MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: JOSIP ŠIMIČEV **Broj indeksa:** 17-1-0101-2011

Vrijeme: od____________ **do**_________ ♣**2** **Broj bodova:** 60

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot \cos(3x^2+4)dx$$

b)

$$\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3+x^2}{x^2-3x+2}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y=-x^2+x+4$ i pravac $y=1-x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y)=-2x^2-y^2+2y-1$$

b) Odredi domenu funkcije:

$$f(x,y)=\sqrt{4-x^2-y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy'-y=4x^3$$

b)

$$y''-y'+2y=xe^x.$$

3.

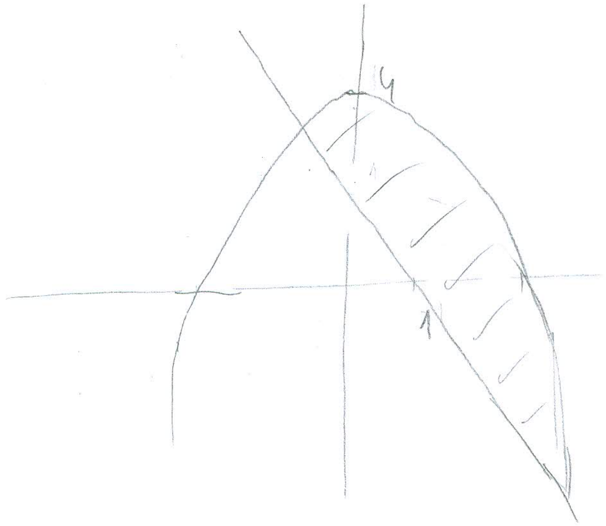
$y = -x^2 + x + 4 \qquad y = 1 - x$

$-x^2 + x + 4 = 1 - x$

$-x^2 + 2x + 3 = 0$

$x_{1,2} = \dfrac{-2 \pm \sqrt{4 - 4(-1)\cdot 3}}{-2} = \dfrac{-2 \pm \sqrt{16}}{-2}$

$x_1 = \dfrac{-2+4}{-2} = \dfrac{2}{-2} = -1 \qquad x_2 = \dfrac{-2-4}{-2} = \dfrac{-6}{-2} = 3$



$P = \int_{-1}^{3} (-x^2 + x + 4) - (1 - x) = \int_{-1}^{3} -x^2 + 2x + 3$

15

$= \int_{-1}^{3} -x^2\,dx + \int_{-1}^{3} 2x\,dx + \int_{-1}^{3} 3\,dx$

$= -\dfrac{x^3}{3}\Big|_{-1}^{3} + 2\dfrac{x^2}{2}\Big|_{-1}^{3} + 3x\Big|_{-1}^{3} = -\dfrac{1}{3}\left(3^3 - (-1)^3\right) + \left(3^2 - (-1)^2\right) + 3\left(3 - (-1)\right)$

$P = \dfrac{-28}{3} + 8 + 12 = \dfrac{-28 + 24 + 36}{3} = \dfrac{32}{3}$

5. a) $xy' - y = 4x^3 \;/ : x$ JOSIP ŠIMIČEV

$$y' - \frac{1}{x} y = 4x^2$$

$P(x)$ $Q(x)$

$$y = e^{-\int p(x)dx} \left(Q(x) \cdot e^{\int p(x)} dx \right)$$

$$\int p(x)\,dx = \int \frac{1}{x} dx = \ln|x| + c$$

$$Q(x) \cdot e^{\int p(x)} = \int 4x^2 \cdot \cancel{e}^{\ln|x|} dx$$

$$= \int \frac{4}{3} x^2 \cdot x = \int \frac{4}{3} x^3 = \frac{4}{4} x^4 + c$$

$$y = e^{-\ln|x|} \cdot (x^4 + c)$$

$$y = \cancel{e}^{\ln|x^{-1}|} \cdot (x^4 + c)$$

$$y = \frac{1}{x} \cdot (x^4 + c)$$

4. b)

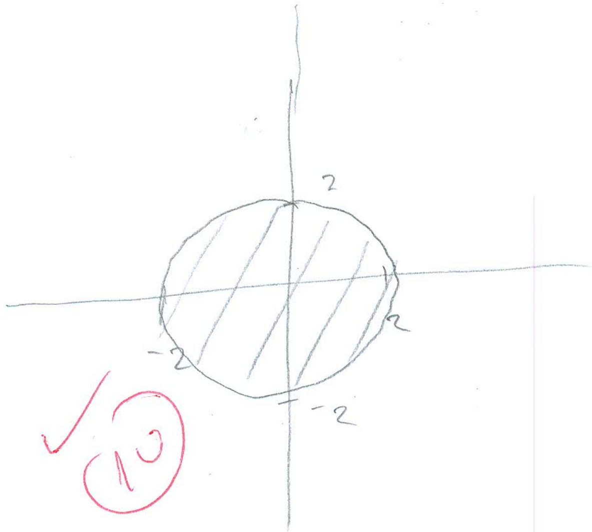
$f(x,y)=\sqrt{4-x^2-y^2}$

$\sqrt{4-x^2-y^2} \geq 0$

$4-x^2-y^2 \geq 0$

$-x^2-y^2 \geq -4 \quad /:(-1)$

$x^2+y^2 \leq 4 \qquad r=2$



10

a) $f(x,y)=-2x^2-y^2+2y-1$

$\frac{\partial f}{\partial x}=-4x-0+0-0=-4x$

$\frac{\partial^2 f}{\partial x^2}=-4$

$\frac{\partial f}{\partial y}=0-2y+2-0=-2y+2$

$\frac{\partial^2 f}{\partial y^2}=-2$

$\frac{\partial^2 f}{\partial x \partial y}=(-2y+2)=0$

$\frac{\partial f}{\partial x}=0 \qquad \frac{\partial f}{\partial y}=0$

STACIONARNA TOČKA

$T(0,1,-1)$

$-4x=0 \qquad -2y+2=0$

$x=0 \qquad -2y=-2$

$y=1$

10

$\Delta=\begin{vmatrix} -4 & 0 \\ 0 & -2 \end{vmatrix}=8>0$

$\frac{\partial^2 f}{\partial x^2}(T)=-4<0$

TOČKA T JE MAKSIMUM

JOSIP ŠIMIČEV

♣2|

1. a) $\int x \cdot \cos(3x^2+4)dx$

$$\left[\begin{array}{c} 3x^2+4=t \,/\,' \\ 6x\,dx = dt \\ x\,dx = \frac{dt}{6} \end{array}\right]$$

$$= \int \cos t \frac{dt}{6} = \frac{1}{6}\int \cos t\, dt = \frac{1}{6}\sin t = \frac{1}{6}\sin(3x^2+4)+c$$

12,5

b) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}}$

$$\left[\begin{array}{c} 1-x^2=t^2 \,/\,' \\ -2x\,dx = 2t\,dt \,/\,:-2 \\ x\,dx = t\,dt \end{array}\right]$$

$$= \int_0^1 \frac{t\,dt}{t^2} = \int_0^1 \frac{dt}{t} = \int_0^1 t^{-1}dt$$

$$= \lim_{\varepsilon \to 0} \int_\varepsilon^1 t^{-1}dt = \lim_{\varepsilon \to 0} t^0 \Big|_\varepsilon^1$$

$$= \lim_{\varepsilon \to 0} (1^0 - \varepsilon^0) = 1 + c$$

7,5

2. $\int \frac{x^3+x^2}{x^2-3x+2}\,dx$

$(x^3+x^2):(x^2-3x+2)=x-4$

x^3-3x^2+2x

$4x^2+2x$

$4x^2-12x+8$

$14x+8$

$\int x\,dx-4\int dx+\int\frac{14x+8}{x^2-3x+2}\,dx$

$x_{1/2}=\frac{3\pm\sqrt{9-4\cdot1\cdot2}}{2}=\frac{3\pm\sqrt{1}}{2}$

$x_1=\frac{3+1}{2}=2 \quad x_2=\frac{3-1}{2}=1$

$\frac{14x+8}{(x-2)(x-1)}=\frac{A}{(x-2)}+\frac{B}{(x-1)}$ / nazivnik

$14x+8=A(x-1)+B(x-2)$

$14x+8=Ax-A+Bx-2B$

$14x+8=x(A+B)+(-A-2B)$

$A+B=14 \Rightarrow A=14-B=14-(-22)=36$

$-A-2B=8 \Rightarrow -B=14+8=22$

$B=-22$

$=\int x\,dx-4\int dx+\int\frac{36}{(x-2)}dx+\int\frac{2}{(x-1)}dx=\frac{x^2}{2}-4x+36\ln|x-2|-22\ln|x-1|+C$

OBRATI U RAČUNU

10

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1-\frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣2

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: AUGUSTIN PTIČAR __________ Broj indeksa: ____________________

Vrijeme: od 08$^{\underline{00}}$h ________ do ________ ♣2 **Broj bodova:** 42.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

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a)

$$xy'-y=4x^3$$

b)

$$y''-y'+2y=xe^x.$$

4.
a) $f(x,y) = -2x^2 - y^2 + 2y - 1$

$\frac{\partial f}{\partial x} = -4x - 0 + 0 - 0 = -4x$ $\frac{\partial f}{\partial y} = 0 - 2y + 2 - 0 = -2y + 2$

$\frac{\partial^2 f}{\partial x^2} = -4$ $\frac{\partial^2 f}{\partial y^2} = -2 + 0 = -2$

$\frac{\partial f}{\partial x} = 0$ $\frac{\partial f}{\partial y} = 0$ $\frac{\partial^2 f}{\partial x \partial y} = -2y + 2 = 0$

$-4x = 0$ $-2y + 2 = 0$ $\frac{\partial^2 f}{\partial y \partial x} = -4x = 0$

$x = 0$ $-2y = -2 \quad /:(-2)$

$y = \frac{-2}{-2}$

$y = 1$

$T(0,1)$

$$\Delta = \begin{vmatrix} -4 & 0 \\ 0 & -2 \end{vmatrix} = -4 \cdot (-2) - 0 \cdot 0 = 8 - 0 = 8$$

$\Delta = 8 > 0$

$\frac{\partial^2 f}{\partial x^2} = -4 < 0$

MAX. FUNKCIJE

10

1.
a) $\int x \cdot \cos(3x^2+4)\,dx$ $\left[\begin{array}{l} 3x^2+4=t \,/\,' \\ 6x\,dx = dt \\ x\,dx = \frac{dt}{6} \end{array}\right.$

✓ 12.5

$\int \cos t \, \frac{1}{6}\, dt = \frac{1}{6} \int \cos t \, dt = \frac{1}{6} \sin t = \frac{1}{6} \sin(3x^2+4) + C$

b) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}}$ $\left[\begin{array}{l} 1-x^2 = t \,/\,' \\ -2x\,dx = dt \,/\,:(-2) \\ x\,dx = -\frac{1}{2}\,dt \end{array}\right]$

x	$t\,(1-x^2)$
0	1
1	0

$-\frac{1}{2}\int_1^0 \frac{dt}{\sqrt{t}} = -\frac{1}{2}\int_1^0 t^{-\frac{1}{2}}\,dt = -\frac{1}{2}\, \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Big|_1^0$

$= -\frac{1}{2}\, \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\Big|_1^0 = -\frac{1}{\cancel{2}}\, \cancel{2}\, t^{\frac{1}{2}}\Big|_1^0 = -t^{\frac{1}{2}}\Big|_1^0$

$= -(1-x^2)^{\frac{1}{2}}\Big|_1^0 = \sqrt{x^2-1}\,\Big|_1^0 = \sqrt{(1^2-0)-1} = \sqrt{1-1} = \sqrt{0} = 0$

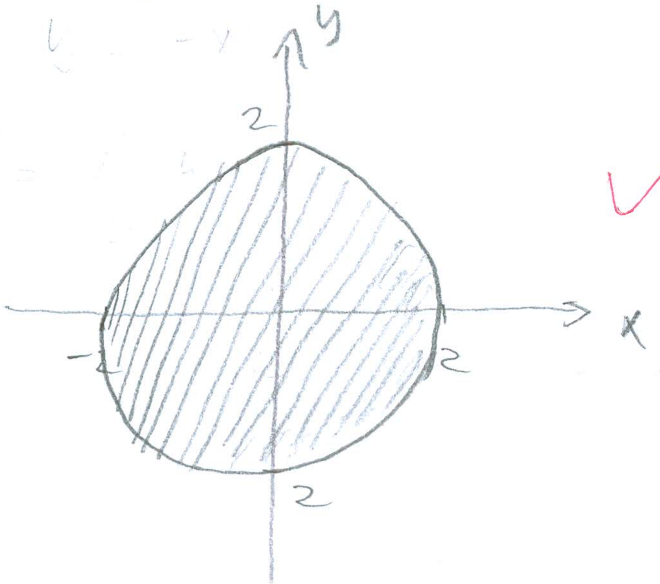
ε♣

$f(x,y) = \sqrt{4-x^2-y^2}$

$D_f \rightarrow$ svi brojevi unutar kružnice, uključujući i brojeve na kružnici

$4 - x^2 - y^2 \geq 0$

$x^2 + y^2 \leq 4$



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$y = -x^2 + x + 4$

$y = 1 - x$

$y=1-x$	x
1	0
0	1
-1	2

$-x^2 + x + 4 = 1 - x$

$\cancel{x}^2 - x - x + 1 - 4 = 0$

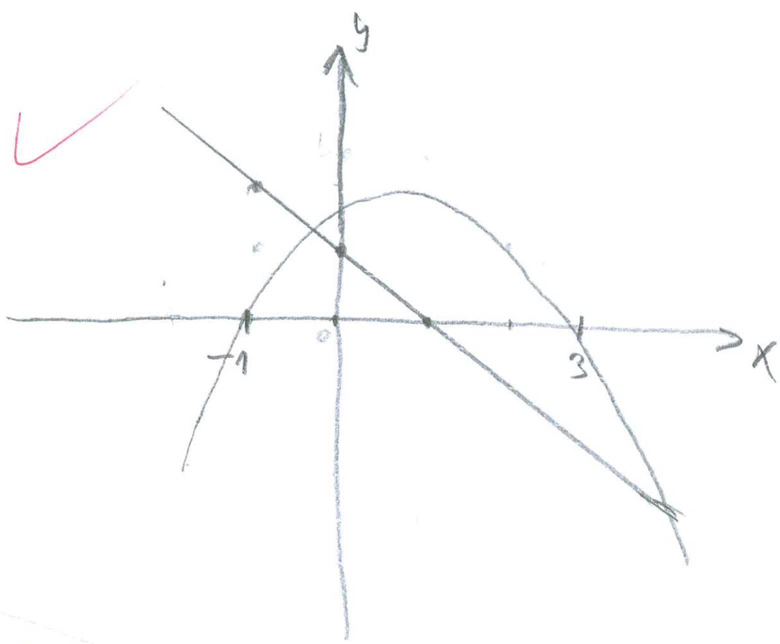
$x^2 - 2x - 3 = 0$

$x_{1/2} = \dfrac{2 \pm \sqrt{4+12}}{2}$

$x_{1/2} = \dfrac{2 \pm \sqrt{16}}{2}$

$x_{1/2} = \dfrac{2 \pm 4}{2}$

$x_1 = \dfrac{2+4}{2} = \dfrac{6}{2} = 3,\quad x_2 = \dfrac{2-4}{2} = \dfrac{-2}{2} = -1,$



$$\int_{-1}^{3} -x^2+x+4-(1-x)\,dx = \int_{-1}^{3} -x^2+x+4-1+x\,dx =$$

$$= \int_{-1}^{3} -x^2 + \int_{-1}^{3} 2x + \int_{-1}^{3} 3\,dx = -1\int_{-1}^{3} x^2\,dx + 2\int_{-1}^{3} x\,dx + 3\int_{-1}^{3} dx$$

$$= -1 \cdot \frac{x^3}{3}\Big|_{-1}^{3} + \frac{2x^2}{2}\Big|_{-1}^{3} + 3x\Big|_{-1}^{3}$$

$$= -\frac{1}{3}x^3\Big|_{-1}^{3} + x^2\Big|_{-1}^{3} + 3x\Big|_{-1}^{3}$$

$$= -\frac{1}{3}\left(3^3-(-1)^3\right) + \left(3^2-(-1)^2\right) + 3\left(3-(-1)\right)$$

$$= -\frac{1}{3}(27+1) + (9-1) + 3(4)$$

$$= -\frac{1}{3} \cdot 28 + 8 + 12$$

$$= -\frac{28}{3} + 8 + 12$$

$$= \frac{-28+24+24}{3} = \frac{20}{3} \qquad 10$$

Tablica osnovnih derivacija

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$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

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♣2

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: NIKOLINA KOMLJENOVIĆ Broj indeksa: 17-2-0114-2011

Vrijeme: od 8:00 **do** ________ ♣2 **Broj bodova:** 27.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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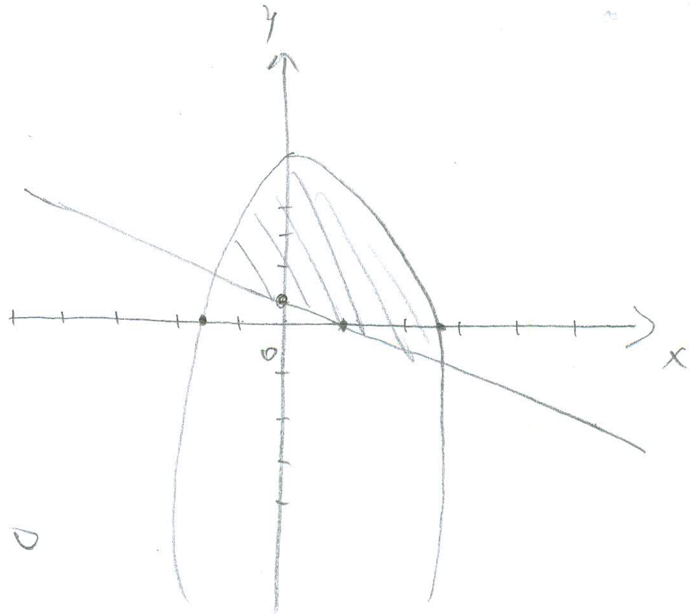
a)

$$xy'-y=4x^3$$

b)

$$y''-y'+2y=xe^x.$$

(3.) parab. $y=-x^2+x+4$ $a<0\ \cap$
prosta $y=1-x$



$y=-x^2+x+4$

$a=-1$
$b=1$
$c=4$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{-1\pm\sqrt{1-4\cdot(-1)\cdot 4}}{-2}$

$x_{1,2}=\frac{-1\pm\sqrt{17}}{-2}$

$x_1=-1{,}56$
$x_2=2{,}56$

$y=1-x$

x	0	1
y	1	0

$-x^2+x+4=1-x$

$-x^2+x+4-1+x=0$

$-x^2+2x+3=0$

$x_{1,2}=\frac{-2\pm\sqrt{4-4\cdot(-1)\cdot 3}}{-2}$

$x_{1,2}=\frac{-2\pm 4}{-2}$, $x_1=-1$, $x_2=3$

$$P=\int_{-1}^{3}-x^2+x+4-(1-x)\,dx=\int_{-1}^{3}\left(-x^2+x+4-1+x\right)dx=\int_{-1}^{3}-x^2+2x+3\,dx$$

$$=-\frac{x^3}{3}+x^2+3x\Big|_{-1}^{3}=\left(-\frac{3^3}{3}+3^2+3\cdot 3\right)-\left(-\frac{(-1)^3}{3}+(-1)^2+3\cdot(-1)\right)$$

$$=9-\left(\frac{1}{3}+1-3\right)=9-\frac{1}{3}-1+3=\frac{32}{3}$$

15

(4.) a) $f(x,y)=-2x^2-y^2+2y-1$

$f'(x,y)_{xx}=-4x+2y$

$f'(x,y)_{yy}=-2y+2x+2$

$-2x$

$-4x+2y=0$
$2x-2y=-2$
$-2x=-2\ /:(-2)$
$x=1$

$f''(x,y)_{xy}=-4$

$f''(x,y)_{yx}=-2$

$$\Delta=\begin{vmatrix}1 & -4\\ -2 & 1\end{vmatrix}=1-8=-7 \qquad -4<0$$

$$\begin{vmatrix}0 & -4\\ -2 & 0\end{vmatrix}=0-8=-8 \qquad \begin{vmatrix}0 & -2\\ -2 & 0\end{vmatrix}=0-4=-4$$

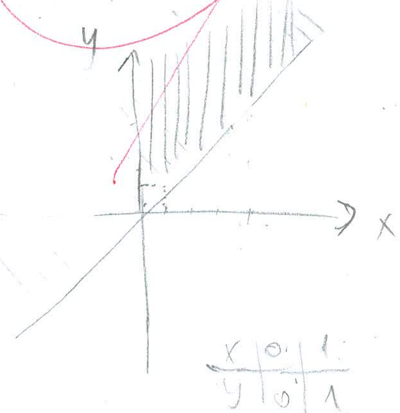
nema mogućih lokalnih ekstrema

b) $f(x,y)=\sqrt{4-x^2-y^2}$

$4-x^2-y^2\geq 0$

$-x^2-y^2\geq -4\ /:(-1)$

$x^2+y^2\geq 4$



x	0	1
y	0	1

Tablica osnovnih derivacija

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$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
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$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
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$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

1.
a) $\int x \cdot \cos(3x^2+4)\,dx = \int \cos(3x^2+4) \cdot x\,dx = \left[\begin{array}{l} 3x^2+4 = t \,/' \\ 6x\,dx = dt \\ x\,dx = \frac{dt}{6} \end{array}\right]$

$= \int \cos t \cdot \frac{dt}{6} = \frac{1}{6} \sin t\, dt$

$= \frac{1}{6} \sin x(3x^2+4) + c$

✓ 2.5

---

b) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} = x \int_0^1 \frac{dx}{\sqrt{1-x^2}} = x \cdot \arcsin x + c \Big|_0^1$

1,57

0

2.) $\int \frac{x^3+x^2}{x^2-3x+2}\,dx = \int \frac{x^3+x^2}{(x+2)(x+1)}\,dx = \frac{A}{(x+2)} + \frac{B}{(x+1)} \Big| \cdot (x+2)(x+1)$

$\int \frac{2}{(x-2)}\,dx + \int \frac{3}{(x+1)}\,dx =$

$x^3+x^2 = A(x+1) + B(x+2)$

$x^3+x^2 = Ax + A + Bx + B2$

$A = 1+1 = 2$

$B = 3$

$c =$

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: IVAN STOJANOV Broj indeksa: 17-2-0061-2010

Vrijeme: od____________ **do**_________ ♣**2** **Broj bodova**: 22.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot \cos(3x^2 + 4)dx$$

b)

$$\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3 + x^2}{x^2 - 3x + 2}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = -x^2 + x + 4$ i pravac $y = 1 - x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = -2x^2 - y^2 + 2y - 1$$

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - y = 4x^3$$

b)

$$y'' - y' + 2y = xe^x.$$

♣2| (1.) a) $\int x\cdot\cos(3x^2+4)\,dx = \left\{\begin{matrix} 3x^2+4=t & 6x=dt \\ \cancel{6x+4=dt} \end{matrix}\right\} =$

$= \left\{\begin{matrix} 3x^2+4=t \\ 6x\,dx=dt \\ x\,dx=\frac{dt}{6} \end{matrix}\right\} = \frac{1}{6}\int \cos t\,dt = \frac{1}{6}\cdot(-\sin t)+C = -\frac{1}{6}\sin(3x^2+4)+C$ 12.5

(2.) $\int \frac{x^3+x^2}{x^2-3x+2}\,dx = \int x\,dx + 4\int dx + \int \frac{10x-8}{x^2-3x+2}$

$x^3+x^2 : (x^2-3x+2) = x$
$-(x^3-3x^2+2x)$
$4x^2-2x : (x^2-3x+2) = 4$
$-(4x^2-12x+8)$
$10x-8$

$= \frac{x^2}{2}+4x+22\cdot\ln|x-1|-12\ln|x-2|+C$ 10

~~$x^2-3x+2=$~~

~~$x_{1,2} = \frac{-b\pm\sqrt{4ac}}{2a} = \frac{3\pm\sqrt{4\cdot1\cdot2}}{2\cdot1} = \frac{3\pm\sqrt{8}}{2}$~~

~~$(x-1)(x+3)=x^2+3x$~~
~~$(x+1)(x-3)=x^2-3x+x-3$~~
~~$(x+2)(x-2)=x^2-2x+2x-4$~~
~~$(x+2)(x-3)=x^2-3x+2x-6$~~
$(x-1)(x-2)=x^2-2x-x+2$

$\frac{10x-8}{x^2-3x+2} = \frac{A}{(x-1)} + \frac{B}{(x-2)} \quad /\cdot(x^2-3x+2)$

GREŠKA U ZBRAJANJU

$10x-8 = A(x-2)+B(x-1)$
$10x-8 = Ax-2A+Bx-B$

SA x: $10=A+B \Rightarrow A=10-B$
bez x: $-8=-2A-B$

$-8=-2\cdot(10-B)-B$
$-8=-20+2B-B$
$-8=-20+B$
$B=-20+8$
$B=-12$

$A=10-(-12)$
$A=10+12$
$A=22$

$\frac{10x-8}{x^2-3x+2} = \int\frac{22}{(x-1)} + \int\frac{-12}{(x-2)}$

$= 22\cdot\ln|x-1| - 12\cdot\ln|x-2|+C$

(3.) $y=-x^2+x+4$
$y=1-x$

~~$\int(-x^2+x+4)-(1-x)\,dx$~~
~~$\int -x^2+2x$~~

$\int -x^2+x+4-(1-x)\,dx =$
$\int(-x^2+x+4-1+x)\,dx =$
$\int(-x^2+2x+3)\,dx =$
$\int -x^2\,dx + 2\int x\,dx + 3\int dx =$
$-\frac{x^3}{3}+2\frac{x^2}{2}+3x+C$
$= -\frac{1}{3}x^3+x^2+3x+C$

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: Ivan Kovačević **Broj indeksa:** 6019831102692257

10

Vrijeme: od_____________ **do**_________ ♣2 **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot \cos(3x^2 + 4)dx$$

b)

$$\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3 + x^2}{x^2 - 3x + 2}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = -x^2 + x + 4$ i pravac $y = 1 - x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = -2x^2 - y^2 + 2y - 1$$

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - y = 4x^3$$

b)

$$y'' - y' + 2y = xe^x.$$

1.)

$$\int x\cdot\cos(3x^2+4)\,dx$$

$u = x \rightarrow du = 1$

$dv = \cos 3x^2 + 4 \quad v = \int \cos 3x^2 + 4\,dx = \int \cos 3x^2\,dx + \int 4\,dx$

$v = \dfrac{\sin(3x^2)}{6}$

$u \cdot v - \int v\cdot du$

$x\cdot\dfrac{\sin(3x^2)}{6} - \int \dfrac{\sin(3x^2)}{6}\cdot 1 \Rightarrow$

$\dfrac{x\sin(3x^2)}{6} + \dfrac{\cos 3x^2}{6} = \dfrac{x\sin(3x^2)+\cos 3x^2}{6}$

$\int \cos 3x^2\,dx \quad \left[\begin{array}{l} 3x^2 = t \quad /' \\ 6x\,dx = dt \quad /:6 \\ x\,dx = \frac{dt}{6} \end{array}\right.$

$\int \cos(t)\,\dfrac{dt}{6}$

$\int \dfrac{1}{6}\cos t\,dt = \dfrac{1}{6}\int \cos t\,dt$

$\dfrac{1}{6}\cdot\sin(t) = \dfrac{1}{6}\sin(3x^2)$

$\int \dfrac{\sin 3x^2}{6} \quad \left[\begin{array}{l} 3x^2 = t \quad /' \\ 6x = t\,dt \end{array}\right]$

$\dfrac{1}{6}\int \sin t\,dt$

$\dfrac{1}{6}-\cos(3x^2)$

b)

$$\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} \quad \left[\begin{array}{l} 1-x^2 = t^2 \quad /' \\ -2x\,dx = 2t\,dt \quad /:2 \\ -x\,dx = t\,dt \end{array}\right]$$

$$\int \frac{-t\,dt}{\sqrt{t^2}} = -1\cdot\int\frac{\cancel{t}\,dt}{\cancel{t}} = -1\cdot\int dt = -1\cdot t = -1\cdot\sqrt{1-x^2}$$

u granicama $\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} = -1\cdot\sqrt{1-x^2}\Big|_0^1 = -1\sqrt{1-1^2} + 1\sqrt{1-0^2} = 1$

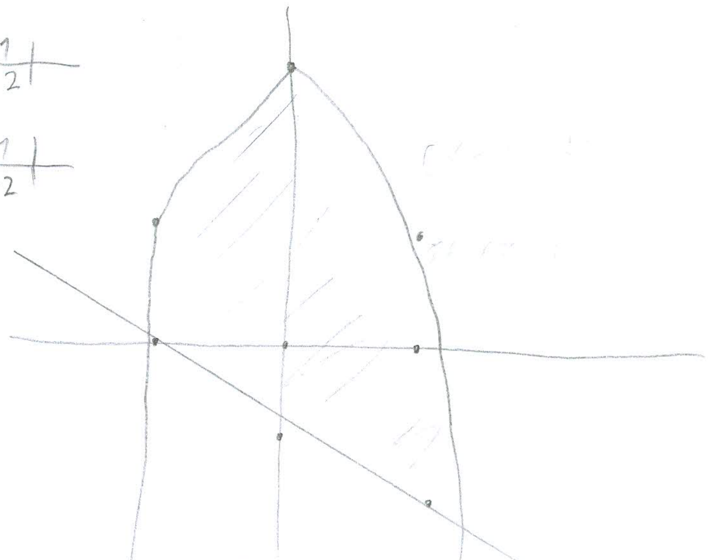
3.)

$y = -x^2 + x + 4$

x	-1	0	1
y	2	4	2

$g = -1 - x$

x	-1	0	1
g	0	-1	-2



$-x^2 + x + 4 = -1 - x$

$-x^2 + x + x + 4 + 1 = 0$

$-x^2 + 2x + 5 = 0$

$x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a}$

$x_{1,2} = \dfrac{-2 \pm \sqrt{4 - 4\cdot -1\cdot 5}}{-2}$

$x_1 = 1-\sqrt{6}$

$x = 1+\sqrt{6}$

$$P = \int^{1+\sqrt{6}} -x^2 + x + 4 - (-1-x)\,dx$$

$$\int_{1-\sqrt{6}}^{1+\sqrt{6}} -x^2 + 2x + 5\,dx = -1\int_{1-\sqrt{6}}^{1+\sqrt{6}} x^2\,dx + 2\cdot\int_{1-\sqrt{6}}^{1+\sqrt{6}} x\,dx + \int_{1-\sqrt{6}}^{1+\sqrt{6}} 5\,dx$$

$$= -\frac{x^3}{3}\Big|_{1-\sqrt{6}}^{1+\sqrt{6}} + 2\cdot\frac{x^2}{2}\Big|_{1-\sqrt{6}}^{1+\sqrt{6}} + 5\cdot x\Big|_{1-\sqrt{6}}^{1+\sqrt{6}}$$

$$= -\frac{(1+\sqrt{6})^3}{3} - \frac{(1-\sqrt{6})^3}{3} + 2\cdot\frac{(1+\sqrt{6})^2}{2} - 2\cdot\frac{(1-\sqrt{6})^2}{2} + 5\cdot 1+\sqrt{6} - 5\cdot 1-\sqrt{6}) = 11{,}333$$

4) a Ispitaj extreme

$$f(x, y) = -2x^2 - y^2 + 2y - 1$$

$$\frac{df}{dx} = -4x \qquad \frac{df}{dy} = 2y + 2 \qquad \frac{df}{dx\,dy} = 0$$

$$\frac{d^2f}{dx^2} = -4 \qquad \frac{d^2f}{dy^2} = 2 \qquad -2x^2 - y^2 + 2y - 1$$

$$\Delta = \begin{vmatrix} \frac{df}{dx} & \frac{df}{dy\,dx} \\ \frac{df}{dx\,dy} & \frac{df}{dy} \end{vmatrix}$$

Stacijonarne točke

$-4x = 0$ $\qquad$ $2y + 2 = 0$ $\qquad$ $z_0 = -2 \cdot 4^2 - \left(\frac{-2}{2}\right)^2 + 2 \cdot \frac{2}{2} \cdot 1$

$x_1 = 4$ $\qquad$ $y = -\frac{2}{2}$ $\qquad$ $z_0 = -36$

$$\Delta = \begin{vmatrix} -4 & 0 \\ 0 & 2 \end{vmatrix} = -4 \cdot 2 - 0 \cdot 0 = -8$$

$\Delta < 0$ $\qquad$ $T(x_0, y_0, z_0)$ JE sedlastu točku

b) odredi domenu

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

$4 - x^2 - y^2 \geq 0$

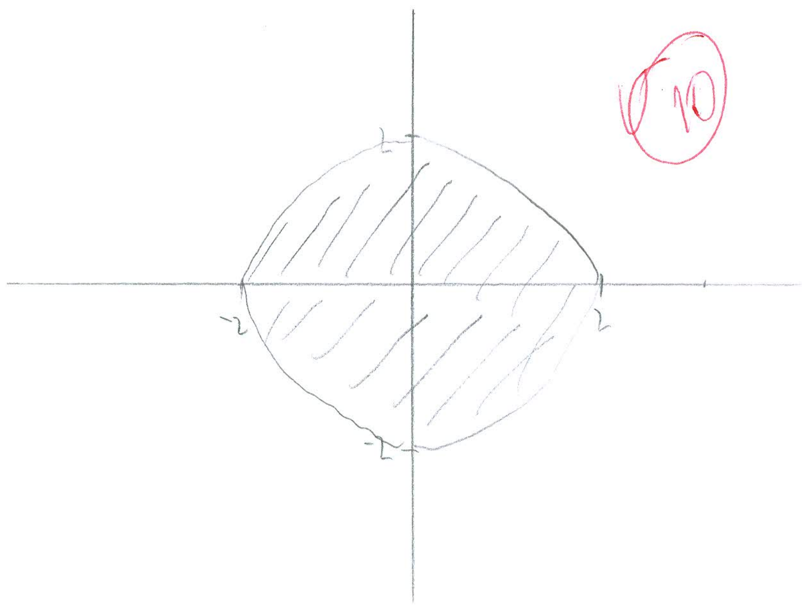
$4 - x^2 - y^2 > 0$

$x^2 + y^2 < 4 \quad /\sqrt{\ }$

$x + y < 2$

$r = 2$

0/10



Domena je unutar kružnice radijusa $r = 2$

Tablica osnovnih derivacija

f	f'	f	f'
$x^{\alpha} (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: Matija Miočić Broj indeksa: 17-1-0110-2012

Vrijeme: od 08:00h do 10:30h ♣2 Broj bodova: 12.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot \cos(3x^2 + 4)dx$$

b)

$$\int_0^1 \frac{xdx}{\sqrt{1-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^3 + x^2}{x^2 - 3x + 2}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = -x^2 + x + 4$ i pravac $y = 1 - x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = -2x^2 - y^2 + 2y - 1$$

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{4 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - y = 4x^3$$

b)

$$y'' - y' + 2y = xe^x.$$

1.a) $\int x \cdot \cos(3x^2+4)dx = \int \cos(3x^2+4) \cdot x\,dx$

$= \left[\begin{array}{l} 3x^2+4 = t \\ 6x\,dx = dt \\ x\,dx = \frac{dt}{6} \end{array} \right] = \int \cos(t) \cdot \frac{dt}{6} = \frac{1}{6}\int \cos(t)\,dt$

12.5

$= \frac{1}{6} \cdot \sin(t) + C = \frac{1}{6} \cdot \sin(3x^2+4) + C$

3. $y = -x^2 + x + 4 \qquad y = 1 - x$

$1 - x = -x^2 + x + 4 \qquad y_1 = 1 - 3$

$x^2 - x - 4 + 1 - x = 0 \qquad y_1 = -2$

$x^2 - 2x - 3 = 0 \qquad y_2 = 1 + 1$

$x_{1,2} = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} \qquad y_2 = 2$

$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_{1,2} = \frac{2 \pm \sqrt{16}}{2}$

$x_1 = 3;\ x_2 = -1$

2. $\int \frac{x^3 + x^2}{x^2 - 3x + 2}\,dx$

$x^3 + x^2 : (x^2 - 3x + 2) = x + 4$
$-x^3 \mp 3x^2 \pm 2x$
$0 + 4x^2 + 2x$
$-4x^2 \mp 12x \pm 8$
$0 + 14x - 8$

1.b) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} = \int_0^1 \frac{x\,dx}{\arcsin x}$

$= \frac{1}{\arcsin(1)} = \frac{1}{\frac{1}{2}\pi} = \frac{2}{\pi}$

4. b) $f(x,y) = \sqrt{4 - x^2 - y^2}$

$4 - x^2 - y^2 \geq 0$

$-x^2 \geq y^2 - 4 \ /\cdot(-1)$

$x^2 \leq -y^2 + 4 \ /\sqrt{\ }$

$x \leq -y + 2$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
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$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos (1 - \frac{x}{a}) + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣2