MATEMATIKA 2

29. lipnja 2013.

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Vrijeme: od 08:15 **do** _____ **♣1** **Broj bodova:** ~~42~~ 47

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12+8)* Integriraj

a)

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx$$

✓ 12

b)

$$\int_0^{+\infty} \frac{1}{(x+1)^2} dx$$

2. *(15)* Integriraj

$$\int \frac{x}{(x+2)(x^2+1)} dx$$

3. *(15)* Odredi površinu koju zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.

✓ 15

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + y^2 - 9$$

10

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2 - 1)$$

10

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + \frac{2}{x} y = x^3$$

b)

$$y'' + 3y' + 2y = e^{2x}$$

④ a) Ispitaj ekstreme funkcije: $f(x,y) = x^2 - 2x + 1 + y^2 - 9$

$\frac{\partial f}{\partial x} = 2x - 2$ $\quad 2x - 2 = 0$ $\quad 2x = 2 \,/:2$ $\quad x = 1$,

$\frac{\partial f}{\partial y} = 2y$ $\quad 2y = 0 \,/:2$ $\quad y = 0$,

Stacionarne točke su $T(1,0)$

$\frac{\partial^2 f}{\partial x^2} = 2 = A$

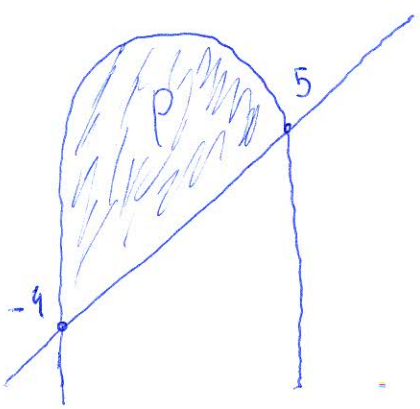
$\frac{\partial^2 f}{\partial y \partial x} = 0 = B$

$\frac{\partial^2 f}{\partial y^2} = 2 = C$

$AC - B^2 = 2 \cdot 2 - 0^2 = 4 > 0$ IMA EKSTREMA

$A = 2 > 0$ MINIMUM

③ Odredi površinu koja zatvaraju $y = 2x - 1$ i parabola $y = 1 + 3x - x^2$.



$1 + 3x - x^2 - 2x + 1 = 0$

$2 + x - x^2 = 0$

$a = -1,\ b = 1,\ c = 2$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4(-1)\cdot 2}}{2 \cdot (-1)}$

$x_{1,2} = \frac{-1 \pm \sqrt{1 - (-8)}}{-2}$

$x_{1,2} = \frac{-1 \pm 9}{-2}$

$x_1 = \frac{-1 + 9}{-2} = -4$,

$x_2 = \frac{-1 - 9}{-2} = 5$,

$$\int_{-4}^{5} \left(1 + 3x - x^2 - 2x + 1\right) dx =$$

$$= \int_{-4}^{5} \left(2 + x - x^2\right) dx = \int_{-4}^{5} 2\,dx + \int_{-4}^{5} x\,dx - \int_{-4}^{5} x^2\,dx = \left(2x + \frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_{-4}^{5}$$

$$= \left(2 \cdot 5 + \frac{(5)^2}{2} - \frac{(5)^3}{3}\right) - \left(2 \cdot (-4) + \frac{(-4)^2}{2} - \frac{(-4)^3}{3}\right) = \left(2 \cdot 5 + \frac{25}{2} - \frac{125}{3}\right) - \left(2 \cdot (-4) + \frac{16}{2} - \frac{(-64)}{3}\right) =$$

$= -19.166$

KRIVO

ISPRAVLJENI ZADATAK NA DRUGOM PAPIRU

FRAUO 210600CC

♣1

① a) $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \left[\begin{array}{l} \frac{1}{x} = t \,/\,' \\ x^{-1} = t \,/\,' \\ -1x^{-2}\,dx = dt \\ -x^{-2}\,dx = dt \,/\cdot(-1) \\ x^{-2}\,dx = -dt \\ \frac{1}{x^2}\,dx = -dt \end{array}\right] = \cancel{e^t}$

$= -\int e^t\,dt = -e^t + C = -e^{\frac{1}{x}} + C$ //

② $\int \frac{x}{(x+2)(x^2+1)}\,dx = \int \frac{A}{x+2}\,dx + \int \frac{Bx+C}{\cancel{x^2+1}}\,dx$

$x(x+1)$

③

$1+3x-x^2=2x+1=0$

$2+x-x^2=0$

$x_{1,2}=\frac{-1\pm\sqrt{9}}{-2}$

$x_1=\frac{-1+3}{-2}=-1$ //

$x_2=\frac{-1-3}{-2}=2$ //

$$\int_{-1}^{2}(2+x-x^2)\,dx=\left(2x+\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_{-1}^{2}$$

$$=\left(2\cdot 2+\frac{(2)^2}{2}-\frac{(2)^3}{3}\right)-\left(2\cdot(-1)+\frac{(-1)^2}{2}-\frac{(-1)^3}{3}\right)=3.33-(-1.166)=$$

$P=4.496$ € Žurio sam se ï

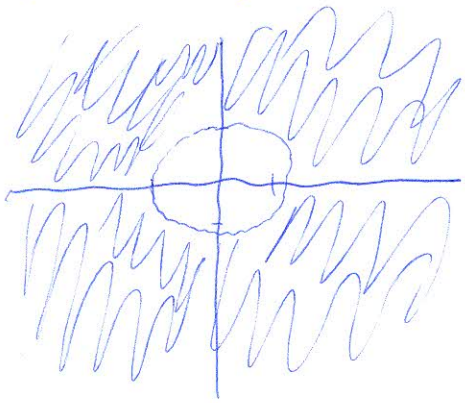
✓ (15)

④ a) Odredi domenu funkcije: $f(x,y)=\ln(x^2+y^2-1)$

$x^2+y^2-1>0$

$x^2+y^2>1$

$S(0,0)\quad r=1$



Domena se nalazi na vanjskoj strani kružnice.

✓ (10)

①
$$\int_0^{+\infty}\frac{1}{(x+1)^2}\,dx=\left[\begin{matrix}x+1=t\\dx=dt\end{matrix}\right]=\int_0^{+\infty}\frac{dt}{t^2}=\int_0^{+\infty}t^{-2}\,dt=\frac{t^{-1}}{-1}\Big|_0^{+\infty}$$

$$=\frac{-1}{t}\Big|_0^{+\infty}=\frac{-1}{+\infty}-\frac{(-1)}{0}=\frac{-1}{x+1}\Big|_0^{+\infty}=\frac{-1}{+\infty+1}-\frac{(-1)}{0+1}=$$

$$=\frac{-1}{+\infty+1}+1=\frac{-1}{+\infty+1}\ //$$

?

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

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