MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: Antun Žunetić **Broj indeksa:** 17-2-0169-2012

Vrijeme: od 8:00 **do** 11:00 ♣4 **Broj bodova:** 62.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int \frac{x}{\cos^2(x^2-4)}dx$$

b)

$$\int_1^2 \frac{dx}{\sqrt{1-x}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x^2+x}{x(x^2+1)}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=-x^2+3$ i pravac $y=-2x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=x^3+xy^2+6xy$$

b) Odredi domenu funkcije:

$$f(x,y)=\ln(2-x)+\sqrt{y+x}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y'+4y=2x+3e^{3x}$$

b)

$$y''-4y'+3y=\sin x.$$

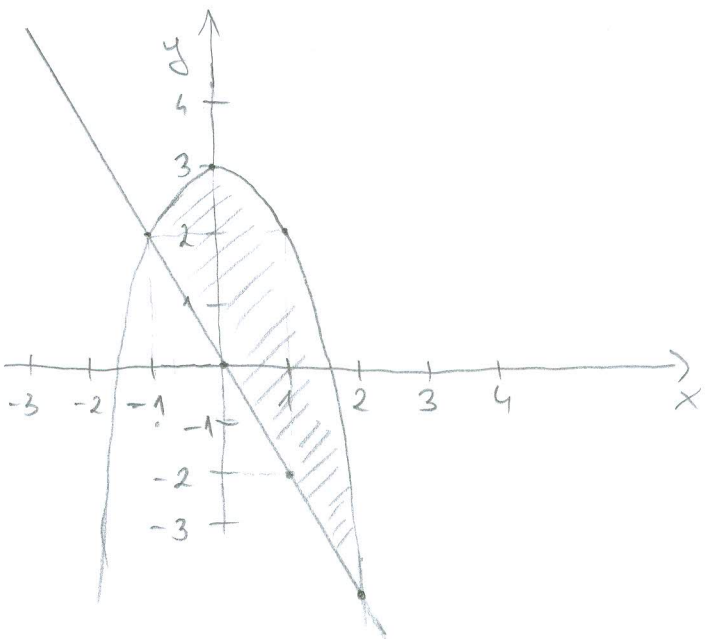
3. $y = -x^2 + 3$ – gornja funkcija

$y = -2x$ – donja funkcija

za $y = -2x$

x	-1	0	1
y	2	0	-2

x	-1	0	1
y	2	3	2



$-x^2 + 3 = -2x$

$-x^2 + 2x + 3 = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{-2}$

$x_{1,2} = \frac{-2 \pm \sqrt{16}}{-2}$

$x_1 = \frac{-2+4}{-2} = -\frac{2}{2} = -1$

$x_2 = \frac{-2-4}{-2} = \frac{-6}{-2} = 3$

$$P = \int_{-1}^{3} \left[(-x^2+3) - (-2x)\right] dx = \int_{-1}^{3} (-x^2 + 2x + 3)\, dx =$$

$$= -\int_{-1}^{3} x^2 dx + 2\int_{-1}^{3} x\, dx + 3\int_{-1}^{3} dx = \left(-\frac{x^3}{3} + \frac{x^2}{2} + 3x\right)\Big|_{-1}^{3} =$$

$$= \left(-\frac{x^3}{3} + x^2 + 3x\right)\Big|_{-1}^{3} = (-9+9+9) - \left(\frac{1}{3} + 1 - 3\right) = 9 - \left(-\frac{5}{3}\right) = \frac{32}{3}$$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$