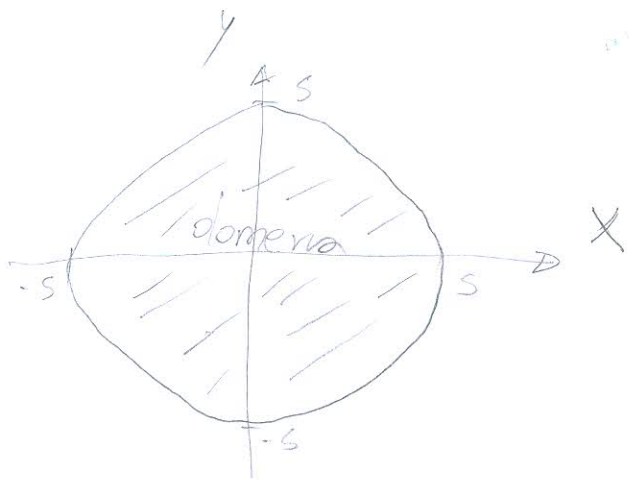
$$f(x,y) = \sqrt{25 - x^2 - y^2}$$

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$$-x^2 - y^2 \geq -25 \quad | \cdot (-1)$$

$$x^2 + y^2 \leq 25$$

$$r = 5$$



a) $f(x,y) = y^2 + xy + 3y + 2x^2 + 3$

$$\frac{\partial f}{\partial x} = 4x + y$$

$$\frac{\partial^2 f}{\partial x^2} = 4$$

$$\frac{\partial f}{\partial y} = 2y + 3 + x$$

$$\frac{\partial^2 f}{\partial^2 y} = 2$$

$$\frac{\partial f}{\partial x \partial y} = (2y + x + 3) = 1$$

$$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$$

$$\rightarrow 4x + y = 0$$

$$y = -4x$$

$$y = -4 \cdot \frac{3}{7}$$

$$y = \frac{31}{7}$$

$$T = \left(\frac{3}{7}, \frac{31}{7}, 3 \right)$$

$$2y + x + 3 = 0$$

$$2 \cdot (-4x) + x + 3 = 0$$

$$-8x + x + 3 = 0$$

$$-7x = -3$$
$$x = \frac{3}{7}$$

$$\Delta = \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} = 8 - 1 = 7 > 0$$

$$\frac{\partial^2 f}{\partial x^2} = 4 > 0 \quad T \text{ je minimum}$$