MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: Antun Žunetić Broj indeksa: 17-2-0169-2012

Vrijeme: od 8:00 do 11:00 ♣4 Broj bodova: 62.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int \frac{x}{\cos^2(x^2-4)}dx$$

b)

$$\int_1^2 \frac{dx}{\sqrt{1-x}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x^2+x}{x(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=-x^2+3$ i pravac $y=-2x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^3+xy^2+6xy$$

b) Odredi domenu funkcije:

$$f(x,y)=\ln(2-x)+\sqrt{y+x}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'+4y=2x+3e^{3x}$$

b)

$$y''-4y'+3y=\sin x.$$

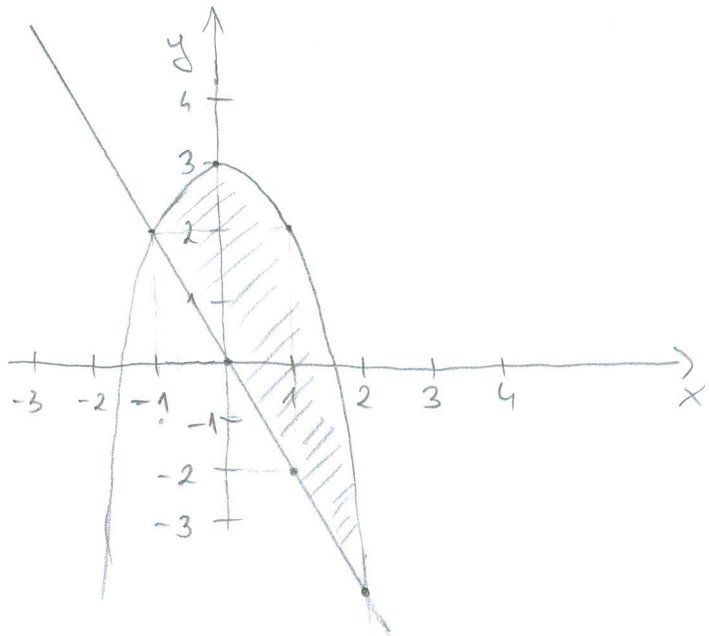
3. $y = -x^2 + 3$ – gornja funkcija

$y = -2x$ – donja funkcija

za $y = -2x$

x	-1	0	1
y	2	0	-2

x	-1	0	1
y	2	3	2



$$-x^2 + 3 = -2x$$

$$-x^2 + 2x + 3 = 0$$

15

$$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{-2}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{16}}{-2}$$

$$x_1 = \frac{-2+4}{-2} = -\frac{2}{2} = -1$$

$$x_2 = \frac{-2-4}{-2} = \frac{-6}{-2} = 3$$

$$P = \int_{-1}^{3} \left[(-x^2+3) - (-2x)\right] dx = \int_{-1}^{3} (-x^2 + 2x + 3)\, dx =$$

$$= -\int_{-1}^{3} x^2 dx + 2\int_{-1}^{3} x\, dx + 3\int_{-1}^{3} dx = \left(-\frac{x^3}{3} + \cancel{2} \cdot \frac{x^2}{\cancel{2}} + 3x\right)\Bigg|_{-1}^{3} =$$

$$= \left(-\frac{x^3}{3} + x^2 + 3x\right)\Bigg|_{-1}^{3} = (\cancel{-9} \cancel{+9} + 9) - \left(\frac{1}{3} + 1 - 3\right) = 9 - \left(-\frac{5}{3}\right) = \frac{32}{3}$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

Antun Žunetić

4. b) $f(x,y) = \ln(2-x) + \sqrt{y+x}$

1° uvjet: $2 - x > 0$

$-x > -2 \quad / \cdot (-1)$

$\boxed{x < 2}$

A SKICA

2° uvjet: $y + x \geq 0$

$\boxed{x \geq -y}$

$\boxed{y \geq -x}$

$y = -2$ $\quad$ $x \geq 2$

$y = 4$ $\quad$ $x \geq -4$

5

$Df(x,y) = \{(x,y) : x < 2, y \geq -x\}$

5. b) $y'' - 4y' + 3y = \sin x$

$a = 1$

$b = -4$

$c = 3$

$k^2 - 4k + 3 = 0$

$k_{1,2} = \frac{4 \pm \sqrt{16 - 12}}{2}$

$k_{1,2} = \frac{4 \pm \sqrt{4}}{2}$

$k_1 = \frac{4+2}{2} = 3$

$k_2 = \frac{4-2}{2} = 1$

$y_1 = e^{3x}$

$y_2 = e^x$

$y = c_1 \cdot y_1 + c_2 \cdot y_2$

$y = c_1 \cdot e^{k_1 x} + c_2 \cdot e^{k_2 x}$

$\boxed{y_H = c_1 \cdot e^{3x} + c_2 \cdot e^x}$ ✓ 5

$f(x) = \sin x$ $\quad$ $\alpha = 1$

$y_p = Ax \sin x$

$y_p' = (Ax \sin x)' = A \cdot \sin x + Ax \cdot \cos x \cdot 1 = \boxed{A \cdot \sin x + Ax \cdot \cos x}$

$y_p'' = \cos x + A \cdot \cos x + Ax \cdot (-\sin x)$

$\cos x + A \cdot \cos x + Ax \cdot (-\sin x) - 4(A \cdot \sin x + Ax \cdot \cos x)$

$+ 3 \cdot Ax \sin x$

...

♣1

4. a) $f(x,y) = x^3 + xy^2 + 6xy$

Antun Žanetić

$$\frac{\partial f}{\partial x} = 3x^2 + y^2 + 6y$$

$$\frac{\partial f}{\partial y} = 2xy + 6x$$

$$\frac{\partial^2 f}{\partial x^2} = 6x$$

$$\frac{\partial^2 f}{\partial y^2} = 2x$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

x_0 - stacionarna točka:

$$3x^2 + y^2 + 6y = 0$$

$$3x^2 = -y^2 - 6y \quad /:3$$

$$x^2 = \frac{-y^2 - 6y}{3}$$

$$x_1 = +\sqrt{\frac{-y^2 - 6y}{3}}$$

$$x_2 = -\sqrt{\frac{-y^2 - 6y}{3}}$$

y_0 - stacionarna točka

$$2xy + 6x = 0$$

$$2xy = -6x \quad /:2x$$

$$y = -\frac{6\cancel{x}}{2\cancel{x}}$$

$$\boxed{y = -3}$$

$$x_1 = \sqrt{3}$$

$$x_2 = -\sqrt{3}$$

$T_1(\sqrt{3}, -3, z_1)$

$T_2(-\sqrt{3}, -3, z_2)$

$$\frac{\partial^2 f}{\partial x^2} > 0$$

$$\Delta_1 = \begin{vmatrix} 6\sqrt{3} & 0 \\ 0 & 2\sqrt{3} \end{vmatrix} = 36 > 0$$

minimum

10

$$z_1 = (\sqrt{3})^3 + \sqrt{3} \cdot (-3)^2 + 6 \cdot \sqrt{3} \cdot (-3) = 3^{\frac{3}{2}} + 9\sqrt{3} - 18\sqrt{3} = 3^{\frac{3}{2}} - 9\sqrt{3}$$

$$\Delta_2 = \begin{vmatrix} -6\sqrt{3} & 0 \\ 0 & -2\sqrt{3} \end{vmatrix} = 36 > 0$$

maksimum

$$z_2 = (-\sqrt{3})^3 + (-\sqrt{3}) \cdot (-3)^2 + 6 \cdot (-\sqrt{3}) \cdot (-3) = -3^{\frac{3}{2}} - 9\sqrt{3} + 18\sqrt{3} =$$

$$= -3^{\frac{3}{2}} + 9\sqrt{3}$$

sve skupa: ???

1. b) $\int_1^2 \frac{dx}{\sqrt{1-x}} = \left| \begin{array}{l} 1-x=t \;/' \\ -dx = dt \;/\cdot(-1) \\ dx = -dt \end{array}\right| = \int_0^{-1} -\frac{dt}{\sqrt{t}} = -\int_0^{-1} \frac{dt}{\sqrt{t}} =$

x	1	2
t	0	-1

$= -\int_0^{-1} t^{-\frac{1}{2}}\, dt = -\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\Big|_0^{-1} = -2t^{\frac{1}{2}}\Big|_0^{-1} = -2\sqrt{t}\Big|_0^{-1}$

$(-2\cdot\sqrt{-1}) - (\;0\;) = -2i$ //

a) $\int \frac{x}{\cos^2(x^2-4)}\, dx = \left| \begin{array}{l} x^2-4=t \;/' \\ 2x\,dx = dt \;/:2 \\ x\,dx = \frac{dt}{2} \end{array}\right| =$

$= \int \frac{1}{\cos^2 t}\cdot\frac{dt}{2} = \frac{1}{2}\int \frac{1}{\cos^2 t}\, dt = \frac{1}{2}\cdot \operatorname{tg} t + C =$

$\frac{1}{2}\cdot \operatorname{tg}(x^2-4) + C$ // ✓ 12.5

2. $\int \frac{x^3+x^2+x}{x(x^2+1)}\, dx = \int \frac{x^3+x^2+x}{x^3+x}\, dx = *$

$(x^3+x^2+x):(x^3+x) = 1 + \frac{x^2}{x^3+x} = 1 + \frac{x^{\cancel{2}}}{\cancel{x}(x^2+1)} =$

$\underline{-(x^3 \quad\quad + x)}$

$0 + x^2$

$= \boxed{1 + \frac{x}{x^2+1}}$

$* = \int 1 + \frac{x}{x^2+1}\, dx = \int dx + \boxed{\int \frac{x}{x^2+1}\, dx} = ♡$

$\int \frac{x}{x^2+1}\, dx = \left| \begin{array}{l} x^2+1=t \;/' \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{array}\right| = \int \frac{1}{t}\cdot\frac{dt}{2} = \frac{1}{2}\int \frac{dt}{t} =$

$\frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|x^2+1| + C$ ✓ 15

$♡ = x + \frac{1}{2}\ln|x^2+1| + C$ //

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: PETAR PERICA Broj indeksa: 026 906 82 02

Vrijeme: od ____________ **do** _________ ♣4 **Broj bodova:** ~~57.5~~ 65 Kosor

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a)

$$y'+4y=2x+3e^{3x}$$

b)

$$y''-4y'+3y=\sin x.$$

17.5

1\. a) $\int \frac{x}{\cos^2(x^2-4)}dx = \begin{vmatrix} x^2-4=t \\ dt=2x\,dx \\ dx=\frac{dt}{2x}\end{vmatrix} = \int \frac{\cancel{x}}{\cos^2(t)}\frac{dt}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{\cos^2 t}dt = \frac{1}{2}\tan t = \frac{1}{2}\tan(x^2-4)+C$

7.5 Kosor

b) $\int_1^2 \frac{dx}{\sqrt{1-x}} = \lim_{y\to 1}\int_y^2 \frac{dx}{\sqrt{1-x}} = \lim_{y\to 1}\left(-2\sqrt{1-x}\,\Big|_y^2\right) = \left(-2\sqrt{-1}-0\right) = -2\sqrt{-1}$

$= -2\sqrt{1-2}+2\sqrt{1-y}$

$\sqrt{1-x} > 0$

$1-x>0$

$-x>-1$

$x>1$

– POVRŠINA SE NE MOŽE IZRAČUNATI NITI PREKO SINGULARITETA

IMAMO SINGULARITET U TOČKI 1.

$\int \frac{dx}{\sqrt{1-x}} = \begin{bmatrix} 1-x=t \\ dt=-dx \end{bmatrix} = -\int \frac{1}{\sqrt{t}}dt = -\int t^{-\frac{1}{2}}dt = -\frac{t^{\frac{1}{2}}}{\frac{1}{2}} = -2t^{\frac{1}{2}} = -2(1-x)^{\frac{1}{2}} = -2\sqrt{1-x}+C$

2. $$\int \frac{x^3+x^2+x}{x(x^2+1)}\,dx = \int dx + \int \frac{x^2}{x^3+x} = \boxed{x + \frac{1}{3}\ln|x^3+x| + C}$$

$$\frac{x^3+x^2+x}{x^3+x} = \frac{\cancel{x^3+x}}{\cancel{x^3+x}} + \frac{x^2}{x^3+x} = 1 + \frac{x^2}{x^3+x}$$

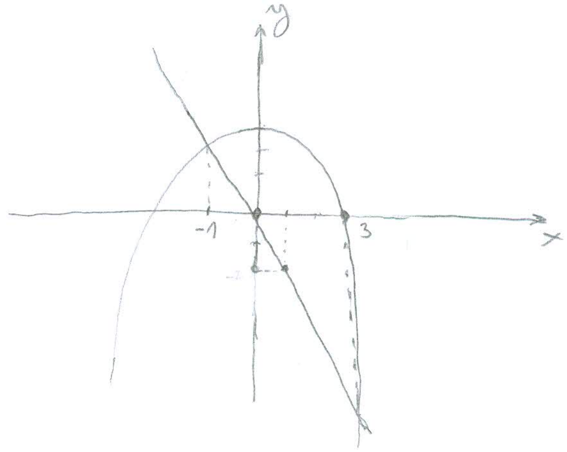
NISTE PODIJELILI BROJNIK S NAZIVNIKOM

$$\int \frac{x^2}{x^3+x} = \left[\begin{matrix} x^3+x = t \\ dt = 3x^2\,dx \end{matrix}\right] = \int \frac{\cancel{x^2}}{t} \cdot \frac{dt}{3\cancel{x^2}} = \frac{1}{3}\int \frac{1}{t}\,dt = \frac{1}{3}\ln|t| = \frac{1}{3}\ln|x^3+x| + C$$

$$\int dx = x + C \qquad \boxed{\int \frac{x^2}{x^3+x}dx = \int \frac{x^2}{x(x^2+1)}dx}$$

3. $y = -x^2+3$
$y = -2x$

$$-x^2+3 = -2x$$
$$x^2-3-2x = 0$$
$$x^2-2x-3 = 0$$
$$x_{1,2} = \frac{2 \pm \sqrt{4-4\cdot 1\cdot(-3)}}{2}$$
$$x_{1,2} = \frac{2 \pm \sqrt{16}}{2}$$
$$x_{1,2} = \frac{2 \pm 4}{2} \quad x_1 = 3, \quad x_2 = -1$$



$$\int_{-1}^{3} \left(-x^2+3-(-2x)\right)dx = \int_{-1}^{3}\left(-x^2+3+2x\right)dx = -\frac{x^3}{3}\Big|_{-1}^{3} + 3x\Big|_{-1}^{3} + x^2\Big|_{-1}^{3} =$$

15

$$= \left(-9-\frac{1}{3}\right) + (9+3) + (9-1) = -\frac{28}{3} + 12 + 8 = -\frac{28}{3} + 20 = \frac{32}{3} \approx 10{,}66$$

4. a) $f(x,y) = x^3 + xy^2 + 6x\cdot y$

$$\frac{\partial f}{\partial x} = 3x^2 + y^2 + 6y$$

$$\frac{\partial f}{\partial y} = 2y\cdot x + 6x$$

$$2y\cdot x + 6x \,/ : x$$
$$2y\cdot x = -6x \,/ : x$$
$$2y = -\frac{6\cancel{x}}{\cancel{x}}$$
$$2y = -6 \,/ : 2$$
$$y = -3$$

$$3x^2 + 9 - 18 = 0$$
$$3x^2 = 18 - 9$$
$$3x^2 = 9 \,/ : 3$$
$$x^2 = 3 \,/ \sqrt{\ }$$
$$x = \sqrt{3} \approx 1{,}73$$

STACIONARNE TOČKE

$T_0(\sqrt{3}, -3)$

$$\frac{\partial^2 f}{\partial x^2} = 6x = 6\sqrt{3} \approx 10{,}39$$

$$\frac{\partial^2 f}{\partial y^2} = 2x = 2\sqrt{3} \approx 3{,}46$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2y + 6 = 2\cdot(-3) + 6 = 0$$

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 6\sqrt{3} & 0 \\ 0 & 2\sqrt{3} \end{vmatrix} =$$

$$= 6\sqrt{3} \cdot 2\sqrt{3} = 36$$

$$\Delta > 0, \quad \frac{\partial^2 f}{\partial x^2} > 0$$

ALI IMA MAKSIMUM!

✓ $T_0(\sqrt{3}, -3)$ je minimum funkcije.

5

e) $f(x,y) = \ln(2-x) + \sqrt{y+x}$

$2 - x > 0$

$-x > -2$

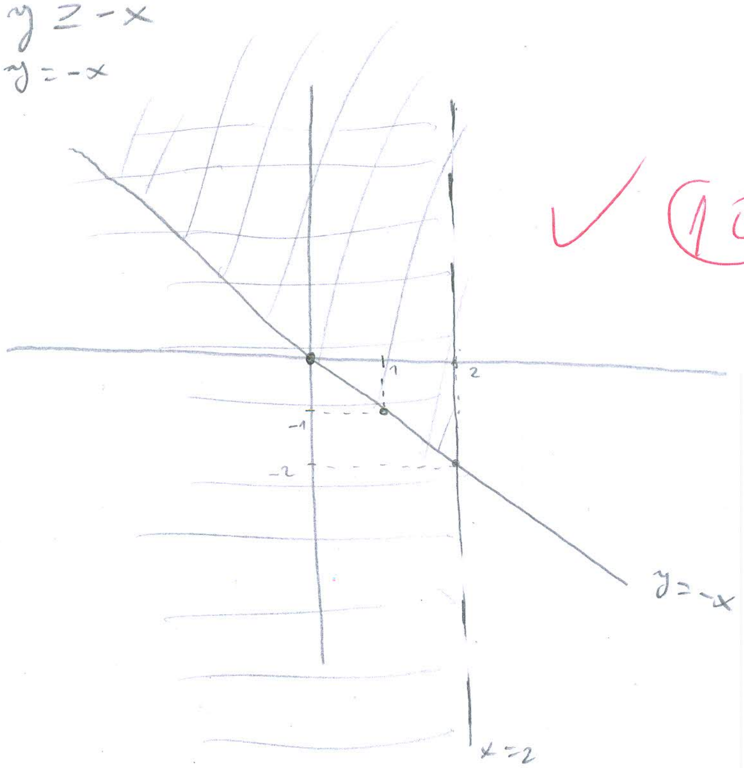
$x < 2$

$x = 2$

$y + x \geq 0$

$y \geq -x$

$y = -x$



✓ 10

5. a) $y' + 4y = 2x + 3e^{3x}$

$y' + 4y - 2x = 3e^{3x}$

$y' + 4y - 2x = 0$

$y' = 2x - 4y$

$\frac{y'}{2x} = 2\frac{y}{x}$

$\frac{y'}{2x} = 2z$

$2x\,dy = 2z\,dz$

$y = \left(\frac{y}{x}\right)^2 + n(x)$

$y' = \frac{2y}{x}$

9

$\frac{y}{x} = z \qquad y' = z' \cdot x + z$

$y = z \cdot x \mid : z$

$x = \frac{y}{z}$

$y' = 2x - 4y$

$y' - 2x = 4y$

$\frac{y'}{2x} = 2\frac{y}{x}$

$z \cdot 2\frac{y}{z}$

5. k) $y'' - 4y' + 3y = \sin x$

$\sin x = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x))$

$\alpha = 0$

$\beta = 1$

$P_m = 0$

$Q_n = 0$ $\quad > N = 0$

$\alpha + \beta i = k$

i $\quad k = 0$

$r^2 - 4y + 3 = 0$

$$r_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2}$$

$$r_{1,2} = \frac{4 \pm \sqrt{4}}{2}$$

$r_1 = 3$

$r_2 = 1$

$$\boxed{y_H = C_1 e^{3x} + C_2 e^{1x}}$$

$$y_p = x^k \cdot e^{\alpha x}(S_N(x)\cos(\beta x) + R_N(x)\sin \beta x)$$

$$y_p = A \cdot \cos x + B \cdot \sin x = \boxed{\frac{1}{5}\cos x + \frac{1}{10}\sin x}$$

$$y_p' = -A \cdot \sin x + B\cos x$$

$$y_p'' = -A\cos x - B\sin x$$

$$-A\cos x - B\sin x - 4(-A\sin x + B\cos x) + 3(A\cos x + B\sin x) = \sin x$$

$$(-A - 4B + 3A)\cos x + (-B + 4A + 3B)\sin x = \sin x$$

$-A - 4B + 3A = 0$

$2A - 4B = 0$

$2A - 4\left(\frac{1}{2} - 2A\right) = 0$

$2A - 2 + 8A = 0$

$10A = 2 \quad /:10$

$A = \frac{2}{10} = \frac{1}{5}$

$-B + 4A + 3B = 1$

$2B + 4A = 1$

$2B = 1 - 4A$

$B = \frac{1}{2} - 2A$

$B = \frac{1}{2} - 2\left(\frac{1}{5}\right)$

$B = \frac{1}{2} - \frac{2}{5}$

$B = \frac{1}{10}$

$\frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10}$

✓ 15

$$y(x) = y_H + y_p = C_1 e^{3x} + C_2 e^{x} + \frac{1}{5}\cos x + \frac{1}{10}\sin x$$

♣1

Tablica osnovnih derivacija

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$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
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Tablica osnovnih integrala

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

$\int^m$

62.5

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: TIN LOBOREC Broj indeksa: 17-2-0188-2012

Vrijeme: od____________ **do**_________ ♣4 **Broj bodova**:

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int \frac{x}{\cos^2(x^2-4)}dx$$

b)

$$\int_1^2 \frac{dx}{\sqrt{1-x}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x^2+x}{x(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=-x^2+3$ i pravac $y=-2x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^3+xy^2+6xy$$

b) Odredi domenu funkcije:

$$f(x,y)=\ln(2-x)+\sqrt{y+x}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'+4y=2x+3e^{3x}$$

b)

$$y''-4y'+3y=\sin x.$$

(1.) a) $\int \frac{x}{\cos^2(x^2-4)}\,dx = \left[\begin{array}{l} x^2-4=t \\ 2x\,dx=dt \\ dx=\frac{dt}{2x} \end{array}\right]$ $x\,dx=\frac{dt}{2}$

(12)

$\frac{1}{2}\int \frac{\cancel{x}}{\cos^2 t}\cdot\frac{dt}{\cancel{x}} = \frac{1}{2}\int\frac{dt}{\cos^2 t} = \frac{1}{2}\tan t + C$

$= \frac{1}{2}\tan(x^2-4) + C$

(1) b $\int_1^2 \frac{dx}{\sqrt{1-x}} = \left[\begin{array}{l} 1.\text{ uvjet} \\ 1-x\geq 0 \\ 2.\text{ uvjet} \\ 1-x\neq 0 \\ -x > -1 \,/\!:(-1) \end{array}\right]$ uvjet je ⊖ nepravom integralu

$\underline{x<1}$

$\int_1^2 \frac{dx}{\sqrt{(1-x)}} = \int_1^2 \frac{dx}{\sqrt{1-x}} = \left[\begin{array}{l} 1-x=t \\ -dx=dt \end{array}\right] = -\int\frac{dt}{\sqrt{t}} = -\int t^{-\frac{1}{2}}\,dt$

$= -\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) = -\left(\frac{\sqrt{1-x}}{\frac{1}{2}}\right) = -\left[2\sqrt{1-x}\right]_1^2$

$= \lim\limits_{x\to 2}(-2\sqrt{1-x}) + 2\sqrt{2} = 2\sqrt{2}$ 0

(2.) $\int \frac{x^3+x^2+x}{x(x^2+1)}\,dx = \int\frac{x^3+x^2+x}{x^3+x}\,dx = \left[\begin{array}{l} x^3+x^2+x : x^3+x = 1+\frac{x^2}{x^3+x} \\ -x^3 \quad -x \\ \hline x^2 \end{array}\right]$

$= \int dx + \int\frac{x^2}{x^3+x}\,dx = x + \int \frac{x^2}{x^3+x}\,dx$ *

* $\int\frac{x^2}{x^3+x}\,dx = \int\frac{x^2}{x(x^2+1)}\,dx = \int\frac{x}{x^2+1}\,dx = \left[\begin{array}{l} x^2+1=t \\ 2x\,dx=dt/:2 \\ x\,dx=\frac{dt}{2} \end{array}\right] = \frac{1}{2}\int\frac{dt}{t}$

$= \frac{1}{2}\ln|x^2+1|$

** $\frac{x^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ $|\cdot(\text{nz})$

$x^2 = A(x^2+1) + (Bx+C)(x)$

$x^2 = Ax^2 + A + Bx^2 + Cx$

$x^2 = x^2(A+B) + x(C) + A$

$A+B=1$ $\boxed{C=0}$ $\boxed{A=0}$

$B=1$

KONAČNO RJEŠENJE

$I = x + \frac{1}{2}\ln|x^2+1| + C$

(15)

TIN LOBOREC

(3) $y_1 = -x^2 + 3$

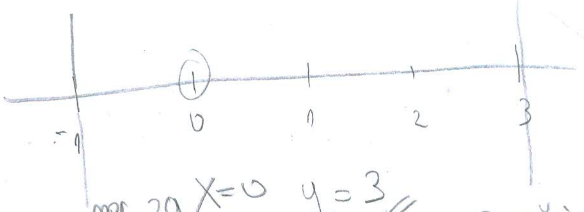
$y = -2x$

$-x^2 + 3 = -2x$

$x^2 - 2x - 3 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$

$x_1 = 3$ $x_2 = -1$



npr. za $x=0$ $y_1 = 3$
$y_2 = 0$ znači parabola je između pravca

$$\int_{-1}^{3} (-x^2 + 3 + 2x)\,dx = -\int_{-1}^{3} x^2\,dx + 3\int_{-1}^{3} dx + 2\int_{-1}^{3} x\,dx$$

$$= \left[\frac{-x^3}{3}\right]_{-1}^{3} + \left[3x\right]_{-1}^{3} + \left[\frac{2x^2}{2}\right]_{-1}^{3}$$

$$= \left[\frac{-x^3}{3} + 3x + x^2\right]_{-1}^{3} = \left[\frac{-27}{3} + 9 + 9\right] - \left[\frac{1}{3} - 3 + 1\right]$$

$$= \left[\cancel{-9} + \cancel{9} + 9\right] - \left[\frac{1 - 9 + 3}{3}\right]$$

$$= 9 + \frac{5}{3} = \frac{32}{3} \approx 10{,}7$$

15

(4) a) $f(x,y) = x^3 + xy^2 + 6xy$

$\frac{\partial f}{\partial x} = 3x^2 + y^2 + 6y$

$\frac{\partial f}{\partial y} = 2yx + 6x$

$3x^2 + y^2 + 6y = 0$

$3x^2 + 9 - 18 = 0$

$3x^2 - 9 = 0$

$3x^2 = 9 \,|:3$

$x^2 = 3 \,/\sqrt{\ }$

$x_1 = -\sqrt{3}$, $x_2 = \sqrt{3}$

$2yx + 6x = 0$

$2yx = -6x \,|:(2x)$

$y = -3$

$T_1(-\sqrt{3}, -3)$, $T_2(\sqrt{3}, -3)$ stacionarne točke

$\frac{\partial^2 f}{\partial^2 x} = 6x$

$\frac{\partial^2 f}{\partial^2 y} = 2x$

$\frac{\partial^2 f}{\partial x \partial y} = 2$

$$\Delta = \begin{vmatrix} 6x & 2 \\ 2 & 2x \end{vmatrix} = 12x^2 - 4$$

A

sljedeća stran. nastavak

nastavak 4.a)

$\Delta = 12x^2 - 4$

$T_1(-\sqrt{3}, -3) = (12 \cdot (-\sqrt{3})^2) - 4 = 36 - 4 = 32$

$T_2(\sqrt{3}, -3) = (12 \cdot (\sqrt{3})^2) - 4 = (12 \cdot 3) - 4 = 32$

za $T_1(-\sqrt{3}, -3)$ $\Delta > 0$ $\frac{\partial^2 f}{\partial x^2} > 0$ pa je $T_1(-\sqrt{3}, -3)$ LOK. MINIMUM

5

za $T_2(\sqrt{3}, -3)$ $\Delta > 0$ a $\frac{\partial^2 f}{\partial x^2} = 6x = (6 \cdot \sqrt{3}) = 6\sqrt{3} > 0$

$\rightarrow$ $\Delta > 0$; $A > 0$ pa je $T_2(\sqrt{3}, -3)$ LOKALNI MINIMUM

4 b) $f(x, y) = \ln(2 - x) + \sqrt{y + x}$

1. UVJET $2 - x > 0$ } $(2-x)$ zbog ln mora biti strogo veći od 0

2. UVJET $y + x \geq 0$

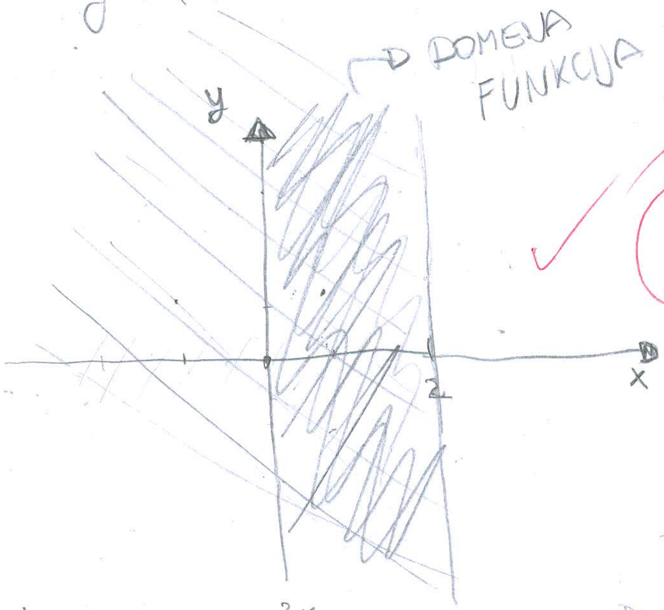
ZAJEDNIČKI UVJETI:

$2 - x > 0 \Rightarrow x < 2$

$y + x > 0$

$x > y$

$y < x < 2$



10

5. a) $y' + 4y = 2x + 3e^{3x}$

$y' + 4y = 0$

$\frac{dy}{dx} = -4y \,/\cdot dx$

$dy = -4y\,dx \,/ :(y)$

$\int \frac{dy}{y} = -4\int dx$

$\ln|y| = -4x - 4C \,/ e$

$y = e^{-4x + 4C} = e^{-4x} + e^{-4C} = C$ jer je konstanta

pa je $y = e^{-4x} + C \Rightarrow y(x) = e^{-4x} + C(x)$ $\quad y(x)' = -4e^{-4x} + C'(x)$

$\cancel{-4e^{-4x}} + C(x) + \cancel{4e^{-4x}} + 4C(x) = 2x + e^{3x}$

$5C(x) = 2x + e^{3x} \,/ :5$

$C(x) = \frac{2x + e^{3x}}{5}$

KONAČNO RJEŠENJE

$y = e^{-4x} + \frac{2x + e^{3x}}{5}$

TIN LOBOREC

♣2|

5) (b) $y'' - 4y' + 3y = \sin x$

$r^2 - 4r + 3 = 0$

$r_{1,2} = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} \Rightarrow r_1 = 3, \; r_2 = 1$ } 2 realna rješenja

$$\boxed{y_{h(x)} = C_1 e^{3x} + C_2 e^{x}}$$

5

$y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$

$y_p'' = -A\sin x - B\cos x$

$-A\sin x - B\cos x - 4A\cos x + 4B\sin x + 3A\sin x + 3B\cos x = \sin x$

$\sin x(-A + 4B + 3A) + \cos x(-B - 4A + 3B) = \sin x$

$\sin x(2A + 4B) + \cos x(2B - 4A) = \sin x$

$2A + 4B = 1$

$2A = 1 - 4B \;/\cdot 2$

$4A = 2 - 8B$

$4A = 2 - \frac{8}{3}$

$4A = \frac{6-8}{3} = -\frac{2}{3} \;/\cdot 3$

$12A = -2 \;/:(12)$

$A = -\frac{1}{6} \Rightarrow y_p = -\frac{1}{6}\sin x + \frac{1}{3}\cos x$

$2B - 4A = 0$

$2B - 2 + 8B = 0$

$6B - 2 = 0$

$6B = 2 \;/:6$

$B = \frac{1}{3}$

GREŠKA U RAČUNU

KONAČNO RJEŠENJE

$y = y_h + y_p = C_1 e^{3x} + C_2 e^{x} - \frac{1}{6}\sin x + \frac{1}{3}\cos x + K$ → konstanta

$$~~y' + 4y = 2x + 3e^{3x} \; \Big| :~~$$

$$~~\frac{dy}{dx} + 4y = 2x + 3e^{3x} \; \Big| \, dx~~$$

$$~~dy + 4y\,dx = (2x + 3e^{3x})\,dx \; \Big| : x~~$$

$$~~dy~~$$

$$y' + 4y = 2x + 3e^{3x}$$

$$y' + 4y = 0$$

$$\frac{dy}{dx} = -4y \; \Big| \cdot dx$$

$$dy = -4y\,dx \; \Big| : (y)$$

$$\int \frac{dy}{y} = -4 \int dx$$

$$\ln|y| = -4x + C$$

$$\ln|y| = x^{-4} + C \; \Big| \, e$$

$$y = e^{x^{-4}} + e^{C} \to C = y = e^{x^{-4}} + C(x)$$

$$y(x)' = e^{x^{-4}} \cdot (x^{-4})' + C'(x)$$

$$y(x) = e^{x^{-4}} \cdot (-4x^{-3}) + C'(x)$$

$$-\sin x - \cos x - 4\cos x + 4\sin x + 3A\sin x + 3B\cos x = \sin$$

$$\sin x(-1 + 4 + 3A) + \cos x(-1 - 4 + 3B) = 0$$

$$3 + 3A = 1$$

$$3A = 1 - 3 \; \Big|$$

$$A = -\frac{2}{3}$$

$$-3 + 3B = 0$$

$$3B = 3 \; \Big| : 3$$

$$B = 1$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \cdot \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: MARKO FRANIĆ Broj indeksa: 55661

Vrijeme: od 08:36 **do** ________ ♣4 **Broj bodova**: 47.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int \frac{x}{\cos^2(x^2-4)}dx$$

b)

$$\int_1^2 \frac{dx}{\sqrt{1-x}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x^2+x}{x(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=-x^2+3$ i pravac $y=-2x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^3+xy^2+6xy$$

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5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'+4y=2x+3e^{3x}$$

b)

$$y''-4y'+3y=\sin x.$$

15.06.2013 MARKO FRANIĆ 55661

1) b) $\int_1^2 \frac{1}{\sqrt{1-x}}dx = \left| \begin{array}{l} u = 1-x \\ du = -dx \end{array} \right| = -2\sqrt{u} = -2\sqrt{1-x} \Big|_1^2 =$

♣3 $= -2\sqrt{1-2} + 2\sqrt{1-1} = -2\sqrt{-1} = -2i$

2) $\int \frac{x^{\cancel{3}^2} + x^{\cancel{2}^1} + \cancel{x}}{\cancel{x}(x^2 \cdot 1)}dx = \int \frac{x^2+x+1}{x^2+1}dx = \int \left(1 + \frac{x}{1+x^2}\right)dx =$

$= \int 1\,dx + \int \frac{x}{1+x^2}dx = \left| \begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \end{array} \right| = x + \frac{1}{2}\int \frac{1}{u}du = x + \frac{\ln u}{2} =$

$= x + \frac{1}{2}\ln(1+x^2) + C \to$ KONSTANTA C

1.5

4) a) $f(x,y) = x^3 + xy^2 + 6xy$

$f_x = 3x^2 - y^2 - 6y$

$f_y = 2xy + 6x$

$3x^2 + y^2 + 6y = 0$

$2xy + 6x = 0$

$3x^2 + 9 - 18 = 0$

$2xy = -6x$

$\boxed{y = -3}$

$x^2 = 9$

$\boxed{x = \pm\sqrt{3}}$

STACIONARNE TOČKE SU:

$(\sqrt{3}, -3)$ i $(-\sqrt{3}, -3)$

1

$f_{xx}=6x$ $\quad f_{yy}=2x$

$f_{xy}=2y+6$ $\quad f_{yx}=2y+6$

$(\sqrt{3},-3)$ ♣3

$\Delta=\begin{vmatrix}6\sqrt{3} & 0\\ 0 & 2\sqrt{3}\end{vmatrix}>0$

$f_{xx}(\sqrt{3},-3)=6\sqrt{3}>0$

$\rightarrow$ MINIMUM FUNKCIJE

$(-\sqrt{3},-3)$

$\Delta=\begin{vmatrix}-6\sqrt{3} & 0\\ 0 & -2\sqrt{3}\end{vmatrix}>0$

$f_{xx}(-\sqrt{3},-3)=-6\sqrt{3}<0$

$\rightarrow$ MAKSIMUM FUNKCIJE

✓ 10

b) $f(x,y)=\ln(2-x)+\sqrt{y+x}$

$2-y>0$ $\quad \rightarrow \boxed{y+x\geq 0}$

$-x>-2 \,/\cdot(-1)$

$\boxed{x<2}$

SKICA ??

5

$Df=\{(x,y)\in\mathbb{R}^2 : x<2 \text{ i } x+y\geq 0\}$

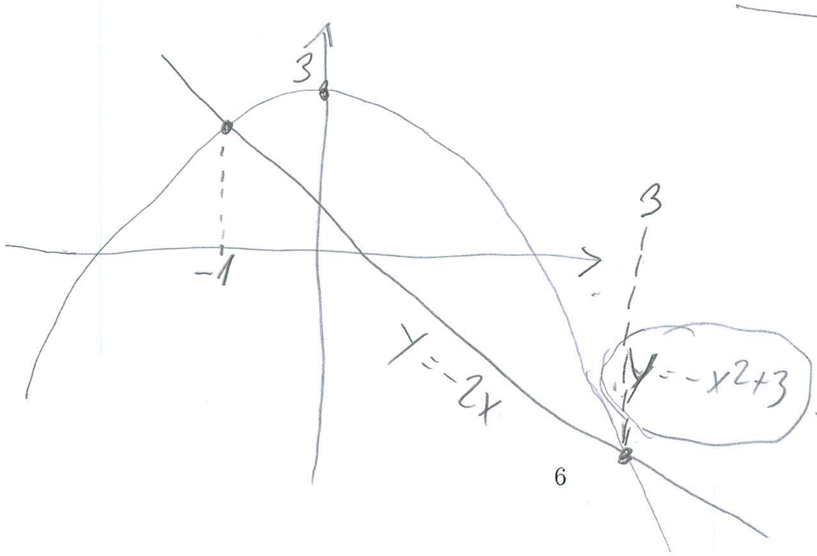
3) $y=-x^2+3$ i $y=-2x$

$x^2-2x-3=0$

$x_{1,2}=\dfrac{2\pm\sqrt{4-12}}{2}=\dfrac{2\pm 4}{2}$

$x_1=-1$

$x_2=3$



$\rightarrow$ OVO JE 0 PARABOLE

2

♣3

$$\dots y\,dx = \int_{-1}^{3} (-x^2+3+2x)\,dx = \left(-\frac{x^3}{3}+x^2+3x\right)\Big|_{-1}^{3} = (\cancel{-9}+\cancel{9}+9)-\left(\frac{1}{3}+1-3\right)=$$

$$= 9-\frac{1}{3}+2 = 11-\frac{1}{3} = \frac{33-1}{3} = \frac{32}{3}$$

1 7

$$\int \frac{x}{\cos^2(x^2-4)}\,dx = \left| \begin{matrix} u = x^2-4 \\ du = 2x\,dx \end{matrix} \right| =$$

$$\frac{x}{\cos^2 u}\cdot\frac{du}{2x} = \frac{1}{2}\int \frac{1}{\cos^2 u}\,du = \frac{1}{2}\operatorname{tg} u = \frac{1}{2}\operatorname{tg}(x^2-4)+C$$

KONSTANTA

12.5

3

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
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f	f'
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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Tablica osnovnih integrala

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣4

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: MLADEN BULIĆ Broj indeksa: 17-1-0018-2010

Vrijeme: od 8⁰⁰ **do** ________ ♣4 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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1.) a) $\int \frac{x}{\cos^2(x^2-4)}\,dx = \left[\begin{matrix} x^2-4=t \\ 2x\,dx=dt \\ x\,dx=\frac{1}{2}dt \end{matrix}\right] = \int \frac{\frac{1}{2}dt}{\cos^2(t)} = \frac{1}{2}\int \frac{dt}{\cos^2(t)} =$

b) $\int_1^2 \frac{dx}{\sqrt{1-x}} = \left[\begin{matrix} 1-x=t^2 \\ dx=2dt \end{matrix}\right] = \int_1^2 \frac{2dt}{\sqrt{(t)^2}} = \int \frac{2\,dt}{t} = 2\int \frac{dt}{t} = 2\ln|t|$

$= 2\ln|1-x| \Big|_1^2 = 2\ln|1-2| - 2\ln|1-1| = 2\ln|3| - 2\ln|0|$

2.) $\int \frac{x^3+x^2+x}{x(x^2+1)}\,dx = \int \frac{x^3+x^2+x}{x^3+x}$

$x^3+x^2+x : x^3+x = 1+\frac{2x}{x^3+x}$

$\cancel{x^3} + \cancel{x}$

$2x$

3.) $y=-x^2+3$

$y=-2x$

$-2x=-x^2+3$

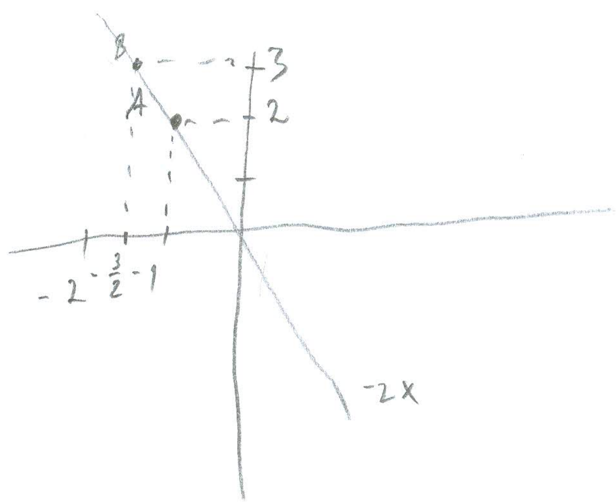
$-2x+x^2-3=$

$x_{1,2} = \frac{-1\pm\sqrt{25}}{-4} = \frac{-1\pm 5}{-4} = \frac{-1+5}{-4} = \frac{4}{-4} = -1$

$x_2 = \frac{-1-5}{-4} = -\frac{3}{2}$

$A(-1, 2)$

$B(-\frac{3}{2}, 3)$



$A=-2\cdot(-1)=2$

$B=-2\cdot\left(-\frac{3}{2}\right)=$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

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4.) $f(x,y) = x^3 + xy^2 + 6xy$

$z_x = 3x + 2y + 6$ $z_y = 2x + 6x$

5.) a) $y' + 4y = 2x + 3e^{3x}$

b) $y'' - 4y' + 3y = \sin x$

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: FRANE JORDAN **Broj indeksa:** 55161-2007

Vrijeme: od 8 **do** 11 ♣4 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int \frac{x}{\cos^2(x^2-4)}dx$$

b)

$$\int_1^2 \frac{dx}{\sqrt{1-x}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x^2+x}{x(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=-x^2+3$ i pravac $y=-2x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^3+xy^2+6xy$$

b) Odredi domenu funkcije:

$$f(x,y)=\ln(2-x)+\sqrt{y+x}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'+4y=2x+3e^{3x}$$

b)

$$y''-4y'+3y=\sin x.$$

$y''-4y'+3y=\sin x$

$$②\quad \int \frac{x^3+x^2+x}{x(x^2+1)} = \left| \begin{array}{l} x^2+1=t \,' \\ 2x = dt \,/\cdot \frac{1}{2} \\ \frac{1}{2}x = \frac{1}{2}dt \end{array} \right| = \int \frac{x^3+x^2+x}{x\cdot t}\cdot\frac{1}{2}dt = \cancel{\frac{1}{2}}\cdot\frac{x^4+x^3+x^2}{\cancel{4}+3+2\cdot x(t)}$$

$$= \frac{x^4+x^3+x^2}{7x(t)} = \frac{x^4+x^3+x^2}{7x(x^2+1)} + C$$

1) a) $\int \frac{x}{\cos^2(x^2-4)}\,dx = \left| \begin{array}{l} x^2-4=t \\ 2x = dt \,/\cdot \frac{1}{2} \\ \frac{1}{2}x = \frac{1}{2}dt \end{array} \right| = \int \frac{x}{\cos^2 t}\frac{1}{2}dt = \frac{x^2\cdot\frac{1}{2}}{\sin^2 x\, t} =$

$= \frac{\frac{1}{2}x^2}{\sin^2 x(x^2-4)} + C$

b) $\int_1^2 \frac{dx}{\sqrt{1-x}} = \left| \sqrt{1-x} = t \right.$

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