

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: LUKA STIPIĆ Broj indeksa: 14-2-0083-2011

Vrijeme: od _____ do _____ ♣1

Broj bodova:

100

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (12.5+7.5) Integriraj

a)

$$\int x \cdot \operatorname{ctg}(x^2 + 1) dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. (15) Integriraj

$$\int \frac{x^2 + x + 3}{x^2 - 1} dx$$

3. (15) Odredi površinu koju zatvaraju parabola
- $y = 2x^2 - 3$
- i pravac
- $y = x$
- .

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x, y) = \sqrt{16 - x^2 - y^2}$$

5. (15+15) Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

$$1. a) \int x \cdot \operatorname{ctg}(x^2+1) dx = \left| \begin{array}{l} x^2+1 = t \quad | \\ 2x dx = dt \\ dx = \frac{dt}{2x} \end{array} \right| = \int x \cdot \operatorname{ctg}(t) \frac{dt}{2x} dx = \frac{1}{2} \int \operatorname{ctg}(t) dt$$

$$= \frac{1}{2} \cdot \ln |\sin t| + C$$

$$= \frac{1}{2} \cdot \ln |\sin(x^2+1)| + C$$

✓ 12.5

$$b) \int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

$$\int \frac{dx}{\sqrt{3^2-x^2}} = \arcsin \frac{x}{3}$$

$$\lim_{x \rightarrow 0} \arcsin \frac{x}{3} \Big|_x^3 = \lim_{x \rightarrow 0} \arcsin \frac{3}{3} - \arcsin \frac{x}{3}$$

$$= \lim_{x \rightarrow 0} \frac{1}{2} \pi - \arcsin \frac{x}{3}$$

$$= \frac{1}{2} \pi - \arcsin \frac{0}{3} = \frac{1}{2} \pi - 0 = \frac{1}{2} \pi$$

✓ 7.5

$$2. \int \frac{x^2+x+3}{x^2-1} dx = \frac{x^2+x+3 = (x^2-1) + 4}{x^2-1} = 1 + \frac{4}{x^2-1}$$

$$= \int_1 dx + \int_2 \frac{x+4}{x^2-1} dx$$

$$J_1 = \int 1 dx = x$$

$$J_2 = \int \frac{x+4}{x^2-1} dx$$

$$x^2-1 = (x-1)(x+1) \quad \begin{array}{l} x_1=1 \\ x_2=-1 \end{array}$$

$$\frac{x+4}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \quad | \cdot (x-1)(x+1) \rightarrow x+4 = A(x+1) + B(x-1)$$

$$x+4 = Ax + A + Bx - B$$

?

$$4. b) f(x,y) = \sqrt{16-x^2-y^2}$$

$$16-x^2-y^2 \geq 0$$

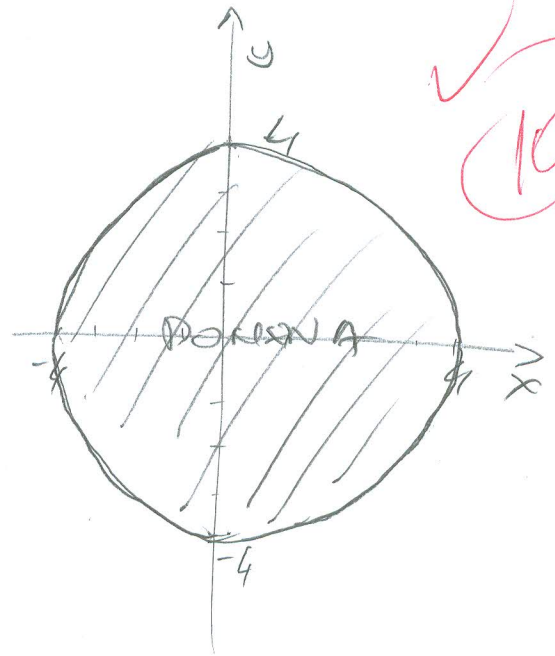
$$\clubsuit 2 | \quad -x^2-y^2 \geq -16 \quad | \cdot (-1)$$

$$x^2+y^2 \leq 16$$

$$\underbrace{a^2+b^2=c^2}$$

$$r^2 = 16 \quad | \sqrt{\quad}$$

$$r = 4$$



$$a) f(x,y) = x^2 - 2x + 1 + 2y^2$$

$$f_x = 2x - 2 \quad f_{xx} = 2 \quad f_{xy} = 0$$

$$f_y = 4y \quad f_{yy} = 4 \quad f_{yx} = 0$$

$$2x - 2 = 0 \quad \rightarrow \quad 2x = 2$$

$$4y = 0$$

$$\downarrow$$

$$y = 0$$

$$x = 1$$

$$T. (1, 0)$$

$$f_{xx} = 2 > 0 \quad \text{MINIMUM}$$

✓ 10

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8 - 0 = 8 > 0$$

IMAMO
ESTREM
U TOČKI' (1, 0)
MINIMUM

