

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: PETAR PERICA

Broj indeksa: 026 906 8202

Vrijeme: od _____ do _____ ♣2

Broj bodova:

85

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (12+8) Integriraj

a)

$$\int \cos^5 x \sin x dx$$

✓ (12)

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

✓ (8)

2. (15) Integriraj

$$\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$$

✓ (15)

3. (15) Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

✓ (11)

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{x} - \sqrt{x+y}$$

✓ (10)

5. (15+15) Rijesi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2y' = 0$$

✓ (15)

b)

$$y'' + 2y' + y = e^{3x}$$

✓ (10)

1. a) $\int \cos^5 x \sin x dx = \left[\begin{array}{l} \cos x = t \\ dt = -\sin x dx \\ dx = -\frac{dt}{\sin x} \end{array} \right] = -\int t^5 \sin x \cdot \frac{dt}{\sin x} = -\int t^5 dt = -\frac{t^6}{6} = -\frac{\cos^6 x}{6} + C$

b) $\int_0^{+\infty} \frac{1}{x^2} dx = \lim_{y \rightarrow 0} \left(-\frac{1}{x} \Big|_y^{+\infty} \right) = 2 \cdot \infty$

$$\int x^{-2} = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$$

$x^2 \neq 0$
 $x \neq 0$
SING.

NEMA POUŠINE

$$= \int dx + \int \frac{\frac{5}{3}}{x-1} dx - \int \frac{\frac{2}{3}}{x+2} dx$$

$$= \left[x + \frac{5}{3} \ln|x-1| - \frac{2}{3} \ln|x+2| \right] + C$$

15

$$\int \frac{\frac{5}{3}}{x-1} dx = \frac{5}{3} \int \frac{1}{x-1} dx = \frac{5}{3} \ln|x-1| + C$$

$$\int \frac{2}{3} \frac{1}{x+2} dx = \frac{2}{3} \ln|x+2| + C$$

$$x^2+x-2 = (x-1)(x+2)$$

$$x^2+x-2 = (x-1)(x+2)$$

$$x^2+x-2 = (x-1)(x+2)$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-4 \cdot 1 \cdot (-2)}}{2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{9}}{2}$$

$$x_1 = 1$$

$$x_2 = -2$$

$$\frac{x+4}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \quad | \cdot (x-1)(x+2)$$

$$x+4 = A(x+2) + B(x-1)$$

$$x+4 = Ax + 2A + Bx - B$$

$$x+4 = x(A+B) + (2A-B)$$

$$A+B = 1$$

$$A = 1-B$$

$$A = 1 + \frac{2}{3} = \frac{5}{3}$$

$$2A - B = 4$$

$$2(1-B) - B = 4$$

$$2 - 2B - B = 4$$

$$-3B = 2 \quad | : (-3)$$

$$B = -\frac{2}{3}$$

$$y = 2x^2 + 9, y = 9x$$

$$2x^2 + 9 = 9x$$

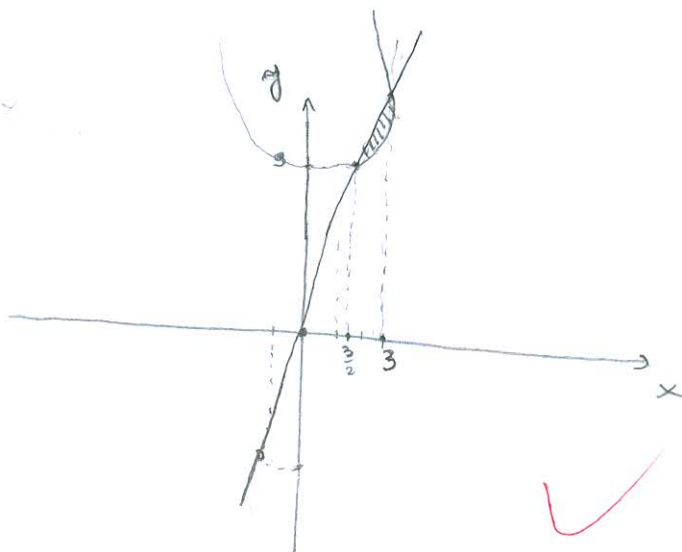
$$2x^2 + 9 - 9x = 0$$

$$2x^2 - 9x + 9 = 0$$

$$x_{1,2} = \frac{9 \pm \sqrt{81 - 8 \cdot 9}}{4}$$

$$x_{1,2} = \frac{9 \pm \sqrt{9}}{4}$$

$$x_1 = 3, x_2 = \frac{3}{2}$$



$$\frac{x}{0} \mid \frac{1}{0} \mid \frac{-1}{91-9}$$

9x

15

$$\int_{\frac{3}{2}}^3 (9x - (2x^2 + 9)) dx = \int_{\frac{3}{2}}^3 (9x - 2x^2 - 9) dx = \left. \frac{9x^2}{2} \right|_{\frac{3}{2}}^3 - \left. \frac{2x^3}{3} \right|_{\frac{3}{2}}^3 - \left. 9x \right|_{\frac{3}{2}}^3 =$$

$$= \left(\frac{81}{2} - \frac{81}{8} \right) - \left(18 - \frac{9}{4} \right) - \left(27 - \frac{27}{2} \right) = -\frac{243}{8} - \frac{63}{4} - \frac{27}{2} = \frac{243 - 126 - 108}{8} =$$

$$= \frac{9}{8} \approx 1,125$$

$$1. f(x, y) = x^3 - 3xy - y^2$$

$$\frac{\partial f}{\partial x} = 3x^2 - 3y \Rightarrow 3x^2 = 3y$$
$$x^2 = \frac{3y}{3} = y \quad | \sqrt{\quad}$$
$$x = \sqrt{y}$$

$$\frac{\partial f}{\partial y} = -3x - 3y^2 \rightarrow -3\sqrt{y} - 3y^2$$
$$-3\sqrt{y} = 3y^2$$

b.) $f(x, y)$

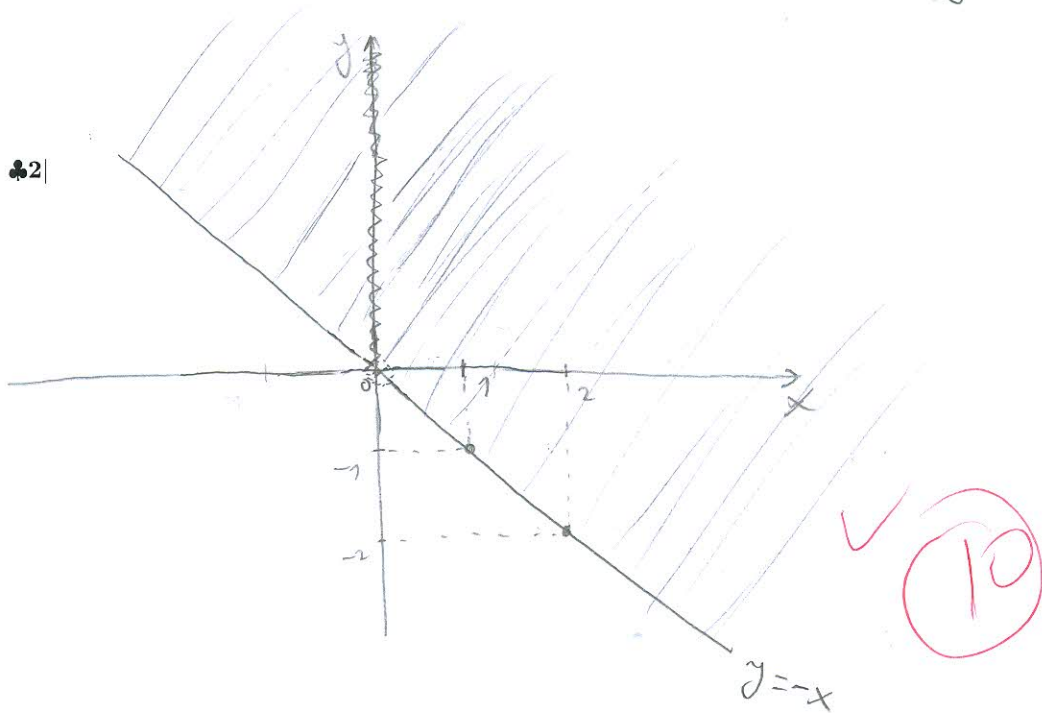
$$e) f(x,y) = \frac{1}{x} - \sqrt{x+y}$$

$$x \neq 0$$

$$x=0$$

$$x+y \geq 0$$

$$y \geq -x$$



5. a)

$$(x-1)y - x^2 y' = 0$$

$$x^2 y' = (x-1)y$$

$$x^2 \cdot \frac{dy}{dx} = (x-1)y$$

$$x^2 \cdot dy = (x-1)y dx \quad | : y$$

$$x^2 \cdot \frac{dy}{y} = (x-1) dx \quad | : x^2$$

$$\int \frac{dy}{y} = \int \frac{x-1}{x^2} dx$$

$$\ln|y| = \ln|x| + \frac{1}{x} + c \quad | e^{\dots}$$

$$y = x \cdot e^{\frac{1}{x}} \cdot c$$



$$\int \frac{x-1}{x^2} = \int \frac{x}{x^2} dx - \int \frac{1}{x^2} dx$$

$$= \int \frac{1}{x} dx - \int x^{-2} dx =$$

$$= \ln|x| + \frac{1}{x} + c$$

e)

$$y'' + 2y' + y = e^{3x}$$

$$y_H = c_1 e^{-x} + c_2 x e^{-x}$$

$$r^2 + 2r + 1 = 0$$

$$r_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$$

$$r_1 = -1$$

$$e^{3x} = e^{2x} (P_m(x) \cos(2x) + Q_n(x) \sin(2x))$$

$$L = 3$$

$$L + B = 3, k$$

$$B = 0$$

$$3 + 0 = 3, k = 0$$

$$P_m = 0$$

$$Q_n = 0$$

$$y_p = e^{3x} \cdot A$$

$$y_p' = 3e^{3x} \cdot A +$$

$$y_p'' = 9e^{3x} \cdot A$$

$$3e^{3x} \cdot A + 2(3e^{3x} \cdot A) + e^{3x} \cdot A = e^{3x}$$

$$e^{3x}(9A + 12A + A) = e^{3x}$$

$$y_p = \frac{e^{3x}}{22}$$

$$22A = 1$$

$$A = \frac{1}{22} \clubsuit 1$$

$$y = y_p + y_H$$

$$y = \frac{e^{3x}}{22} + C_1 e^{-x} + C_2 x e^{-x} + \frac{e^{3x}}{22}$$

o result
u račun
10

$$6e^{3x} \cdot 2A$$

$$8e^{3x} \cdot A$$

$$2(3e^{3x} \cdot A)$$

$$12Ae^{3x}$$

$$8Ae^{3x}$$

$$6e^{3x} \cdot 2A$$

$$e^{3x}(6 \cdot 2A) = 12A$$

$$2(6Ae^{3x})$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$-\frac{1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$-\frac{1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \cos x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left \frac{x+a}{x-a} \right + C$
$\int \frac{1}{x} dx = \ln x + C$	$\int \cot x dx = \ln \sin x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣2

-3,3,3

$$\int x^{-2}$$

$$\frac{3 \cdot \left(\frac{3}{2}\right)^2}{2} = \frac{9 \cdot \frac{9}{4}}{2} = \frac{81}{8}$$

$$\frac{9 \cdot 3}{2}$$

$$2 \cdot \frac{\left(\frac{3}{2}\right)^3}{3} = \frac{2 \cdot \frac{27}{8}}{3} = \frac{54}{24} = \frac{3}{2}$$

$$9 \cdot \frac{3}{2}$$

$$(3x)' \quad \frac{81}{8} - \frac{81}{2} = \frac{81-324}{8} = -\frac{243}{8}$$

=3

$$3 \cdot 3e^{3x}$$

$$-\frac{27}{2}$$

$$-3x - 3y^2$$

3.

$$-3x = 3y^2$$

3x

$$3x = -3y^2$$

$$-3y^2$$

4

$$3y^2 = -3x$$

$$3y = -3y^2$$

$$x = \frac{-3y^2}{3} = -y^2$$

$$\frac{9 \cdot \frac{9}{4}}{2} = \frac{\frac{81}{4}}{2} = \frac{81}{8}$$

-30,375

$$\frac{324 - 81}{8} = \frac{63}{4} = \frac{27}{2}$$

$$\frac{253}{8} - \frac{63}{4} - \frac{27}{2} = \frac{253 - 126 - 108}{8} =$$

$$30,375 - 15,75 - 13,5$$

