MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: LUKA MILIN **Broj indeksa:** 17-2-0177-2012

Vrijeme: od 08:15 **do** 10:35 ♣3 **Broj bodova:** 52.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}.$$

LUKA MILIN

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x+a}{x-a}\right\| + C$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\int \cot x dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\|x + \sqrt{x^2 \pm a^2}\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1-\frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣3

1 a) $\int 3x^2 \sqrt{x^3+4}\, dx = \left| \begin{array}{l} x^3+4=t \\ 3x^2dx = dt \end{array} \right| = \int \sqrt{t}\, dt = \int t^{\frac{1}{2}} dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3} \cdot t^{\frac{3}{2}} + C$

$= \frac{2}{3} \cdot (x^3+4)^{\frac{3}{2}} + C$

2.5

b) $\int_0^8 \frac{dx}{\sqrt[3]{x^2}} = \int_0^8 \frac{1}{x^{\frac{2}{3}}} dx = \int_0^8 x^{-\frac{3}{2}} dx$

$= \frac{x^{-\frac{1}{2}}}{\frac{1}{2}} + C = 2 \cdot x^{-\frac{1}{2}} \Big|_0^8$

$= 2 \cdot (8)^{-\frac{1}{2}} - 0 = 0,707107$

(2) $\int \frac{x^3+x+2}{x^2-1}\,dx$

$(x^3+x+2):(x^2-1) = x + \frac{2x+2}{x^2-1}$

$-\underline{x^3 \mp x}$

$2x+2$

$= \int x\,dx + \int \frac{2x+2}{x^2-1}\,dx \Rightarrow I$

$\frac{2x+2}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} \quad / \cdot (x+1)(x-1)$

$2x+2 = A(x-1) + B(x+1)$

$I = 2\int \frac{dx}{x-1} = 2\ln|x-1|$

$x=-1$ $\quad$ $x=1$

$0=-2A$ $\quad$ $4=2B$

$-2A=0$ $\quad$ $2B=4$

$A=0$ $\quad$ $B=2$

$= \int x\,dx + 2\int \frac{dx}{x-1} = \frac{x^2}{2} + 2\cdot\ln|x-1| + C$

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(3) $y = x^2-8x+16$ $\quad a>0, \cup$ $\quad , \quad y=-x+6$

$x^2-8x+16 = -x+6$

$x^2-8x+x+16-6=0$

$x^2-7x+10=0$

$x_{1,2} = \frac{7\pm\sqrt{49-40}}{2} = \frac{7\pm 3}{2}$

$x_1 = \frac{7-3}{2} = \frac{4}{2} = 2 \quad , \quad x_2 = \frac{7+3}{2} = \frac{10}{2} = 5$

$y_1 = -2+6 = 4$

$y_2 = -5+6 = 1$

$S_1(2,4) \quad , \quad S_2(5,1)$

$x^2-8x+16=0$

$x_{1,2} = \frac{8\pm\sqrt{64-64}}{2} = \frac{8}{2} = 4$

$y=-x+6$

$x_0 = -\frac{b}{2a} = \frac{8}{2} = 4$

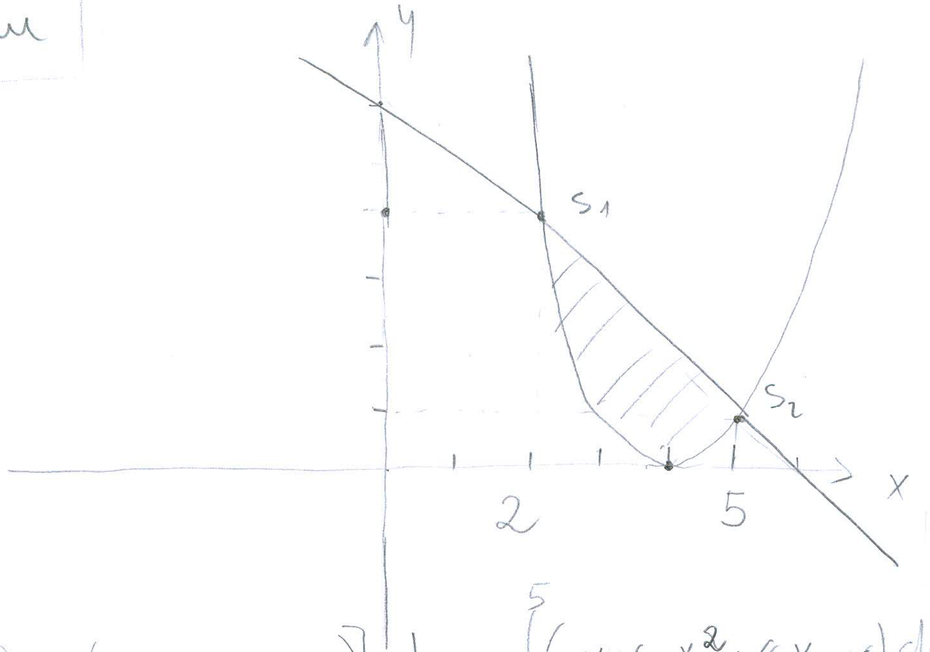
$T(4,0)$

$y_0 = \frac{4ac-b^2}{4a} = \frac{64-64}{4} = \frac{0}{4} = 0$

x	0	6
y	6	0

Luka Milin

♣1



$$P=\int_2^5\left[(-x+6)-(x^2-8x+16)\right]dx=\int_2^5(-x+6-x^2+8x-16)\,dx$$

$$-x^2+7x-10$$

$$=\int_2^5(-x^2+7x-10)\,dx=-\frac{x^3}{3}+7\cdot\frac{x^2}{2}-10x\Big|_2^5=-\frac{125}{3}+7\cdot\frac{25}{2}-50-\left(-\frac{8}{3}+7\cdot2-20\right)$$

$$=-\frac{125}{3}+\frac{175}{2}-50+\frac{8}{3}-14+20$$

$$=-\frac{125}{3}+\frac{175}{2}-44+\frac{8}{3}=\frac{9}{2}=4{,}5$$

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4 a) $f(x,y)=x^2+4x+4+y^2$

$$\frac{\partial f}{\partial x}=2x+4 \qquad 2x+4=0 \Rightarrow 2x=-4 \quad \boxed{x=-2}$$

$$\frac{\partial f}{\partial y}=2y \qquad 2y=0 \quad \boxed{y=0}$$

$T(-2,0)$

A $\frac{\partial^2 f}{\partial^2 x}=2$ $\qquad \frac{\partial^2 f}{\partial^2 y}=2$ C

$\frac{\partial^2 f}{\partial x\partial y}=0$ B

$$\Delta=\begin{vmatrix}A & B\\ B & C\end{vmatrix}=\begin{vmatrix}2 & 0\\ 0 & 2\end{vmatrix}=4-0=4>0$$

funkcija ima ekstrem

$A>0$

$2>0$ funkcija ima minimum

$z_{min}=f(x,y)=4-8+4+0$
$=0$

$T(-2,0,0)$

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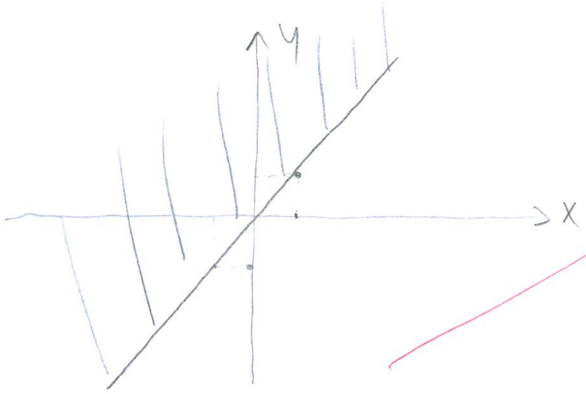
b) $f(x,y)=\frac{1}{\ln(y-x)}$ U: $\ln(y-x)\neq 0$ LUKA MILIN

$y-x>0$

$y>x$

$y=x$

$Df=\{(x,y)\in\mathbb{R}^2 \mid y>x,\ \ln(y-x)\neq 0\}$



5 a) $y'+4y=3e^x$

$f(x)=4$, $g(x)=3e^x$

$yp=e^{-\int f(x)dx}\cdot\left[\int e^{\int f(x)}dx \cdot g(x)\,dx+C\right]$

$\int f(x)=\int 4dx=4x$

$\int g(x)=\int 3e^x dx=3\int e^x dx=3e^x$

$y=e^{-4x}\cdot\left[(e^{4x}\cdot 3e^x)+C\right]$

$=e^{-4x}\cdot e^{4x}\cdot 3e^x+C$

b) $y''-y'+6y=7e^{5x}$ $b=5\neq 1$

$r^2-r+6=0$

$r_{1,2}=\frac{1\pm\sqrt{1-24}}{2}=\frac{1\pm\sqrt{-23}}{2}=\frac{1\pm 4,8i}{2}\Big/\cdot 2$

$=2\pm 9,6i$

$a=2,\ b=9,6$

$yH=e^{ax}(c_1\cos bx+c_2\sin bx)$

$=e^{2x}(c_1\cos 9,6x+c_2\sin 9,6x)$

$yp=\frac{k\cdot e^{bx}}{P(b)}=\frac{7\cdot e^{5x}}{26}$

$P(b)=b^2-b+6$

$P(5)=25-5+6=26$

$y=e^{2x}(c_1\cos 9,6x+c_2\sin 9,6x)+\frac{7\cdot e^{5x}}{26}$

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: MARIO MATEK Broj indeksa: 17-1-0111-12

Vrijeme: od 08:10 do ♣3 Broj bodova: 45

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}.$$

1)a) $\int 3x^2\sqrt{x^3+4}\,dx = \int 3x^2 (x^3+4)^{\frac{1}{2}} dx = \left[\begin{matrix} x^3+4 = t\ /' \\ 3x^2 dx = dt \end{matrix}\right.$

$= \int (t)^{\frac{1}{2}} dt$

$= \left(\dfrac{(x^3+4)^{\frac{3}{2}}}{\frac{3}{2}}\right) = \dfrac{2}{3}(x^3+4)^{\frac{3}{2}} + C\ //$ 12.5

b) $\int_0^8 \dfrac{dx}{\sqrt[3]{x^2}} \Rightarrow$ SING. $\lim\limits_{\varepsilon \to 0} \int_\varepsilon^8 x^{-\frac{2}{3}} dx = \dfrac{8^{\frac{1}{3}}}{\frac{1}{3}} - \dfrac{0^{\frac{1}{3}}}{\frac{1}{3}} = 6 - 0 = 6\ //$ 7.5

2. $\int \dfrac{x^3+x+2}{x^2-1}$

$(x^3+x+2) : (x^2-1) = x$
$\underline{-x^3 \mp x}$
$2x+2$

$= \int x\,dx + \int \dfrac{2\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} dx = \left[\begin{matrix} x-1 = t\ /' \\ dx = dt \end{matrix}\right]$

$= \dfrac{x^2}{2} + 2\ln|x-1| + C\ //$ 15

$\frac{6}{4}$

3. $y = -x+6$
$y = x^2 - 8x + 16$

x	-1	0	1	2
$-x+6$	7	6	5	4

x	-1	0	1	2	3	4	5
$x^2-8x+16$	25	16	8	4	1	0	-19

$x^2 - 8x + 16 = 0$

$x_{1,2} = \dfrac{8 \pm \sqrt{64-64}}{2}$

$x_{1,2} = \dfrac{8}{2} = 4$

$\int_4^4 (-x+6) + (x^2 - 8x + 16)$

$= \int_4^4 x^2 - 9x + 22$

$\dfrac{x^3}{3}\Big|_4^4 - 9\dfrac{x^2}{2}\Big|_4^4 + 22x\Big|_4^4$

$\left(\dfrac{4^3}{3} - \dfrac{4^3}{3}\right) - \left(\dfrac{9\cdot 4^2}{2} - \dfrac{9\cdot 4^2}{2}\right) + (22\cdot 4 - 22\cdot 4) = 0\ //$

$\int 22\,dx$

NEMA POVRŠINE

5) $y'' - y' + 6y = 7e^{5x}$

♣1

$r^2 - r + 6 = 0$

$$r_{1,2} = \frac{1 \pm \sqrt{1-24}}{2}$$

$$r_{1,} = \frac{1 \pm \sqrt{-23}}{2}$$

$$r_1 = \frac{1 + \sqrt{23}\, i}{2}$$

$$r_2 = \frac{1 - \sqrt{23}\, i}{2}$$

4. a) $f(x,y) = x^2 + 4x + 4 + y^2$

MARIN MATEK

STAC. TOČKE

$$\frac{\partial f}{\partial x} = 2x + 4 \qquad \frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$$\frac{\partial f}{\partial y} = 2y \qquad \frac{\partial^2 f}{\partial y^2} = 2$$

$2x + 4 = 0$

$x = -2$

$y = 0$

$$\frac{\partial f}{\partial x \partial f} = 0$$

10

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial x \partial y} \\ \frac{\partial f}{\partial x \partial y} & \frac{\partial f}{\partial y^2} \end{vmatrix} \qquad \Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4$$

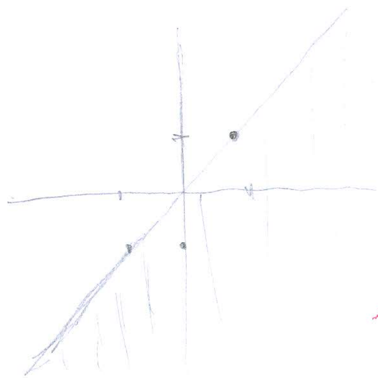
$\Rightarrow$ EKSTREM JE MINIMUM

b) $f(x,y) = \dfrac{1}{\ln(y-x)}$

$y - x > 0$

$y > x$

$y = x$



Tablica osnovnih derivacija

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Tablica osnovnih integrala

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$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x \sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

Mauer

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: MARIN GVOZDEN **Broj indeksa:** 17-2-0137-2011

Vrijeme: od 08:10 **do** 10:20 ♣3 **Broj bodova:** 42.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}.$$

1. a)

$$\int 3x^2\sqrt{x^3+4}\,dx = \left| \begin{matrix} x^3+4 = t \\ 3x^2 dx = dt \\ dx = \frac{dt}{3x^2} \end{matrix} \right| = \int \cancel{3x^2}\sqrt{t}\cdot\frac{dt}{\cancel{3x^2}} =$$

(12.5)

$$= \int \sqrt{t}\,dt = \int t^{\frac{1}{2}}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2t^{\frac{3}{2}}}{3} = \boxed{\frac{2}{3}\sqrt[2]{(x^3+4)^3}+C}$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}} = \int_0^8 x^{-\frac{2}{3}} = \frac{x^{\frac{1}{3}}}{\frac{1}{3}}\Big|_0^8 = 3x^{\frac{1}{3}}\Big|_0^8 = \left(3\cdot 8^{\frac{1}{3}}\right)-\left(3\cdot 0^{\frac{1}{3}}\right)$$

$$= \boxed{6}$$

NEISPRAVAN RAČUN

4.

a) $f(x,y) = x^2+4x+4+y^2$

$Z_x = 2x+4$ $\quad Z_{xx}=2 \quad Z_{xy}=1$

$Z_y = 2y$ $\quad Z_{yy}=2 \quad Z_{yx}=1$

$Z_x = 0$, $Z_y = 0$

$2x+4=0$

$2x=-2$

$\boxed{x=-1}$

GREŠKA U RAČUNU

$2y=0$

$\boxed{y=0}$

$Z_{xx}>0 \to 2$

$\boxed{T(-1,0)\text{ MIN}}$

(5)

$$\Delta = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-2=2 \Longrightarrow \text{MINIMUM}$$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln\|\cos x\| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x+a}{x-a}\right\| + C$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\int \cot x dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\|x + \sqrt{x^2 \pm a^2}\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

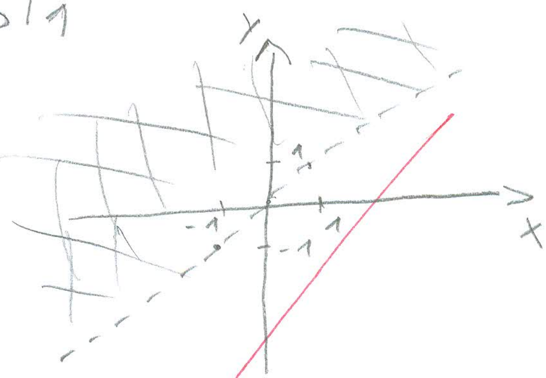
♣3

4^*

b)

$$f(x,y)=\frac{1}{\ln(y-x)}$$

x	-1	0	1
y	-1	0	1



$y-x\neq 0$

$y>x$

→ DOMENA NE UKLJUČUJE PRAVAC !

2.

$$\int \frac{x^3+x+2}{x^2-1}\,dx$$

$$(x^3+x+2):(x^2-1)=x$$
$$\frac{-(x^3-x)}{2x+2}$$

$$\int x\,dx+\int\frac{2x+2}{x^2-1}\,dx$$

$$\frac{x^2}{2}+\underbrace{\int\frac{2x+2}{(x+1)(x-1)}}_{*}$$

*

$$\int\frac{2x+2}{(x+1)(x-1)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}\quad /\cdot(x+1)(x-1)$$

$$2x+2=A(x-1)+B(x+1)$$
$$2x+2=Ax-A+Bx+B$$

$$\cancel{A}+B=2 \Rightarrow B=2$$
$$-\cancel{A}+B=2\Rightarrow A=0$$

$$\int\frac{0}{(x+1)}dx+\int\frac{2}{(x-1)}\,dx$$
$$=2\int\frac{dx}{(x-1)}=2\ln|x-1|+C$$

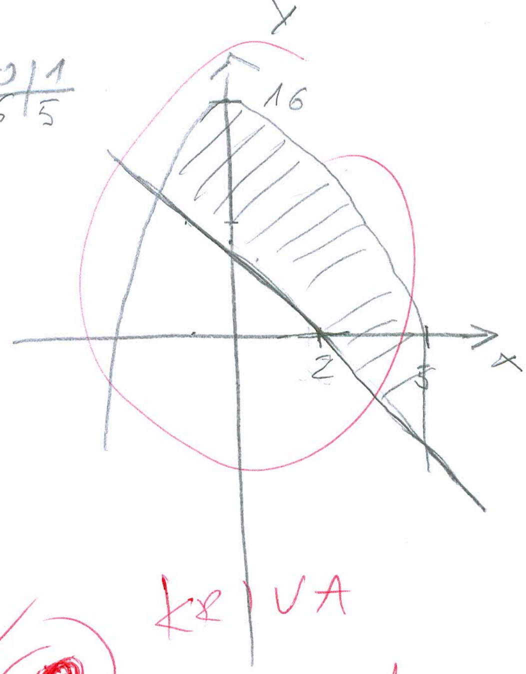
$$\int\frac{x^3+x+2}{x^2-1}\,dx=\frac{x^2}{2}+2\ln|x-1|+C$$

1,5

3.

$y = x^2 - 8x + 16 \qquad y = -x + 6$

x	-1	0	1
y	7	6	5



$x^2 - 8x + 16 = -x + 6$

$x^2 - 8x + 16 + x - 6 = 0$ $\quad a = 1$

$x^2 - 7x + 10 = 0$ $\quad b = -7$

$\quad c = 10$

$x_{1,2} = \frac{7 \pm \sqrt{49-40}}{2} \qquad x_1 = \frac{7+3}{2} = 5$

$x_2 = \frac{7-3}{2} = 2$

KRIVA

SLIKA

10

$$P = \int_2^5 \left[(x^2 - 8x + 16) - (-x + 6) \right] dx$$

$$= \int_2^5 (x^2 - 8x + 16 + x - 6)\, dx$$

$$= \int_2^5 (x^2 - 7x + 10)\, dx$$

$$= \frac{x^3}{3}\Big|_2^5 - \frac{7x^2}{2}\Big|_2^5 + 10x\Big|_2^5$$

$$= \left(\frac{125}{3} - \frac{7 \cdot 25}{2} + 50 \right) - \left(\frac{8}{3} - \frac{7 \cdot 4}{2} + 20 \right) =$$

$$= (41.66 - 87.5 + 50) - (2.66 - 14 + 20) =$$

$$= 4.16 - 8.66 = 4.5 = \underline{\frac{9}{2}}$$

♣3

Tena Krupotić

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: TENA KRUPOTIĆ Broj indeksa: 5970

Vrijeme: od_____________ **do**_________ **♣3** **Broj bodova:** 22.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + 4y = 3e^x$$

b)

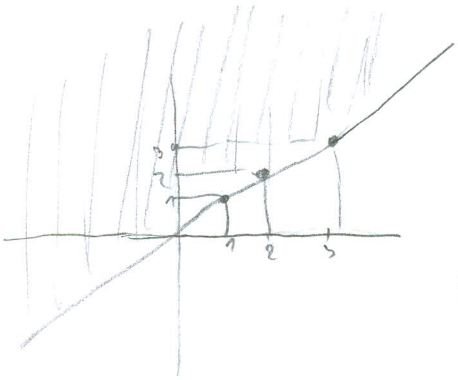
$$y'' - y' + 6y = 7e^{5x}.$$

(4) $f(x,y) = \frac{1}{\ln(y-x)}$

$\ln(y-x)$

1° $y - x > 0$

$y > x$



$y-x$	x
1	1
2	2
3	3

Pravac ne ulazi u domenu!

(5) a) $y' + \underbrace{4y}_{P} = \underbrace{3e^{x}}_{Q}$

$$y = e^{-\int P_x dx} \left[Q(x) \cdot e^{\int P_x dx} + C \right]$$

$\int 4y$

TENA KRMPOTIĆ

① a) $\int 3x^2 \sqrt{x^3+4}\,dx$ $\quad\left|\begin{array}{l} x^3+4=t \,/' \\ 3x^2 dx = dt \\ dx = \frac{dt}{3x^2}\end{array}\right.$

$\int \cancel{3x^2} \sqrt{t}\ \frac{dt}{\cancel{3x^2}}$

$\int \sqrt{t} = \int t^{\frac{1}{2}} = \int \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{(x^3+4)^{\frac{3}{2}}}{\frac{3}{2}} + C$

b) $\int_0^8 \frac{dx}{\sqrt[3]{x^2}} = \lim_{\varepsilon\to 0} \int_\varepsilon^8 \frac{dx}{\sqrt[3]{x^2}} = \lim_{\varepsilon\to 0} \int_\varepsilon^8 \frac{dx}{x^{\frac{2}{3}}} = \lim_{\varepsilon\to 0} \int_\varepsilon^8 x^{-\frac{2}{3}}dx = \lim_{\varepsilon\to 0} \left.\frac{x^{\frac{1}{3}}}{\frac{1}{3}}\right|_\varepsilon^8$

$= \lim_{\varepsilon\to 0} \left(\frac{8^{\frac{1}{3}}}{\frac{1}{3}} - \frac{\varepsilon^{\frac{1}{3}}}{\frac{1}{3}}\right) = 6 - 0 = 6$ ✓ 2.5

③ $y = x^2 - 8x + 16$ $\quad y = -x + 6$

$x^2 - 8x + 16 = 0$

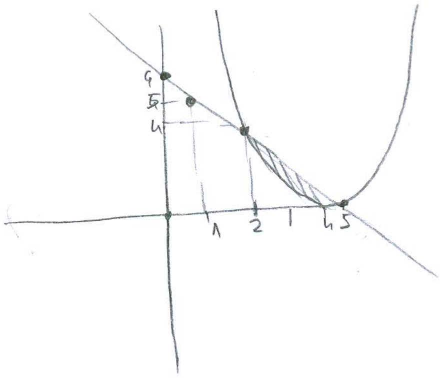
$x_{12} = \frac{8 \pm \sqrt{64-64}}{2}$

$x_{12} = \frac{8 \pm \sqrt{0}}{2}$

$x_{12} = \frac{8 \pm 0}{2}$

$x_1 = \frac{8-0}{2} = 4$

$y = -x+6$	x
6	0
5	1
4	2



$x^2 - 8x + 16 = -x + 6$

$x^2 - 8x + 16 + x - 6 = 0$

$x^2 - 7x + 10 = 0$

$x_{12} = \frac{7 \pm \sqrt{49-40}}{2}$

$x_{12} = \frac{7 \pm \sqrt{9}}{2}$

$x_1 = \frac{7-3}{2} = \frac{4}{2} = 2$

$x_2 = \frac{7+3}{2} = \frac{10}{2} = 5$

$P = \int_2^5 (-x+6) - (x^2 - 8x + 16)$

$P = \int_2^5 -x + 6 - x^2 + 8x - 16$

$P = \int_2^5 -x^2 + 7x - 10$

$P = \int_2^5 -x^2 dx + \int_2^5 7x\,dx - \int_2^5 10\,dx$

$P = -\left.\frac{x^3}{3}\right|_2^5 + 7\left.\frac{x^2}{2}\right|_2^5 - 10x\Big|_2^5$

$P = -\frac{1}{3}(5^3 - 2^3) + \frac{7}{2}(5^2 - 2^2) - (10\cdot 5 - 10\cdot 2)$

$P = -\frac{1}{3}(125 - 8) + \frac{7}{2}(25 - 4) - (50 - 20)$

$P = -\frac{1}{3}(117) + \frac{7}{2}(21) - (30)$

$P = -39 + \frac{147}{2} - 30$ ✓ 1.5

$P = \frac{9}{2}$

④ $f(x,y) = x^2 + 4x + 4 + y^2$

$\frac{df}{dx} = 2x + 4$

$\frac{df}{dy} = 2y$

$\frac{d^2f}{d^2x} = 2$

$\frac{d^2f}{d^2y} = 2$

$\frac{df}{dy\,dx} = 0$

$\frac{df}{dx\,dy} = 0$

$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 2\cdot 2 - 0\cdot 0 = 4 - 0 = 4$

$\Delta > 4$

Max extrem

(2) $\int \frac{x^3+x+2}{x^2-1}\,dx = \frac{x^3+x+2}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} \Big/ (x-1)(x+1)$

$= \frac{3}{(x-1)} - \frac{2}{(x+1)}$

$x^3+x+2 = A(x+1)+B(x-1)$

$= Ax+A+Bx-B$

$= x(A+B)+A-B$

$A+B=1$

$A-B=2 \quad /+$

$\boxed{A=3}$

$3+B=1$

$B=1-3$

$\boxed{B=-2}$

$= \int \frac{3}{(x-1)}\,dx - \int \frac{2}{(x+1)}\,dx \qquad \left| \begin{array}{l} x-1=t \,/' \\ dx=dt \end{array} \right.$

$= 3\int \frac{dx}{t} - 2\int \frac{dx}{t} \qquad \left| \begin{array}{l} x+1=t \,/' \\ dx=dt \end{array} \right.$

$= 3\ln|t| - 2\ln|t|$

$= 3\ln|x-1| - 2\ln|x+1| + C$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

Lllp

MATEMATIKA 2

~~15.~~ lipnja 2013.

Ime i prezime: LUKA PEROS **Broj indeksa:** 02184

Vrijeme: od ____________ **do** _________ ♣3 **Broj bodova:** 20

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}.$$

1. a) $\int 3x^2\sqrt{x^3+4}\,dx = \left| \begin{matrix} x^3+4=t \\ 3x^2dx=dt \end{matrix} \right| = \int \sqrt{t}\,dt = \frac{t^2}{2} = \frac{1}{2}(x^3+4)^2 + C$

b) $\int_0^8 \frac{dx}{\sqrt[3]{x^2}} =$ ovaj integral je nepravi jer ako uvrstimo jednu od granica u nazivnik podintegralne funkcije rješenje je nula.

$\int \frac{dx}{\sqrt[3]{x^2}} = \int x^{-\frac{2}{3}}dx = \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} = 3\,x^{\frac{1}{3}} = \left[3\sqrt[3]{x}\right]\Big|_0^8$

$= \left[3\sqrt[3]{8}\right] - \underbrace{\left[3\sqrt[3]{0}\right]}_{0} = 3\sqrt[3]{4\cdot 2} = 6\sqrt{2}$

2. $\int \frac{x^3+x+2}{x^2-1}dx =$

$(\cancel{x^3}+x+2):(x^2-1) = x + \frac{2x+2}{x^2-1}$

$\cancel{x^3} - x$

$2x+2$

$\int x\,dx + \int \frac{2x+2}{x^2-1}dx = x + I_2$

$I = x + \ln(x^2-1) + \ln(x^2-1) + C$

$I_2 = \int \frac{2x+2}{x^2-1}dx = 2\int \frac{x}{x^2-1}dx + 2\int \frac{1}{x^2-1}dx = 2\,I_3 + \cancel{2}\frac{1}{\cancel{2}}\ln(x^2-1)$

$I_3 = 2\int \frac{x}{x^2-1}dx = \left| \begin{matrix} x^2-1=t \\ 2x\,dx=dt \quad /:2 \\ x\,dx=\frac{dt}{2} \end{matrix} \right| = \cancel{2}\cdot\frac{1}{\cancel{2}}\int\frac{dt}{t} = \ln t = \ln(x^2-1)$

3.) $y = x^2 - 8x + 16$, pravac: $y = -x + 6$

$x^2 - 8x + 16 = 0$ $\qquad x_{1,2} = \frac{8 \pm \sqrt{64-64}}{2} = \frac{8 \pm 0}{2}$

$x_1 = 4, \; x_2 = 4$

$x_0 = -\frac{b}{2a}$

$x_0 = -\frac{-8}{2} = \boxed{4}$

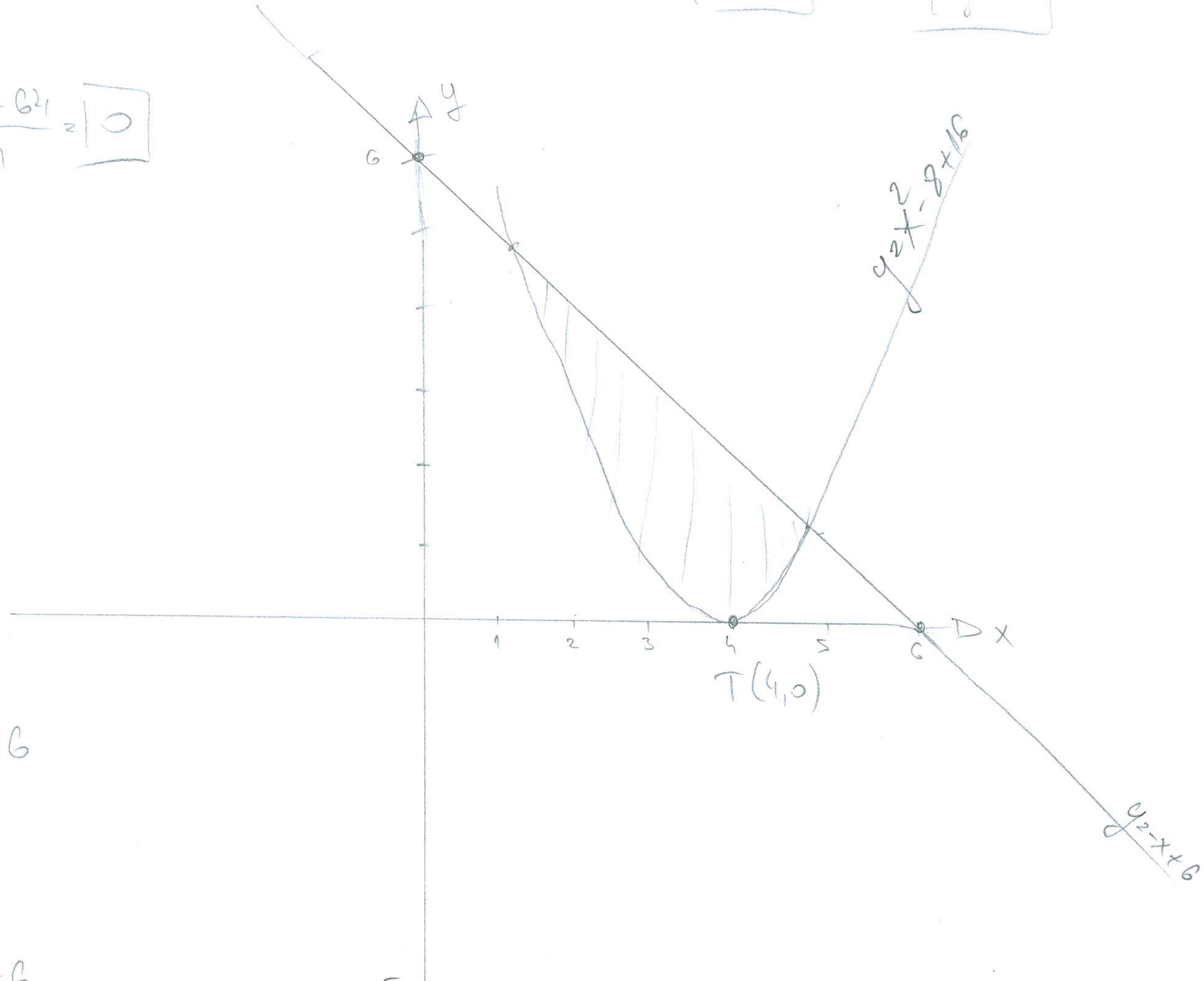
$y_0 = \frac{4ac - b^2}{4a} = \frac{64-64}{4} = \boxed{0}$

Pravac: $y = -x + 6$

$y = 0$ $\qquad x = 0$

$0 = -x + 6$ $\qquad y = -0 + 6$

$\boxed{x = 6}$ $\qquad \boxed{y = 6}$



$y = x^2 - 8x + 16$

$y = -x + 6$

$y = y$

$x^2 - 8x + 16 = -x + 6$

$x^2 - 8x + x + 16 - 6 = 0$

$x^2 - 7x + 10 = 0$

$x_{1,2} = \frac{7 \pm \sqrt{49-40}}{2}$

$x_{1,2} = \frac{7 \pm 3}{2}$

$x_1 = 5, \; x_2 = 2$

$$P = \int_2^5 (-x+6)\,dx - \int_2^5 (x^2 - 8x + 16)\,dx =$$

$$P = \left[-\frac{x^2}{2} + 6x\right]_2^5 - \left[\left(-\frac{x^3}{3} - 4x^2 + 16x\right)\right]_2^5 \Rightarrow$$

$$\int(-x+6)\,dx = -\int x\,dx + 6\int dx = -\frac{x^2}{2} + 6x$$

$$\int(x^2 - 8x + 16)\,dx = -\int x^2\,dx - 8\int x\,dx + 16\int dx = -\frac{x^3}{3} - \cancel{8}\frac{x^2}{\cancel{2}} + 16x$$

$$P = \left\{\left[-\frac{25}{2}+30\right]-\left[-\frac{4}{2}+12\right]\right\} - \left\{\left[-\frac{125}{3}-100+80\right]-\left[-\frac{8}{3}-16+32\right]\right\}$$

GREŠKA U RAČUNU

$$P = \left[\frac{35}{2}-\frac{20}{2}\right] - \left[-\frac{185}{3}+\frac{40}{3}\right] = \frac{15}{2}+\frac{145}{3} = \frac{45+290}{6} = \boxed{\frac{335}{6}}$$

10

1.

a) $f(x,y) = x^2+4x+4+y^2$

$$2x+4=0 \qquad 2x=-4 \qquad \boxed{x=-2}$$

$$2y=0 \qquad \boxed{y=0}$$

$$z_x = \frac{\partial f}{\partial x} = 2x+4+0+0 = 2x+4$$

$$z_y = \frac{\partial f}{\partial y} = 0+0+0+2y = 2y$$

$$z_{xx} = \frac{\partial^2 f}{\partial^2 x} = 2 \quad \Rightarrow A = 2>0 \quad \Rightarrow \text{MINIMUM.}$$

$$z_{xy} = \frac{\partial f}{\partial x \partial y} = 0 \quad \Rightarrow B = 0$$

$$z_{yy} = \frac{\partial^2 f}{\partial^2 y} = 2 \quad \Rightarrow C = 2>0$$

10

$$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4-0 = 4>0$$

postojanje ekstrema treba ispitati!

b) $f(x,y) = \dfrac{1}{\ln(y-x)}$

$\ln(y-x) \neq 0$

$y-x \geq 0$

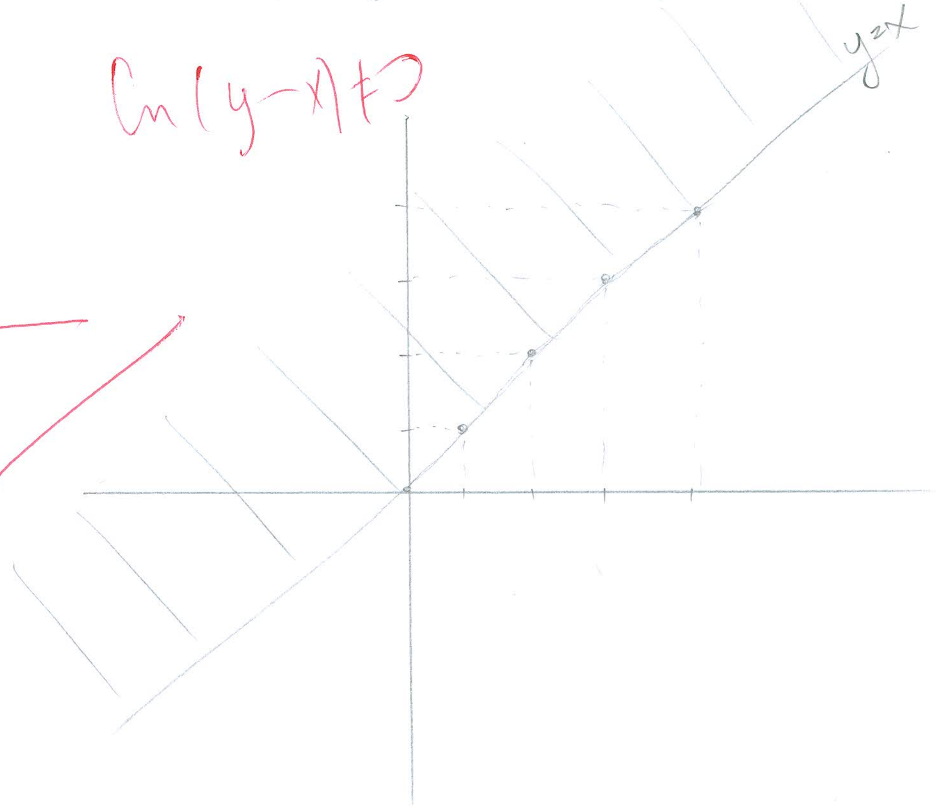
$y \geq x$

$y = x$

x	0	1	2	3	4
y	0	1	2	3	4

$(0,1)$ $1-0>0$

$1>0$ ✓



5. a) $y' + 4y = 3e^x$

$$\frac{dy}{dx} + 4y = 3e^x \quad / \cdot dx$$

$$dy + 4y = 3e^x dx$$

$$\cancel{4y\,dy = 3e^x dx} \quad / \int$$

$$\int 4y\,dy = \int 3e^x dx$$

$$4\frac{y^2}{2} = 3e^x + C$$

$$2y^2 - 3e^x = C$$

---

b) $y'' - y' + 6y = 7e^{5x}$

$$r^2 - r + 6 = 0$$

$$r_{1,2} = \frac{1 \pm \sqrt{1-24}}{2}$$

$$r_{1,2} = \frac{1 \pm 23i}{2}$$

$$r_1 = \frac{1}{2} + \frac{23i}{2}, \quad r_2 = \frac{1}{2} - \frac{23i}{2}$$

$$y = y_0 + h$$

$$y_0 = e^{\frac{1}{2}x}\left[C_1 e^{\frac{23}{2}x} + C_2 e^{-\frac{23}{2}x}\right]$$

$$h = \frac{ke^{nx}}{P'(b)} = \frac{7e^{5x}}{35e^{5x}}$$

?

$$y = e^{\frac{1}{2}x}\left[C_1^{\frac{23}{2}x} + C_2 e^{-\frac{23}{2}x}\right] + \frac{7e^{5x}}{35e^{5x}} + C$$

$$P = 7e^{5x}$$

$$P' = 35e^{5x}$$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: JASMIN NEKIĆ Broj indeksa: 0226

Vrijeme: od ____________ **do** _________ **♣3** **Broj bodova**: 12.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}.$$

1. a) $\int 3x^2\sqrt{x^3+4}\,dx$

$x^3+4=t \,/'$

$3x^2 dx = dt$

$dx = \frac{dt}{3x^2}$

$\int \cancel{3x^2}\sqrt{t}\,\frac{dt}{\cancel{3x^2}}$

$\int \sqrt{t}\,dt =$

$\int t^{\frac{1}{2}}dt =$

$\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2t^{\frac{3}{2}}}{3} + C =$

$= \frac{2\cdot(x^3+4)^{\frac{3}{2}}}{3} + C$

$= \frac{2\sqrt{(x^3+4)^3}}{3} + C$

✓ (1,5)

b) $\int_0^8 \frac{dx}{\sqrt[3]{x^2}} =$

$x^2 = t^3 \,/'$ $\quad t = x^{\frac{2}{3}}$

$t^{\frac{1}{2}} = x^{\frac{2}{6}=\frac{1}{3}}$

$x\,dx = 2t^2\,dt$

$dx = \frac{2t^2}{x}dt$

$dx = \frac{2t^2}{t^{\frac{3}{2}}}dt$

$dx = \frac{2t}{t^{\frac{1}{2}}}dt$

$\int_0^8 \frac{\frac{2t}{t^{\frac{1}{2}}}}{\sqrt[3]{t^3}}dt$

$\int_0^8 \frac{\frac{2t}{t^{\frac{1}{2}}}}{\frac{t}{1}}dt$

$\int_0^8 2\cdot\frac{1}{t^{\frac{1}{2}}}dt$

$\int_0^8 2t^{-\frac{1}{2}}dt = 2\cdot\int_0^8 t^{-\frac{1}{2}}dt = 2\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} = 4t^{\frac{1}{2}}$

$= 4\cdot x^{\frac{1}{3}}$

$= 4\cdot\sqrt[3]{8} - 4\cdot\sqrt[3]{0}$

$= 4\cdot 2 - 0 = \underline{\underline{8}}$

2. $\int \frac{x^3+x+2}{x^2-1}dx$

$\int \frac{x^3+2x-x+2}{x^2-1}dx$

$\int \frac{x(x^2-1)+2x+2}{x^2-1}dx$

$\int\left(\frac{x\cancel{(x^2-1)}}{\cancel{x^2-1}} + \frac{2x+2}{x^2-1}\right)dx$

$\int x\,dx + \int\frac{2x+2}{x^2-1}dx$

$\underline{x + \int\frac{2x+2}{x^2-1}dx + C}$

♣3

5. a) $y' + 4y = 3e^x$

$\frac{dy}{dx} + 4y = 3e^x \,/ \cdot dx$

$4y + dy = 3e^x dx \,/ \int$

$\int 4y + \int dy = \int 3e^x dx$

$\cancel{4}^2 \cdot \frac{y^2}{\cancel{2}_1} + y = 3e^x$

$2y^2 + y = 3e^x + C$

b) $y'' - y' + 6y = 7e^{5x} \,/ \int$

$y' - y + 3y^2 = 7 \cdot \frac{1}{5} e^{5x} + C_1$

$y' - y + 3y^2 = \frac{7}{5} e^{5x} \,/ \int$

$y - \frac{y^2}{2} + 2y^3 = \frac{7}{25} e^{5x} + C_1 x + C_2$

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sinh^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: MARINO ZUBČIĆ Broj indeksa: 17-2-0216-2012

Vrijeme: od ______ do ______ ♣3 Broj bodova: 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + 4x + 4 + y^2$$

b) Odredi domenu funkcije:

$$f(x, y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' + 4y = 3e^x$$

b)

$$y'' - y' + 6y = 7e^{5x}.$$

3

$y = x^2 - 8x + 16$; $y = -x + 6$.

SJECIŠTE : $y = y$

$x^2 - 8x + 16 = -x + 6$

$x^2 - 8x + 16 + x - 6 = 0$

$x^2 - 7x + 10 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{7 \pm \sqrt{49 - 40}}{2}$

$x_{1,2} = \frac{7 \pm \sqrt{9}}{2}$

$x_{1,2} = \frac{7 \pm 3}{2}$

$\boxed{x_1 = 5}$

$\boxed{x_2 = 2}$

$y_1 = -5 + 6$

$\boxed{y_1 = 1}$

$y_2 = -2 + 6$

$\boxed{y_2 = 4}$

$\Rightarrow S_1(5, 1)$

$S_2(2, 4)$

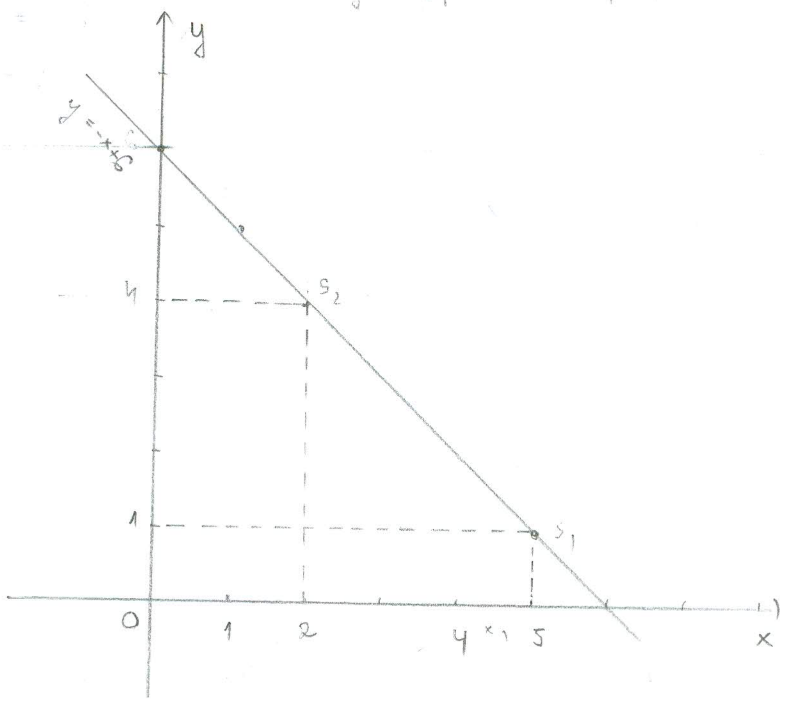
STACIONARNE TOČKE :

$$x^2 - 8x + 16 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{8 \pm \sqrt{64-64}}{2}$$

x	0	1	2	3
y = -x+6	6	5	4	3



④ a) $f(x, y) = x^2 + 4x + 4 + y^2$

$\partial_x f = 2x + 4$ $\qquad$ $\partial_y f = 2y$

$\partial_{xx} f = 2$ $\qquad$ $\partial_{yy} f = 2$

$\partial_{xy} f = 1$

$\partial_x f = 0$ $\qquad$ $2x + 4 = 0 \Rightarrow 2x = -4$

$\partial_y f = 0$ $\qquad$ $2y = 0$ $\qquad$ $\boxed{x = -2}$

$\boxed{y = 0}$

b) $f(x, y) = \dfrac{1}{\ln(x-y)}$ $\qquad$ $D_f = \{ x, y \in \mathbb{R}^2 : (x \neq 0$

UVJET :

$\ln(x-y) \neq 0$ $\qquad$ $x > y$

$x - y > 0$

Tablica osnovnih derivacija

f	f'
$x^{\alpha} (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: STIPE DUŠEVIĆ **Broj indeksa:** 17-2-0051-2010

Vrijeme: od 09:00 **do** ________ ♣3 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

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b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2 - 8x + 16$ i pravac $y = -x + 6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + 4x + 4 + y^2$$

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Tablica osnovnih derivacija

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$x^{\alpha}(\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
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e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
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$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x+a}{x-a} \right\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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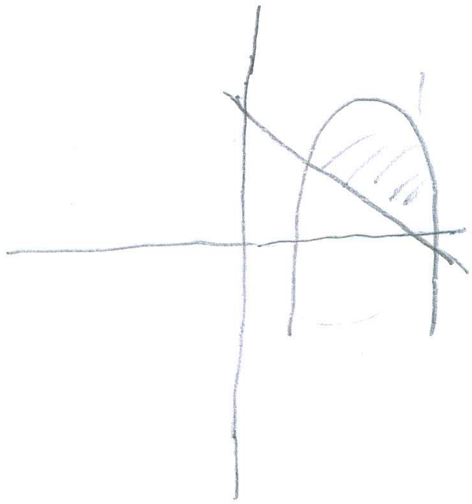
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3

3. $y = x^2 - 8x + 16$

$y = -x + 6$



$x^2 - 8x + 16 = -x + 6$

$x^2 - 7x + 10 = 0$

$x_{1,2} = \frac{7 \pm \sqrt{49-40}}{2}$

$x_{1,2} = \frac{7 \pm 3}{2}$

$x_1 = 2$

$x_2 = 5$

$$P = \int_2^5 \left(x^2 - 8x + 16 - (-x+6) \right) dx = \int_2^5 (x^2 - 8x + 16 + x - 6)\, dx =$$

$$= \int_2^5 (x^2 - 7x + 10)\, dx = \int_2^5 x^2 dx - \int_2^5 7x\, dx + \int_2^5 10\, dx =$$

$$= \frac{x^3}{3} \Big|_2^5 - 7 \frac{x^2}{2} \Big|_2^5 + 10x \Big|_2^5 =$$

$$= \frac{125}{3} - \frac{8}{3} - 7 \frac{25}{2} - \left(-7 \frac{4}{2}\right) + 50 - 20 =$$

$$= \frac{118}{3} - \frac{39}{2} + \frac{18}{2} + 30 =$$

$$= \frac{118}{3} - \frac{21}{2} + 30 = \frac{236 - 63 + 180}{6} = \frac{353}{6}$$

♣1

4.

a) $f(x,y) = x^2 + 4x + 4 + y^2$

$f(x) = x^2 + 4x$ $\qquad$ $f'_{xx} = 2x + 4$

$f(y) = y^2$ $\qquad$ $f'_{xy} = 0$

$f'_{yy} = 2y$

$f'_{yx} = 0$

$$D = \begin{vmatrix} 2x+4 & 0 \\ 2y & 0 \end{vmatrix} = 0$$

$\Rightarrow$ funkcija nema lokalnog minimuma ni maksimuma

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: MARKO PARANCIN Broj indeksa: 17-1-0062-2011

Vrijeme: od 08:00 **do** ______ ♣3 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int 3x^2\sqrt{x^3+4}dx$$

b)

$$\int_0^8 \frac{dx}{\sqrt[3]{x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x+2}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju prabola $y = x^2-8x+16$ i pravac $y = -x+6$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2+4x+4+y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \frac{1}{\ln(y-x)}.$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$+y^2+4x+y^2$

$$y'+4y = 3e^x$$

b)

$$y''-y'+6y = 7e^{5x}.$$

4.a. $f(x,y) = x^2+4x+4+y^2$

$\frac{Df}{Dx} = 2x+4xy^2$

$\frac{Df}{DxDy} = -8xy$

$\frac{Df}{Dx^2} = 2-4y^2$

$\frac{Df}{Dy,Dx} = -8xy$

$\frac{Df}{Dy} = 2y-4x^2y$

$\frac{Df}{Dy^2} = 2-4x^2$

(1.a) $f(x,y) = \dfrac{1}{\ln(y-x)}$

$\ln(y-x) \neq 0$

$(y-x) > 0$

Tablica osnovnih derivacija

f	f'
$x^{\alpha}(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^{x}	e^{x}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^{x} dx = e^{x} + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^{x} dx = \frac{a^{x}}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣3