MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: JOSIP ŠIMIČEV **Broj indeksa:** 17-1-0101-2011

Vrijeme: od _____ **do** _____ **♣2** **Broj bodova:** 60

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot \cos(3x^2 + 4)dx$$

b)

$$\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3 + x^2}{x^2 - 3x + 2}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = -x^2 + x + 4$ i pravac $y = 1 - x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = -2x^2 - y^2 + 2y - 1$$

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - y = 4x^3$$

b)

$$y'' - y' + 2y = xe^x.$$

3.

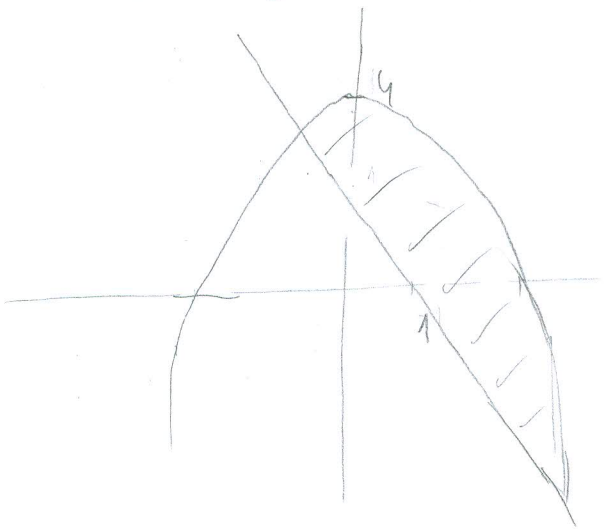
$y = -x^2 + x + 4 \quad y = 1 - x$

$-x^2 + x + 4 = 1 - x$

$-x^2 + 2x + 3 = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4 - 4(-1)\cdot 3}}{-2} = \frac{-2 \pm \sqrt{16}}{-2}$

$x_1 = \frac{-2+4}{-2} = \frac{2}{-2} = -1 \qquad x_2 = \frac{-2-4}{-2} = \frac{-6}{-2} = 3$



$P = \int_{-1}^{3} (-x^2 + x + 4) - (1 - x) = \int_{-1}^{3} -x^2 + 2x + 3$ 15

$= \int_{-1}^{3} -x^2\,dx + \int_{-1}^{3} 2x\,dx + \int_{-1}^{3} 3\,dx$

$= -\frac{x^3}{3}\Big|_{-1}^{3} + 2\frac{x^2}{2}\Big|_{-1}^{3} + 3x\Big|_{-1}^{3} = \frac{-1}{3}\left(3^3 - (-1)^3\right) + \left(3^2 - (-1)^2\right) + 3(3 - (-1))$

$P = \frac{-28}{3} + 8 + 12 = \frac{-28 + 24 + 36}{3} = \frac{32}{3}$

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5. a) $xy' - y = 4x^3 \mid \cdot x$

$y' - \frac{1}{x} y = 4x^2$

$p(x)$ $\quad$ $Q(x)$

$y = e^{-\int p(x)dx} \left(Q(x) \cdot e^{\int p(x) dx} \right)$

$\int p(x) dx = \int \frac{1}{x} dx = \ln|x| + c$

$Q(x) \cdot e^{\int p(x)} = \int 4x^2 \cdot e^{\ln|x|} dx$

$= \int \frac{4}{3} x^2 \cdot x = \int \frac{4}{3} x^3 = \frac{4}{4} x^4 + c$

$y = e^{-\ln|x|} \cdot (x^4 + c)$

$y = e^{\ln|x^{-1}|} \cdot (x^4 + c)$

$y = \frac{1}{x} \cdot (x^4 + c)$

4. 5)

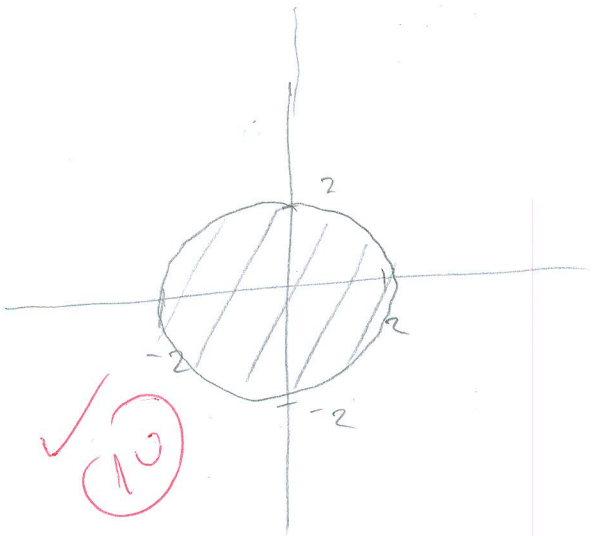
$f(x,y)=\sqrt{4-x^2-y^2}$

$\sqrt{4-x^2-y^2} \geq 0$

$4-x^2-y^2 \geq 0$

$-x^2-y^2 \geq -4 \quad /:(-1)$

$x^2+y^2 \leq 4 \qquad r=2$



✓ (1)

a) $f(x,y)=-2x^2-y^2+2y-1$

$\frac{\partial f}{\partial x} = -4x - 0 + 0 - 0 = -4x$

$\frac{\partial^2 f}{\partial x^2} = -4$

$\frac{\partial f}{\partial y} = 0 - 2y + 2 - 0 = -2y+2$

$\frac{\partial^2 f}{\partial y^2} = -2$

$\frac{\partial^2 f}{\partial x \partial y} = (-2y+2) = 0$

$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$

$-4x = 0 \qquad -2y+2=0$

$x=0 \qquad -2y=-2$

$y=1$

STACIONARNA TOČKA $T(0,1,-1)$

✓ (1)

$\Delta = \begin{vmatrix} -4 & 0 \\ 0 & -2 \end{vmatrix} = 8 > 0$

$\frac{\partial^2 f}{\partial x^2}(T) = -4 < 0$

TOČKA (T) JE MAKSIMUM