MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: FRANO ŽIVKOVIĆ **Broj indeksa:** 54958-2007

Vrijeme: od 08:15 **do** ______ ♣1 **Broj bodova:** ~~35~~ 47

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx$$ ✓ 12

b)

$$\int_0^{+\infty} \frac{1}{(x+1)^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)} dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$. ✓ 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$ 10

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2 - 1)$$ 10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x} y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

4\. a) Ispitaj ekstreme funkcije: $f(x,y) = x^2 - 2x + 1 + y^2 - 9$

$$\frac{\partial f}{\partial x} = 2x - 2 \qquad 2x - 2 = 0 \qquad 2y = 0 \;/:2$$

$$2x = 2 \;/:2 \qquad y = 0$$

$$\frac{\partial f}{\partial y} = 2y \qquad x = 1$$

Stacionarne točke su $T(1,0)$

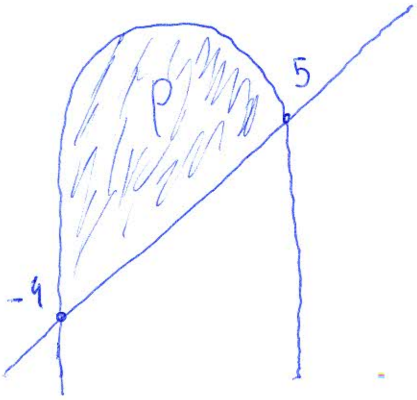
$$\frac{\partial^2 f}{\partial x^2} = 2 = A \qquad AC - B^2 = 2\cdot 2 - 0^2 = 4 > 0 \quad \text{IMA EKSTREMA}$$

$$A = 2 > 0 \quad \text{MINIMUM}$$

$$\frac{\partial^2 f}{\partial y \partial x} = 0 = B$$

$$\frac{\partial^2 f}{\partial y^2} = 2 = C$$

3\. Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.



$$1 + 3x - x^2 - 2x + 1 = 0$$

$$2 + x - x^2 = 0$$

$$a = -1,\; b = 1,\; c = 2$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4(-1)\cdot 2}}{2\cdot(-1)} \qquad x_{1,2} = \frac{-1 \pm 9}{-2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-(-8)}}{-2} \qquad x_1 = \frac{-1+9}{-2} = -4$$

$$x_2 = \frac{-1-9}{-2} = 5$$

$$\int_{-4}^{5} \left(1 + 3x - x^2 - 2x + 1\right) dx =$$

$$= \int_{-4}^{5} \left(2 + x - x^2\right) dx = \int_{-4}^{5} 2\,dx + \int_{-4}^{5} x\,dx - \int_{-4}^{5} x^2\,dx = \left(2x + \frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_{-4}^{5}$$

$$= \left(2\cdot 5 + \frac{(5)^2}{2} - \frac{(5)^3}{3}\right) - \left(2\cdot(-4) + \frac{(-4)^2}{2} - \frac{(-4)^3}{3}\right) = \left(2\cdot 5 + \frac{25}{2} - \frac{125}{3}\right) - \left(2\cdot(-4) + \frac{16}{2} - \frac{-(64)}{3}\right) =$$

$$= -19.166$$

KRIVO

ISPRAVLJENI ZADATAK NA DRUGOM PAPIRU

FRANO ZIVKOVIC

♣1

① a) $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \left[\begin{array}{l} \frac{1}{x} = t \,/\,' \\ x^{-1} = t \,/\,' \\ -1x^{-2}dx = dt \\ -x^{-2}dx = dt \,/\cdot(-1) \\ x^{-2}dx = -dt \\ \frac{1}{x^2}\,dx = -dt \end{array}\right] = \cancel{\int e^{t}} = -\int e^{t}dt = -e^{t} + C = -e^{\frac{1}{x}} + C$

② $\int \frac{x}{(x+2)(x^2+1)}\,dx = \int \frac{A}{x+2}dx + \cancel{\int \frac{Bx+C}{x(x+1)}}\,dx$

③

$1+3x-x^2-2x+1=0$

$2+x-x^2=0$

$x_{1,2}=\frac{-1\pm\sqrt{9}}{-2}$

$x_1=\frac{-1+3}{-2}=-1$

$x_2=\frac{-1-3}{-2}=2$

$$\int_{-1}^{2}\left(2+x-x^2\right)dx=\left(2x+\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_{-1}^{2}$$

$$=\left(2\cdot 2+\frac{(2)^2}{2}-\frac{(2)^3}{3}\right)-\left(2\cdot(-1)+\frac{(-1)^2}{2}-\frac{(-1)^3}{3}\right)=3.33-(-1.166)=$$

$P=4.496$ ← Žurio sam se ̈n

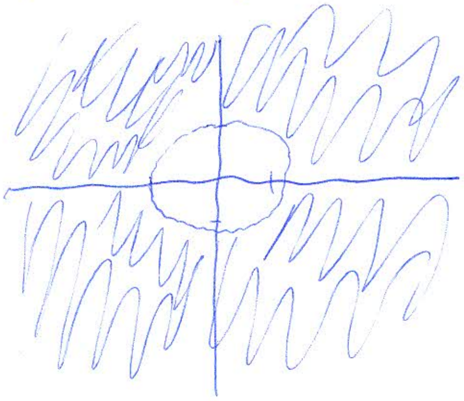
15

④ a) Odredi domenu funkcije: $f(x,y)=\ln(x^2+y^2-1)$

$x^2+y^2-1>0$

$x^2+y^2>1$

$S(0,0)\quad r=1$



Domena se nalazi na ravnini izvan kružnice.

10

①
$$\int_0^{+\infty}\frac{1}{(x+1)^2}dx=\begin{bmatrix}x+1=t \\ dx=dt\end{bmatrix}=\int_0^{+\infty}\frac{dt}{t^2}=\int_0^{+\infty}t^{-2}dt=\frac{t^{-1}}{-1}\Big|_0^{+\infty}$$

$$=\frac{-1}{t}\Big|_0^{+\infty}=\cancel{\frac{-1}{+\infty}-\frac{(-1)}{0}}=\frac{-1}{x+1}\Big|_0^{+\infty}=\frac{-1}{+\infty+1}-\frac{(-1)}{0+1}=$$

$$=\frac{-1}{+\infty+1}+1=\frac{-1}{+\infty+1}$$

?

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1

Kristi Jozi

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: KRISTIAN JOZIĆ **Broj indeksa:** 17-1-0012-2010

Vrijeme: od 8:14 **do** ________ ♣1 **Broj bodova:** 25

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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4. *(10+10)*

a) ~~Ispitaj ekstreme funkcije~~

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5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x} y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$ 15 ✓ 15

$x + y > 0$

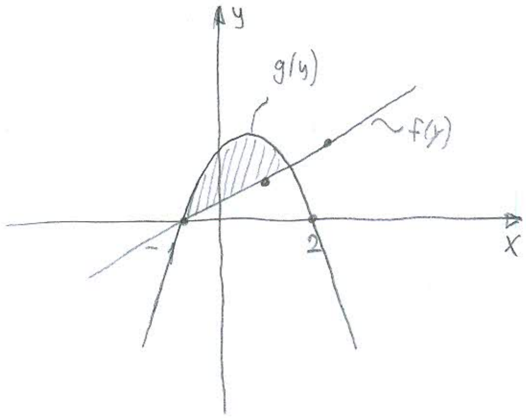
$y > -x$

3. $f(y) = 2x - 1$

$g(y) = 1 + 3x - x^2$

$P = ?$

$y = 2x-1$	x
-1	0
1	1
3	2



$$2x - 1 = 1 + 3x - x^2$$

$$2x - 1 - 1 - 3x + x^2 = 0$$

$$x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2}$$

$$x_{1,2} = \frac{1 \pm 3}{2}$$

$$x_1 = \frac{1+3}{2} = \frac{4}{2} = 2$$

$$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1$$

$$P = \int_{-1}^{2} \left(g(x) - f(x)\right) dx$$

$$P = \int_{-1}^{2} \left((1 + 3x - x^2) - (2x - 1)\right) dx$$

$$= \int_{-1}^{2} (1 + 3x - x^2 - 2x + 1)\, dx$$

$$= \int_{-1}^{2} (-x^2 + x + 2)\, dx$$

$$= \int_{-1}^{2} -x^2\, dx + \int_{-1}^{2} x\, dx + \int_{-1}^{2} 2\, dx$$

$$= \int_{-1}^{2} -\frac{x^3}{3}\, dx + \int_{-1}^{2} \frac{x^2}{2} + \int_{-1}^{2} 2\, dx$$

$$= -\frac{1}{3} x^3\, dx \Big|_{-1}^{2} + \frac{1}{2} x^2 \Big|_{-1}^{2} + 2x \Big|_{-1}^{2}$$

$$= -\frac{1}{3} \cdot \left(2^3 - (-1)^3\right) + \frac{1}{2} \cdot \left(2^2 - (-1)^2\right) + 2 \cdot \left(2 - (-1)^2\right)$$

$$= -\frac{1}{3} \cdot (8 + 1) + \frac{1}{2} \cdot (4 - 1) + 2 \cdot (2 - 1)$$

$$= -\frac{1}{3} \cdot 9 + \frac{1}{2} \cdot 3 + 2 \cdot 1$$

$$= -3 + \frac{3}{2} + 2 = \frac{1}{2}$$

$$\boxed{P = \frac{1}{2}}$$

4. a) $f(x,y) = x^2 - 2x + 1 + y^2 - 9$

$\frac{\partial f}{\partial x} = 2x - 2 \Rightarrow 2x - 2 = 0 \Rightarrow 2x = 2 \; /:2 \quad x = 1$

$T(1,0)$

MINIMUM

$\frac{\partial f}{\partial y} = 2y \Rightarrow 2y = 0 \Rightarrow y = 0$

$\frac{\partial^2 f}{\partial x^2} = \boxed{2} \qquad \frac{\partial f}{\partial y \partial x} = 0$

$\frac{\partial^2 f}{\partial y^2} = \boxed{2} \qquad \frac{\partial f}{\partial x \partial y} = 0$

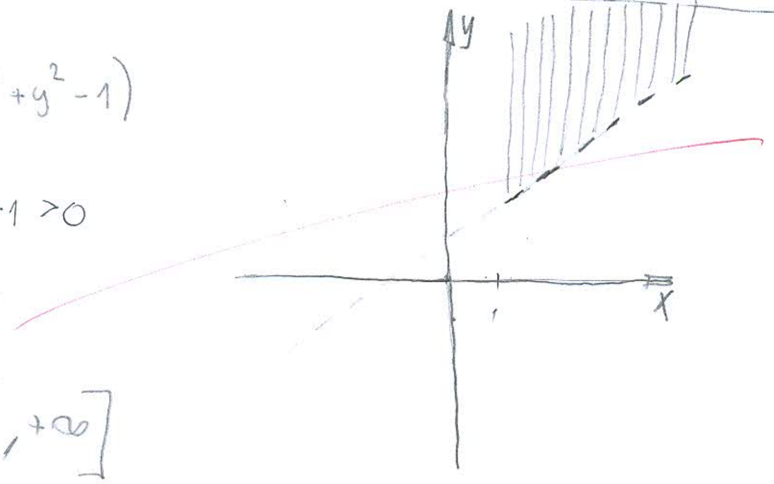
$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{df}{dydx} \\ \frac{df}{dydx} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

$\Delta = 2 \cdot 2 - 0 \cdot 0 = 4$

$T(1,0)$ = MINIMUM

4. b) $f(x,y) = \ln(x^2 + y^2 - 1)$

$x^2 + y^2 - 1 > 0$



$\mathcal{D} = \mathbb{R} \; [1, +\infty]$

$x^2 - 1$	y
-1	0
0	1
3	2

$x^2 + y^2 - 1 > 0$

$y^2 > -x^2 - 1$

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$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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♣1

5. b) $y'' + 3y' + 2y = e^{2x}$

$y = y_0 + y$

$y'' + 3y' + 2y = 0$

$\lambda_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\lambda_{1,2} = \dfrac{-3 \pm \sqrt{9-8}}{2}$

$\lambda_{1,2} = \dfrac{-3 \pm 1}{2} = \quad \lambda_1 = \dfrac{-3-1}{2} = \boxed{-2} \qquad \lambda_2 = \dfrac{-3+1}{2} = \boxed{-1}$

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$\boxed{y = A \cdot e^{2x}}$

$y' = (A \cdot e^{2x})$

$y' = A \cdot e^{2x} \cdot 2$

$y' = 2A\,e^{2x}$

$y'' = 2Ae^{2x} \cdot 2$

$y'' = 4Ae^{2x}$

$y'' + 3y' + 2y = e^{2x}$

$4Ae^{2x} + 3 \cdot (2Ae^{2x}) + 2 \cdot (A \cdot e^{2x}) = e^{2x}$

$4Ae^{2x} + 6Ae^{2x} + 2Ae^{2x} = e^{2x}$

$12Ae^{2x} = e^{2x} \quad / : e^{2x}$

$12A = 1$

$\boxed{A = \dfrac{1}{12}}$

$y = A \cdot e^{2x}$

$y = \dfrac{1}{12} e^{2x}$ ✓

$\lambda_1 \neq \lambda_2 \Rightarrow y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

$C_1, C_2 \in \mathbb{R}$

$\boxed{y = y_0 + y = C_1 e^{-2x} + C_2 e^{-1x}, \quad C_1, C_2 \in \mathbb{R}}$

ꝛ

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1.5) $\int_0^{+\infty} \frac{1}{(x+1)^2}\,dx = \lim_{\varepsilon \to \infty} \int_0^{\varepsilon} \frac{1}{(x+1)^2}\,dx$

$= \lim_{\varepsilon \to \infty} \int_0^{\varepsilon}$

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: MARKO PARANCIN **Broj indeksa:** 17-1-0062-2011

Vrijeme: od 08:00 **do** 09:20 ♣1 **Broj bodova:** 20

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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$$\int \frac{x}{(x+2)(x^2+1)}dx$$

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4. *(10+10)*

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a)

$$y' + \frac{2}{x}y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

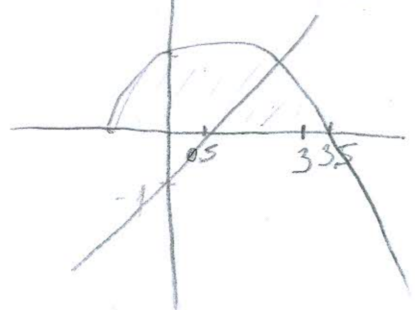
$y = 2x - 1 \quad y = 1 + 3x - x^2$

$2x - 1 = 1 + 3x - x^2$

$-x + x^2 - 2 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 1(-2)}}{2} = \frac{1 + \sqrt{9}}{2}$ $\quad x_1 = \frac{1+3}{2} = \frac{4}{2} = 2$

$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1$



$1 = \int_{-1}^{2} 1+3x+x^2-(2x-1) = \int_{-1}^{2} 2+x-x^2$

$1 = \int_{-1}^{2} 2dx + \int_{-1}^{2} x\,dx - \int_{-1}^{2} x^2\,dx$

$= 2x\Big|_{-1}^{2} + x\Big|_{-1}^{2} - \frac{1}{3}x\Big|_{-1}^{2}$

$= 2\cdot(2-(-1)) + (2-(-1)) - \frac{1}{3}(2^3-(-1)^3)$

$= 6+\frac{3}{2}-\frac{9}{3} = 6+\frac{3}{2}-3 = \frac{12+3-3}{2} = \frac{12}{2} = 6$

GREŠKA U ZBRAJANJU

(10)

4 a) $f(x,y) = x^2-2x+1+y^2-9$

b) $f(x,y) = \ln(x^2+y^2-1)$

a) $\frac{\partial f}{\partial x} = 2x-2$, $\frac{\partial f}{\partial y} = 2y$

$\frac{\partial^2 f}{\partial x^2} = 2$ $\quad \frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial f}{\partial x \partial y} = (2y) = 0$

$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$

$2x-2=0 \qquad 2y=0 \qquad T(1,0,-9)$

$2x=2 \qquad y=0$

$x=1$

$D = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4>0 \qquad T$ (MINIMUM)

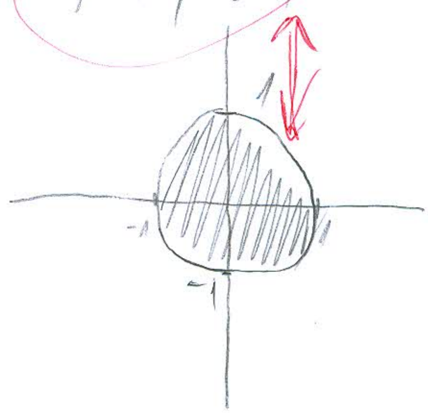
$\frac{\partial^2 f}{\partial x^2} > 0$

b) $\ln(x^2+y^2-1)$

$\ln x > 0$

$x^2+y^2-1>0$

$x^2+y^2>1$



LIST 2

MARKO PARANCIN

(5a) $y' = \frac{2}{x}y = x^3$

$y = e^{-\int P(x)} \cdot \left(Q(x) \cdot e^{\int P(x)} dt \right)$

$\int P(x) = \int \frac{2}{x} dx = 2\ln(x)$

$Q(x) \cdot e^{\int P(x)} dx = \int x^3 - e^{2\ln(x)} = \int x^3 \cancel{e}^{\ln} (x+2)$?

$= \int x^3 - x^{+2} = \frac{x^4}{4}$?

$y = e^{-2\ln(x)} \cdot \left(\frac{x^4}{4} \cdot \frac{x^3}{3} + c \right)$

$y = x^{-2} \cdot \left(\frac{x^4}{4} \cdot \frac{x^3}{3} + c \right)$

(1a) $\int_0^{+\infty} \frac{1}{(x+1)^2} dx$

$\left[\begin{array}{l} x+1 = t^2 \,/\, ' = \\ dx = 2dt \end{array} \right]$

$= \int_0^{+\infty} \frac{2dt}{t^2} = 2\int_0^{+\infty} t^{-2} dt$

$= 2 \cdot \frac{t^1}{1} \Big|_0^{\infty} = 2 \lim_{x \to \infty} \frac{t^1}{1} \Big|_{\infty}^{0}$

$= 2 \cdot (0^1 - \infty) = 0 + C$

$0 - \infty =$

♣1

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$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x\rvert + C$	$\int \cot x dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1

Antun Žanetić

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: Antun Žanetić **Broj indeksa:** 17-2-0169-2012

Vrijeme: od 08:00 **do** 11:00 ♣1 **Broj bodova:** 18

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx$$

~~b)~~

$$\int_0^{+\infty} \frac{1}{(x+1)^2}dx$$ ✓ 8

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

4. *(10+10)*

~~a)~~ Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$ ✓ 10

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x}y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

1. b)

$$\int_0^{+\infty} \frac{1}{(1+x)^2}\,dx$$

$$F(x) = \int \frac{1}{(1+x)^2}\,dx = \left| \begin{matrix} 1+x = t /' \\ dx = dt \end{matrix} \right| = \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt =$$

$$= -t^{-1} + C = -(1+x)^{-1} + C = -\frac{1}{1+x} + C$$

$$\lim_{b\to+\infty} \int_0^b \frac{1}{(1+x)^2}\,dx = \lim_{b\to+\infty} \left(-\frac{1}{1+x}\right)\Bigg|_0^b = \lim_{b\to+\infty} \left(\frac{-1}{1+b}\right) - \left(-\frac{1}{1+0}\right) =$$

$$= 0 + 1 = 1$$

4. a) $f(x,y) = x^2 - 2x + 1 + y^2 - 9$

$$\frac{\partial f}{\partial x} = 2x - 2 \qquad \frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$2x - 2 = 0$

$2x = 2$

$x = 1$

$2y = 0$

$y = 0$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$$

$\frac{\partial^2 f}{\partial x^2} > 0 \Rightarrow$ minimum

$T(1, 0, z_0)$

$\Downarrow$

$T(1, 0, -9)$

$z_0 = 1 - 2 + 1 + 0 - 9$

$z_0 = -9$

10

3. $y = 2x - 1$ – gornja f

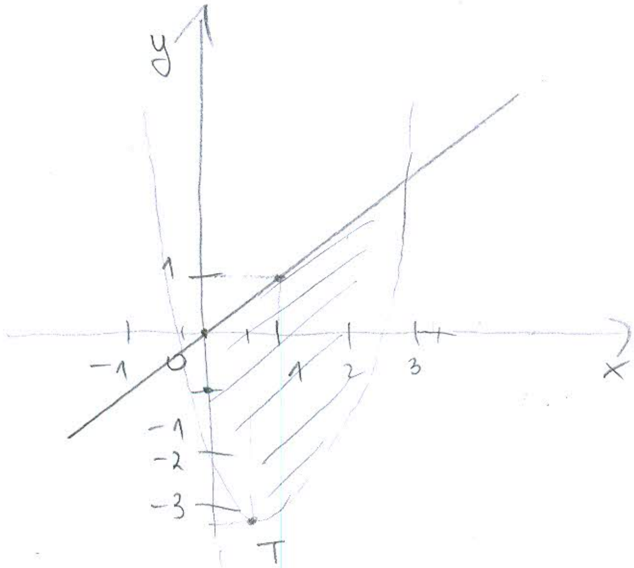
$y = 1 + 3x - x^2$ – donja f.

$y = 2x - 1$

x	0	1
y	-1	1

$y = 1 + 3x - x^2$ ♣2|

$y = -x^2 + 3x + 1$



$-x^2 + 3x + 1 = 0 \quad / \cdot (-1)$

$x^2 - 3x - 1 = 0$

$x_{1,2} = \dfrac{3 \pm \sqrt{9+4}}{2}$

$x_{1,2} = \dfrac{3 \pm \sqrt{13}}{2}$

$x_1 = \dfrac{3 + \sqrt{13}}{2}$ // 3.3

$x_2 = \dfrac{3 - \sqrt{13}}{2}$ // -0.30

$T\left(-\dfrac{2a}{b}, \dfrac{4ac - b^2}{4a}\right)$

$T\left(\dfrac{2}{3}, \dfrac{-4-9}{4}\right)$

$T\left(\dfrac{2}{3}, -\dfrac{13}{4}\right)$

$P = \int_{-1}^{2} (2x-1) - (1+3x-x^2)$

$P = \int_{-1}^{2} 2x - 1 - 1 - 3x + x^2$

$P = \int_{-1}^{2} x^2 - x - 2$

$F(x) = \int (x^2 - x - 2)\,dx$

$F(x) = \int x^2 dx - \int x dx - 2\int dx$

$F(x) = \dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x + c$

$2x - 1 = -x^2 + 3x + 1$

$2x - 1 + x^2 - 3x - 1 = 0$

$x^2 - x - 2 = 0$

$x_{1,2} = \dfrac{1 \pm \sqrt{1+8}}{2}$

$x_{1,2} = \dfrac{1 \pm \sqrt{9}}{2}$

$x_{1,2} = \dfrac{1 \pm 3}{2}$

$x_1 = \dfrac{4}{2} = 2$

$x_2 = \dfrac{-2}{2} = -1$

$P = \left(\dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x\right)\Big|_{-1}^{2}$

$P = \left(\dfrac{8}{3} - 2 - 4\right) - \left(-\dfrac{1}{3} - \dfrac{1}{2} + 2\right)$

$P = -\dfrac{9}{2}$??

1\. a) $\int \frac{e^{\frac{1}{x}}}{x^2}dx = \left| \begin{array}{l} \frac{1}{x} = t \\ x^{-1} = t \;/' \\ -x^{-2}dx = dt \\ x^{-2}dx = -dt \end{array} \right| = \int e^t(-dt) = -\int e^t dt =$

$= -e^t + C = -e^{\frac{1}{x}} + C$ //

2\. $\int \frac{x}{(x+2)(x^2+1)}dx = \int \frac{x}{x^3+x+2x^2+2} = \int \frac{x}{x^3+2x^2+x+2}$ *

* $\frac{x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{B}{x^2+1} \quad / \cdot (x+2)(x^2+1)$

$x = A(x^2+1) + B(x+2)$

$x = Ax^2 + A + Bx + 2B$

$x = Ax^2 + Bx + A + 2B$

$A + 2B = 0$

5. a) $y' + \frac{2}{x} y = x^3$

$y' + \frac{2y}{x} = x^3$

$\frac{dy}{dx} + \frac{2y}{x} = x^3$ ♣1

$\frac{dy}{dx} = x^3 - \frac{2y}{x} \quad / \cdot dx$

$dy = \left(x^3 - \frac{2y}{x}\right) \cdot dx$

$dy = \left(\frac{x^4 - 2y}{x}\right) dx \quad / : y$

$\frac{dy}{y} = \frac{x^4 - 2}{x} dx \quad / \int$

$\int \frac{dy}{y} = \boxed{\int \frac{x^4 - 2}{x} dx}$ *

* $\int \frac{x^4 - 2}{x} dx = \int \frac{x^4}{x} dx - \int \frac{2}{x} dx = \int x^3 dx - 2\int \frac{dx}{x} =$

$= \frac{x^4}{4} - 2 \cdot \ln|x| + C$

$\ln|y| = \frac{x^4}{4} - 2 \cdot \ln|x| + C \quad / e^{\square}$

$y = e^{\frac{x^4}{4} - 2\ln|x| + C}$

$y = \frac{e^{\frac{x^4}{4}}}{e^{2\ln|x|}} \cdot e^C$

$y = \frac{e^{\frac{x^4}{4}}}{x^2} \cdot C$ //

$C = C(x)$

$y = \frac{e^{\frac{x^4}{4}}}{x^2} \cdot C(x) \quad /'$

$y' = \frac{e^{\frac{x^4}{4}} \cdot x^2 - e^{\frac{x^4}{4}} \cdot 2x}{x^4} \cdot C(x) + \frac{e^{\frac{x^4}{4}}}{x^2} \cdot C'(x) =$

?

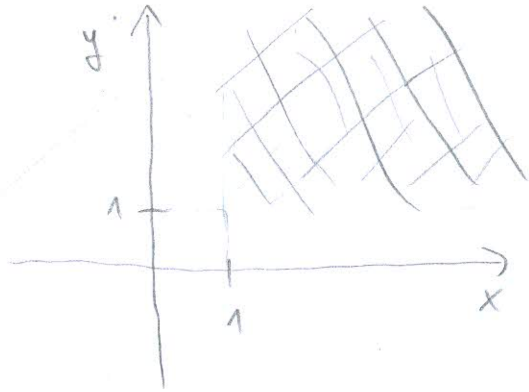
Antun Žanetić

$f(x,y) = \ln(x^2+y^2-1)$

$x^2+y^2-1>0$

$x^2+y^2>1$

$$\boxed{\begin{matrix} x^2>1 \\ y^2>1 \end{matrix}}$$



$D_f = \{(x,y): x^2>1, y^2>1\}$

5. b) $y''+3y'+2y = e^{2x}$

$a=1$
$b=3$
$c=2$

$k^2+3k+2=0$

$k_{1,2} = \frac{-3\pm\sqrt{9-8}}{2}$

$k_{1,2} = \frac{-3\pm 1}{2}$

$k_1 = \frac{-3+1}{2} = -2$

$k_2 = \frac{-3-1}{2} = -4$

$y_1 = e^{-2x}$

$y_2 = e^{-4x}$

$$\boxed{y_H = C_1\cdot e^{-2x} + C_2\cdot e^{-4x}}$$

PARTIKULARNO RJEŠENJE:

$f(x) = e^{2x}$

$y_p = 2Ax^2+Bx+C$ $/'$

$y_p' = 4Ax+B$ $/''$

$y_p'' = 4A$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: BRANIMIR PIJACA **Broj indeksa:** 17-2-0086-2011

Vrijeme: od ____________ **do** _________ ♣1 **Broj bodova:** 10

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx$$

b)

$$\int_0^{+\infty} \frac{1}{(x+1)^2}dx$$

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$ ✓ 10

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x}y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

1a

$$\int \frac{x^2}{e^{-x}}dx \quad \left| \begin{array}{l} t = e^{-x} \\ dt = e^{-x}dx \\ dx = \frac{dt}{e^{-x}} \end{array} \right.$$

$$\int \frac{x^2\,dx}{t^2}$$

$$\int t^{-2}x^2\,dt$$

$$\int x^2\,dt = \frac{x^{2+1}}{2+1} + C \quad \alpha \neq -1$$

$$= \frac{x^3}{3} + C$$

5a

$$\cancel{y' = \frac{2}{x}y = x^3} \qquad y' = \frac{dy}{dx}$$

$$\cancel{\frac{dy}{dx} = -\frac{2y}{x}} \;/dx$$

$$\cancel{dy = \frac{-2y}{x}dx} \;|\,y$$

$$\cancel{\frac{dy}{y} = -2\frac{dx}{x}} \;/\int$$

$$\cancel{\int\frac{dy}{y} = \int -\frac{2dx}{x}}$$

$$\cancel{\ln|y| = \ln|a| + C}$$

③ parabola $y = 1+3x-x^2$

prava $y = 2x-1$

a) -1

$b = 3$

$c = 1$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{3^2+4}}{-2}$

$x_1 =$

$2x-1 = 1+3x-x^2$

$x^2-3x-1+2x-1=0$

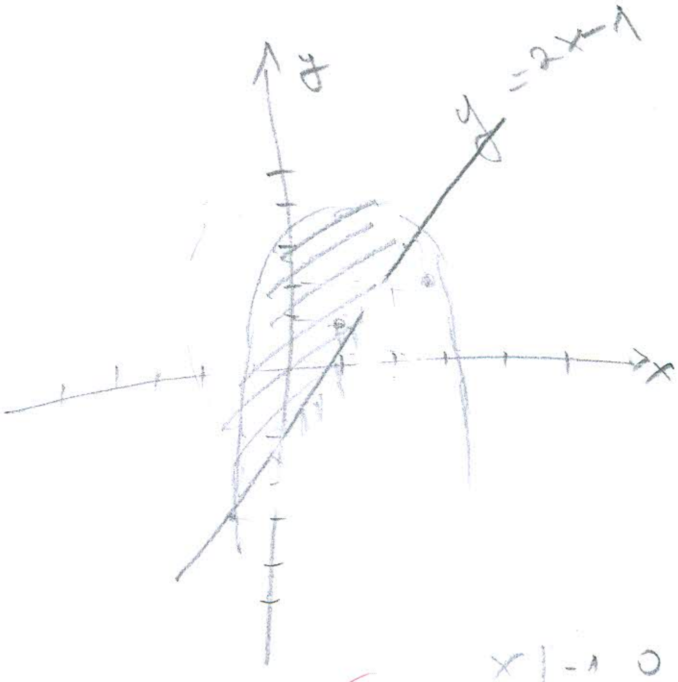
$x^2-x-2=0$ $\quad a=1 \quad b=-1 \quad c=-2$

$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{12} = \frac{1 \pm \sqrt{1+8}}{2}$

$= \frac{1+\sqrt{9}}{2} \qquad = \frac{1-\sqrt{9}}{2}$

$x_1 = 2 \qquad x_2 = -1$



x	-1	0	1	2
y	-3	-1	1	3

$\int_{-1}^{2} x^2-x-2 \, dx$

$\frac{x^3}{3}\Big|_{-1}^{2} - \frac{x^2}{2}\Big|_{-1}^{2} - 2x\Big|_{-1}^{2}$

$P = \left(\frac{2^3}{3} - \frac{-1^3}{3}\right) - \left(\frac{2^2}{2} - \frac{+1^2}{2}\right) - (-4-2)$

$P = \frac{23}{2} = 11{,}5$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln\lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \cot x dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\lvert x+\sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1-\frac{x}{a}) + C$
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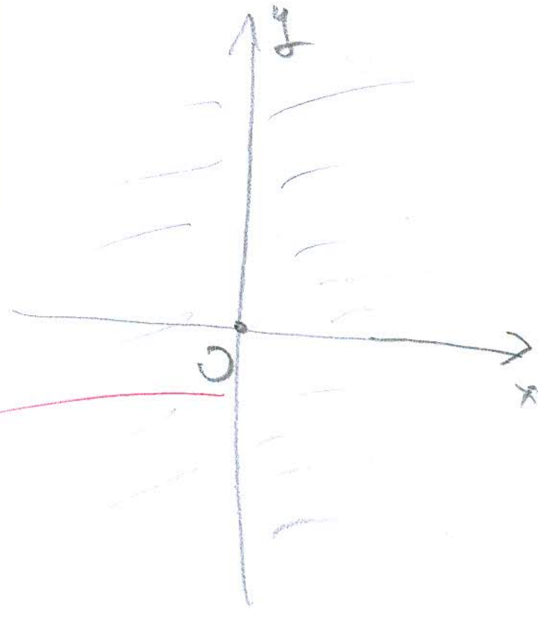
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

4b $f(x,y) = \ln(x^2+y^2-1)$

$x^2+y^2-1 \neq 0$

$D\{x y;\ \mathbb{R}/0\}$



4a) $f(x,y) = x^2 - 2x + 1 + y^2 = 0$

$2x - 2 = 0$ $\quad 2y = 0$

$2x = 2$ $\quad y = 0$

$x = 1$

$Z_x = 2x - 2$ $\quad Z_y = 2y$

$T(1,0)$

$Z_{x^2} = 2$ $\quad Z_{y^2} = 2$

✓ 10

$Z_{xy} = 0$

$$\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = (4 - 0) = 4 > 0$$

imamo ekstrem (minimum)

5a $y' + \frac{2}{x} y = x^3$

~~$\frac{dy}{dx} + \frac{2}{x} y = x^3$~~

~~$y' = \frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{-2y}{x} \mid dx$~~

~~$\int \frac{dy}{y} = \int \frac{-2\,dx}{x}$ $\quad dy = \frac{-2y}{x} dx \;/\; y$~~

~~$\frac{dy}{y} = -\frac{2}{x} dx \;/\int$~~

---

1) b $\int_0^{+\infty} \frac{1}{(x+1)^2} dx$ $\quad \left[\begin{array}{l} t = (x+1) \\ dt = dx \end{array} \right.$ $\quad x^2 + 2ax + 1$

$$\int_1^{+\infty} \frac{dt}{t^2} = \int_1^{+\infty} t^{-2} dt$$

$$\int_1^{\infty} (x+1)^{-2} dt = \frac{(x+1)^{-1}}{-1} \Big|_1^{\infty}$$

$$= \frac{1}{x+1} \Big|_1^{\infty} = \frac{1}{\infty} - \frac{1}{2} = -\frac{1}{2}$$

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: ANĐELO ŽMIRĆ **Broj indeksa:** 0269065578

Vrijeme: od 8:14 **do** 9:43 ♣1 **Broj bodova:** 10

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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$$\int \frac{x}{(x+2)(x^2+1)}dx$$

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b)

$$y'' + 3y' + 2y = e^{2x}$$

(1.) a) $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \begin{bmatrix} e^{\frac{1}{x}} = t \,/' \\ \frac{1}{2}\frac{e^{\frac{1}{x}}}{x^2}\,dx = dt \end{bmatrix} = \frac{1}{2}\int dt = \frac{1}{2}t + c$

$= \frac{1}{2}e^{\frac{1}{x}} + c = \frac{e^{\frac{1}{x}}}{2} + c$

b) $\int_0^{+\infty} \frac{1}{(x+1)^2}\,dx = \begin{bmatrix} t = x+1 \,/' \\ dt = dx \end{bmatrix} = \int_0^{+\infty} \frac{dt}{t^2} = \int_0^{+\infty} t^{-2}\,dt = \frac{t^{-1}}{-1}\Big|_0^{+\infty} = -\frac{1}{t}\Big|_0^{+\infty} = -\frac{1}{(x+1)^2}\Big|_0^{+\infty} =$

$= -\frac{1}{+\infty} + 0 = 0$

(2.)

$\int \frac{x}{(x+2)(x^2+1)}\,dx = \int \frac{dx}{x^2+1^2} = \frac{1}{1}\arctan\frac{x}{1} + c = \arctan x + c$

$\frac{x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{B}{x^2+1} \quad / \cdot (x+2)(x^2+1)$

$x = A(x^2+1) + B(x+2)$

$x = Ax^2 + A + Bx + 2B$

$x = Ax^2 + Bx + A + 2B$

$A = 0$

$B = 1$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\| \frac{x+a}{x-a} \right\| + C$
$\int \frac{1}{x} dx = \ln \|x\| + C$	$\int \cot x dx = \ln \|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \|x + \sqrt{x^2 \pm a^2}\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1

ANĐELO ŽMIRE

$$\int_0^{+\infty} \frac{1}{(x+1)^2}\,dx = \begin{bmatrix} t = x+1 \,/' \\ dt = dx \end{bmatrix} = \int_0^{+\infty} \frac{dt}{t^2} = \int_0^{+\infty} t^{-2}\,dt = \frac{t^{-1}}{-1}\Big|_0^{+\infty} =$$

$$= -\frac{1}{t}\Big|_0^{+\infty} = -\frac{1}{(x+1)^2}\Big|_0^{+\infty} = -\frac{1}{+\infty} - 0 = 0$$

1. a)

$e^{\frac{1}{x}} = t \,/'$

$$e^{\frac{1}{x}} \cdot \left(-\frac{x^{-2}}{2}\right) = -e^{\frac{1}{x}} \frac{1}{2x^2} = \frac{e^{\frac{1}{x}}}{2x^2} = \frac{1}{2}\,\frac{e^{\frac{1}{x}}}{x^2}\,dx$$

♣2|

ANĐELO ŽMIRE

♣3

(4) a) $f(x,y)=x^2-2x+1+y^2-9$

$$\frac{\partial f}{\partial x}=2x-2$$

$$\frac{\partial f}{\partial y}=2y$$

$$\frac{\partial f}{\partial x\partial y}=0$$

$$\frac{\partial^2 f}{\partial x^2}=2>0$$

$$\frac{\partial^2 f}{\partial y^2}=2$$

$$2x-2=0 \qquad 2y=0$$
$$2x=2 \qquad \boxed{y=0}$$
$$\boxed{x=1}$$

$T_0(1,0)$

$$\Delta=\begin{vmatrix}2 & 0\\ 0 & 2\end{vmatrix}=4>0$$

$T_0(1,0)$ je minimum funkcije.

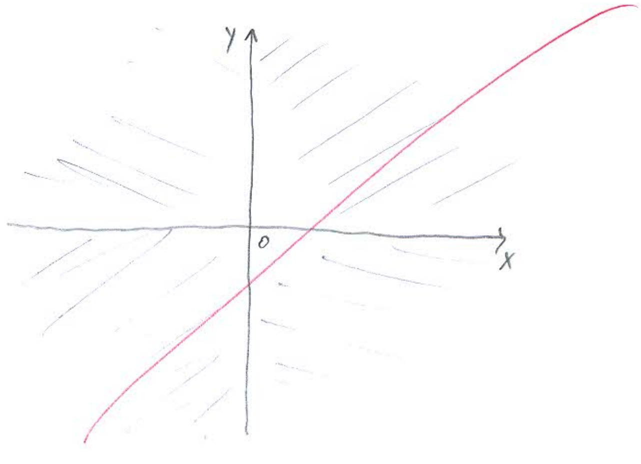
✓ (10)

b) $f(x,y)=\ln(x^2+y^2-1)$

$$x^2+y^2-1>0$$

$$x^2+y^2>1$$

$D_f: \mathbb{R}^2$



3.

$y = 2x - 1$

$y = 1 + 3x - x^2$

x	y
0	-1
1	1
2	3
-2	-5

x	y
0	1
1	3
2	3
-2	-9
-1	-3
3	1
4	-3

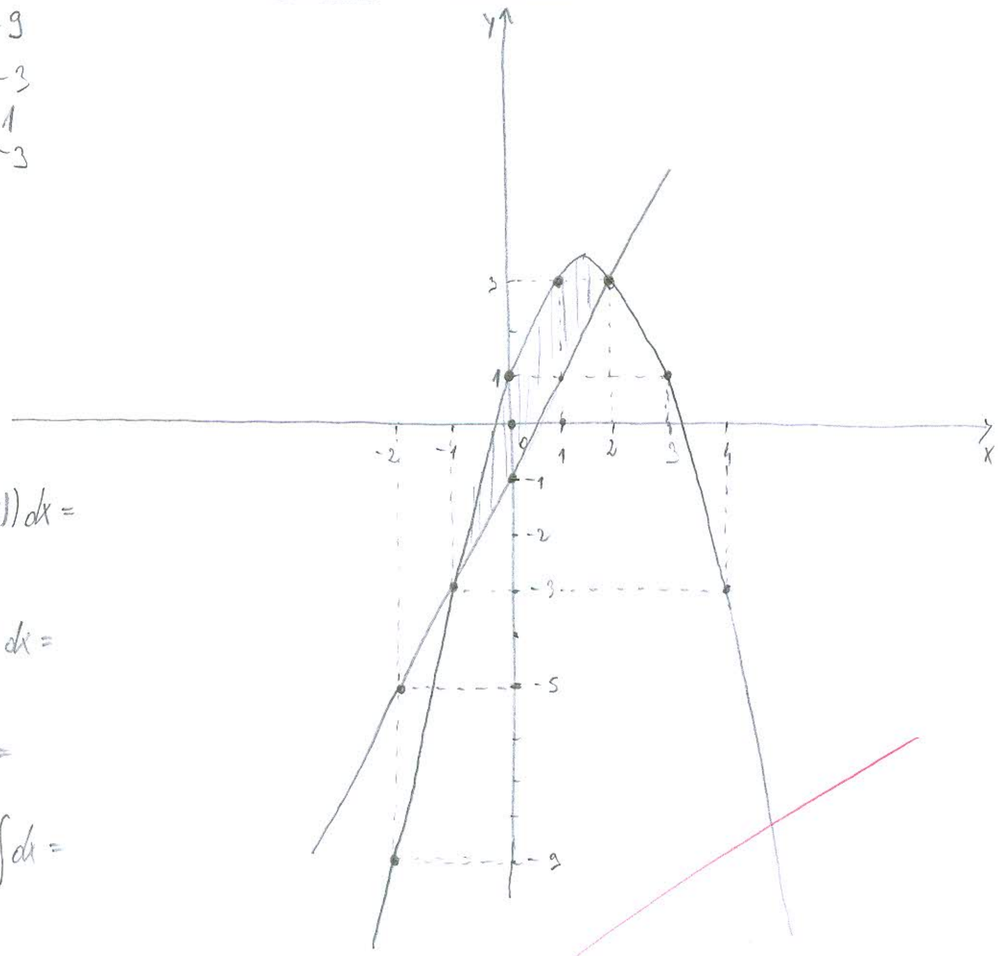
$2x - 1 = 1 + 3x - x^2$

$x^2 + 2x - 3x - 1 - 1 = 0$

$x^2 - x - 2 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$

$x_1 = 2$ $x_2 = -1$



$P = \int_{-1}^{2} (1 + 3x - x^2 - (2x - 1))\, dx =$

$\int_{-1}^{2} (1 + 3x - x^2 - 2x + 1)\, dx =$

$\int_{-1}^{2} (-x^2 + x + 2)\, dx =$

$-\int_{-1}^{2} x^2\, dx + \int x\, dx + 2 \int dx =$

$-\frac{x^3}{3}\Big|_{-1}^{2} + \frac{x^2}{2}\Big|_{-1}^{2} + 2x\Big|_{-1}^{2} =$

$-\frac{8}{3} - \frac{1}{3} + \cancel{2} - \frac{1}{2} + 4 - \cancel{2} =$

$-3 - \frac{1}{2} + 4 = \frac{1}{2}$

MATEMATIKA 2

29. lipnja 2013.

Ime i prezime: Mitrović Martin **Broj indeksa:** 17-2-0033-2010

Vrijeme: od_____________ **do**_________ ♣1 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx$$

b)

$$\int_0^{+\infty} \frac{1}{(x+1)^2}dx$$

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 - 2x + 1 + y^2 - 9$$

b) Odredi domenu funkcije:

$$f(x, y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x}y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

4/ b/ $f(x,y) = \ln(x^2+y^2-1)$

$D_{f(x,y)} \in \langle 1, +\infty\rangle$

3. $y = 2x - 1 \qquad y = 1 + 3x - x^2$

① sjecište

$$y = 2x - 1$$
$$y = 1 + 3x - x^2$$

$$2x - 1 = 1 + 3x - x^2$$
$$2x - 3x + x^2 - 1 + 1 = 0$$
$$-x + x^2 - 2 = 0$$
$$\underset{a}{x^2} \underset{b}{- x} \underset{c}{- 2} = 0$$

② $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$
$$x_{1,2} = \frac{1 \pm \sqrt{1 + 8}}{2}$$
$$x_{1,2} = \frac{1 \pm \sqrt{9}}{2}$$
$$x_{1,2} = \frac{1 \pm 3}{2}$$
$$x_1 = \frac{1+3}{2} = \frac{4}{2} = 2$$
$$x_2 = \frac{1-3}{2} = \frac{-2}{2} = -1$$

③ $y_1 = 2x_1 - 1 \qquad y_2 = 2x_2 - 1$

$$y_1 = 4 - 1 \qquad y_2 = -3$$
$$y_1 = 3$$

$S_1(2, 3)$

$S_2(x_2, y_2)$

$S_2(-1, -3)$

$f(x) = ax^2 + bx + c$

④ $y = 2x - 1$

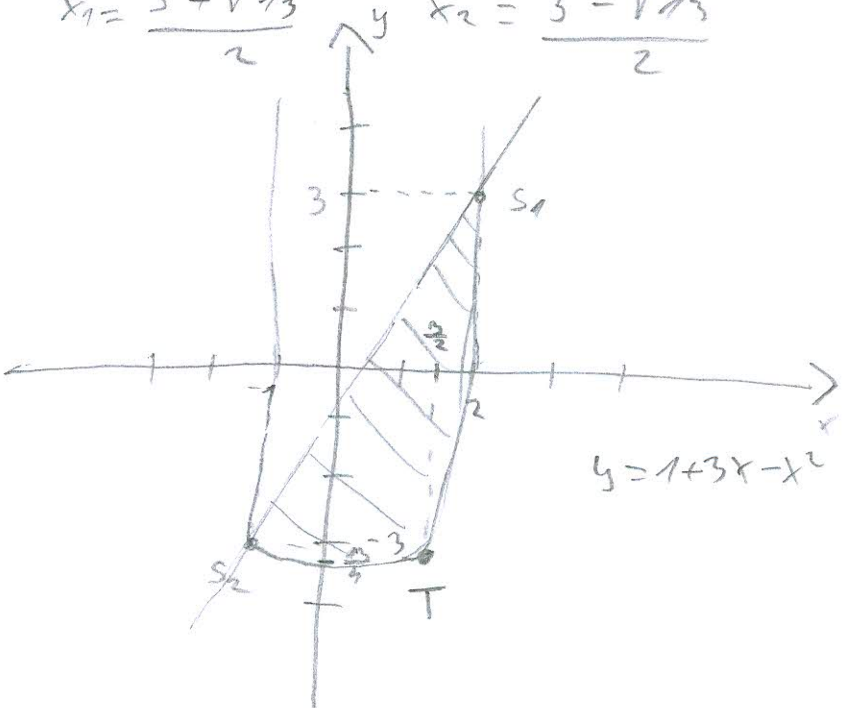
x	0	1
$y=2x-1$	-1	1

⑤ $y = 1 + 3x - x^2$

$$-x^2 + 3x + 1 = 0 \,/\cdot(-1)$$
$$\underset{a}{x^2} \underset{b}{- 3x} \underset{c}{- 1} = 0$$
$$x_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$
$$x_{1,2} = \frac{3 \pm \sqrt{9 + 4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$
$$x_1 = \frac{3 + \sqrt{13}}{2} \qquad x_2 = \frac{3 - \sqrt{13}}{2}$$

⑥ $T\left(-\dfrac{b}{2a}, \dfrac{4ac - b^2}{4a}\right)$

$$T\left(\frac{3}{2}, \frac{4 \cdot 1 \cdot (-1) - (-3)^2}{4 \cdot 1}\right)$$
$$T = \left(\frac{3}{2}, \frac{-4 - 9}{4}\right)$$
$$T = \left(\frac{3}{2}, \frac{-13}{4}\right)$$



(3)

$$P = \int_{-1}^{2} \left(2x-1-(1+3x-x^2)\right) dx$$

$$P = \int_{-1}^{2} (2x-1-1-3x+x^2) dx$$

$$P = \int_{-1}^{2} (-x-2+x^2) dx$$

$$P = \left(-x-2x+\frac{x^3}{3}\right) \Big|^{2}$$

$$P = \left(-\frac{x^2}{2}-4+\frac{8}{3}\right) - \left(-\left(\frac{(-1)^2}{2}\right) - (2\cdot(-1)) + \frac{(-1)^3}{3}\right)$$

$$P = \left(-1-4+\frac{8}{3}\right) - \left(-\frac{1}{2}+2+-\frac{1}{3}\right)$$

$$P = -5+\frac{8}{3}-\left(\frac{7}{6}\right)$$

$$P = -5+\frac{8}{3}-\frac{7}{6}$$

$$P = -\frac{7}{2}$$

♣1

Tablica osnovnih derivacija

f	f'	f	f'
$x^{\alpha}(\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
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Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x\rvert + C$	$\int \cot x dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)] + C$

♣1

MATEMATIKA 2
29. lipnja 2013.

56188 -2008

Ime i prezime: MARKO TKALČEC **Broj indeksa:** 0269024536

Vrijeme: od 09:35 **do** ________ ♣1 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx$$

b)

$$\int_0^{+\infty} \frac{1}{(x+1)^2}dx$$

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$

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$$f(x,y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x}y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

1) $\int \frac{e^{\frac{1}{x}}}{x^2} dx$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: IVAN VIDAKOVIĆ **Broj indeksa:** 57188

Vrijeme: od 0800 **do** 1100 ♣1 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

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2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

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a) Ispitaj ekstreme funkcije

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b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x}y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

1.) a) $\int \frac{e^{\frac{1}{x}}}{x^2}dx = \left[uv - \int v\,du\right] \left[\begin{matrix} u = x^2 & \int dv = e^{\frac{1}{x}}dx \\ du = 2x & v = e^{\frac{1}{x}} \end{matrix}\right] = x^2 \cdot e^{\frac{1}{x}} - \int e^{\frac{1}{x}} \cdot 2x =$

$= \left[\begin{matrix} u = 2x & \int dv = e^{\frac{1}{x}}dx \\ du = 2\,dx & v = e^{\frac{1}{x}} \end{matrix}\right] = x^2 \cdot e^{\frac{1}{x}} \cdot 2x \cdot e^{\frac{1}{x}} - \int e^{\frac{1}{x}} \cdot 2\,dx = x^2 e^{\frac{1}{x}} \cdot 2x \cdot e^{\frac{1}{x}} - 2\int e^{\frac{1}{x}}dx =$

$= x^2 e^{\frac{1}{x}} \cdot 2x \cdot e^{\frac{1}{x}} - 2 \cdot e^{\frac{1}{x}} + C$

1.) b) $\int_0^{+\infty} \frac{1}{(x+1)^2}dx = \left[\begin{matrix} x+1 = u \\ du = dx \end{matrix}\right] = \int_0^{+\infty} \frac{1}{u^2}du = 1\ \ln|u^2| \Big|_0^{+\infty} = \ln|(x+1)^2| \Big|_0^{+\infty}$

$\ln|x^2+2x+1| \Big|_0^{+\infty} = \ln|+\infty + 2 +\infty + 1| - \ln|0+0+1| = \ln|+\infty| - \ln|1| = +\infty$

2) $\int \frac{x}{(x+2)(x^2+1)} dx = \frac{Ax+B}{(x+2)} + \frac{Cx+D}{(x^2+1)} = (Ax+B)(x^2+1) + (Cx+D)(x+2)$

$x = Ax^3 + Ax + Bx^2 + B + Cx^2 + 2Cx + Dx + 2D$

$\boxed{A = 0}$ $\quad B + C = 0$

$A + 2C + D = 1 \quad B + 2D = 0$

$0 + 2C + D = 1 \quad B + \frac{1-D}{2} = 0$

$2C = 1 - D \quad 2D = -B$

$C = \frac{1-D}{2} \quad D = -\frac{B}{2}$

$B + \frac{1 - \frac{B}{2}}{2} = 0$

$B + \frac{1}{2} - \frac{B}{4} = 0$

$B + \frac{4-2B}{8} = 0$

$\frac{8B + 4 - 2B}{8} = 0$

$\frac{6B+4}{8} = 0$

$\frac{6B}{8} + \frac{1}{4} = 0$

$\frac{6B}{8} = -$

3. $y = 2x - 1$

$y = 1 + 3x - x^2 \Rightarrow$

Sjecišta $= 2x - 1 = 1 + 3x - x^2$

$x^2 - 3x + 2x - 1 - 1 = 0$

$x^2 - x - 2 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2}$

$x_{1,2} = \frac{1 \pm \sqrt{9}}{2}$

$X_1 = 2$

$X_2 = 1$

$P = \int (1 + 3x - x^2) - (2x - 1)$

$P = \int_1^2 (1 + 3x - x^2) - (2x - 1)$

$= \int_1^2 1dx + \int_1^2 3x\,dx - \int_1^2 x^2 dx - \int_1^2 2x\,dx + \int_1^2 1dx$

$= (2-1) + 3\left(\frac{x^2}{2}\right)\Big|_1^2 - \left(\frac{x^3}{3}\right)\Big|_1^2 - 2\left(\frac{x^2}{2}\right)\Big|_1^2 + (2-1)$

$= 1 + 3\left(\frac{4}{2} - \frac{1}{2}\right) - \left(\frac{8}{3} - \frac{1}{3}\right) - 2\left(\frac{4}{2} - \frac{1}{2}\right) + 1$

$= 1 + 3\,\frac{3}{2} - \frac{7}{3} - 2\,\frac{3}{2} + 1 = 1 + \frac{9}{2} - \frac{7}{3} - \frac{6}{2} + 1$

$= \frac{6 + 27 - 14 - 18 + 6}{6} = \frac{7}{6}$

4.) a)

$f(x\,y) = x^2 - 2x + 1 + y^2 - 9$

$x^2 - 2x + 1 + y^2 - 9 = 0$

$x^2 + y^2 - 2x - 8 = 0$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: Lovre Kereš **Broj indeksa:** 57933

Vrijeme: od 8^{00} **do** 10^{00} ♣1 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx$$

b)

$$\int_0^{+\infty} \frac{1}{(x+1)^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)} dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x} y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

1)

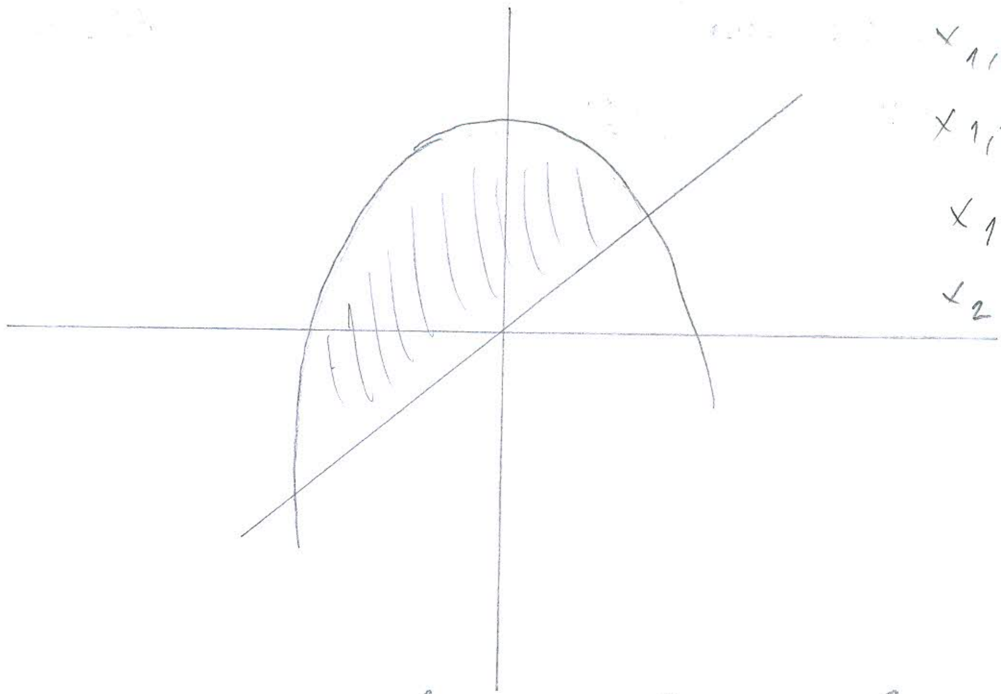
a) $\int \frac{e^{\frac{1}{x}}}{x^2} dx = \left[\begin{array}{c} x^2 = t \;/' \\ 2x = dt \\ dx = dt \end{array} \right] = \int \frac{e^{\frac{1}{x}}}{t} dt$

$\ln t \, dt = \ln x^2 dx + C$

3. $y = 1 + 3x - x^2 \qquad y = 2x - 1$

$1 + 3x - x^2 = 2x - 1$

$2 + x - x^2 = 0$



$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4(-1)2}}{2}$

$x_{1,2} = \frac{-1 \pm \sqrt{9}}{2}$

$x_1 = \frac{-1+3}{2} = 1$

$x_2 = \frac{-1-3}{2} = -2$

$$P = \int_1^{-2} 2 + x - x^2 \, dx = \int_1^{-2} 2 \, dx + \int_1^{-2} x \, dx - \int_1^{-2} x^2 dx =$$

$$= 2 \int_1^{-2} dx + \int_1^{-2} x \, dx + \frac{1}{2} \int_1^{-2} x \, dx + C$$

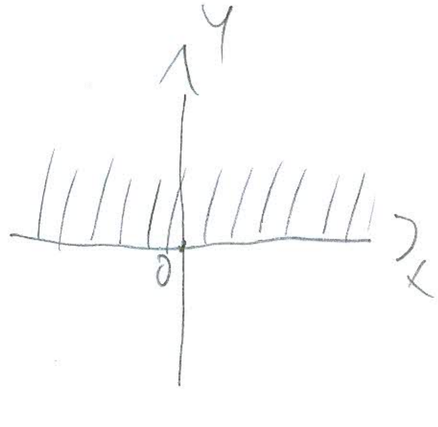
$$= 2(-2-1) + (-2-1) + \frac{1}{2}(-2-1) + C$$

$$= -6 - 3 - \frac{3}{2} + C = -9 - \frac{3}{2} + C = \frac{-18-3}{2} + C = -\frac{21}{2} + C$$

4.

b) $f(x,y) = \ln(x^2 + y^2 - 1)$

$\sqrt{\ } \geq 0$



Tablica osnovnih derivacija

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Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\vert \frac{a+x}{a-x} \right\vert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \vert \cos x \vert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\vert \frac{x+a}{x-a} \right\vert + C$
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♣1

Tena Krupotić

MATEMATIKA 2
29. lipnja 2013.

Ime i prezime: TENA KRUPOTIĆ **Broj indeksa:** 5970

Vrijeme: od________ **do**________ ♣1 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)
$$\int \frac{e^{\frac{1}{x}}}{x^2}dx$$

b)
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2. *(15)* Integriraj
$$\int \frac{x}{(x+2)(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije
$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$

b) Odredi domenu funkcije:
$$f(x,y) = \ln(x^2 + y^2 - 1)$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$y' + \frac{2}{x}y = x^3$$

b)
$$y'' + 3y' + 2y = e^{2x}$$

5 POVRŠINA

TENA FRUDTIĆ

PARABOLA $y=1+3x-x^2$

PRAVAC $y=2x-1$

$f(x)=g(x)$

$-x^2+3x+1=2x-1$

$-x^2+3x+1-2x+1=0$

$-x^2+x+2=0$

$x_{1,2}=\frac{-1\pm\sqrt{1+8}}{-2}$

$x_{1,2}=\frac{-1\pm\sqrt{9}}{-2}$

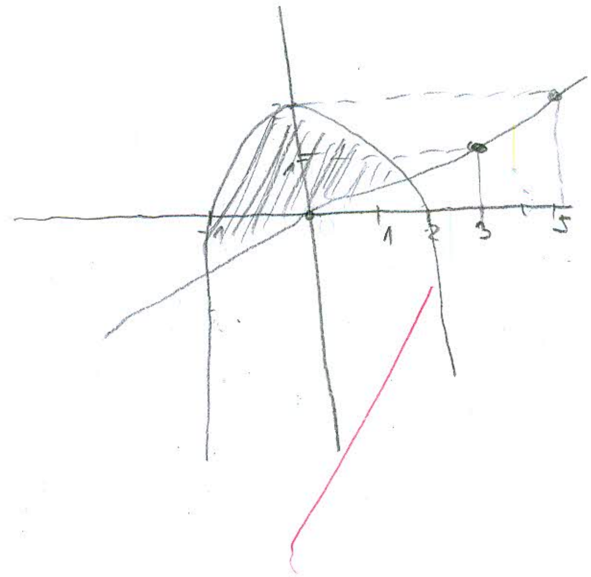
$x_{1,2}=\frac{-1\pm 3}{-2}$

$x_1=\frac{-1+3}{-2}$ $\quad x_2=\frac{-1-3}{-2}$

$x_1=\frac{2}{-2}$ $\quad x_2=\frac{-4}{-2}$

$\boxed{x_1=-1}$ $\quad \boxed{x_2=2}$

$y=2x+1$	x
0	1
1	3
2	5



$\int_{-1}^{2} -x^2+3x+1-(2x-1)$

$\int_{-1}^{2} -x^2+3x+1-2x+1$

$\int_{-1}^{2} -x^2+x+2$

$\int_{-1}^{2} -x^2dx+\int_{-1}^{2} x\,dx+\int_{-1}^{2} 2\,dx$

$-\frac{x^3}{3}\Big|_{-1}^{2}+\frac{x^2}{2}\Big|_{-1}^{2}+2x\Big|_{-1}^{2}$

$-\frac{1}{3}x^3\Big|_{-1}^{2}+\frac{1}{2}x^2\Big|_{-1}^{2}+2x\Big|_{-1}^{2}$

$-\frac{1}{3}(2^3-(-1^3))+\frac{1}{2}(2^2-(-1)^2)+2\cdot 2-2\cdot(-1)$

$-\frac{1}{3}(8-(-1))+\frac{1}{2}(4-1)+4-(-2)$

$-\frac{1}{3}(8+1)+\frac{1}{2}(3)+4+2$

$-\frac{1}{3}(9)+\frac{1}{2}(3)+8 \Rightarrow$

$\Rightarrow -3+\frac{3}{2}+8=\frac{13}{2}$ //

② Integriraj

$$\int \frac{x}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)} = / (x+2)(x^2+1)$$

$$x = A(x^2+1) + Bx+C(x+2)$$

$$= \cancel{Ax^2} + A + \cancel{Bx^2} + 2\cancel{Bx} + \cancel{Cx} + 2C$$

$$= x^2(A+B) + x(2B+C) + A + 2C$$

$A+B=0 \Rightarrow B=-A$

$2B+C=1 \quad -2A+C=1$

$A+2C=0$

$C=2A+1$

$A+2(2A+1)=0$

$A+4A+2=0$

$5A+2=0$

$5A=-2 \,/ :5$

$\boxed{A=\frac{-2}{5}}$ $\boxed{B=\frac{2}{5}}$

$C=2\left(-\frac{2}{5}\right)+1$

$C=\frac{-4}{5}+1$

$\boxed{C=\frac{1}{5}}$

$$\ldots = \frac{-\frac{2}{5}}{(x+2)} + \frac{\frac{2}{5}x+\frac{1}{5}}{(x^2+1)}$$

$$= \int \frac{-\frac{2}{5}}{(x+2)}dx + \underbrace{\int \frac{\frac{2}{5}x+\frac{1}{5}}{(x^2+1)}dx}_{*}$$

$$= -\frac{2}{5}\int \frac{dx}{(x+2)} +$$

$\left| \begin{array}{l} x+2=t \,/' \\ dx=dt \end{array} \right|$

$$= \frac{-2}{5}\int \frac{dt}{t}$$

$$= -\frac{2}{5}\ln|t|$$

$$= \frac{-2}{5}\ln|x+2| + \frac{26}{5}\ln|x^2+1|$$

$$* \int \frac{\frac{2}{5}x+\frac{1}{5}}{(x^2+1)}$$

$\left| \begin{array}{l} x^2+1=t \,' \\ 2x\,dx=dt \\ dx=\frac{dt}{2x} \end{array} \right|$

$$\int \frac{\frac{2}{5}x+\frac{1}{5}}{t}\frac{dt}{2x}$$

$$\int \frac{5+\frac{1}{5}}{t}dt$$

$$\int \frac{\frac{26}{5}}{t}dt$$

$$\frac{26}{5}\int \frac{dt}{t}$$

$$\frac{26}{5}\ln|t|$$

$$\frac{26}{5}\ln|x^2+1|$$

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$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
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♣1

(5) a) $y' + \frac{2}{x}y = x^3$

$P(x)$ Q

$y = e^{-\int P x}\left[Q(x)\cdot e^{\int P x} + C\right]$

$y = e^{-2\ln|x|}\left[\frac{x^4}{4}\cdot e^{2x} + C\right]$

$\int P(x) \int \frac{2}{x}dx = 2\int \frac{dx}{x}$

$= 2\ln|x|$

$y = \cancel{e^{\ln}}|\frac{-2}{x}|\left[\frac{x^4}{4}\cdot e^{2x} + C\right]$

$y = \frac{-2}{x}\cdot\left[\frac{x^4}{4}\cdot e^{2x} + C\right]$ //

$\int Q(x \cdot e^{\int P(x)} dx =$

$= \int x^3 \cdot e^{2\ln|x|}$

$= \frac{x^4}{4}\cdot \cancel{e}^{2\ln|x|}$

$= \frac{x^4}{4}\cdot \cancel{e^{\ln}}|2x|$

$= \frac{x^4}{4}\cdot 2x$

(1) a) $\int \frac{e^{\frac{1}{x}}}{x^2}dx$

$\left|\begin{array}{l} \frac{1}{x} = t \,|' \\ \ln dx = dt \\ dx = \frac{dt}{\ln} \end{array}\right|$

$\int \frac{e^t}{x^2}\frac{dt}{\ln}$

$\int \frac{t}{x^2}dt$

b) $\int_0^{+\infty} \frac{1}{(x+1)^2}dx = \lim_{\varepsilon\to+\infty}\int_0^{\varepsilon}\frac{1}{(x+1)^2}dx = \lim_{\varepsilon\to\infty}\int_0^{\varepsilon}\frac{dx}{t^2} = \lim_{\varepsilon\to\infty} t^{-2}\Big|_0^{\varepsilon} dx = \lim_{\varepsilon\to\infty} \varepsilon^{-2} - 0^{-2}$

$\left|\begin{array}{l} x+1 = t \,|' \\ dx = dt \end{array}\right|$

$= +\infty - 0 = +\infty$ //

TENA KRMPOTIĆ

(4) EKSTREM

a)

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$

$$\frac{df}{dx} = 2x - 2 \Rightarrow 2x - 2 = 0$$

$$2x = 2 \,/ :2$$

$$\boxed{x = 0}$$

$$\frac{df}{dy} = 2y \Rightarrow 2y = 0 \,/ :2$$

$$\boxed{y = 0}$$

$T(0,0)$

$$\frac{df^2}{dx^2} = 2$$

$$\frac{df^2}{dy^2} = 2$$

$$\frac{df}{dy\,dx} = 0$$

$$\frac{df}{dx\,dy} = 0$$

$$\Delta = \begin{vmatrix} \frac{df}{dx^2}^2 & \frac{df}{dxdy} \\ \frac{df}{dxdy} & \frac{df}{dy^2} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

$$\Delta = 2 \cdot 2 - 0 \cdot 0$$

$$= 4 - 0$$

$$= 4$$

$$\Delta > 0$$

TOČKA $T(0,0)$ je minimum funkcije.

b) DOMENA

$$f(x,y) = \ln(x^2 + y^2 - 1)$$

1° $\ln > 0$

$$x^2 + y^2 - 1 > 0$$

$$x^2 + y^2 > 1$$

KRUŽNICA

$r = 1$

Domena su svi uređeni parovi (x,y) u kružnici koji ne uključuju par $(0,0)$ i one na obodu kružnice