MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: PETAR PERICA **Broj indeksa:** 0026 9068202

Vrijeme: od____________ **do**_________ ♣2 **Broj bodova:** 85

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$ 12

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$ 8

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$. 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{x} - \sqrt{x+y}$$ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2y' = 0$$ 15

b)

$$y'' + 2y' + y = e^{3x}$$ 10

1. a) $\int \cos^5 x \sin x \, dx = \left[\begin{array}{l} \cos x = t \\ dt = -\sin x \, dx \\ dx = -\frac{dt}{\sin x} \end{array}\right] = -\int t^5 \cancel{\sin x} \cdot \frac{dt}{\cancel{\sin x}} = -\int t^5 dt = -\frac{t^6}{6} = -\frac{\cos^6 x}{6} + C$

b) $\int_0^{+\infty} \frac{1}{x^2} dx = \lim_{y \to 0} \left(-\frac{1}{x} \Big|_y^{+\infty} \right) = +\infty$

$\int x^{-2} = \frac{x^{-1}}{-1} = -\frac{1}{x} + c$

$x^2 \neq 0$

$x \neq 0$

sing.

NEMA POVRŠINE

$$\int \frac{\dots}{x^2+x-2}\,dx =$$

$$= \int dx + \int \frac{\frac{5}{3}}{x-1}\,dx - \int \frac{\frac{2}{3}}{x+2}\,dx$$

$$= \boxed{x + \frac{5}{3}\ln|x-1| - \frac{2}{3}\ln|x+2| + C}$$

15

$$\circledast \int \frac{\frac{5}{3}}{x-1}\,dx = \frac{5}{3}\int \frac{1}{x-1}\,dx = \frac{5}{3}\ln|x-1| + C$$

$$\circledast \ \frac{2}{3}\int \frac{1}{x+2}\,dx = \frac{2}{3}\ln|x+2| + C$$

$$x^2+2x+2 : x^2+x-2 = 1 + \frac{x+4}{x^2+x-2}$$

$$\frac{\mp x^2 \mp x \pm 2}{\quad x+4}$$

$$x^2+x-2 = (x-1)(x+2)$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 1\cdot(-2)}}{2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{9}}{2}$$

$$x_1 = 1$$

$$x_2 = -2$$

$$\frac{x+4}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \quad / \cdot (x-1)(x+2)$$

$$x+4 = A(x+2) + B(x-1)$$

$$x+4 = Ax + 2A + Bx - B$$

$$x+4 = x(A+B) + (2A-B)$$

$$A+B = 1 \qquad 2A - B = 4$$

$$A = 1-B \qquad 2(1-B) - B = 4$$

$$A = 1 + \frac{2}{3} = \frac{5}{3} \qquad 2 - 2B - B = 4$$

$$-3B = 2 \quad /:(-3)$$

$$B = -\frac{2}{3}$$

3. $y = 2x^2 + 9$, $y = 9x$

$$2x^2 + 9 = 9x$$

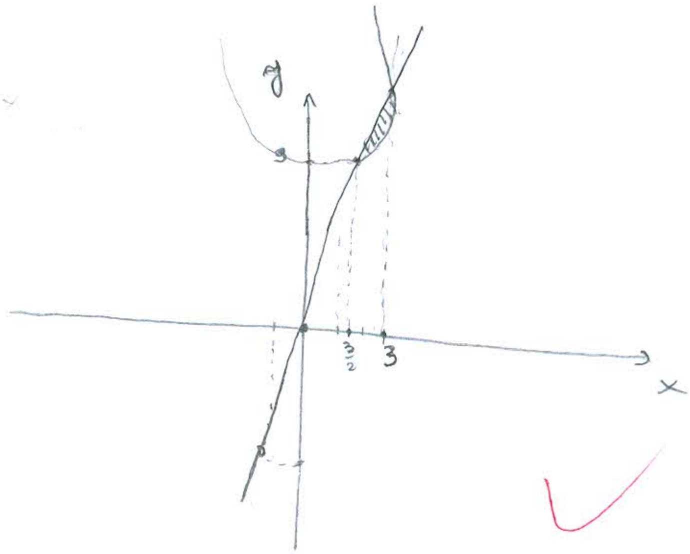
$$2x^2 + 9 - 9x = 0$$

$$2x^2 - 9x + 9 = 0$$

$$x_{1,2} = \frac{9 \pm \sqrt{81 - 8\cdot 9}}{4}$$

$$x_{1,2} = \frac{9 \pm \sqrt{9}}{4}$$

$$x_1 = 3, \quad x_2 = \frac{3}{2}$$



x	0	1	-1
y	0	9	-9

$9x$

10

$$\int_{\frac{3}{2}}^{3} \left(9x - (2x^2+9)\right) dx = \int_{\frac{3}{2}}^{3} (9x - 2x^2 - 9)\,dx = \left.\frac{9x^2}{2}\right|_{\frac{3}{2}}^{3} - \left.\frac{2x^3}{3}\right|_{\frac{3}{2}}^{3} - \left.9x\right|_{\frac{3}{2}}^{3} =$$

$$= \left(\frac{81}{2} - \frac{81}{8}\right) - \left(18 - \frac{9}{4}\right) - \left(27 - \frac{27}{2}\right) = \frac{243}{8} - \frac{63}{4} - \frac{27}{2} = \frac{243 - 126 - 108}{8} =$$

$$= \frac{9}{8} \approx 1{,}125$$

4. $f(x,y) = x^3 - 3xy - y^3$

$$\frac{\partial f}{\partial x} = 3x^2 - 3y \Rightarrow 3x^2 = 3y$$

$$x^2 = \frac{3y}{3} = y$$

$$x = \sqrt{y}$$

$$\frac{\partial f}{\partial y} = -3x - 3y^2 \rightarrow -3\sqrt{y} - 3y^2$$

$$-3\sqrt{y} = 3y^2$$

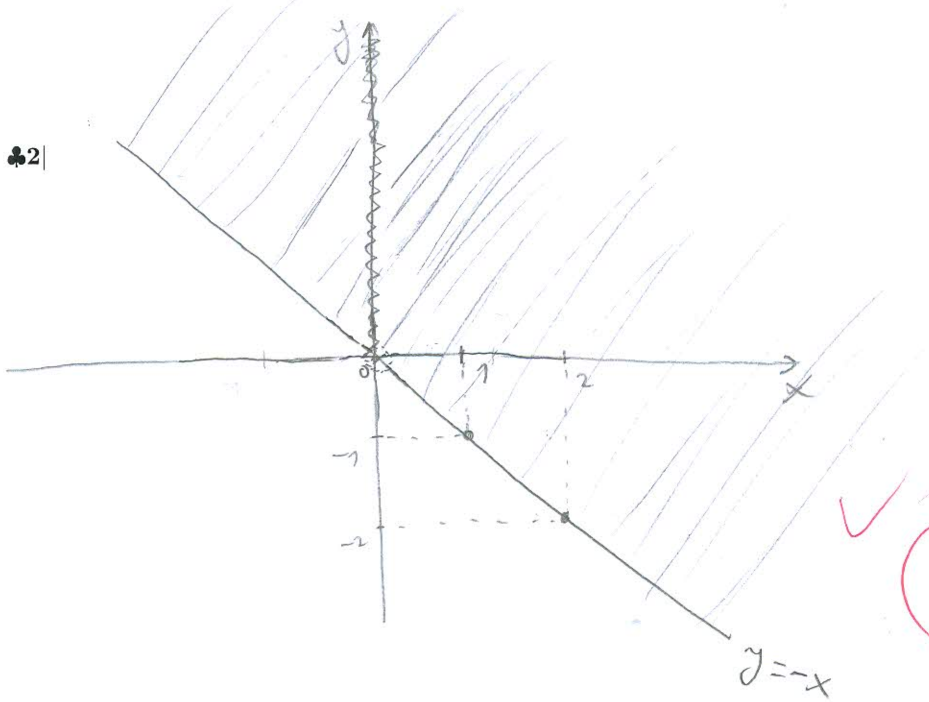
b) $f(x,y$

b) $f(x,y) = \frac{1}{x} - \sqrt{x+y}$

$x \neq 0$ $\quad$ $x+y \geq 0$

$x = 0$ $\quad$ $y \geq -x$

♣2|



10

5. a)

$(x-1)y - x^2 y' = 0$

$x^2 y' = (x-1)y$

$x^2 \cdot \frac{dy}{dx} = (x-1)y$

$x^2 \cdot dy = (x-1)y\,dx \quad / : y$

$x^2 \cdot \frac{dy}{y} = (x-1)\,dx \quad / : x^2$

$\int \frac{dy}{y} = \int \frac{x-1}{x^2}\,dx$

$\ln|y| = \ln|x| + \frac{1}{x} + c \quad / e^{\square}$

$\boxed{y = x \cdot e^{\frac{1}{x}} \cdot C}$ ✓

$\int \frac{x-1}{x^2} = \int \frac{x}{x^2}dx - \int \frac{1}{x^2}\,dx$

$= \int \frac{1}{x}\,dx - \int x^{-2}\,dx =$

$= \ln|x| + \frac{1}{x} + c$

b)

$y'' + 2y' + y = e^{3x}$

$y_H = C_1 e^{-x} + C_2 x e^{-x}$

$r^2 + 2r + 1 = 0$

$r_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$

$r_1 = -1$

$e^{3x} = e^{\alpha x}\left(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x)\right)$

$\alpha = 3 \qquad \alpha + \beta i,\ k$

$\beta = 0 \qquad 3 + 0 = 3,\ k = 0$

$P_m = 0$

$Q_n = 0$

$y_p = e^{3x} \cdot A$

$y_p' = 3e^{3x} \cdot A +$

$y_p'' = 9e^{3x} \cdot A$

$9e^{3x}\cdot A + 2(3e^{3x}\cdot A) + e^{3x}\cdot A = e^{3x}$

$e^{3x}(9A + 12A + A) = 1e^{3x}$

$y_p = \frac{e^{3x}}{22}$

$22A = 1$

$A = \frac{1}{22}$ ♣1

$y = y_p + y_H$

$y = \frac{e^{3x}}{22} + C_1 e^{-x} + C_2 x e^{-x} + \frac{e^{3x}}{22}$

GREŠKA U RAČUNU

1

$6e^{3x}\cdot 2A$ $9e^{3x}\cdot A$ $2(3e^{3x}\cdot A)$ $12Ae^{3x}$

$9Ae^{3x}$ $6e^{3x}\cdot 2A$

$e^{3x}(6\cdot 2A = 12A$ $2(3e^{3x}\cdot 2A)$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

$\frac{x^2\,dy}{y}$ $\frac{x^{-1}}{-1}$ $-\frac{x}{1}$ $-\frac{1}{x}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\vert \frac{a+x}{a-x} \right\vert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \vert \cos x \vert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\vert \frac{x+a}{x-a} \right\vert + C$
$\int \frac{1}{x} dx = \ln \vert x \vert + C$	$\int \cot x dx = \ln \vert \sin x \vert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \vert x + \sqrt{x^2 \pm a^2} \vert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

x^{-2}

$-35{,}43$ $\int x^-$

$(3\cdot e^{3x})'$ $3\cdot 3e^{3x}$ $= 9e^{3x}$

$\frac{9\cdot(\frac{3}{2})^2}{2} = \frac{9\cdot\frac{9}{4}}{2} = \frac{\frac{81}{4}}{2} = \frac{81}{8}$

$\frac{9\cdot 9}{2}$

$2\cdot\frac{(\frac{3}{2})^3}{3} = \frac{2\cdot\frac{27}{8}}{3} = \frac{\frac{54}{8}}{3}$ $\frac{\cancel{54}\,27}{\cancel{24}\,12} = \frac{9}{4}$

$9\cdot\frac{3}{2}$

$(3x)'$ $=3$

$\frac{81}{8} - \frac{81}{2} = \frac{81-324}{8} = -\frac{243}{8}$

$3\cdot 3e^{3x}$ $-\frac{27}{2}$

$3\cdot$

$3x$

$-3x-3y^2$

$-3x=3y^2$

$3x=-3y^2$

$-3y^2$

$3y^2=-3x$

$3x=-3y^2$

$x=\frac{-3y^2}{3}=-y^2$

$-30{,}375$

$$\frac{9\cdot\frac{9}{4}}{2}=\frac{\frac{81}{4}}{\frac{2}{1}}$$

$$\frac{324-81}{8}-\frac{63}{4}-\frac{27}{2}$$

$$\frac{243}{8}-\frac{63}{4}-\frac{27}{2}=\frac{243-126-108}{8}=$$

$30{,}375-15{,}75-13{,}5$

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: IVAN STOJANOV **Broj indeksa:** 17-2-0061-2010

Vrijeme: od________ **do**________ ♣2 **Broj bodova:** 15

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

✓ 15

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2 y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

1. b) $\int_0^{+\infty} \frac{1}{x^2} dx$

$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$

$-\frac{1}{x}\Big|_0^{+\infty} = -\frac{1}{\infty} - \frac{1}{0} = -\frac{1}{\infty}$

2. $\int \frac{x^2+2x+2}{x^2+x-2} dx = \int 1 dx + \int \frac{x+4}{x^2+x-2}$

$x^2+2x+2 : (x^2+x-2) = 1$
$-(x^2+x-2)$
$x+4$

$\frac{x+4}{x^2+x-2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} \quad / \cdot x^2+x-2$

$(x+1)(x-2) = x^2-2x+x-2$
$(x-1)(x+2) = x^2+2x-x-2$

$x+4 = A(x+2) + B(x-1)$
$x+4 = Ax+2A+Bx-B$
SA x: $1 = A+B \Rightarrow A = 1-B$
bez x: $4 = 2A - B$

$4 = 2\cdot(1-B) - B$
$4 = 2-2B-B$
$4 = 2-3B$
$-3B = 4-2$
$-3B = 2 \Rightarrow B = \frac{-2}{3}$

$1 = A - \frac{2}{3}$
$A = 1 + \frac{2}{3}$
$A = \frac{5}{3}$

$\Rightarrow$

(1.) a) $\int \cos^5 x \sin x \, dx = \left\{ \begin{matrix} u = \cos^5 x & \sin x = dv \\ du = 5\cos^4 x \, dx & -\cos x = v \end{matrix} \right\} = \cos^5 x \cdot (-\cos x) - \int -\cos x \cdot 5\cos^4 x \, dx$

$\cancel{= \cos^5 x} = -\cos^6 x + \int \cos x \cdot 5 \cos^4 dx = -\cos^6 x + 5 \int \cos^5 x \, dx = -\cos^6 x + 5 \frac{\sin^6 x}{6} + C$

$= -\cos^6 x + \frac{5}{6} \cdot \sin^6 x + C$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

(2.) nastavak $\int \frac{x+4}{x^2+x-2} = \int \frac{\frac{5}{3}}{(x-1)} dx + \int \frac{-\frac{2}{3}}{(x+2)} dx = \frac{5}{3} \ln|x-1| - \frac{2}{3} \ln|x+2|$

$\int \frac{x^2+2x+2}{x^2+x-2} = x + \frac{5}{3} \ln|x-1| - \frac{2}{3} \ln|x+2| + C$ ✓ 15

(3.) $y = 2x^2 + 9 \quad y = 9x$

$y = (2x^2 + 9) - 9x = 2x^2 - 9x + 9$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{2 \cdot 2} = \frac{9 \pm \sqrt{81 - 36}}{4} = \frac{9 \pm \sqrt{45}}{4}$

$x_1 = \frac{9 + \sqrt{45}}{4} = 10.677 \approx 10.68$

$x_2 = \frac{9 - \sqrt{45}}{4} = 7.322 \approx 7.32$

$\int_{7,32}^{10,68} (2x^2 - 9x + 9) dx = 308.6$

$\cancel{= \int 2x^2} = 2\int x^2 dx - 9 \int x dx + 9 \int dx$

$= 2 \cdot \frac{x^3}{3} - 9 \cdot \frac{x^2}{2} + 9x + C$

$\cancel{= 2 \cdot \frac{x^3}{3} +} = \left| 2 \cdot \frac{x^3}{3} - 9 \cdot \frac{x^2}{2} + 9x \right|^{10,68} - \left| 2 \cdot \frac{x^3}{3} - 9 \cdot \frac{x^2}{2} + 9x \right|_{7,32}$

$\cancel{= \frac{2}{3} \cdot 1218.2 + 9}$ $I_2 = \frac{2}{3} \cdot 392.2 - \frac{9}{2} \cdot 53.6 + 9 \cdot 7.32 = 86.2$

$P = I_1 - I_2 = 394.8 - 86.2 = 308.6$

$I_1 = \frac{2}{3} \cdot 1218.2 - \frac{9}{2} \cdot 114.1 + 9 \cdot 10.68 = 394.8$

Mar

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MARIN GVOZDEN **Broj indeksa:** 17-2-0137-2011

Vrijeme: od 08:20 **do** ________ ♣2 **Broj bodova:** 15

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$ V 15

5. b) $y'' + 2y' + y = e^{3x}$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda_{1,2} = \dfrac{-2 \pm \sqrt{4-4}}{2}$

$\lambda_{1,2} = \dfrac{-2}{2} = -1$

$\underline{y_0 = c_1 e^{-x} + x c_2 e^{-x}}$

$\underline{k = 1 \quad b = 3}$

$\eta = \dfrac{k \cdot e^{bx}}{P(b)}$

$\eta = \dfrac{e^{3x}}{16}$

$P(b) = 3^2 + 2 \cdot 3 + 1$

$P(b) = 16$

✓ 15

$\underline{y = c_1 e^{-x} + x c_2 e^{-x} + \dfrac{e^{3x}}{16}}$

a)

$(x-1)y - x^2 y' = 0 \quad / :(x-1)$

$y - x^2 y' = 0$

$y' x^2 - y = 0 \quad / : x^2$

$y' - \dfrac{1}{x^2} \cdot y = 0$

$P(x) = \dfrac{-1}{x^2}$

$g(x) = 0$

$P(x) = -\int x^{-2} dx = \dfrac{+x^{-1}}{+1} = \dfrac{1}{x}$

$y = e^{-\frac{1}{x}} \cdot \left[e^{\frac{1}{x}} + c \right]$

$y = e^{-\frac{1}{x} + \frac{1}{x}}$

$\underline{= e}$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
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e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣2

4. a)

$f(x,y) = x^3 - 3xy - y^3$

$Z_x = 3x^2 - 3y \qquad Z_{xx} = 6x \qquad Z_{xy} = 0$

$Z_y = -3x - 3y^2 \qquad Z_{yy} = -6y \qquad Z_{yx} = 0$

NUŽAN UVJET:

$3x^2 - 3y = 0$

$-3x - 3y^2 = 0 \Rightarrow y^2 = x$

$y = \sqrt{x}$

$3x^2 = 3y$

$x^2 = y$

$x = \sqrt{y}$

$-3y = 0$

$y = 0$

$-3x - 3(\sqrt{x})^2 = 0$

$-3x - 3x = 0$

$-6x = 0 \Rightarrow x = 0$

$T(0,0)$

$Z_{xx}(6) > 0 \Rightarrow$ MINIMUM

DOVOLJAN UVJET:

$\Delta = \begin{vmatrix} 6 & 0 \\ 0 & -6 \end{vmatrix} = -36$

SEDLASTA TOČKA $T(0,0)$

1. a)

$$\int \cos^5 x \sin x \, dx = \left| \begin{matrix} u = \cos^5 x & dv = \sin x \, dx \\ du = -5\sin^4 x \, dx & v = -\cos x \end{matrix} \right| = u \cdot v - \int v \cdot du =$$

$$= \cos^5 x \cdot (-\cos x) - \int -\cos x \cdot (-5\sin^4 x)\, dx$$

$$= -\cos^6 x - \int 5\sin^4 x \cdot \cos x \, dx$$

$$= -\cos^6 x -$$

$$\left| \begin{matrix} u = 5\sin^4 x & dv = \cos x \\ du = 20\cos^3 x & v = \sin x \end{matrix} \right|$$

$$= 5\sin^4 x \cdot \sin x - \int \sin x \cdot 20\cos^3 x$$

$$= 5\sin^5 x - \int \sin x \cdot 20\cos^3 x$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} = \lim_{x\to\infty} \frac{x^{-1}}{-1}\Big|_0^{+\infty} = -\frac{1}{x}\Big|_0^{-\infty} = \lim_{x\to\infty} -\frac{1}{x} + \frac{1}{0} = -0 + 0$$

$$= 0$$

2.

$$\int \frac{x^2+2x+2}{x^2+x-2}$$

$$(x^2+2x+2) : (x^2+x-2) = 1$$

$$\frac{-(x^2+x-2)}{x+4}$$

$$\int 1\,dx + \int \frac{x+4}{x^2+2x-2}$$

$$= x + \int \frac{x+4}{(x-1)} + \frac{x+4}{(x+1)-1}$$

$$\frac{x+4}{(x-1)(x+1)-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)-1} \quad \Big/ \cdot (x-1)(x+1)-1$$

$$x+4 = Ax + A\cdot(-1) + Bx - B$$

$$A + B = 1$$

$$-A + B = 4$$

$$2B = 5$$

$$B = \frac{5}{2}$$

$$A = -\frac{3}{2}$$

$$\int \frac{-\frac{3}{2}}{(x-1)} + \frac{\frac{5}{2}}{(x+1)-1} = -\frac{3}{2}\ln|x-1| + \frac{5}{2}\ln|(x+1)-1|$$

$$\int \frac{x^2+2x+2}{x^2+x-2} = x - \frac{3}{2}\ln|x-1| + \frac{5}{2}\ln|(x+1)-1|$$

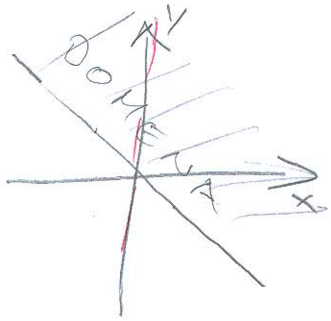
4. b)

$f(x,y) = \frac{1}{x} - \sqrt{x+y}$

x	-1	0	1
	1	0	-1

$x + y \geq 0$

$y \geq -x$



3.

$y = 2x^2 + 9$ $\quad y = 9x$

x	-1	0	1
y	-9	0	9

SKICA:

$2x^2 + 9 + 9x = 0$ $\quad a = 2,\ b = 9,\ c = 9$

$x_{1,2} = \frac{-9 \pm \sqrt{81 - 72}}{4}$

$x_{1,2} = \frac{-9 \pm 3}{4}$ $\qquad \underline{x_1 = -\frac{3}{2} \quad x_2 = -3}$

$x_1 = -\frac{\cancel{6}^{3}}{\cancel{4}_{2}} \qquad x_2 = -\frac{12}{4} = -3$

$P = 9x - 2x^2 + 9$

$P = -\int_{-3}^{-\frac{3}{2}} 2x^2\,dx + \int_{-3}^{-\frac{3}{2}} 9x\,dx + \int_{-3}^{-\frac{3}{2}} 9\,dx$

$= -2\frac{x^3}{3}\Big|_{-3}^{-\frac{3}{2}} + 9\frac{x^2}{2}\Big|_{-3}^{-\frac{3}{2}} + 9x\Big|_{-3}^{-\frac{3}{2}}$

$= -2\left(\frac{\frac{\cancel{-27}^{-9}}{8}}{\cancel{3}_{1}} - \frac{\cancel{-27}^{-9}}{\cancel{3}}\right) + 9\left(\frac{\frac{9}{4}}{\frac{2}{1}} - \frac{9}{2}\right) + 9\left(-\frac{3}{2} - 3\right)$

$-2\cdot\left(-\frac{9}{8} - 9\right) + 9\left(\frac{9}{8} - \frac{9}{2}\right) + 9\left(-\frac{9}{2}\right)$

$\cancel{15.75} - 3.375 - 40.5$

$20.25 - 30.375 - 40.5$

$= -50.625$

$$\ldots x\,dx = \left| \begin{array}{l} \cos x = t \\ -\sin x\,dx = dt \\ dx = \dfrac{dt}{\sin x} \end{array} \right| =$$

$$\ldots \frac{dt}{\sin x} = \int t^5\,dt = \frac{t^6}{6} = \frac{(\cos x)^6}{6}$$

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: JASMIN NEKIĆ **Broj indeksa:** 17-1-0050-2011

Vrijeme: od ____________ **do** _________ ♣2 **Broj bodova:** 15

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2 y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

3. $y = 2x^2 + 9$
$y = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{81-72}}{4}$

$x_{1,2} = \frac{9 \pm 3}{4}$

$x_1 = 3, \; x_2 = \frac{3}{2}$

$P = \int_{\frac{3}{2}}^{3} (9x - 2x^2 - 9)$

$P = \frac{9x^2}{2}\Big|_{\frac{3}{2}}^{3} - \frac{2x^3}{3}\Big|_{\frac{3}{2}}^{3} - 9x\Big|_{\frac{3}{2}}^{3}$

$= \frac{81}{2} - \frac{81}{8} - 18 + \frac{9}{4} - 27 + \frac{27}{2}$

$= 54 - \frac{63}{8} - 45 = 9 - \frac{63}{8}$

$P = \frac{72-63}{8} = \frac{9}{8}$ ✓ 1,5

2. $\int \frac{x^2+2x+2}{x^2+x-2}\, dx$

$\int \frac{x^2+x-2+x+4}{x^2+x-2}$

$\int \frac{x^2+x-2}{x^2+x-2} + \int \frac{x+4}{x^2+x-2}$

$\int dx + \int \frac{x+4}{x^2+x-2}$

4. a) $f(x,y) = x^3 - 3xy - y^3$

$\frac{\partial f}{\partial x} = 3x^2 - 3 \qquad \frac{\partial f}{\partial y} = -3y^2 - 3$

$\frac{\partial f}{\partial x} = 6x \qquad \frac{\partial f}{\partial y} = -6y$

$\frac{\partial f}{\partial x} = 6 \qquad \frac{\partial f}{\partial y} = -6$

$\frac{d^2 f}{dx\,dy} = -1$

1. b) $\int_0^{+\infty} \frac{1}{x^2}\, dx = \int_0^{+\infty} x^{-2}\, dx$

$= \frac{x^{-1}}{-1}\Big|_0^{+\infty} = -\frac{1}{x}\Big|_0^{+\infty}$

$= -\frac{1}{\infty} - \frac{1}{0}$

$\downarrow \qquad \downarrow$

$0 \qquad$ N.O.

5. a) $(x-1)y - x^2 y' = 0$

$x^2 y' = xy - y$

$y' = \frac{xy}{x^2} - \frac{y}{x^2}$

$y' = \frac{y}{x} - \frac{y}{x^2}$

$u' = u - u \cdot \frac{1}{x}$

$u' + u \cdot \frac{1}{x} = u$

$u' + u \cdot \frac{1}{x} = 0$

$u' = -u \cdot \frac{1}{x}$

$\frac{du}{u} = -\frac{1}{x}\, dx \,\Big|\int$

$\frac{y}{x} = u$

$y = u \cdot x \,|'$

$y' = u'(x)$

$\int \frac{du}{u} = -\int \frac{1}{x}\, dx$

$\ln|u| = -\ln|x| + c \,\Big|\, e$

$u = e^{-\ln|x| + c}$

$u = -x \cdot e^c$

$\frac{y}{x} = -x \cdot e^c$

$y = -\frac{x}{x} \cdot \frac{e^c}{x}$

$y = -\frac{e^c}{x}$

4. b) $f(x,y) = \frac{1}{x} - \sqrt{x+y}$

$x + y \geq 0$

$1 \geq 0$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2-x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: Matija Miočić **Broj indeksa:** 17-1-0110-2012

Vrijeme: od____________ **do**_________ ♣2 **Broj bodova:** 10

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

10

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

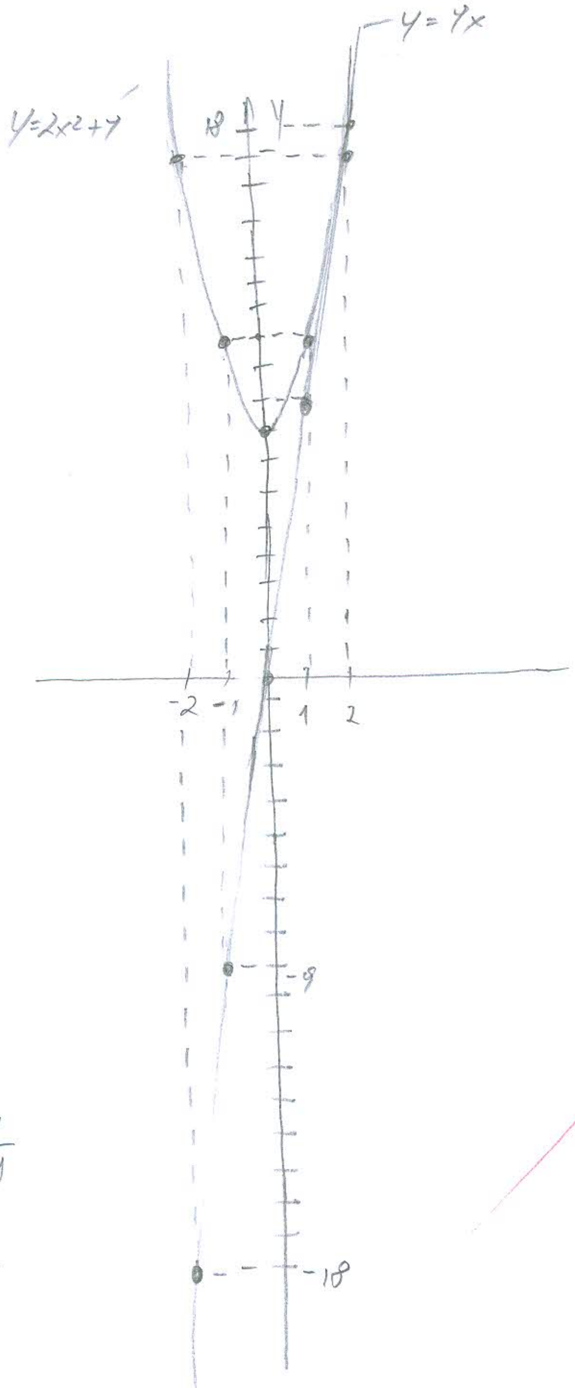
1\. a) $\int \cos^5 x \sin x \, dx = \left[\begin{matrix} \cos x = t \\ -\sin x \, dx = dt \end{matrix} \right]$

$= \int t^5 dt = \frac{t^6}{6} = -\frac{\cos^6 x}{6} + c$ ✓ 1

4\. b) $f(x,y) = \frac{1}{x} - \sqrt{x+y}$

$x + y > 0$

$Df: \langle 0, +\infty \rangle$



3\. $y = 2x^2 + 9 \qquad y = 9x$

$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1/2} = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{4}$

$x_{1/2} = \frac{9 \pm \sqrt{81 - 72}}{4}$

$x_{1/2} = \frac{9 \pm \sqrt{9}}{4}$

$x_1 = \frac{9+3}{4} = 3$

$x_2 = \frac{9-3}{4} = \frac{3}{2}$

x	-2	-1	0	1	2	3	4
y	17	11	9	11	17	27	41

x	-2	-1	0	1	2	3	4
y	-18	-9	0	9	18	27	36

$= \int_{\frac{3}{2}}^{3} (2x^2+9) - (9x) = \left| (2x^2+9) - (9x) \right|_{\frac{3}{2}}^{3} = \left[(2 \cdot 3^2 + 9) - (9 \cdot 3) \right] - \left[(2 \cdot (\tfrac{3}{2})^2 + 9) - (9 \cdot \tfrac{3}{2}) \right]$

$= \left[27 - 27 \right] - \left[\frac{27}{2} - \frac{27}{2} \right] = 0 - 0 = 0$

Pravac $y = 9x$ je tangenta na parabolu $y = 2x^2 + 9$

4. a) $f(x,y) = x^3 - 3xy - y^3$

$$\frac{\partial f}{\partial x} = 3x^2 - 3y \qquad \frac{\partial f}{\partial^2 x} = 6x$$

$$\frac{\partial f}{\partial y} = -3x - 3y^2 \qquad \frac{\partial f}{\partial^2 y} = -6y$$

STACIONARNE TOČKE

$$\frac{\partial f}{\partial x} = 0 \Rightarrow 3x^2 - 3y = 0 \Rightarrow -3y = -3x^2 /:(-3)$$

$$y = x^2$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow -3x - 3y^2 = 0$$

$$-3x - 3(x^2)^2 = 0$$

$$-3x - 3x^4 = 0$$

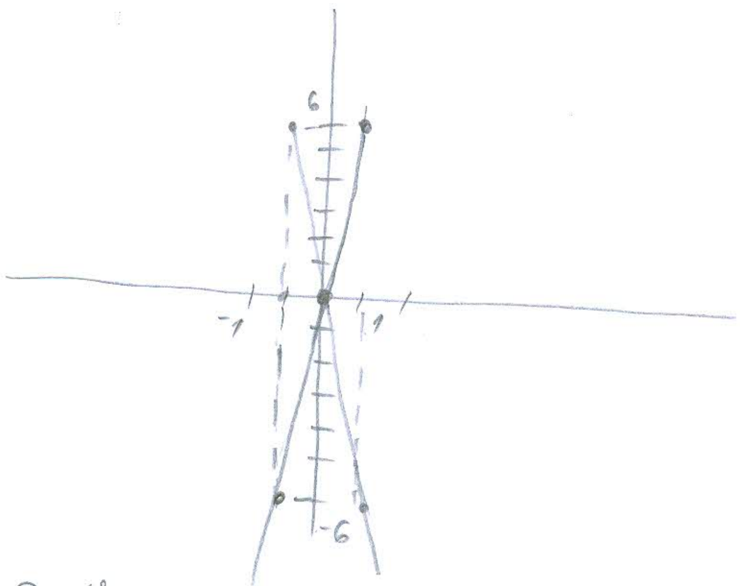
$$-6x^5 = 0 /:(-6)$$

$$x^5 = 0$$

$$x = 0$$

x	-1	0	1
y	-6	0	6

x	-1	0	1
y	6	0	-6



$$3 \cdot 0^2 - 3y = 0$$

$$-3y = 0 /:(-3)$$

$$y = 0$$

$T(0,0)$

$$\frac{\partial f}{\partial x \partial y} = -3$$

$$\frac{\partial f}{\partial y \partial x} = -3$$

$$\Delta = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} = 0 - 9 = -9$$

$\Delta < 0$

Točka $T(0,0)$ je sedlasta točka

$$2.\ \int \frac{x^2+2x+2}{x^2+x-2}\,dx = \int \frac{x^2+2(x+1)}{x(x+1)-2} = \int \frac{x^2}{x(x+1)-2} + 2\int \frac{x+1}{x(x+1)-2}$$

♣2| $$= \int \frac{x}{x+1-2} + 2\int \frac{1}{x-2}$$

$$1.\ b)\ \int_0^{+\infty} \frac{1}{x^2}\,dx = \int_0^{+\infty} x^{-2}\,dx = \frac{x^{-1}}{-1}\Big|_0^{+\infty} = \frac{1}{x}\,dx\Big|_0^{+\infty}$$

$$= \ln|x|\Big|_0^{+\infty}$$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x+a}{x-a} \right\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: MARIN MATEK **Broj indeksa:** 17-1-011-12

Vrijeme: od 08:00 **do** ________ ♣2 **Broj bodova:** 5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$ 5

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{x} - \sqrt{x + y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x - 1)y - x^2 y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

1\. a) $\int \cos^5 x \sin x\, dx = \begin{bmatrix} \cos x = t /' \\ \sin x\, dx = dt \end{bmatrix}$

$= \int t^5 dt = \frac{t^6}{6} = \frac{\cos^6 x}{6} + C$ //

$-\frac{1}{6}\cos^6 x + C$

5

b) $\int_0^{+\infty} \frac{1}{x^2} dx = \lim_{\varepsilon \to 0} \frac{1}{\infty^2} dx - \lim_{\varepsilon \to 0} \frac{1}{0^2} dx = d/P$

∞

2\) $\int \frac{x^2+2x+2}{x^2+x-2} =$

$\cancel{x^2}+2x+2 : (x^2+x-2) = 1$

$-\cancel{x^2} \pm x \mp 2$

$x+4$

$= \int dx + \int \frac{x+4}{x^2+x-2}$

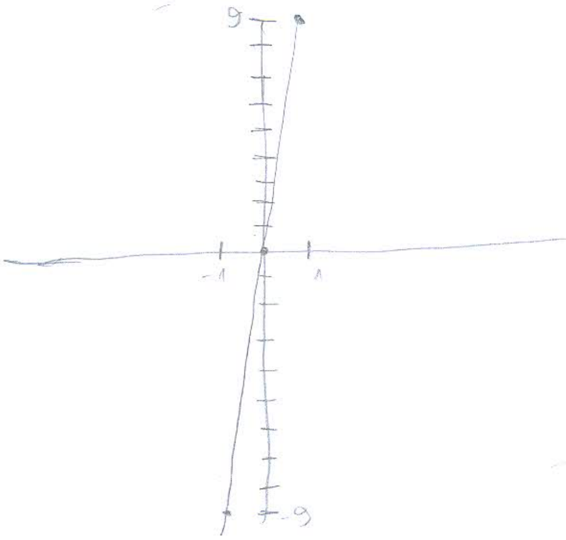
$\int \frac{x+4}{x^2+x-2} = \frac{A}{x^2+x-2} \quad / \cdot x^2+x-2$

$= x +$

$x+4 = A$

3\) $y = 2x^2 + 9$

$y = 9x$



x	-1	0	1
y	-9	0	9

$2x^2 + 9 = 0$

$x^2 = -\frac{9}{2} \quad / \sqrt{\ }$

$x = \pm\sqrt{-\frac{9}{2}}$

POVRŠINA NE POSTOJI //

4\)

$f(x,y) = x^3 - 3xy - y^3$

STAC. TOČKE

$\frac{\partial f}{\partial x} = 3x^2 - 3y$ $\quad \frac{\partial f}{\partial y} = -3x - 3y^2$ $\quad \frac{\partial^2 f}{\partial x^2} = 0$ $\quad \frac{\partial^2 f}{\partial y^2} = 0$

$\frac{\partial^2 f}{\partial x^2} = 6x - 3$ $\quad \frac{\partial^2 f}{\partial y^2} = -3 - 6y$ $\quad 6x - 3 = 0$ $\quad -3 - 6y = 0$

$6x = 3$ $\quad -6y = 3$

$x = \frac{1}{2}$ $\quad y = -\frac{1}{2}$ //

$\frac{\partial^3 f}{\partial x^3} = 6$ $\quad \frac{\partial^3 f}{\partial y^3} = -6$ $\quad \frac{\partial f}{\partial x \partial y} = -3$

4)

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial x \partial y} \\ \frac{\partial f}{\partial x \partial y} & \frac{\partial f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 6 & -3 \\ -3 & -6 \end{vmatrix} = -36 - 9 = -45$$

♣2

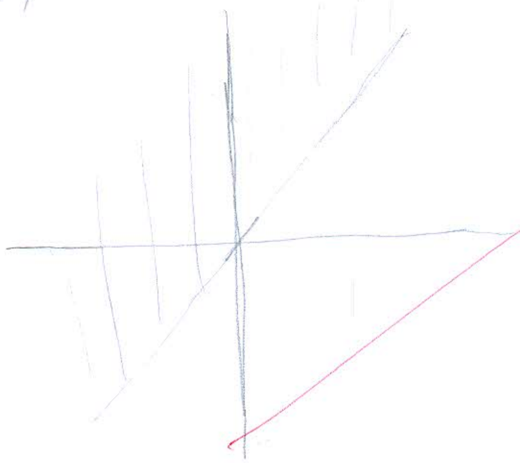
$\Downarrow$ SEDLASTA

b) $f(x,y) = \frac{1}{x} - \sqrt{x+y}$

$\sqrt{x+y} > 0$

$x + y > 0$

$x > y$



5) a) $(x-1)y - x^2 y' = 0 \;/\, (x-1)$

$y - x^2 y' = 0 \quad / -x^2$

$y y' = 0$

$y \frac{dy}{dx} = 0 \quad / \cdot dx$

$\int y\, dy = \int 0$

$\frac{y^2}{2} = C$

b) $y'' + 2y' + y = e^{3x}$

$r^2 + 2r + 1 = 0$

$r_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$

$r_{1,2} = \frac{-2 \pm 0}{2} = -1$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: NIKOLINA KOMLJENOVIĆ **Broj indeksa:** 17-2-0114-2011

Vrijeme: od 8:00 **do** ________ ♣2 **Broj bodova:** Ø

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

① a) $\int \cos^5 x \sin x \, dx = \left[\begin{array}{l} \cos x = t \\ -\sin x \, dx = dt \end{array} \right]$

$\int t^5 \cdot dt = \frac{1}{5} \int t \cdot dt = \frac{1}{5} \int \cos x + c$ //

② $\int \frac{x^2+2x+2}{x^2+x-2} dx = \int dx + \int \frac{-x^2-2}{x^2+x-2} dx = 1 + \int \frac{-x^2-2}{\underset{(x+4)(x-4)}{x^2+x-2}} dx$

$(x^2+2x+2) : (x^2+x-2) = 1$

x^2+2

$-x^2-2$

$= 1 + \int \frac{-x^2-2}{(x+4)(x-4)} dx$

$\frac{-x^2-2}{(x+4)(x-4)} = \frac{A}{(x+4)} + \frac{B}{(x-4)} = / : (x+4)(x-4)$

$-x^2-2 = A(x-4) + B(x-4)$

$-x^2-2 = Ax - A4 + Bx - B4$

$-2 = A + B - 1$ $\quad$ $\cancel{A+B=-1}$

$A = x-4$ $\quad$ $A+B=0$

$B = x-4$ $\quad$ $-4B-4B = -\cancel{8} / : -2$ $\quad$ $B=4$

$A = B$

$-1 =$

$= 1 + \int \frac{4}{(x+4)} dx + \int \frac{4}{(x-4)} dx$

?

$= 1 + 4 \ln|x+4| + 4 \ln|x-4| + C$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \cot x dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1-\frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣2

2

3.) parabola... $y = 2x^2 + 9$

pravac... $y = 9x$

$2x^2 + 9 = 0$

$x^2 + \frac{9}{2} = 0 \,/\sqrt{\ }$

$x + \sqrt{\frac{9}{2}} = 0$

$x + \frac{3}{\sqrt{2}} = 0$

$x = -\frac{3}{\sqrt{2}} \approx -2{,}12$

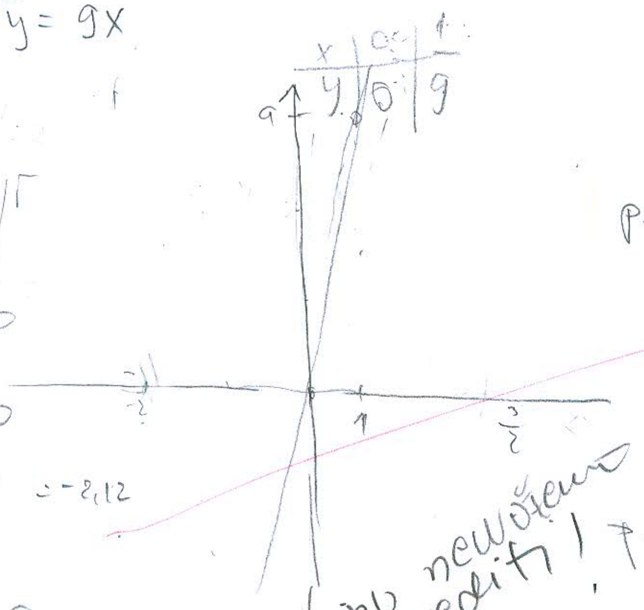
$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{4}$

$x_{1,2} = \frac{9 \pm 3}{4}$ $\quad x_1 = 3$ $\quad x_2 = \frac{3}{2} \approx 1{,}5$



$P = \int_{\frac{3}{2}}^{3} \left(9x - (2x^2 + 9)\right) dx =$

$P = \int_{\frac{3}{2}}^{3} 9x - 2x^2 - 9 \, dx =$

$P = \int_{\frac{3}{2}}^{3} -2x^2 + 9x - 9 \, dx$

$= -\frac{2x^3}{x^3} + \frac{9x^2}{x^2} - 9x \Big|_{\frac{3}{2}}^{3}$

$= \left(-\frac{2 \cdot 3^3}{3^3} + \frac{9 \cdot 3^2}{3^2} - 9 \cdot 3\right) - \left(-\frac{2(\frac{3}{2})^3}{\frac{3}{2}^3} + \frac{9 \cdot \frac{3}{2}^2}{\frac{3}{2}^2} - 9 \cdot \frac{3}{2}\right)$

$= -20 - \left(-\frac{13}{2}\right) = -\frac{27}{2}$ //

površinu nemožemo odrediti!

4

4) a) $f(x,y) = x^3 - 3xy - y^3$

$f'(x,y)_x = 3x^2 - 3y = 0$

$f'(x,y)_y = -3x - 3y = 0$

$3x^2 - 3y = 0 \Rightarrow 3x^2 = 3y \,/ :3$

$x^2 = y$

$3x - 3y = 0$

$3x - 3(x^2)^2 = 0$

$3x - 3x^4 = 0$

$x(3 - 3x^3) = 0$

$x_1 = 0$

$3 - 3x^3 = 0$

$y_1 = 0$ $T_1(0,0)$ $T_2($

$f''(x,y)_x = 6x$

$f''(x,y)_y = -3$

$\frac{df}{dxdy} = 3$

$\frac{df}{dydx} = -3$

$\Delta = \begin{vmatrix} 6x & -3 \\ -3 & 3 \end{vmatrix} = \cancel{6 \cdot 0 \cdot 3} - (-3 \cdot -3) \quad -9 < 0$

ima max u $T(0,5)$

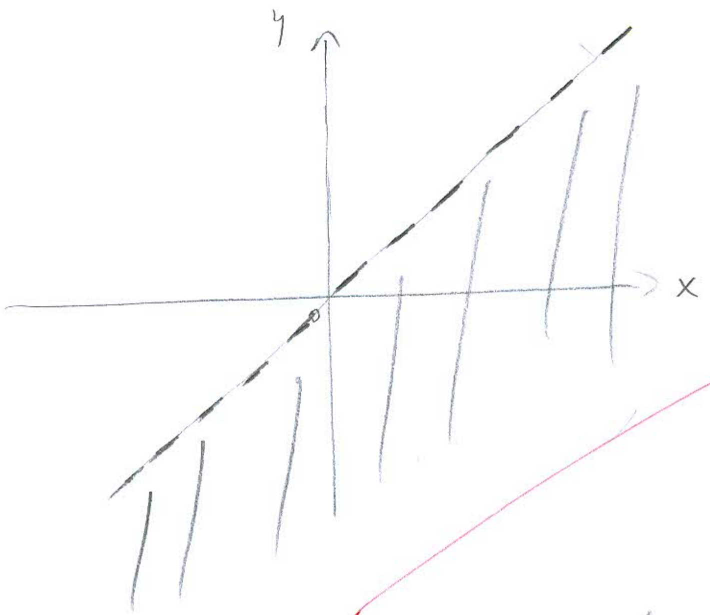
b) $f(x,y) = \frac{1}{x} - \sqrt{x+y}$

I° $x \neq 0$

II° $x + y \geq 0$

$x + y > 0$

$y > -x$



U domenu ne ulazi pravac

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: Ivan Colić **Broj indeksa:** 17-2-0152-2011

Vrijeme: od 09:00 **do** ______ ♣2 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

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b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{x} - \sqrt{x + y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x - 1)y - x^2 y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

b)

$$\int_0^{+\infty} \frac{1}{x^2}\,dx = \int_0^{+\infty} x^{-1-2}\,dx = \int_0^{+\infty} \frac{x^{-2+1}}{-2+1}\,dx = \frac{x^{-1}}{-1}\Big|_0^{+\infty} =$$

$$= \frac{+\infty^{-1}}{-1} - \frac{0^{-1}}{-1} = \frac{+\infty}{-1} = -\infty$$

$$\cos^5 x = 5\cos^4 x \cdot \sin x\,dx = dt$$

$$\sin x\,dx = \frac{dt}{5\cos^4 x}$$

$$\sin^6 x = dt$$

$$6\sin^5 x \cdot \cos\,dx = dt$$

1. a)

$$\int \cos^5 x \,\sin x\,dx = \left|\begin{array}{l} \cos^5 x = t \,|' \\ 5\cos^4 x \cdot \sin x\,dx = dt \\ \sin x\,dx = \dfrac{dt}{5\cos^4 x} \end{array}\right| =$$

$$\int -$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
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$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

4. b)

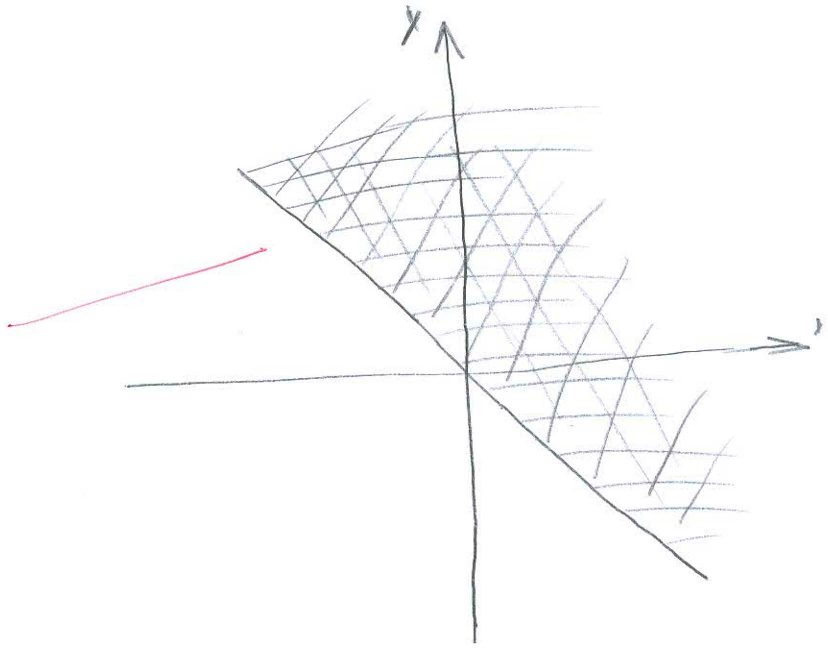
domena

$f(x,y) = \frac{1}{x} - \sqrt{x+y}$

$x + y > 0 \quad x \neq 0$

$x > -y$

$x = -y$



3. $y = 2x^2 + 9$ $\quad 2x^2 + 9 = 9x$

$y = 9x$ $\quad 2x^2 - 9x + 9 = 0$

$$x_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} = \frac{9 \mp \sqrt{(-9)^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 2} = \frac{9 \mp \sqrt{81-72}}{4} = \frac{9 \mp \sqrt{9}}{4} = \frac{9 \mp 3}{4}$$

$x_1 = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} = \frac{3}{2}$

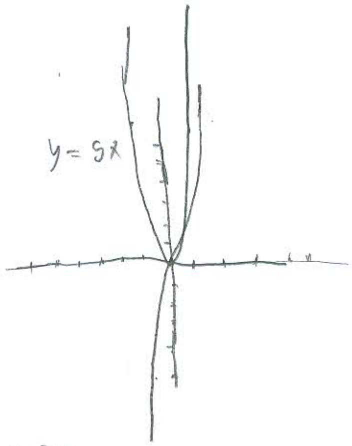
$x_2 = \frac{9+3}{4}$

$x_2 = 3$

$$\int_{\frac{3}{2}}^{3} (2x^2 - 9x + 9)\,dx = \int_{\frac{3}{2}}^{3} 2x^2 dx - \int_{\frac{3}{2}}^{3} 9x\,dx + \int_{\frac{3}{2}}^{3} 9\,dx = 2\frac{x^3}{3}\Big|_{\frac{3}{2}}^{3} - 9\frac{x^2}{2}\Big|_{\frac{3}{2}}^{3} + 9x\Big|_{\frac{3}{2}}^{3} =$$

$$= \left(2\frac{3^3}{3} - 2\cdot\frac{\left(\frac{3}{2}\right)^3}{3}\right) - \left(9\frac{3^2}{2} - 9\frac{\left(\frac{3}{2}\right)^2}{2}\right) + \left(9\cdot 3 - 9\cdot\frac{3}{2}\right) =$$

$$= \frac{2}{3}\left(3^3 - \left(\frac{3}{2}\right)^3\right) - \frac{9}{2}\left(9 - \frac{9}{4}\right) + 9\left(3 - \frac{3}{2}\right) = -\frac{99}{8}$$



$-\frac{207}{8} +$

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: TOMISLAV TUTA **Broj indeksa:** 172 0071 2010

Vrijeme: od ____________ **do** _________ ♣2 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

4.) a) $f(x,y) = x^3 - 3xy - y^3$

$\frac{df}{dx} = 3x^2 - 3y$ $\qquad \frac{d^2f}{dx^2} = 6x$ $\qquad \Delta = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = (6x \cdot 6y) - (-3 \cdot -3)$

$36xy - 9$

$\frac{df}{dy} = -3x - 3y^2$ $\qquad \frac{d^2f}{dy^2} = 6y$

$\frac{df}{dx\,dy} = -3$

2.) $\int \frac{1}{x^2} + \frac{0}{x} + \frac{1}{-2}\, dx$

$\frac{1}{2} \int x^2 \cdot (-1)$

$\frac{1}{2} \int \frac{x^4}{4} \cdot -x + C$

T. TUTA

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \|\frac{a+x}{a-x}\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \|\frac{x+a}{x-a}\| + C$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\int \cot x dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\|x + \sqrt{x^2 \pm a^2}\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣2

3.) PARABOLA – $y = 2x^2 + 9$

PRAVAC – $y = 9x$

$$2x^2 + 9 = 9x$$

$$2x^2 + 9 - 9x$$

$a = 2$

$b = -9$

$c = +9$

$$\int_{\frac{3}{2}}^{3} (2x^2+9) - (9x)\, dx$$

$$\int_{\frac{3}{2}}^{3} (2 \cdot 3^2 + 9) - (9 \cdot 3)\, dx$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{+9 \pm \sqrt{(-9)^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 2}$$

$$x_{1,2} = \frac{+9 \pm \sqrt{81 - 72}}{4}$$

$$x_{1,2} = \frac{+9 \pm 3}{4}$$

$x_1 = 3$

$x_2 = \frac{3}{2}$

$$\int_{\frac{3}{2}}^{3} (27 - 21) = 6 //$$

$$\int_{\frac{3}{2}}^{3} \left(2\left(\frac{3}{2}\right)^2 + 9\right) - \left(9 \cdot \frac{3}{2}\right) = \frac{27}{2} - \frac{27}{2} = 0$$

?

4.) a) $3x^2 - 3y = 0$ $\qquad -3x - 3y^2 = 0$ $\qquad y = x^2$

$3x^2 = 3y \ /:3$ $\qquad -3x - 3x^4 = 0$

$x^2 = y$ $\qquad 3x + 3x^4 = 0$

$x = \sqrt{y}$ $\qquad 6x^4 = 0$

$3(\sqrt{y})^2 - 3y = 0$ $\qquad x^4 = 0$

$\qquad x = 0$ $\qquad 3x^2 - 3y^2 = 0$

$3y - 3y = 0$ $\qquad 3 \cdot 0 - 3y = 0$

$3y = 3y$ $\qquad 0 - 3y = 0$

$y = y$ $\qquad y = 0$

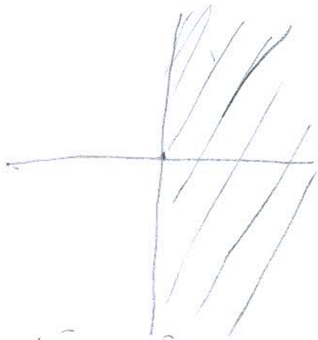
$36 \cdot 0 \cdot 0 - 9 = -9$ - NIJE EKSTREM

$6 \cdot 0 = 0$

b) $f(x, y) = \frac{1}{x} - \sqrt{x+y}$

$1^\circ \ x \neq 0$

$2^\circ \ x + y \geq 0$



DOMENA

2.) $\int \frac{x^2+2x+2}{x^2+x-2} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{+2} \ / \cdot (x^2+x-2)$

$x^2 + 2x + 2 = A(x-2) + B(x^2-2) + C(x^2+x)$

$x^2 + 2x + 2 = Ax - 2A + Bx^2 - 2B + Cx^2 + Cx$

$x^2 + 2x + 2 = x^2(B+C) + x(A+C) + 2(-A-B)$

$1 = (B+C)$ $\qquad 1 = B + C \qquad B = C - 1$ $\qquad A + C = 2$

$2 = (A+C)$ $\qquad 1 = C - 1 + C$ $\qquad A + 1 = 2$

$2 = (-A-B)$ $\qquad C - 1 + C = 1$ $\qquad A = 1$

$C + C = 2$ $\qquad B = 1 - 1$

$2C = 2$ $\qquad B = 0$

$C = 1$

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: DARIAN RAJMAN **Broj indeksa:** 57635 -2005.

Vrijeme: od____________ **do**_________ ♣**2** **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \cos^5 x \sin x dx$$

b)

$$\int_0^{+\infty} \frac{1}{x^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x^2+2x+2}{x^2+x-2} dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 + 9$ i pravac $y = 9x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 - 3xy - y^3$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{x} - \sqrt{x+y}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$(x-1)y - x^2 y' = 0$$

b)

$$y'' + 2y' + y = e^{3x}$$

3 $y = 2x^2 + 9$
$y = 9x$

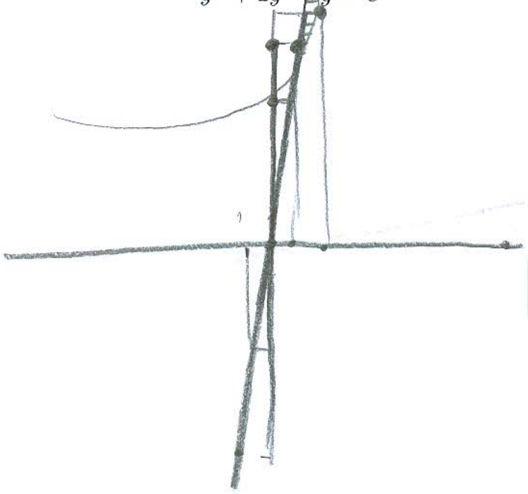
$y = 9x$	y
0	0
1	9
-1	-9

$y = 2x^2 + 9$

$2x^2 + 9 = 0$

x

$y = 2x^2 + 9$	y
0	9
1	11
2	17



(4) b) $f(x,y) = \frac{1}{x} - \sqrt{x+y}$

$x+y \geqslant 0$

$2+2 \geqslant 0$

$4 \geqslant 0$

$D = \mathbb{R}$

$x > 0$

a) $f(x,y) = x^3 - 3xy - y^3$

$\frac{df}{dx} = 3x^2 - 3y \Rightarrow 3x^2 - 3y = 0 \Rightarrow 3x^2 = 3y \quad /:3$

$x^2 = y$

$x = \sqrt{y}$

$\frac{df}{dy} = -3x - 3y^2 \Rightarrow -3x - 3y^2 = 0 \Rightarrow -3y^2 = +3x$

$y^2 = \frac{\cancel{3}x}{-\cancel{3}}$

$y^2 = -x$

$y = \sqrt{-x}$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

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♣2