

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: LUKA MILIĆ

Broj indeksa: 17-2-0177-2012

Vrijeme: od 08:15 do 10:35 ♣3

Broj bodova: 52.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (12.5+7.5) Integriraj

a)

$$\int 3x^2 \sqrt{x^3 + 4} dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. (15) Integriraj

$$\int \frac{x^3 + x + 2}{x^2 - 1} dx$$

3. (15) Odredi površinu koju zatvaraju parabola  $y = x^2 - 8x + 16$  i pravac  $y = -x + 6$ .

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{\ln(y - x)}$$

5. (15+15) Riješi sljedeće diferencijalne jednačbe:

a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}$$



Tablica osnovnih derivacija

$f$	$f'$	$f$	$f'$
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
$e^x$	$e^x$	$\coth x$	$\frac{1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$-\frac{1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln  \cos x  + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left  \frac{x+a}{x-a} \right  + C$
$\int \frac{1}{x} dx = \ln  x  + C$	$\int \cot x dx = \ln  \sin x  + C$	$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln  x+\sqrt{x^2+a^2}  + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right)] + C$

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① a)  $\int 3x^2 \sqrt{x^3+4} dx = \left| \begin{matrix} x^3+4=t \\ 3x^2 dx=dt \end{matrix} \right| = \int \sqrt{t} dt = \int t^{\frac{1}{2}} dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3} \cdot t^{\frac{3}{2}} + C$

$= \frac{2}{3} \cdot (x^3+4)^{\frac{3}{2}} + C$

b)  $\int_0^8 \frac{dx}{\sqrt[3]{x^2}} = \int_0^8 \frac{1}{x^{\frac{2}{3}}} dx = \int_0^8 x^{-\frac{2}{3}} dx$

$= \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} + C = 2 \cdot x^{-\frac{1}{3}} \Big|_0^8$

$= 2 \cdot (8)^{-\frac{1}{3}} - 0 = 0, 707107$

