MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: LUKA STIPIĆ Broj indeksa: 17-2-0083-2011

Vrijeme: od__________ **do**________ ♣**1** **Broj bodova**: 100

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot ctg(x^2+1)dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^2+x+3}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 - 3$ i pravac $y = x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{16 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

1.

a) $\int x \cdot ctg(x^2+1)\,dx = \left| \begin{array}{c} x^2+1 = t \;/d \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \end{array} \right| = \int \cancel{x} \cdot ctg(t) \frac{dt}{2\cancel{x}} dx = \frac{1}{2}\int ctg(t)\,dt$

$= \frac{1}{2} \cdot \ln|\sin t| + C$

$= \frac{1}{2} \cdot \ln|\sin(x^2+1)| + C$

✓ 12.5

b) $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

$\int \frac{dx}{\sqrt{3^2-x^2}} = \arcsin \frac{x}{3}$

$\lim_{x\to 0} = \arcsin \frac{x}{3} \Big|_x^3 = \lim_{x \to 0} \arcsin \frac{3}{3} - \arcsin \frac{x}{3}$

$= \lim_{x\to 0} \frac{1}{2}\pi - \arcsin \frac{x}{3}$

✓ 7.5

$= \frac{1}{2}\pi - \arcsin \frac{0}{3} = \frac{1}{2}\pi - 0 = \frac{1}{2}\pi$

2. $\int \frac{x^2+x+3}{x^2-1} dx =$

$x^2+x+3 : (x^2-1) = 1$

$\underline{\;-x^2 \underset{+}{-} 1\;}$

$x+4$

$= \underbrace{\int 1\,dx}_{J_1} + \underbrace{\int \frac{x+4}{x^2-1} dx}_{J_2}$

?

$J_1 = \int 1\,dx = x$

$J_2 = \int \frac{x+4}{x^2-1} dx$ $\quad x^2 = 1 \quad x = \pm 1 \quad x_1 = 1 \quad x_2 = -1 \rightarrow x^2-1 = (x-1)(x+1)$

$\frac{x+4}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{x+1} \Big| \cdot (x-1)(x+1) \rightarrow x+4 = A\cdot(x+1) + B\cdot(x-1)$

$x+4 = Ax + A + Bx - B$

4. b) $f(x,y)=\sqrt{16-x^2-y^2}$

$16-x^2-y^2\geq 0$

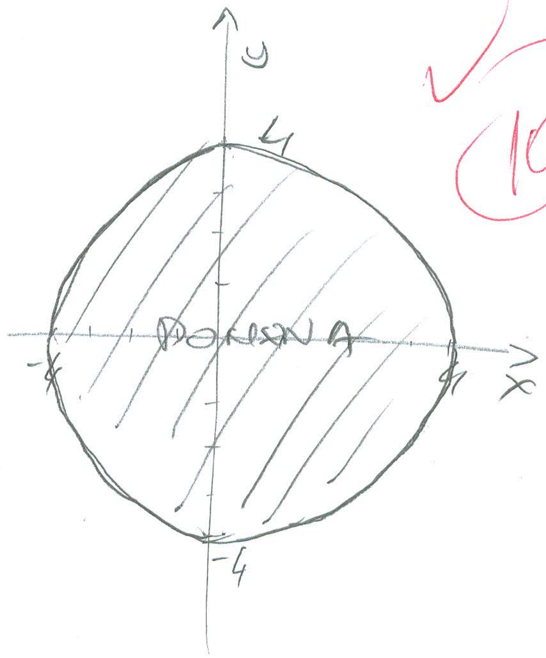
♣2| $-x^2-y^2\geq -16 \,/\cdot(-1)$

$x^2+y^2\leq 16$

$a^2+b^2=r^2$

$r^2=16 \,/\sqrt{}$

$r=4$



✓ 10

a) $f(x,y)=x^2-2x+1+2y^2$

$Z_x=2x-2 \qquad Z_{xx}=2 \qquad Z_{xy}=0$

$Z_y=4y \qquad Z_{yy}=4 \qquad Z_{yx}=0$

$2x-2=0 \rightarrow 2x=2$

$4y=0 \qquad x=1$

$\downarrow$

$y=0$

$T.(1,0)$

$Z_{xx}=2>0$ MINIMUM

✓ 10

$D=\begin{vmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{vmatrix}=\begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix}=8-0=8>0$ IMAMO EKSTREM U TOČKI $(1,0)$ MINIMUM

5.

b) $y'' + 2y' + y = \sin x$

$\boxed{y_0}$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$

$\lambda_{1,2} = -1$

$y_0 = C_1 e^{-x} + x C_2 e^{-x}$

$y = y_0 + y_p$

$y_p = A\cos x + B\sin x \,/'$

$y_p' = A\sin x + B\cos x \,/'$

$y_p'' = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2A\sin x + 2B\cos x + A\cos x + B\sin x = \sin x$

$-B - 2A + B = 1$

$-2A = 1$

$-A = \frac{1}{2}$

$A = -\frac{1}{2}$

$-A + 2B + A = 0$

$2B = 0$

$B = 0$

$y_p = -\frac{1}{2}\cos x + 0 \cdot \sin x$

$y = y_0 + y_p$

$y = C_1 e^{-x} + x C_2 e^{-x} - \frac{1}{2}\cos x$ ✓ 15

5. a) $xy' + y = \cos x \,/ \cdot \frac{1}{x}$

$y' + \frac{1}{x} y = \frac{\cos x}{x}$ — $P(x)$, $Q(x)$

Linearna

$y = e^{-\int P(x)\,dx} \cdot \left[\int Q(x) \cdot e^{\int P(x)\,dx} + C\right]$

$\int \frac{1}{x}\,dx = \ln|x|$

$\int \frac{\cos(x)}{x} \cdot e^{\ln|x|}\,dx = \int \frac{\cos(x)}{x} \cdot x\,dx = \int \cos x\,dx = \sin x$

$y = e^{-\ln|x|} \cdot [\sin x + C]$

$y = x^{-1} \cdot (\sin x + C)$

$y = \frac{1}{x} \cdot (\sin x + C)$ ✓ 15

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

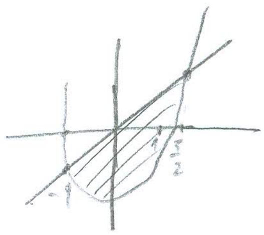
$$A+B=1 \rightarrow \frac{5}{2}+B=1$$
$$A-B=4 \qquad B=1-\frac{5}{2}$$
$$\cancel{A=\frac{\;}{2}} \qquad B=\frac{2}{2}-\frac{5}{2}$$
$$B=\frac{2-5}{2}$$
$$2A=5$$
$$A=\frac{5}{2} \qquad B=-\frac{3}{2}$$

15

$$J_2=\int\frac{\frac{5}{2}}{x-1}\,dx+\int\frac{-\frac{3}{2}}{x+1}\,dx$$
$$=\frac{5}{2}\ln|x-1|-\frac{3}{2}\ln|x+1|+C$$

2. $y=2x^2-3$

$y=x$



10

$$2x^2-3=x$$
$$2x^2-x-3=0$$
$$x_{1,2}=\frac{1\pm\sqrt{1+24}}{4}$$
$$x_1=\frac{1+5}{4}=\frac{6}{4}=\frac{3}{2}$$
$$x_2=\frac{1-5}{4}=-1$$

$$P=\int_{-1}^{\frac{3}{2}} x-2x^2+3\,dx$$
$$P=\int_{-1}^{\frac{3}{2}} x\,dx-\int_{-1}^{\frac{3}{2}}2x^2\,dx+\int_{-1}^{\frac{3}{2}}3\,dx$$
$$P=\frac{x^2}{2}\Big|_{-1}^{\frac{3}{2}}-\frac{2x^3}{3}\Big|_{-1}^{\frac{3}{2}}+3x\Big|_{-1}^{\frac{3}{2}}$$
$$P=\frac{5}{8}-\frac{35}{12}+\frac{15}{2}$$
$$P=\frac{125}{24}$$

Cvetkovski

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: KRISTIJAN CVETKOVSKI Broj indeksa:

Vrijeme: od __________ **do** ________ ♣1 **Broj bodova:** 57.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$xy' + y = \cos x$$

b)
$$y'' + 2y' + y = \sin x$$

1.) a.) $x^2+1 = t \,/'$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$\frac{1}{2}\int ctg\,t\,dt$ (12.5)

$= \frac{1}{2}\ln(\sin(x^2+1)) + C$ //

1.) b) $\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\left(\frac{x}{3}\right)\Big|_0^3 = \frac{\pi}{2}$ //

2.) $x^2+x+3 : (x^2-1) = 1 + \frac{x+4}{x^2-1}$

$-x^2 \quad +1$

$x+4$

$(x-1)(x+1)$

$x^2 + \cancel{x} - \cancel{x} - 1$

$\frac{x+4}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)} \,/ \cdot (x+1)(x-1)$

$x+4 = A(x-1) + B(x+1)$

$x+4 = Ax - A + Bx + B$

$x+4 = x(A+B) - A + B$

$x(A+B)-A+B = x+4$

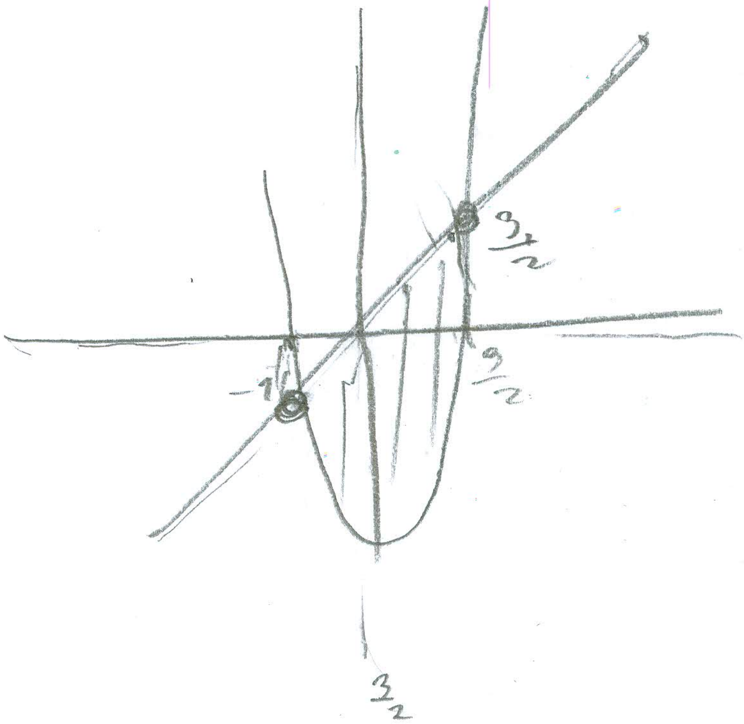
$A+B = 1$ $\quad 2B = 5$

$-A+B = 4$ $\quad B = \frac{5}{2} \quad A = -\frac{3}{2}$

$$\int 1 - \frac{\frac{3}{2}}{x+1} + \frac{\frac{5}{2}}{x-1} = x - \frac{3}{2}\ln|x+1| + \frac{5}{2}\ln|x-1| + C$$

1б

③ $y = 2x^2 - 3 \qquad y = x$



$2x^2 - 3 = x$

$2x^2 - x - 3 = 0$

$$\frac{1 \pm \sqrt{1+24}}{4}$$

$$\frac{1 \pm 5}{4} = \begin{matrix} x_1 = \frac{3}{2} \\ x_2 = -1 \end{matrix}$$

1б

$$\int_{-1}^{\frac{3}{2}} x - 2x^2 + 3 = \frac{x^2}{2} - \frac{2}{3}x^3 + 3x \Big|_{-1}^{\frac{3}{2}} = \frac{125}{24}$$

$\frac{9}{8}$

$P = \frac{125}{24}$

$y'' + 2y' + y = \sin x$

$r^2 + 2r + r = 0$

$\frac{-2 \pm \sqrt{4-4}}{2} = -1$

♣1

$y_h = e^{-x} C_1 + x e^{-x} C_2$

KRISTIJAN CVETKOV

$y_p = (Ax + B) \sin x$

5

$y_p' = A \sin x + (Ax + B) \cos x$

$y_p'' = A \cos x + A \cos x - (Ax + B) \sin x$

$A \cos x + A \cos x + (-Ax - B) \sin x + 2A \sin x + (2Ax + 2B) \cos x$

$(A + A + 2Ax + 2B) \cos x + (-Ax - B + 2A) \sin x = \sin x$

$(2A + 2Ax + 2B) \cos x + (-Ax - B + 2A) \sin x = \sin x$

$2A + 2B = 0$

$2A - B = 1 \; / -1$

$2A + 2B = 0$

$-2A + B = -1$

$4B = -1$

$B = \frac{-1}{4}$ $\quad A = \frac{-2B}{2}$

$A = \frac{1}{4}$

$y = e^{-x} C_1 + x e^{-x} C_2 + \left(\frac{1}{4} x - \frac{1}{4}\right) \sin x$

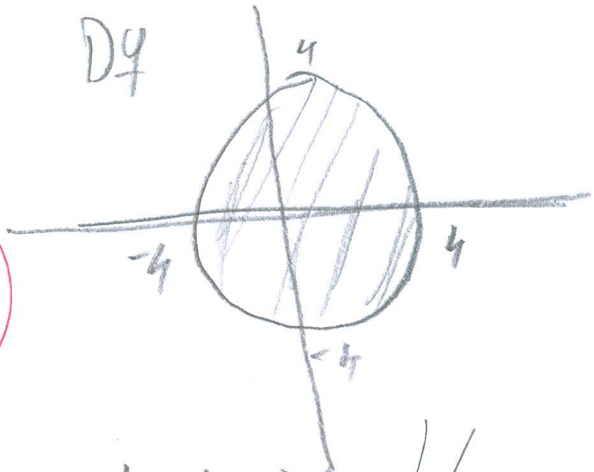
4) $f(x) = \sqrt{16 - x^2 - y^2}$

$16 - x^2 - y^2 \geq 0$

$-x^2 - y^2 \geq -16 \quad / \cdot (-1)$

$x^2 + y^2 \leq 16$

D_f



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Sve točke unutar kružnice

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
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$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
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$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \cot x dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
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$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2-x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

$\frac{1}{y} = x \cdot c$

$y(x \cdot c) = 1$

$y = \frac{1}{x \cdot c}$

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: ZLATKO LALIĆ Broj indeksa: 57676 - 2009

Vrijeme: od____________ **do**_________ **♣1** **Broj bodova:** 42.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

4. $f(x,y) = x^2 - 2x + 1 + 2y^2$

$z_x = 2x - 2$

$z_y = 4y$

$2x - 2 = 0$
$4y = 0$

$4y = 0 \quad /:4$

$y = 0$

$2x - 2 = 0$

$2x = 2 \quad /:2$

$x = 1$

$z_{xx} = 2$

$z_{xy} = 0$

$z_{yx} = 0$

$z_{yy} = 4$

$T(1,0)$

$D = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8 > 0 \; \exists$ u $T(1,0)$

$z_{xx} = 2 > 0 \Rightarrow$ min

$z\,min(1,0) = 1^2 - 2\cdot 1 + 1 + \cancel{2}\cdot 0$
$= 1 - 2 + 1$
$= 0$

$T(1,0,0)$

10

♣**2**|

1. a) $\int x \operatorname{ctg}(x^2+1)\,dx = \left[\begin{array}{l} x^2+1=t \\ 2x\,dx = dt \;/ :2 \\ x\,dx = \frac{dt}{2} \end{array}\right]$

$= \frac{1}{2}\int \operatorname{ctg} t\,dt$

$= \frac{1}{2} \ln|\sin t| + C$

$= \frac{1}{2} \ln|\sin(x^2+1)| + C$

2.5

~~2.~~ $\cancel{\int \frac{x^2+x+3}{x^2-1}\,dx}$

$\cancel{\frac{x^2+x+3}{x^2-1} = \frac{Ax}{x^2-1} + \frac{B}{x-1}}$

$\cancel{x^2+x+3 = Ax+}$

5. a) $xy' + y = \cos x \quad | :x$

$$y' + \frac{1}{x}y = \frac{\cos x}{x}$$

$$y = e^{-\int f(x)dx}\left[\int e^{+\int f(x)dx} \cdot g(x) + C\right]$$

$$y = e^{-\int \frac{dx}{x}}\left[\int e^{\int \frac{dx}{x}} \cdot \left(\frac{\cos x}{x}\right)dx + C\right]$$

$$y = e^{-\ln x}\left(\int e^{\ln x} \cdot \left(\frac{\cos x}{x}\right)dx + C\right)$$

$$y = e^{\ln x^{-1}}\left(\int x \cdot \left(\frac{\cos x}{x}\right)dx + C\right)$$

$$y = e^{\ln \frac{1}{x}}\left[\int \cancel{x} \cdot \left(\frac{\cos x}{\cancel{x}}\right)dx + C\right]$$

$$y = \frac{1}{x}\left[\int \cos x \, dx + C\right]$$

$$y = \frac{1}{x}\left[\sin x + C\right]$$

15

b) $y'' + 2y' + y = \sin x$

$$r^2 + 2r + 1 = 0$$

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2}$$

$$\alpha = \frac{-2}{2} = -1$$

$$r_1 = r_2$$

$$y_H = C_1 e^{rx} + C_2 x e^{rx}$$

$$y_H = C_1 e^{-x} + C_2 x e^{-x}$$

15

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
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$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \| \cos x \| + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\| \frac{x+a}{x-a} \right\| + C$
$\int \frac{1}{x} dx = \ln \|x\| + C$	$\int \cot x dx = \ln \| \sin x \| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \|x + \sqrt{x^2 \pm a^2}\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1-\frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

A. Vipotnik

MATEMATIKA 2

15. lipnja 2013.

Ime i prezime: Adriano Vipotnik Broj indeksa: 17-2-0138-2011

Vrijeme: od 09:00 do ________ ♣1 Broj bodova: 32.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot ctg(x^2+1)dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^2+x+3}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 - 3$ i pravac $y = x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{16 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

3.) $y = 2x^2 - 3 \qquad y = x$

$2x^2 - 3 = x$

$2x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1+24}}{4}$

$x = \frac{1 \pm 5}{4}$

$x_1 = \frac{3}{2} \qquad x_2 = \frac{4}{4} = 1$

$y = x$

$y_1 = \frac{3}{2} \qquad S_1\left(\frac{3}{2}, \frac{3}{2}\right)$

$y_2 = 1 \qquad S_2(1, 1)$

$y = 2x^2 - 3 \Rightarrow a = 2 \quad \cup$

$2x^2 - 3 = 0$

$2x^2 = 3 \,/:2$

$x^2 = \frac{3}{2} \,/\sqrt{}$

$x = \pm\sqrt{\frac{3}{2}}$

$x_1 = \sqrt{\frac{3}{2}} \qquad x_2 = -\sqrt{\frac{3}{2}}$

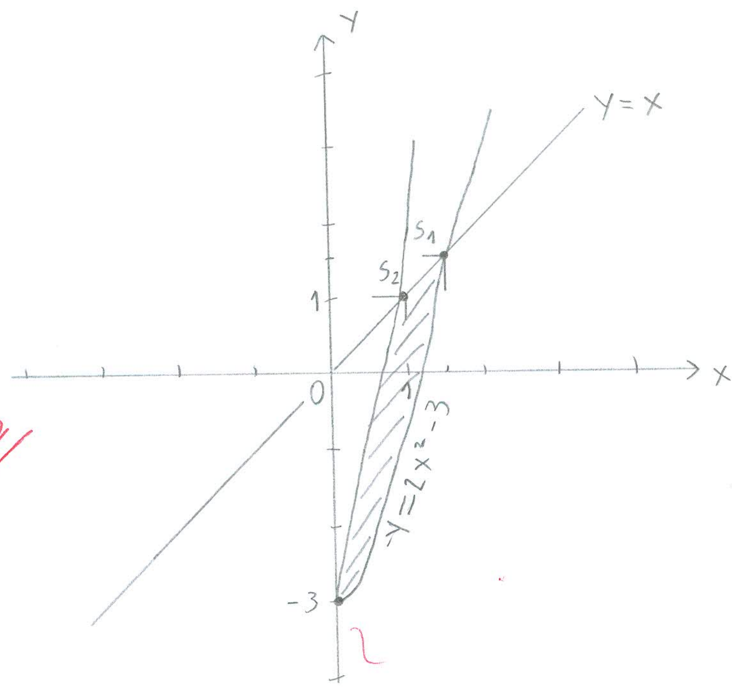
$y = x$

x	0	1
y	0	1

$T\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right) \qquad a = 2,\ b = 0,\ c = -3$

$T\left(\frac{0}{4}, \frac{-24-0}{8}\right)$

$T(0, -3)$



$P = \int_1^{\frac{3}{2}} \left[x - (2x^2 - 3)\right] dx$

$P = \int_1^{\frac{3}{2}} (-2x^2 + x + 3)\, dx = -2\int x^2\, dx + \int x\, dx + 3\int dx$

$= -2\frac{x^3}{3} + \frac{x^2}{2} + 3x$

$P = \left(-\frac{2x^3}{3} + \frac{x^2}{2} + 3x\right)\Big|_1^{\frac{3}{2}} = -\frac{2}{3} + \frac{1}{2} + 3 - \left(-\frac{9}{4} + \frac{9}{8} + \frac{9}{2}\right)$

$= -\frac{2}{3} + \frac{1}{2} + 3 + \frac{9}{4} - \frac{9}{8} - \frac{9}{2}$

$P = \int_{-3}^{1} \left[2x^2 - 3 - (x)\right] dx = \int_{-3}^{1} (2x^2 - 3 - x)\, dx$

$2\int x^2\, dx - \int x\, dx - 3\int dx = 2\frac{x^3}{3} - \frac{x^2}{2} - 3x$

$P = \left(2\frac{x^3}{3} - \frac{x^2}{2} - 3x\right)\Big|_{-3}^{1} = -18 - \frac{9}{2} + 9 - \left(\frac{2}{3} - \frac{1}{2} - 3\right)$

$= -18 - \frac{9}{2} + 9 - \frac{2}{3} + \frac{1}{2} + 3$

KRIVO

4.) a) $f(x,y) = x^2 - 2x + 1 + 2y^2$

$\partial_x f = 2x - 2$ $\qquad$ $\partial_y f = 4y$

$\partial_{xx} f = 2$ $\qquad$ $\partial_{yy} f = 4$

$\partial_{xy} f = 0$ $\qquad$ $\partial_{yx} f = 0$

$\partial_x f \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$

$\partial_y f \Rightarrow \underline{4y = 0} \Rightarrow y = 0$

$A = \partial_{xx} f = 2$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8$$

$\left.\begin{array}{l} \Delta = 8 > 0 \\ A = 2 > 0 \end{array}\right\}$ minimum

stacionarne točke $(1,0)$

$f(1,0) = x^2 - 2x + 1 + 2y^2$

$f(1,0) = 1 - 2 + 1 + 0 = 0$

$f(1,0) \Rightarrow$ ima minimum u točki $(0,0)$

minimum $= 0$

10

e) $f(x,y) = \sqrt{16 - x^2 + y^2}$

$16 - x^2 + y^2 \geq 0$

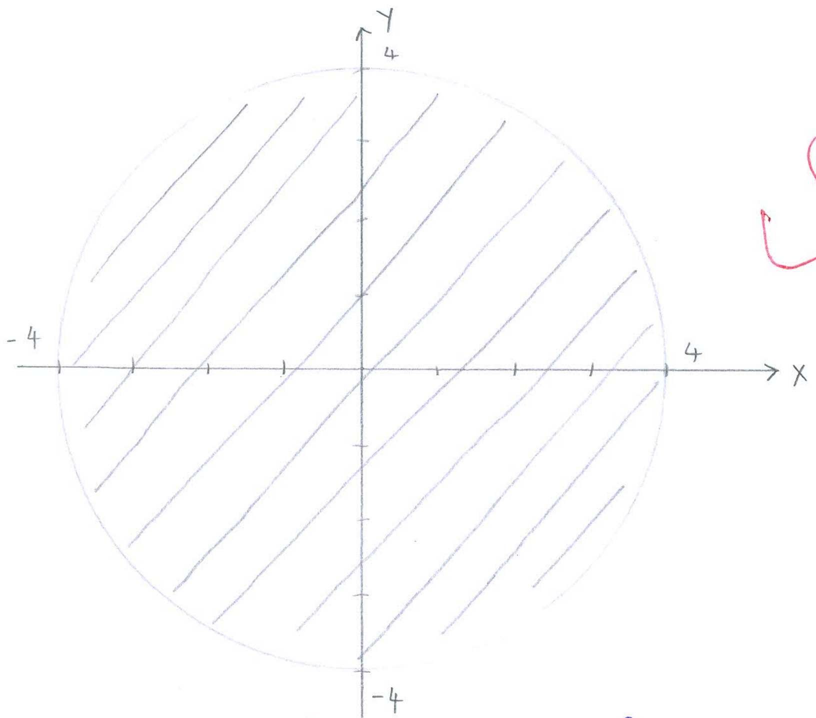
$-x^2 - y^2 = -16 / \cdot(-1)$

$x^2 + y^2 = 16$

$x^2 + y^2 = r \Rightarrow$ formula za kružnicu

kružnica radijusa 4

$r = 4$



10

Domena funkcije su svi realni brojevi u kružnici uključujući i one na obodu kružnice.

$D(f) = \mathbb{R} \in \cup \{x^2 + y^2 = 16 ; x = 4 \text{ i } y = 4\}$

♣2|

1.) $\int x \cdot ctg(x^2+1)\,dx = \begin{bmatrix} x^2+1 = t \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{bmatrix} = \int ctg\, t \cdot \frac{dt}{2} = \frac{1}{2}\int ctg\, t\, dt$

$= \frac{1}{2} \ln|\sin t| + C = \frac{1}{2}\ln|\sin(x^2+1)| + C$ ✓ 12.5

b) $\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \cancel{\begin{bmatrix} 9-x^2 = t \\ -2x\,dx = dt \\ x\,dx = -\frac{1}{2}dt \end{bmatrix}} = \cancel{\int_0^3 \frac{dx}{\sqrt{\;}}}$

$\begin{bmatrix} u = 9-x^2 \,|' & dv = dx \,|^{\int} \\ du = -2x\,dx & v = x \\ x\,dx = -\frac{1}{2}du & \end{bmatrix} = (9-x^2)\cdot x - \int x(-2x)\,dx$

$= -x^3 + 9x + 2\int x^2\,dx$

$= -x^3 + 9x + 2\frac{x^3}{3} = \cancel{\frac{-3x^3}{3}}$

$= \left(-\frac{x^3}{3} + 9x\right)\Big|_0^3$

$= 0 + 0 - (-9+27) = 9 - 27 = -18$

2.) $\int \frac{x^2+x+3}{x^2-1}\,dx = \int \frac{x^2+x+3}{(x-1)(x+1)}\,dx$

$= \int \frac{4\,dx}{(x-1)} + \int \frac{-3\,dx}{x+1} = 4\int\frac{dx}{x-1} - 3\int\frac{dx}{x+1}$

$= 4\ln|x-1) - 3\ln|x+1| + C$.

NISTE PODIJELILI BROJNIK S NAZIVNIKOM!

$\frac{x^2+x+3}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} = \Big/ \cdot (x-1)(x+1)$

$x^2+x+3 = A(x+1) + B(x-1)$

$x^2 + 3 + x = Ax + A + Bx - B$

$x^2 + x + 3 = x(A+B) + A - B$

$A + B = 1 \qquad B = 1 - 4$

$A - B = 3 \qquad B = -3$

$A = 4$

Tablica osnovnih derivacija

f	f'
$x^{\alpha}\,(\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^{x}	e^{x}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: TOMISLAV TUTA Broj indeksa: 17-2-0072-2010

Vrijeme: od __________ **do** ________ **♣1** **Broj bodova**: 25

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot ctg(x^2+1)dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^2+x+3}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 - 3$ i pravac $y = x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{16 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

1. b) $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

$f(x)\,dx \approx h \cdot \left(\frac{y_0 + y_n}{2} + y_1 + y_2 \ldots y_{n-1}\right)$

$b = 3 = y_0$

$a = 0 = x_0$

$n = 6$

$h = \frac{b-a}{n}$

$h = \frac{3-0}{6}$

$h = \frac{3}{6} = \frac{1}{3}$

i	1	2	3	4	5	6
x_i	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
y_i	0.1118	0.2279	0.3536	0.4961	0.6682	0.8944

$x_i = x_0 + i \cdot h \qquad 0 + 1 \cdot \frac{1}{3}$

$y_i = f(x_i)$

$\frac{1}{3} \cdot \left(\frac{3 + 0.8944}{2} + 0.1118 + 0.2279 + 0.3536 + 0.4961 + 0.6682\right)$

$= 1.2683$

5. a) $xy' + y = \cos x \;/ \cdot x$

$y' + \frac{1}{x} y = \frac{\cos}{x} \quad Q(x)$

$y = e^{-\int P(x)dx} \left[\int Q(x) \cdot e^{\int P(x)dx} dx + c\right]$

$\int P(x) = \int \frac{1}{x} dx = \ln(x)$

$\int Q(x) \cdot e^{P(x)dx} dx = \int \frac{\sin x}{x} \cdot e^{\ln(x)}$

$= \int \frac{\sin}{x} \cdot x\,dx = \int \sin x\,dx = + \sin x$

$y = e^{-\ln(x)} \left[+ \sin + c\right]$

$y = e^{\ln(x-1)} \left(\sin + c\right]$

$= \frac{1}{x} \cdot (\sin x + c)$

15

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x+a}{x-a} \right\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a}\right)] + C$

♣1

TOMISLAV TUTA

3.) PARABOLA - $y = 2x^2 - 3$

PRAVAC - $y = x$

$$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$2x^2 - 3 = x$$

$$2x^2 - 3 - x$$

$$\frac{1 \pm \sqrt{1 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} \qquad \frac{1 \pm 3}{4}$$

$A = 2$

$b = -1$

$c = -1$

GREŠKA

$4 \cdot 2 \cdot (-3)$

$x_1 = 1$

$x_2 = -\frac{1}{2}$

$$\int_{-\frac{1}{2}}^{1} \left(x - (2x^2 - 3)\right) dx$$

$$\int_{-\frac{1}{2}}^{1} \left(x - 2x^2 + 3\right) dx \qquad \int_{-\frac{1}{2}}^{1} \frac{x^2}{2}\Big|_{-\frac{1}{2}}^{1} \frac{2x^3}{3} + \Big|_{-\frac{1}{2}}^{1} \frac{3x}{x}$$

$$\frac{1}{2}\left(1 - (-\tfrac{1}{2})^2\right) - \frac{2}{3}\left(1^3 - (-3^3)\right) + 3\left(1 - (-\tfrac{1}{2})\right) =$$

$$\frac{1}{2}\left(1 - \frac{1}{4}\right) - \frac{2}{3}(1 - 27) + 3\left(1 + \frac{1}{2}\right) = 22{,}2083$$

4. $f(x,y) = x^2 - 2x + 1 + 2y^2$

$$\frac{df}{dx} = 2x - 2 \qquad \frac{d^2f}{dx^2} = 2 \qquad \Delta = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix}$$

$A > 0$

LOK MIN

10

$$\frac{df}{dy} = 4y \qquad \frac{d^2}{dx^2} = 4 \qquad \Delta = 2 \cdot 4 - 0 \cdot 0$$

$\Delta = 8$

$$\frac{df}{dx\,dy} = 0$$

$2x - 2 = 0$ $\qquad 4y = 0$

$\qquad\qquad\quad y = 0$

$2x = 2$

$x = 1$

$T(1,0)$ - EKSTREM

4. b) $f(x,y) = \sqrt{16 - x^2 - y^2}$

$16 - x^2 - y^2 > 0$

$x^2 + y^2 = 16$

$r = 4$

-4 4 4

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: ANTONIO SEKULA Broj indeksa: 17-2-0025-2010

Vrijeme: od 08:30 do 09:15 ♣1 Broj bodova: 12.5

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot ctg(x^2+1)dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^2+x+3}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 - 3$ i pravac $y = x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{16 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

1. a) $\int x \cdot \operatorname{ctg}(x^2+1)\,dx = \begin{vmatrix} t = x^2+1 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{vmatrix} =$

$= \int \cancel{x} \cdot \operatorname{ctg} t \cdot \frac{dt}{2\cancel{x}} = \frac{1}{2}\int \operatorname{ctg} t\, dt =$

$= \frac{1}{2}\ln|\sin t| + c = \frac{1}{2}\ln|\sin(x^2+1)| + c$

12.5

b) $\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \int_0^3 \frac{dx}{\sqrt{3^2-x^2}} = \arcsin\frac{x}{3}\Big|_0^3 = \arcsin\frac{3}{3} - \arcsin\frac{0}{3}$

$= \arcsin$

2. $\int \frac{x^2+x+3}{x^2-1}dx = \int 1 + \frac{x+4}{x^2-1}\,dx = 1\int dx + \int \frac{x+4}{x^2-1}\,dx$

$= x + (x^2-1)^2 + 2\ln\left|\frac{x+1}{x-1}\right| + c$

$(x^2+x+3):(x^2-1) = 1$
$\ominus x^2 \quad \oplus 1$
$x+4$

$\int \frac{x+4}{x^2-1} = \int \frac{x}{x^2-1} + \int \frac{4}{x^2-1} = (x^2-1)^2 + 2\ln\left|\frac{x+1}{x-1}\right|$

$\int \frac{x}{x^2-1} = \begin{vmatrix} t = x^2-1 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{vmatrix} = \int \frac{x}{t}\frac{dt}{2x} = 2\int t = \cancel{2}\cdot\frac{t^2}{\cancel{2}} = t^2 = (x^2-1)^2$

$\int \frac{4}{x^2-1} = 4\int \frac{1}{x^2-1} = 4\cdot\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + c = 2\ln\left|\frac{x+1}{x-1}\right|$

ANTONIO SEKULA

3. $y = 2x^2 - 3$
$y = x$

$2x^2 - 3 = x$

$2x^2 - x - 3 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2 \cdot a}$

$x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2}$

$x_{1,2} = \frac{1 \pm \sqrt{25}}{4}$

$x_{1,2} = \frac{1 \pm 5}{4}$

$x_1 = \frac{6}{4} = \frac{3}{2}$

$x_2 = \frac{4}{4} = 1$

$P = \int_1^{\frac{3}{2}} 2x^2 - 3 - x$

$= \int_1^{\frac{3}{2}} 2x^2 dx - \int_1^{\frac{3}{2}} 3\, dx - \int_1^{\frac{3}{2}} x\, dx$

$= 2\int_1^{\frac{3}{2}} x^2 dx - 3\int_1^{\frac{3}{2}} dx - \int_1^{\frac{3}{2}} x\, dx$

$= 2 \cdot \frac{x^3}{3} \Big|_1^{\frac{3}{2}} - 3x \Big|_1^{\frac{3}{2}} - \frac{x^2}{2} \Big|_1^{\frac{3}{2}}$

$P = \frac{19}{12} - \frac{3}{2} - \frac{5}{8} = \frac{17}{24}$

NEMA SKICE
KRIVA POSTAVKA

$2\frac{x^3}{3}\Big|_1^{\frac{3}{2}} = 2 \cdot \frac{\left(\frac{3}{2}\right)^3}{3} - 2 \cdot \frac{1^3}{3} = 2\frac{\frac{27}{8}}{3} = \frac{9}{4} - \frac{2}{3} = \frac{19}{12}$

$3x\Big|_1^{\frac{3}{2}} = 3 \cdot \frac{3}{2} - 3 \cdot 1 = \frac{9}{2} - 3 = \frac{3}{2}$

$\frac{x^2}{2}\Big|_1^{\frac{3}{2}} = \frac{\left(\frac{3}{2}\right)^2}{2} - \frac{1^2}{2} = \frac{9}{8} - \frac{1}{2} = \frac{5}{8}$

Tablica osnovnih derivacija

f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert\frac{a+x}{a-x}\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert\cos x\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert\frac{x+a}{x-a}\rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x\rvert + C$	$\int \cot x dx = \ln \lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2}\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2 - x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: LUKA BORZIĆ **Broj indeksa:** 17-2-0016-2010

Vrijeme: od_____________ **do**_________ ♣1 **Broj bodova:** 10

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot ctg(x^2+1)dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^2+x+3}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 - 3$ i pravac $y = x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{16 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

4. a) $f(x,y) = x^2 - 2x + 1 + 2y^2$

$\frac{\partial f}{\partial x} = 2x - 2 + 0 + 0 = 2x - 2 = 0 \Rightarrow 2x = 2 /:2$

$x = 1$

$\frac{\partial f}{\partial y} = 0 - 0 + 0 + 4y = 4y = 0 /:4$

$y = 0$

$T(1,0)$ 5/20 ?

$A = \frac{\partial f}{\partial x^2} = 2x - 2 + 0 + 0 = 2$

$B = \frac{\partial f}{\partial x \partial y} = 2x - 2 + 0 + 0 = 0$

$C = \frac{\partial f}{\partial y^2} = 0 - 0 + 1 + 4y = 4$

$H = \begin{vmatrix} A & C \\ B & C \end{vmatrix} = AC - B^2$

$H = AC - B^2 = 2 \cdot 4 + 0^2 = 8$ ✓ 10

MINIMUM

5. a) $xy' + y = \cos x \qquad xy' = \frac{y}{x}$

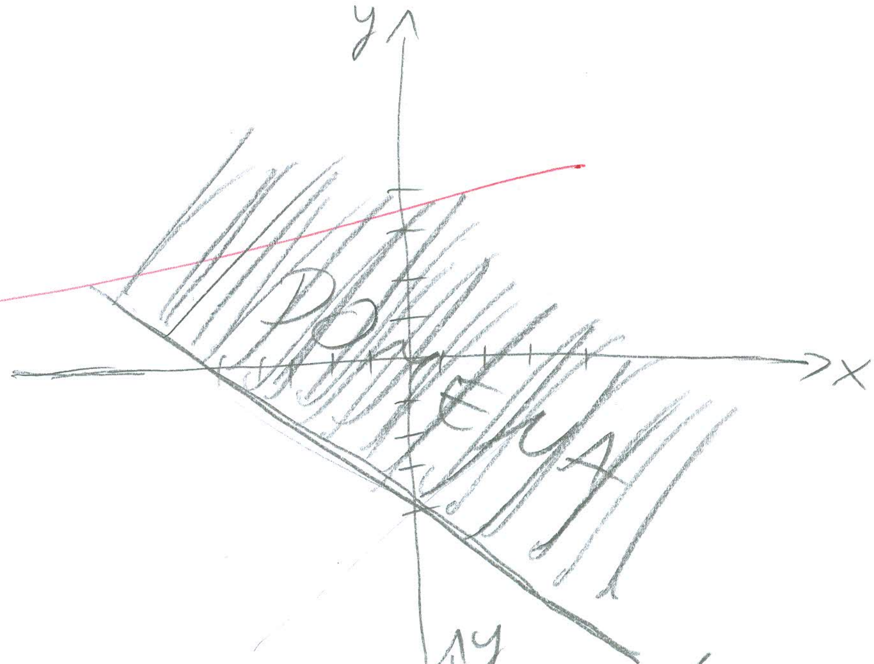
$\frac{y}{x} + y = \cos x$

b) $f(x,y) = -\sqrt{16 - x^2 - y^2}$

$-\sqrt{16 - x^2 - y^2} \geq 0$

$4 - x - y \geq 0$

$-y \leq x - 4$



3. PARABOLA $y = 2x^2 - 3$

PRAVAC $y = x$

$y = 2x^2 - 3 + x$

$P = \int 2x^2 dx - \int 3 dx + \int x dx$

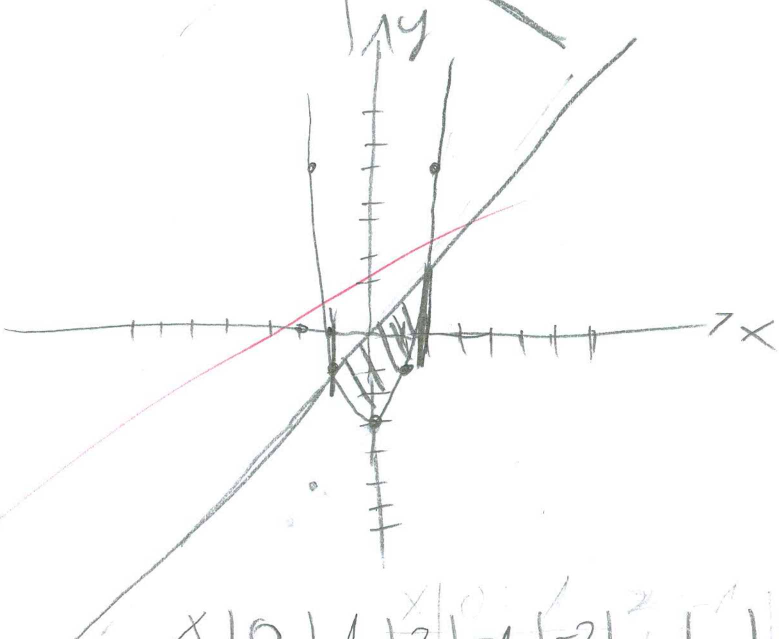
$P = \frac{2x^3}{3} - 3\int dx + \frac{x^2}{2}$

$P = \left. \frac{2x^3}{3} - 3 + \frac{x^2}{2} \right|_{-1}^{2}$

$P = \frac{2 \cdot (-1)^3}{3} - 3 + \frac{(-1)^2}{2} + \frac{2(2)^3}{3} - 3 + \frac{2^2}{2} =$

$P = -\frac{2}{3} - 3 + \frac{1}{2} + \frac{16}{3} - 3 + 2$

$P = \frac{25}{[illegible]}$



x	0	1	2	-1	-2
y	-3	-1	5	-1	5

LUKA BORZIĆ

Tablica osnovnih derivacija

f	f'	f	f'
$x^\alpha (\alpha \neq 0)$	$\alpha x^{\alpha-1}$	$\cosh x$	$\sinh x$
$\ln x$	$\frac{1}{x}$	$\tanh x$	$\frac{1}{\cosh^2 x}$
e^x	e^x	$\coth x$	$\frac{-1}{\sin^2 x}$
$\sin x$	$\cos x$	$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\arctan x$	$\frac{1}{1+x^2}$
$\tan x$	$\frac{1}{\cos^2 x}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cot x$	$\frac{-1}{\sin^2 x}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sinh x$	$\cosh x$	$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \mathrm{arc}\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
$\int \frac{1}{x} dx = \ln \lvert x \rvert + C$	$\int \cot x dx = \ln \lvert \sin x \rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \lvert x + \sqrt{x^2 \pm a^2} \rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos(1 - \frac{x}{a}) + C$
$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}[x\sqrt{a^2-x^2} + a^2 \arcsin(\frac{x}{a})] + C$

♣1

5. b) $y'' + 2y' + y = \sin x$ DIFER JEDNA $= y_0 + Y$

$r^2 + 2r + r = 0$

$r_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2}{2} = -1$

$y_0 = e^x (C_1 + C_2 x)$

$Y = A \sin x \quad Y' = A \cos x \quad Y'' = -A \sin x$

$-A \sin x + 2A \cos x + A \sin x = \sin x$

$2A \cos x = \sin x \ /: \cos x$

$2A = \frac{\sin x}{\cos x} = 2A = \mathrm{tg}\, x \ /: 2$

$A = \frac{\mathrm{tg}\, x}{2}$

$= e^x (C_1 + C_2 x)^2 + \frac{\mathrm{tg}\, x}{2}$

1. a) $\int x \cdot ctg(x^2+1)\,dx =$ $\quad x = u$ $\quad dx = du$ $\quad ctg(x^2+1) = dv$ $\quad v = \int ctg(x^2+1)$ $\quad v = \ln|x^2+1|$

$u \cdot v - \int v\,du$

$x \cdot \ln|x^2+1| - \int \ln|x^2+1|\,dx$ $\quad \ln|x^2+1| = t$ $\quad dx = dt$

$x \cdot \ln|x^2+1| - \int t\,dt$

$x \cdot \ln|x^2+1| - \frac{1}{t}\,dt$

$x \cdot \ln|x^2+1| - \frac{1}{x^2+1} + C$

b) $\int_0^3 \frac{dx}{\sqrt{9-x^2}} = \int \frac{1}{\sqrt{9-x^2}}\,dx$

$= \int (9-x^2)^{-1}\,dx$ $\quad (9-x^2) = t$ $\quad dx = dt$

$= \int t^{-1}\,dt = \int t\,dt =$

$= \Big|_0^3 9-x^2 = 9-0^2+9-3^2 = 9-0+9-9 = 9 + C$

2. $\int \frac{x^2+x+3}{x^2-1}\,dx = \int x^2+x+3 \cdot \int (x^2-1)^{-1}\,dx$

$\int x^2\,dx + \int x\,dx + \int 3\,dx \cdot \int (x^2-1)^{-1}\,dx$

$\frac{x^3}{3} + \frac{x^2}{2} + x \cdot \int (x^2-1)^{-1}\,dx$ $\quad (x^2-1) = t$ $\quad dx = dt$

$\frac{x^3}{3} + \frac{x^2}{2} + x \cdot \int t^{-1}\,dt = \frac{x^3}{3} + \frac{x^2}{2} + x \cdot \int t\,dt$

$\frac{x^3}{3} + \frac{x^2}{2} + x \cdot (x^2-1) + C$

1/8

MATEMATIKA 2
15. lipnja 2013.

Ime i prezime: FRANO ŽIVKOVIĆ Broj indeksa: 54958-2007

Vrijeme: od ____________ **do** ________ ♣**1** **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(12.5+7.5)* Integriraj

a)

$$\int x \cdot ctg(x^2+1)dx$$

b)

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

2. *(15)* Integriraj

$$\int \frac{x^2+x+3}{x^2-1}dx$$

3. *(15)* Odredi površinu koju zatvaraju parabola $y = 2x^2 - 3$ i pravac $y = x$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 - 2x + 1 + 2y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \sqrt{16 - x^2 - y^2}$$

5. *(15+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' + y = \cos x$$

b)

$$y'' + 2y' + y = \sin x$$

4. a) $f(x,y)=x^2-2x+1+2y^2$

$\frac{\partial f}{\partial x}=2x-2=0 \qquad 2x=2 \;/:2 \qquad x=0$

$\frac{\partial f}{\partial y}=4y=0 \qquad 4y=0 \;/:4 \qquad y=0$

$T(0,0)$

$\frac{\partial^2 f}{\partial x^2}=2=A$

$\frac{\partial^2 f}{\partial y\,\partial x}=0=B$

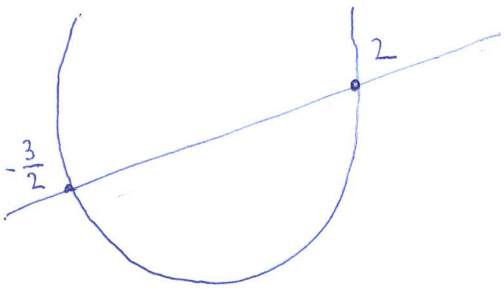
$\frac{\partial^2 f}{\partial y^2}=4=C$

$AC-B^2=2\cdot 4-0^2=8>0$

IMA EKSTREMA

$A>0$ EKSTREM JE MINIMUM.

3) $y=2x^2-3 \qquad y=x$



$y=2x^2-3-x$

$a=2,\ b=-1,\ c=-3$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1-4\cdot2\cdot(-3)}}{4}=\frac{-1\pm\sqrt{25}}{4}$

$x_1=\frac{-1+5}{4}=2$

$x_2=\frac{-1-5}{4}=\frac{-6}{4}=-\frac{3}{2}$

$\int_{-\frac{3}{2}}^{2}(2x^2-3-x)\,dx=\frac{2\cdot x^3}{3}-3x-\frac{x^2}{2}=\frac{2}{3}x^3-3x-\frac{1}{2}x^2\Big|_{-\frac{3}{2}}^{2}$

$=\left(\frac{2}{3}(2)^3-3\cdot2-\frac{1}{2}2^2\right)-\left(\frac{2}{3}(-\frac{3}{2})^3-3(-\frac{3}{2})-\frac{1}{2}(-\frac{3}{2})^2\right)=\left(\frac{2}{3}\cdot8-6-\frac{1}{2}\cdot4\right)-\left(\frac{2}{3}(-3.375)-(-4.5)-1.125\right)=$

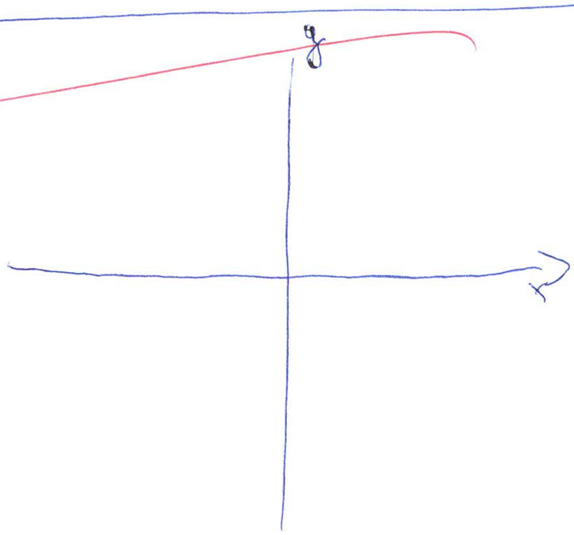
$=-2.666-1.125=-3.791$

1. a) $\int x\cdot ctg(x^2+1)\,dx=\begin{bmatrix} x=u \;/' & ctg(x^2+1)\,dx=dv \;/\int \\ dx=du & \ln|\sin(x^2+1)|\frac{x^3}{3}=v\end{bmatrix}$

$=x\ln|\sin(x^2+1)|\frac{x^3}{3}-\int \ln|\sin(x^2+1)|\frac{x^3}{3}\,dx$

4. b) $f(x,y)=\sqrt{16-x^2-y^2}$

$\sqrt{x}\geq 0$



Tablica osnovnih derivacija

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$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	f'
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$

Tablica osnovnih integrala

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} arc\tan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \lvert \frac{a+x}{a-x} \rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$	$\int \tan x dx = -\ln \lvert \cos x \rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \lvert \frac{x+a}{x-a} \rvert + C$
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$\int \sin x dx = -\cos x + C$	$\int \sinh x dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} [x \sqrt{x^2 \pm a^2} \pm a^2 \ln (x + \sqrt{x^2 \pm a^2})] + C$
$\int \cos x dx = \sin x + C$	$\int \cosh x dx = \sinh x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} [x \sqrt{a^2 - x^2} + a^2 \arcsin (\frac{x}{a})] + C$

♣1