Popuniti odmah!

IME I PREZIME: Marijan Štrk **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

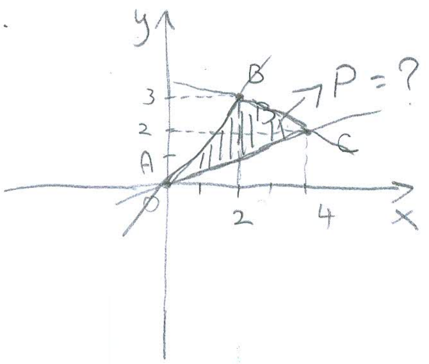
Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,0)$, $B(2,3)$ i $C(4,2)$. 15
2. Zadano je $f(x) = \frac{1}{\sqrt{x+1}}$. Odrediti $\int_{-1}^{1} f(x)\,dx$. Skicirati graf funkcije f i površinu koja je određena integralom $\int_{-1}^{1} f(x)\,dx$. 15
3. Grafički prikazati funkciju $f(x,y) = \dfrac{x^2}{y}$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$? ~~15~~ 10
4. Istražiti domenu i ekstreme funkcije $f(x,y) = x - y + \dfrac{1}{xy}$. 20
5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\,y\,y' = 1 - x^2$ 15
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe: 20

$$y'' + 4y = 4, \quad y(0) = 0,\; y'(0) = 2$$

OKUPNO $\boxed{95}$

1.



$$p_{AB} \equiv (x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$$

$$(2-0)(y-0) = (3-0)(x-0)$$

$$2y = 3x \,/:2$$

$$y = \frac{3}{2}x$$

$$p_{AC} \equiv (4-0)(y-0) = (2-0)(x-0)$$

$$4y = 2x \,/:4$$

$$y = \frac{1}{2}x$$

$$p_{BC} \equiv (4-2)(y-3) = \overbrace{(2-3)}^{=-1}(x-2)$$

$$2y - 6 = -x + 2$$

$$2y = -x + 8 \,/:2$$

$$y = -\frac{1}{2}x + 4$$

$$\int -x+4\,dt = \int -x + 4\int dt$$

$$= -\frac{x^2}{2} + 4x$$

$$P_1 = \int_0^2 \frac{3}{2}x - \frac{1}{2}x\,dt$$

$$= \int_0^2 x\,dt = \frac{x^2}{2}\Big|_0^2$$

$$= 2$$

$$P_2 = \int_2^4 -\frac{1}{2}x + 4 - \frac{1}{2}x\,dt$$

$$= \int_2^4 -x+4\,dt$$

$$= -\frac{x^2}{2} + 4x\Big|_2^4$$

$$= 2$$

$$P_{\Delta ABC} = P_1 + P_2$$

$$= 4 \,//$$

2. $f(x) = \frac{1}{\sqrt{x+1}}$

$\int_{-1}^{1} f(x)\,dx = ?$

$\int \frac{1}{\sqrt{x+1}}\,dx = \left[\begin{array}{l} x+1 = t \,/\, ' \\ dx = dt \end{array}\right]$

$\int_{-1}^{1} \frac{1}{\sqrt{x+1}}\,dx =$

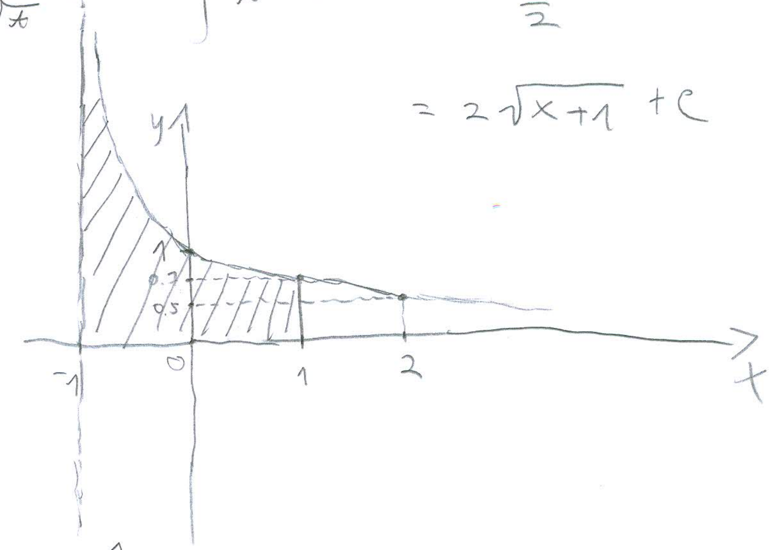
$\int \frac{1}{\sqrt{t}}\,dt = \int t^{-\frac{1}{2}}\,dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}}$

- točka sing. $x = -1$

$= 2\sqrt{x+1} + C$

$= \lim_{t \to -1^+} \left(\int_{t}^{1} \frac{1}{\sqrt{x+1}}\,dx \right)$

$= \lim_{t \to -1^+} \left(2\sqrt{x+1} \Big|_{t}^{1} \right)$



$= 2\sqrt{2} - 0$

$= 2\sqrt{2} \approx 2.83$ ✓

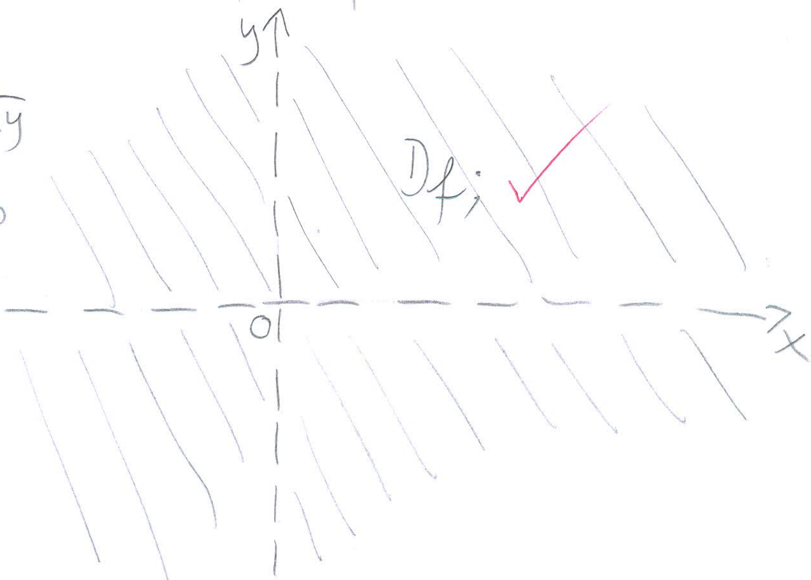
4. $f(x,y) = x - y + \frac{1}{xy}$

Df: uvjet: $x \cdot y \neq 0$

$x \neq 0$

$y \neq 0$

$Df: \mathbb{R}^2 \,|\, x,y \notin \{0\}$



$\frac{\partial f}{\partial x} = 1 + \frac{1}{y} \cdot (x^{-1})'$

$= 1 + \frac{1}{y} \cdot (-1) x^{-2}$

$= 1 - \frac{1}{y} \cdot \frac{1}{x^2}$

$= 1 - \frac{1}{yx^2}$

$\frac{\partial f}{\partial y} = -1 + \frac{-1 \cdot \cancel{x}}{x^{\cancel{2}} y^2}$

$= -1 - \frac{1}{xy^2}$

$\frac{\partial^2 f}{\partial y^2} = - \frac{-1 \cdot x \cdot 2\cancel{y}}{x^{\cancel{2}} y^{\cancel{4}3}}$

$= \frac{2}{xy^3}$

$\frac{\partial^2 f}{\partial y \partial x} = - \frac{-1 \cdot \cancel{x^2}}{y^2 x^{\cancel{4}2}}$

$= \frac{1}{y^2 x^2} = \frac{\partial^2 f}{\partial x \partial y}$

$\frac{\partial^2 f}{\partial x^2} = - \frac{0 \cdot yx^2 - 1 \cdot 2\cancel{y}x}{y^{\cancel{2}} x^{\cancel{4}3}} = \frac{2}{yx^3}$

mat. 4. zad.   Marijan Strk

$$1-\frac{1}{yx^2}=0 \quad \rightarrow \quad 1-\frac{1}{y\cdot\left(-\frac{1}{y^2}\right)^2}=0$$

$$-1-\frac{1}{xy^2}=0 / \cdot xy^2$$

$$1-\frac{1}{y\cdot\frac{1}{y^{4}}}=0$$

$$-xy^2-1=0$$

$$-xy^2=1 / : y^2$$

$$1-\left(\frac{\frac{1}{1}}{\frac{1}{y^3}}\right)=0$$

$$-x=\frac{1}{y^2} / \cdot(-1)$$

$$x=-\frac{1}{y^2}$$

$$1-y^3=0$$

$$y^3=1$$

$$x=-\frac{1}{1^2} \qquad T_0(-1,1)$$

$$y=1$$

$$x=-1$$

$$\Delta=\begin{vmatrix}-2 & 1\\ 1 & -2\end{vmatrix}$$

$$\frac{\partial^2 f}{\partial x^2}(T_0)=-2<0 \qquad \frac{\partial^2 f}{\partial x\partial y}=1$$

$$\Delta=4-1$$

$$\Delta=3>0$$

$$\frac{\partial^2 f}{\partial y^2}(T_0)=-2$$

To je maksimum funkcije.

$$t=\sqrt[3]{x}=x^{\frac{1}{3}}$$

5. $\sqrt[3]{x}\, yy'=1-x^2 / : \sqrt[3]{x}$

$$\int\frac{1-x^2}{\sqrt[3]{x}}\,dt=\left[\begin{array}{l}x=t^3 /'\\ dt=3t^2\,dt\end{array}\right.$$

$$yy'=\frac{1-x^2}{\sqrt[3]{x}}$$

$$y\cdot\frac{dy}{dt}=\frac{1-x^2}{\sqrt[3]{x}} / \cdot dt$$

$$=\int\frac{1-t^6}{\sqrt[3]{t^3}}\cdot 3t^2\,dt$$

$$y\,dy=\frac{1-x^2}{\sqrt[3]{x}}\,dt / \int$$

$$=\int\frac{3t^2-3t^8}{t}\,dt$$

$$\int y\,dy=\int\frac{1-x^2}{\sqrt[3]{x}}\,dt$$

$$=\int 3t-3t^7\,dt$$

$$\frac{y^2}{2}=\frac{3\cdot x^{\frac{2}{3}}}{2}-\frac{3x^{\frac{8}{3}}}{8} / \cdot 2$$

$$=\int 3t\,dt-3\int t^7\,dt$$

$$y^2=3x^{\frac{2}{3}}-\frac{3x^{\frac{8}{3}}}{4}+C$$

$$=3\cdot\frac{t^2}{2}-3\cdot\frac{t^8}{8}$$

$$y=\mp\sqrt{3x^{\frac{2}{3}}-\frac{3x^{\frac{8}{3}}}{4}+C}$$

$$=\frac{3\cdot x^{\frac{2}{3}}}{2}-\frac{3\cdot x^{\frac{8}{3}}}{8}+C$$

6. $y'' + 4y = 4 \qquad y(0) = 0 \qquad y'(0) = 2$

$r^2 + 4 = 0$

$r^2 = -4$

$r_1 = -2i \qquad \alpha = 0$

$r_2 = 2i \qquad \beta = 2$

$y_H = e^{\alpha x}\left(C_1 \cdot \cos(\beta x) + C_2 \cdot \sin(\beta x)\right)$

$= \left(C_1 \cdot \cos 2x + C_2 \sin 2x\right)$

$4 = e^{\alpha x}\left(P_n(x) \cos(\beta x) + Q_m(x) \sin(\beta x)\right)$

$\alpha = 0$

$\beta = 0$

$\left.\begin{matrix} m=0 \\ n=0 \end{matrix}\right\} N = 0$ - stupanj polinoma

$k = 0 \qquad 0 + 0i \notin r_{1,2}$

$4 = \underbrace{x^0}_{=1} \underbrace{e^{0x}}\left(A \cdot \underbrace{\cos(0)}_{=1} + B \underbrace{\sin(0)}_{=0}\right)$

$y_p = A \qquad 0 + 4A = 4$

$y_p' = 0 \qquad A = 1$

$y_p'' = 0 \qquad y_p = 1$

$y = y_H + y_p$

$y = C_1 \cdot \cos 2x + C_2 \sin 2x + 1$

$y = C_1 \cdot \cos(2 \cdot 0) + \underbrace{C_2 \sin(2 \cdot 0)}_{=0} + 1$

$0 = C_1 + 1$

$y' = C_1 (-\sin(2x)) \cdot 2 + C_2 \cdot \cos(2x) \cdot 2$

$y' = -2C_1 \sin(2x) + 2C_2 \cos(2x)$

$2 = 2C_2 \,/ : 2$

$C_2 = 1 \qquad y = -\cos(2x) + \sin(2x) + 1$

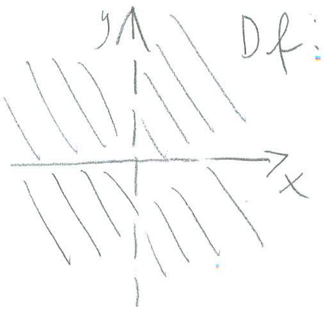
$C_1 + 1 = 0$

$C_1 = -1$

3. Marijan Štrk

$f(x,y) = \dfrac{x^2}{y}$

uvjet $y \neq 0$



$Df: \mathbb{R}^2 \mid y \neq 0$

$Im(f): \mathbb{R}$

$c = \dfrac{x^2}{y}$

$c = 1 \quad 1 = \dfrac{x^2}{y} \Big/ \cdot y$

$y = x^2$

$c = 0$

$\dfrac{x^2}{y} = 0 \Big/ \cdot y$

$x^2 = 0$

$x = 0$

$c = -2$

$-2 = \dfrac{x^2}{y} \Big/ \cdot y$

$-2y = x^2 \quad y = -\dfrac{x^2}{2}$

$c = 2$

$2 = \dfrac{x^2}{y} \Big/ \cdot y$

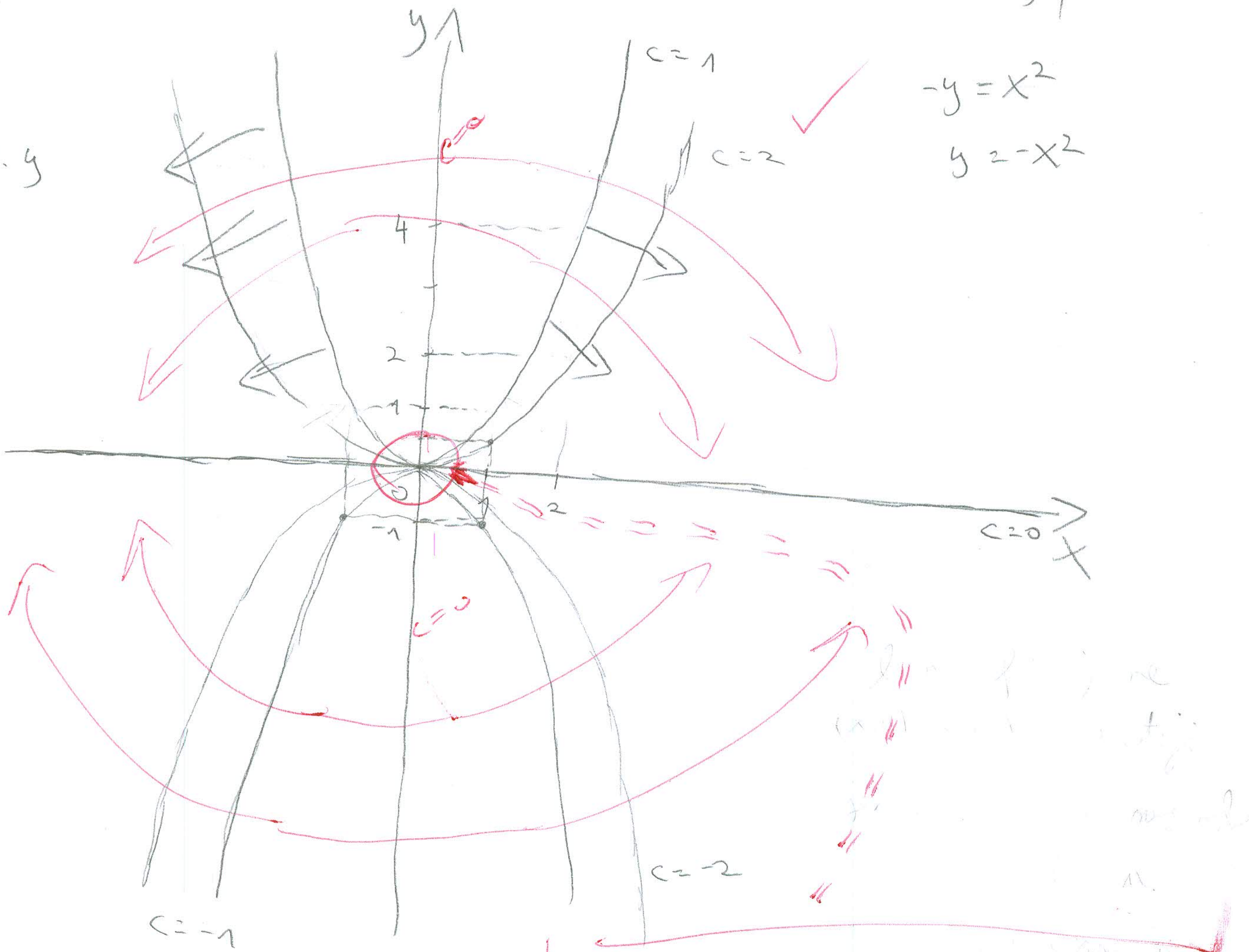
$2y = x^2 \quad y = \dfrac{x^2}{2}$

$c = -1$

$-1 = \dfrac{x^2}{y} \Big/ \cdot y$

$-y = x^2$

$y = -x^2$



$\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2}{y} = \lim\limits_{x\to 0} \left(\lim\limits_{y\to 0} \dfrac{x^2}{y} \right)$

NE POSTOJI JER SE U ISHODIŠTU SIJEKU RAZLIČITE RAZINSKE KRIVULJE (VIDI SLIKU ZA C=1 I C=2)

Popuniti odmah!

Ime i prezime: TONĆI MARINOVIĆ **Broj indeksa:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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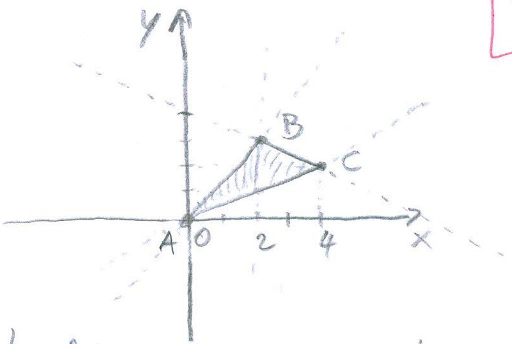
$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

UKUPNO: ~~88~~ 73 Kos

(1)

$$(y-y_1) = \left(\frac{y_2-y_1}{x_2-x_1}\right)\cdot(x-x_1)$$

$A(0,0)$
$B(2,3)$
$C(4,2)$



pravac kroz točke A,B

$$(y-0) = \left(\frac{3-0}{2-0}\right)(x-0)$$

$$y = \frac{3}{2}x$$

pravac kroz točke A,C

$$(y-0) = \left(\frac{2-0}{4-0}\right)(x-0)$$

$$y = \frac{1}{2}x$$

pravac kroz točke B,C

$$(y-3) = \left(\frac{2-3}{4-2}\right)(x-2)$$

$$y-3 = -\frac{1}{2}(x-2)$$

$$y = -\frac{1}{2}x + 1 + 3$$

$$y = -\frac{1}{2}x + 4$$

$$1^\circ\ P_1 = \int_0^2 \left(\frac{3}{2}x - \frac{1}{2}x\right)dx = \int_0^2 x\,dx = \frac{x^2}{2}\Big|_0^2 = 2$$

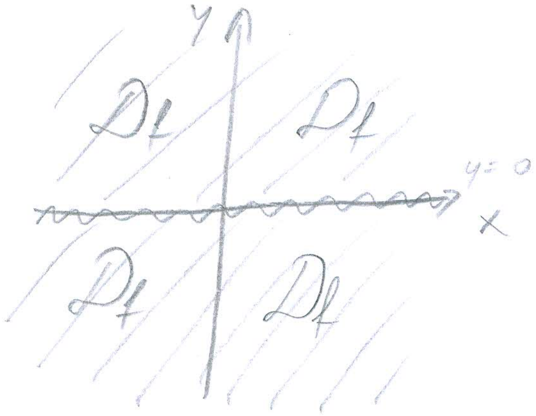
$$2^\circ\ P_2 = \int_2^4 -\frac{1}{2}x + 4 - \frac{1}{2}x = \int_2^4 -x + 4 = -\int x\,dx + 4\int dx = \left(-\frac{x^2}{2} + 4x\right)\Big|_2^4 = 2$$

$$\boxed{P = P_1 + P_2 = 2 + 2 = 4}$$

(3) $f(x,y) = \frac{x^2}{y}$

(1°) DOMENA

$y \neq 0$



$\boxed{Im_f = R^2}$

TONČI MARINOVIĆ

6. $y''+4y=4$, $y(0)=0$, $y'(0)=2$

1° $\lambda^2+4=0$ $\qquad y_H = \underbrace{e^{0\cdot x}}_{=1}(C_1\cos 2x + C_2 \sin 2x)$

$\lambda^2=-4$ $\qquad \boxed{y_H = C_1\cos 2x + C_2\sin 2x}$

$\lambda_{1,2}=\pm 2i$

$\boxed{a=0}$
$\boxed{b=2}$

2° $f(x)=4$ $\qquad y(x)=y_H+Y$

$= e^{0\cdot x}\cdot 4$ $\qquad \boxed{y(x)=C_1\cos 2x + C_2\sin 2x + 1}$ ✓

$\boxed{Y=A}$ $\qquad 4A=4$

$Y'=0$ $\qquad \boxed{A=1} \Rightarrow \boxed{Y=1}$

$Y''=0$

$\boxed{y'(x)=-2C_1\sin 2x + 2C_2\cos 2x}$

1° $0=C_1\cos 0 + C_2\sin 0 + 1$

$0=C_1+1 \Rightarrow \boxed{C_1=-1}$

2° $2=-2C_1\sin 0 + 2C_2\cos 0$

$2=2C_2$

$\boxed{C_2=1}$

KONAČNO RJEŠENJE!

$\boxed{y(x)=-\cos 2x+\sin 2x+1}$ ✓

4. EKSTREMI NASTAVAK!

1° $T_1(1,1)$ $\qquad D=\begin{vmatrix} 2 & 1 \\ 1 & 2\end{vmatrix} = 4-1=3>0$, $\frac{\partial^2 f}{\partial x^2}>0$ → Točka $T_1(1,1)$ je MINIMUM! ✗

NIJE STAC. TOČKA

2° $T_2(-1,1)$ $\qquad D=\begin{vmatrix} -2 & 1 \\ 1 & -2\end{vmatrix} = 4-1=3>0$, $\frac{\partial^2 f}{\partial x^2}<0$ → Točka $T_2(-1,1)$ je MAKSIMUM! ✓

3° $T_3(1,-1)$ $\qquad D=\begin{vmatrix} -2 & 1 \\ 1 & -2\end{vmatrix} = 4-1=3>0$ → Točka $T_3(1,-1)$ je MAKSIMUM ✗

NIJE STAC. TOČKA

4° $T_4(-1,-1)$ $\qquad D=\begin{vmatrix} 2 & 1 \\ 1 & 2\end{vmatrix} = 4-1=3>0$ → Točka $T_4(-1,-1)$ je MINIMUM ✗

④ $f(x,y) = x - y + \frac{1}{xy}$

$(-2y\cdot x^{-2})' = 4y\,x^{-3} = \frac{4y}{x^3}$

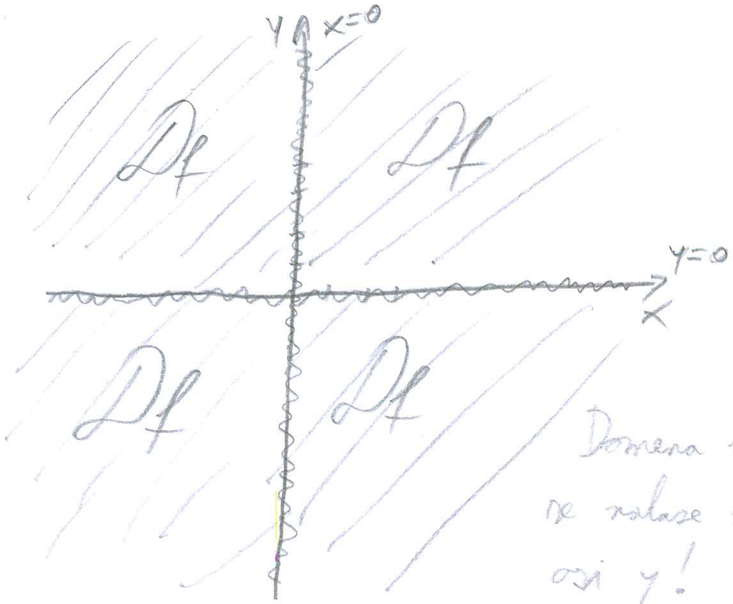
$-\frac{2y}{x^2}$

$\frac{\partial f}{\partial y} = \frac{2}{x} =$

1° DOMENA

$xy \neq 0$

$\boxed{\begin{matrix} x \neq 0 \\ y \neq 0 \end{matrix}}$



Domena ne uključuje točke koje se nalaze na samoj osi x i samoj osi y!

2° EKSTREMI

$$\frac{\partial f}{\partial x} = \left(x - y + \frac{1}{y}\cdot x^{-1}\right)' = 1 - x^{-2}\cdot\frac{1}{y} = 1 - \frac{1}{x^2 y}$$

$$\frac{\partial f}{\partial y} = \left(x - y + \frac{1}{x}\cdot y^{-1}\right)' = -1 - y^{-2}\cdot\frac{1}{x} = -1 - \frac{1}{xy^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \left(-1 - \frac{1}{y^2}\cdot x^{-1}\right)'$$

$$\boxed{\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{y^2}\,x^{-2} = \frac{1}{x^2 y^2}}$$

$$\frac{\partial^2 f}{\partial x^2} = \left(1 - \frac{1}{y}\cdot x^{-2}\right)' = 2\cdot\frac{1}{y}\cdot x^{-3} = \frac{2}{x^3 y}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \left(1 - \frac{1}{x^2}\cdot y^{-1}\right)' = \frac{1}{x^2}\cdot y^{-2} = \frac{1}{x^2 y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \left(-1 - \frac{1}{x}\cdot y^{-2}\right)' = 2\cdot\frac{1}{x}\cdot y^{-3} = \frac{2}{xy^3}$$

1) $1 - \frac{1}{x^2 y} = 0$

$\frac{1}{x^2 y} = \frac{1}{1}$

$\boxed{x^2 y = 1}$

2) $-1 - \frac{1}{xy^2} = 0$

$\frac{1}{xy^2} = -\frac{1}{1}$

$\boxed{xy^2 = -1} \quad /^2$

$x^2 y^4 = 1$

$\boxed{x^2 = \frac{1}{y^4}}$

$y^2 = \frac{-1}{x}$

$y^2 \neq -1$

$y^2 = 1$

$y_1 = 1, \quad y_2 = -1$

$\frac{1}{y^{\cancel{4}3}}\cdot\cancel{y} = \frac{1}{1}$

$y^3 = 1 \Rightarrow \boxed{y_1 = 1} \quad \boxed{y_2 = -1}$

$x^2 = \frac{1}{1}$

$x^2 = 1$

$\boxed{x_1 = 1} \quad \boxed{x_2 = -1}$

(2) $f(x) = \frac{1}{\sqrt{x+1}}$

$t = \sqrt{x+1}$

$$\int_{-1}^{1} \frac{1}{\sqrt{x+1}} dx = \lim_{\varepsilon \to -1^+} \int_{\varepsilon}^{1} \frac{1}{\sqrt{x+1}} dx = \left| \begin{matrix} x+1 = t^2 \\ dx = 2t\,dt \end{matrix} \right| = \int \frac{2t\,dt}{t} = 2\int dt =$$

$$= 2t = 2\sqrt{x+1} \Big|_{\varepsilon}^{1} =$$

$$= (2\sqrt{2}) - \underbrace{(2\sqrt{\varepsilon+1})}_{\downarrow \, 0} =$$

$$= 2\sqrt{2} \approx 2.8284$$

$\downarrow$

KONVERGIRA!

$x + 1 > 0$

$\boxed{x > -1}$

NEPRAVI INTEGRAL!

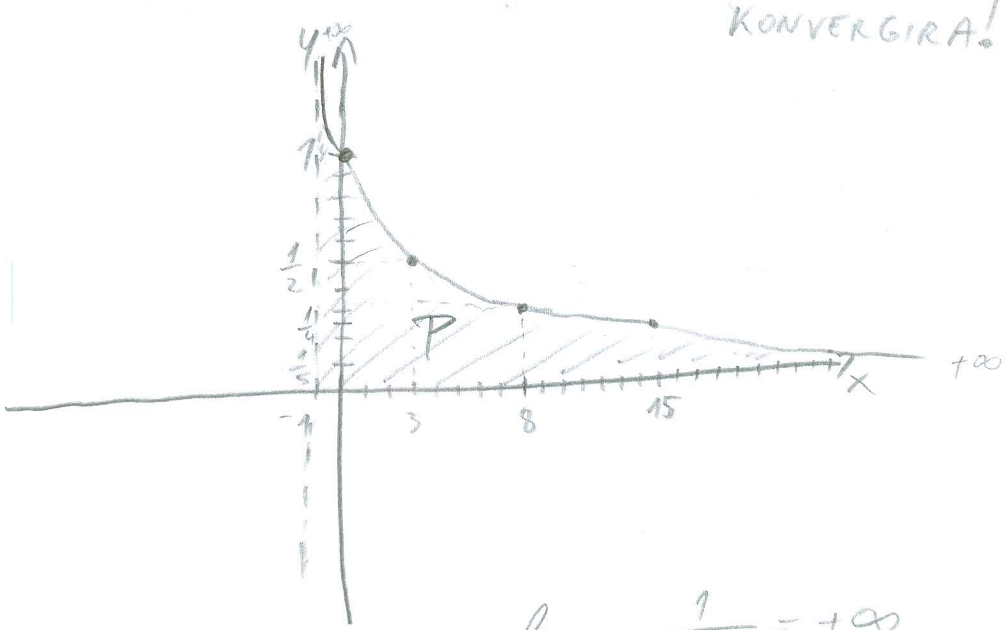
$f(3) = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$f(8) = \frac{1}{\sqrt{9}} = \frac{1}{3}$

$f(15) = \frac{1}{\sqrt{16}} = \frac{1}{4}$

$f(24) = \frac{1}{5}$

$f(0) = \frac{1}{1} = 1$



$$\lim_{x \to -1^+} \frac{1}{\sqrt{x+1}} = +\infty$$

(5) $\sqrt[3]{x}\, y y' = 1 - x^2$

TONCI MARINOVIĆ

$x^{\frac{1}{3}} y y' = 1 - x^2 \quad / : \sqrt[3]{x}$

$y y' = \frac{1}{\sqrt[3]{x}} - \left(x^{2 - \frac{1}{3}}\right)$

$y y' = x^{-\frac{1}{3}} - x^{\frac{5}{3}}$

$y \frac{dy}{dx} = x^{-\frac{1}{3}} - x^{\frac{5}{3}} \quad / \cdot dx$

$\int y\, dy = \int x^{-\frac{1}{3}} dx - \int x^{\frac{5}{3}} dx$

$\frac{y^2}{2} = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + C$

$\frac{y^2}{2} = \frac{3\sqrt[3]{x^2}}{2} - \frac{3\sqrt[3]{x^8}}{8} \quad / \cdot 2$ ✓

$y^2 = 3\sqrt[3]{x^2} - \frac{3\sqrt[3]{x^8}}{4} + C$ //

$y = \sqrt{3\sqrt[3]{x^2} - \frac{3}{4}\sqrt[3]{x^8} + C}$ //

Popuniti odmah!

IME I PREZIME: NIKOLA KNEŽEVIĆ **BROJ INDEKSA:** 17-1-0002-2010

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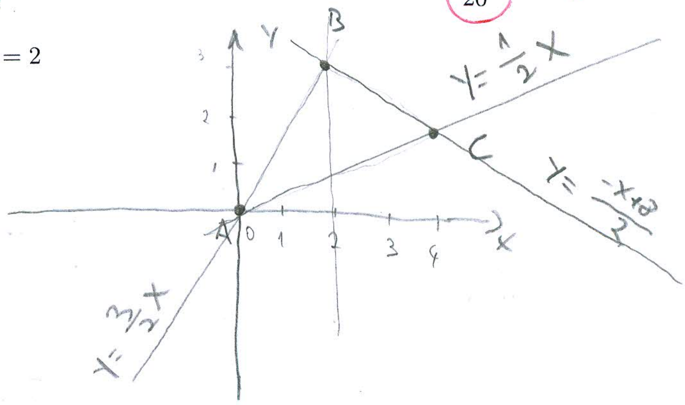
5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\, y\, y' = 1 - x^2$ 15
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe:

$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

UKUPNO $\boxed{50}$



1.

$A(0,0)$

$B(2,3)$

$C(4,2)$

$\overline{AB} \Rightarrow (Y-Y_1)(X_2-X_1) = (X-X_1)(Y_2-Y_1)$

$(Y-0)(2-0) = (X-0)(3-0)$

$2Y = 3X$

$\boxed{Y = \frac{3}{2}X}$

$\overline{AC} \Rightarrow (Y-Y_1)(X_2-X_1) = (X-X_1)(Y_2-Y_1)$

$(Y-0)(4-0) = (X-0)(2-0)$

$4Y = 2X$

$Y = \frac{2}{4}X$

$\boxed{Y = \frac{1}{2}X}$

$\overline{BC} \Rightarrow (Y-Y_1)(X_2-X_1) = (X-X_1)(Y_2-Y_1)$

$(Y-3)(4-2) = (X-2)(2-3)$

$4Y - 2Y - 12 + 6 = 2X - 3X - 4 + 6$

$4Y - 2Y = -1X - 4 + 6 - 6 + 12$

$2Y = -X + 8$

$\boxed{Y = \frac{-X+8}{2}}$

1. NASTAVAK NIKOLA KNEŽEVIĆ

$$P=\left(\int_0^2 \frac{3}{2}x-\frac{1}{2}x\,dx\right)+\left(\int_2^4 \frac{-x+8}{2}-\frac{1}{2}x\,dx\right)$$

$$P=\frac{3}{2}\int_0^2 x\,dx-\frac{1}{2}\int_0^2 x\,dx+\int_2^4 \frac{-x+8}{2}\,dx-\frac{1}{2}\int_2^4 x\,dx$$

$$P=\frac{3}{2}\int_0^2 x\,dx-\frac{1}{2}\int_0^2 x\,dx-\frac{1}{2}\int_2^4 x+8\,dx-\frac{1}{2}\int_2^4 x\,dx$$

$$P=\frac{3}{2}\int_0^2 x\,dx-\frac{1}{2}\int_0^2 x\,dx-\frac{1}{2}\int_2^4 x\,dx-4\int_2^4 dx-\frac{1}{2}\int_2^4 x\,dx$$

$$P=\frac{3}{2}\left(\frac{x^2}{2}\right)\Big|_0^2-\frac{1}{2}\left(\frac{x^2}{2}\right)\Big|_0^2-\frac{1}{2}\left(\frac{x^2}{2}\right)\Big|_2^4-4x\Big|_2^4-\frac{1}{2}\left(\frac{x^2}{2}\right)\Big|_2^4$$

$$P=\frac{3}{2}\cdot\frac{2^2}{2}-\frac{1}{2}\,\frac{2}{2}-\frac{1}{2}\left(\frac{4^2}{2}-\frac{2^2}{2}\right)-4(4-2)-\frac{1}{2}\left(\frac{4^2}{2}-\frac{2^2}{2}\right)$$

$$P=\frac{12}{4}-\frac{2}{4}-\frac{1}{2}\,\frac{12}{2}-8-\frac{1}{2}\,\frac{12}{2}$$

$$P=\frac{12}{4}-\frac{2}{4}-\frac{12}{4}-\frac{32}{4}-\frac{12}{4}$$

$$P=\frac{-46}{4}$$

$$\boxed{P=\frac{46}{4}=\frac{23}{2}}$$

(1.) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{x+1}}\,dx =$ OVO JE NEPRAVI INTEGRAL JER TOČKA -1 NIJE U DOMENI FUNKCIJE $\dfrac{1}{\sqrt{x+1}}$

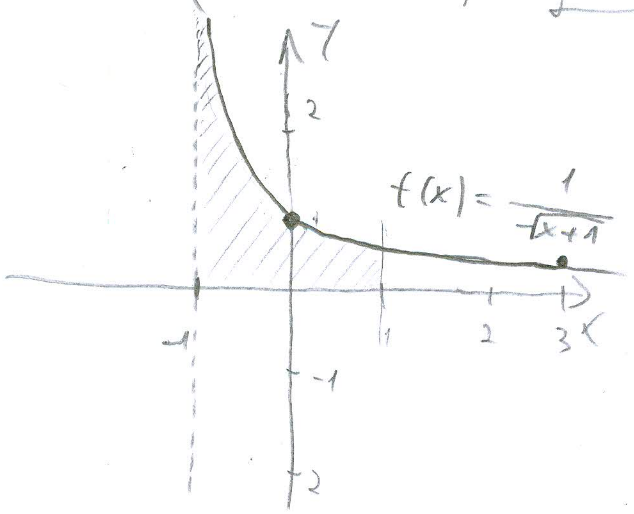
$$\underbrace{\lim_{z\to -1} \int_{z}^{1} \frac{dx}{\sqrt{x+1}}}_{*}$$

$$* \int \frac{dx}{\sqrt{1+x}} = \int \frac{dx}{(1+x)^{\frac{1}{2}}} = \left| \begin{array}{l} 1+x=t^2 \\ dx=2t\,dt \end{array} \right|$$

$$= \int \frac{2t\,dt}{(t^2)^{\frac{1}{2}}} = 2\int dt = 2t = 2\left(\sqrt{x+1}\right)$$

$$\lim_{z\to -1} 2\left(\sqrt{x+1}\right)\Big|_{z}^{1} = \lim_{z\to -1} 2\left[\left(\sqrt{1+1}\right) - \left(\sqrt{z+1}\right)\right]$$

$$= 2\left(\sqrt{2} - 0\right) = \boxed{2\sqrt{2}}$$



$f(x) = \dfrac{1}{\sqrt{x+1}}$

x	-1	0	3	
y	∞	1	$\frac{1}{2}$	

(5.) $\sqrt[3]{x}\, y\, y' = 1 - x^2 \quad \Big/ \cdot \frac{1}{\sqrt[3]{x}}$

$$y\, y' = \frac{1-x^2}{\sqrt[3]{x}}$$

$$y \frac{dy}{dx} = \frac{1-x^2}{\sqrt[3]{x}} \quad \Big/ \cdot dx$$

$$\int y\, dy = \underbrace{\int \frac{1-x^2}{\sqrt[3]{x}}\, dx}_{*}$$

$$* \int \frac{1-x^2}{x^{\frac{1}{3}}}\, dx = \left| \begin{array}{l} x = t^3 \\ dx = 3t^2 dt \end{array} \right| = \int \frac{1-(t^3)^2}{(t^3)^{\frac{1}{3}}}\, 3t^2 dt$$

$$= \int \frac{1-t^6}{t}\, 3t^2 dt = 3 \int \frac{(1-t^6)\, t^{\cancel{2}}}{\cancel{t}}\, dt = 3 \int t - t^7\, dt$$

$$= 3\int t\, dt - 3\int t^7 dt = 3\, \frac{t^2}{2} - 3\, \frac{t^8}{8} = \frac{3}{2} t^2 - \frac{3}{8} t^8$$

$$= \frac{3}{2} \sqrt[3]{x^2} - \frac{3}{8} \sqrt[3]{x^8} + C$$

$$\boxed{\frac{y^2}{2} = \frac{3}{2} \sqrt[3]{x^2} - \frac{3}{8} \sqrt[3]{x^8} + C} \quad \Big/ \cdot 2$$

$$y^2 = 3 \sqrt[3]{x^2} - \frac{3}{4} \sqrt[3]{x^8} + C$$

$$y = \sqrt{3 \sqrt[3]{x^2} - \frac{3}{4} \sqrt[3]{x^8} + C}$$

NIKOLA KNEŽEVIĆ

4. $f(x,y) = x - y + \frac{1}{xy}$

$\partial x f = 1 + \frac{-(xy)'}{(xy)^2} = 1 + \frac{-(x' \cdot y + x \cdot y')}{(xy)^2} = 1 + \frac{-y}{(xy)^2}$

$\partial x f = 1 - \frac{y}{(xy)^2}$ ✓

$\partial xx f = -\frac{-y((xy)^2)'}{(xy)^4} = -\frac{-2y^2}{(xy)^4} = \frac{2y^2}{(xy)^4}$

$\partial y f = -1 + \frac{-x}{(xy)^2} = -1 - \frac{x}{(xy)^2}$ ✓

$\partial yy f = -\frac{-x\,2x}{(xy)^4} = \frac{2x^2}{(xy)^4}$

$\partial xy f = -\frac{(xy)^2 - x\,2y}{(xy)^4} = -\frac{(xy)^2 - 2xy}{(xy)^4}$

$\partial x f = 0$

$1 - \frac{y}{(xy)^2} = 0$

$\frac{y}{(xy)^2} = 1$

$\partial y f = 0$

$-1 - \frac{x}{(xy)^2} = 0$

$\frac{x}{(xy)^2} = -1$

NIKOLA KNEŽEVIĆ

6. $y'' + 4y = 4$

$y'' + 4y = 0$

$r_{1,2} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \underbrace{0}_{\alpha} \pm \underbrace{\frac{\sqrt{-4 \cdot 4}}{2}}_{\beta} = \pm 2i$

$y_H = e^{\alpha x}\left(C_1 \cos(\beta x) + C_2 \sin(\beta x)\right)$

$y_H = C_1 \cos(2x) + C_2 \sin(2x)$

$4 = e^{\alpha x}\left(P_m(x) \cos(\beta x) + Q_n(x) \sin(\beta x)\right)$

$\alpha = 0$ $\qquad \alpha + \beta i = 0 + 0i$

$\beta = 0$ $\qquad k = 0$

$P = 4 \quad m = 0$
$Q = 0 \quad n = 0$ $\Big\}\ N = 0$

$y_p = x^k e^{\alpha x}\left(S_N(x) \cos(\beta x) + T_N \sin(\beta x)\right)$

$y_p = A$

$y'_p = 0$

$y''_p = 0$

$$Y'' + 4Y = 4$$

$$4A = 4$$

$$A = 1$$

$$Y_P = 1$$

$$Y = Y_H + Y_P$$

$$\boxed{Y = C_1 \cos(2x) + C_2 \sin(2x) + 1}$$

$$Y' = C_1 \cdot (-\sin(2x)) \cdot 2 + C_2 \cos(2x) \cdot 2$$

$$\boxed{Y' = -2C_1 \sin(2x) + 2C_2 \cos(2x)}$$

$$Y(0) = 0$$

$$Y = 0 = C_1 \cos 0 + C_2 \sin 0 + 1$$

$$0 = C_1 + 1$$

$$\underline{C_1 = -1}$$

$$Y'(0) = 2$$

$$Y' = 2 = \cancel{-2C_1 \sin 0} + 2C_2 \cos 0$$

$$2 = 2C_2$$

$$\underline{C_2 = 2}$$

PARTIKULARNO

$$\boxed{Y = -\cos(2x) + 2\sin(2x) + 1}$$

Popuniti odmah! Rikardo Radovčić

IME I PREZIME: Rikardo Radovčić **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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① $A(0,0)$
$B(2,3)$
$C(4,2)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$p_{AB} \ldots y - 0 = \frac{3-0}{2-0}(x-0)$$

$$y = \frac{3}{2}x$$

$$p_{BC} \ldots y - 3 = \frac{2-3}{4-2}(x-2)$$

$$y - 3 = \frac{-1}{2}(x-2)$$

$$y - 3 = \frac{-x}{2} + 1$$

$$y = -\frac{x}{2} + 4$$

$$p_{AC} \ldots y - 0 = \frac{2-0}{4-0}(x-0)$$

$$y = \frac{1}{2}x$$

$$P = \int_0^2 (P_{AB} - P_{AC})\,dx + \int_2^4 (P_{BC} - P_{AC})\,dx =$$

$$= \int_0^2 \left(\frac{3}{2}x - \frac{1}{2}x\right) dx + \int_2^4 \left(-\frac{x}{2} + 4 - \frac{x}{2}\right) dx =$$

$$= \int_0^2 x\,dx + \int_0^4 (-x + 4)\,dx =$$

$$= \frac{x^2}{2}\Big|_0^2 - \frac{x^2}{2}\Big|_2^4 + 4x\Big|_2^4 = \left(\frac{4}{2} - 0\right) - \left(\frac{16}{2} - \frac{4}{2}\right) + (16 - 8)$$

$$= 2 - (8 - 2) + 8$$

$$= 2 - 8 + 2 + 8$$

$$= 4_{\,//}$$

④ Domenu i ekstreme

$$f(x,y) = x - y + \frac{1}{xy}$$

$$D(f) = \mathbb{R}^2 \setminus \{x,y\} \mid x = 0 \text{ i/ili } y = 0 \mid$$

DOMENA

EKSTREMI ?

⑤ $\sqrt[3]{x}\, y\, y' = 1 - x^2$

$$y y' = \frac{1-x^2}{\sqrt[3]{x}}$$

$$y\,dy = \frac{1-x^2}{\sqrt[3]{x}}\,dx \;/\int$$

$$\frac{y^2}{2} = -\frac{3}{8}x^{\frac{2}{3}}(x^2-4) + c$$

$$y^2 = -\frac{3}{4}x^{\frac{2}{3}}(x^2-4) + 2c$$

$$y = \pm\sqrt{2c - \frac{3}{4}x^{\frac{2}{3}}(x^2-4)}$$

$$\int \frac{1-x^2}{\sqrt[3]{x}}\,dx = \left\{\begin{array}{l} t = \sqrt[3]{x} \\ x = t^3 \\ x^2 = t^6 \end{array} \Rightarrow dx = 3t^2\,dt\right\} = \int \frac{1-t^6}{\cancel{t}}\,3t^{\cancel{2}}\,dt =$$

$$= 3\int (t - t^7)\,dt = \frac{3}{2}t^2 - \frac{3}{8}t^8 + c = \frac{3}{2}x^{\frac{2}{3}} - \frac{3}{8}x^{\frac{8}{3}} + C$$

$$= -\frac{3}{8}x^{\frac{2}{3}}(x^2-4) + c$$

Rikardo Radovčić

⑥ $y'' + 4y = 4$, $y(0) = 0$

$y'(0) = 2$

$y'' + 4y = 0$

~~$x^2+4=0$~~

$r^2 + 4 = 0$

$r^2 = -4$

$r_{1,2} = \pm 2i$

$y_0 = e^{0 \cdot x}(C_1 \cos 2x + C_2 \sin 2x), C_1, C_2 \in \mathbb{R}$

$f(x) = 4 \cdot e^{0 \cdot x}$

$y = A \cdot e^{0x} = A$

$y' = 0$

$y'' = 0$

$0 + 4A = 4$

$\boxed{A = 1}$

$y_0 = C_1 \cos(2x) + C_2 \sin(2x) + 1$

$y(0) = C_1 \cos(2 \cdot 0) + C_2 \sin(2 \cdot 0) + 1 = 0$

$y(0) = C_1 + 1 = 0 \Rightarrow \boxed{C_1 = -1}$

$y' = -1\, C_1 \sin(2x) + 2\, C_2 \cos(2x)$

$y'(0) = \underbrace{-2 \cdot C_1 \sin(0)}_{=0} + 2\, C_2 \cos(0) = 2$

$2C_2 = 2$

$\boxed{C_2 = 1}$

$y = -\cos(2x) + \sin(2x) + 1$ ✓

Popuniti odmah!

Ime i prezime: ROKO KRALJEV **Broj indeksa:**

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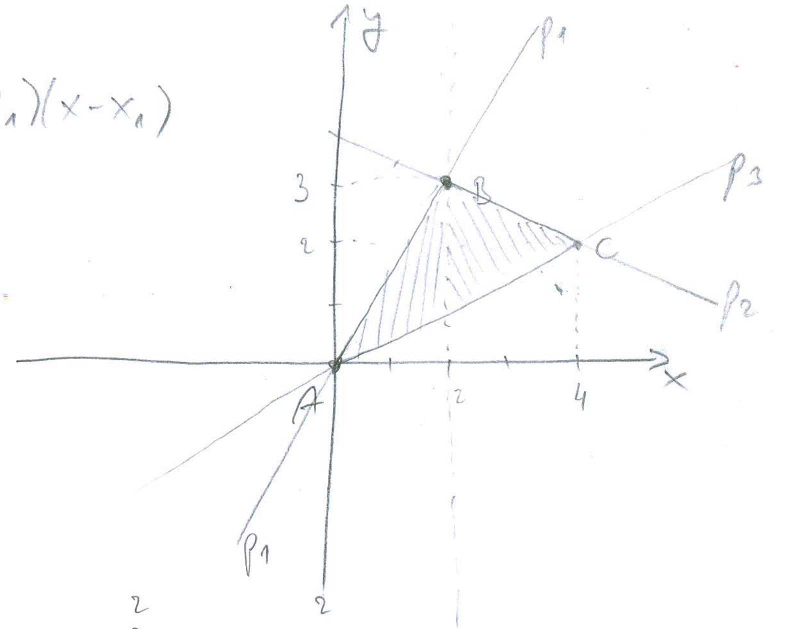
1.

$A(0,0)$ $\quad (y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$

$B(2,3)$

$C(4,2)$



$p_1 \ldots (y-0)(2-0)=(3-0)(x-0)$

$2y=3x$

$p_1 \ldots y=\frac{3}{2}x$

$(y-3)(4-2)=(2-3)(x-2)$

$2y-6=-x+2$

$2y=-x+8$

$p_2 \ldots y=-\frac{1}{2}x+4$

$(y-0)(4-0)=(2-0)(x-0)$

$4y=2x$

$p_3 \ldots y=\frac{1}{2}x$

$$P_1=\int_0^2 (p_1-p_3)\,dx=\int_0^2\left(\frac{3}{2}x-\frac{1}{2}x\right)dx=$$

$$=\int_0^2 x\,dx=\frac{x^2}{2}\Big|_0^2=\frac{2^2}{2}-\cancel{\frac{0}{2}}=2$$

$$P_2=\int_2^4 (p_2-p_3)\,dx=\int_2^4\left(-\frac{1}{2}x+4-\frac{1}{2}x\right)dx=$$

$$=\int_2^4(-x+4)\,dx=-\int_2^4 x\,dx+4\int_2^4 dx=$$

$$=\left(-\frac{x^2}{2}\right)\Big|_2^4+4x\Big|_2^4=-\frac{4^2}{2}+\frac{2^2}{2}+4(4-2)=$$

$$=\cancel{-8}+2\cancel{+16}\cancel{-8}=2$$

1. roztawak

$$P_{uk} = P_1 + P_2 = 2 + 2 = 4$$

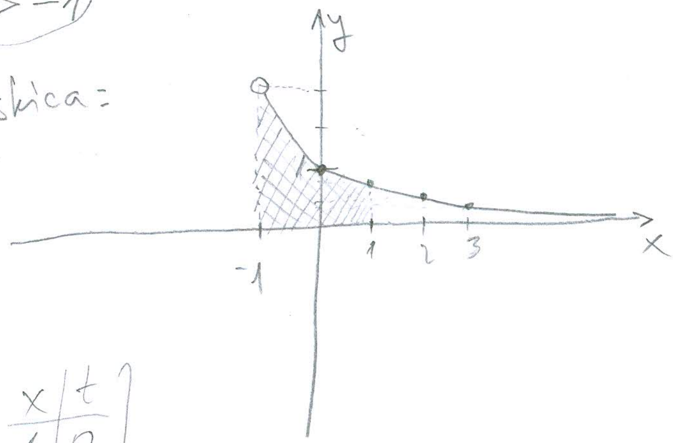
---

2. $f(x) = \frac{1}{\sqrt{x+1}}$ $\quad \sqrt{x+1} \neq 0 \quad x > -1$

$x + 1 \neq 0$

$x \neq -1$

skica:



$$\int_{-1}^{1} f(x)$$

$$\int_{-1}^{1} \frac{dx}{\sqrt{x+1}} = \lim_{\varepsilon \to -1} \int_{\varepsilon}^{1} \frac{dx}{\sqrt{x+1}} = \left\{ \begin{matrix} x+1 = t \,/' \\ dx = dt \end{matrix} \quad \begin{array}{c|c} x & t \\ \hline -1 & 0 \\ 1 & 2 \end{array} \right\}$$

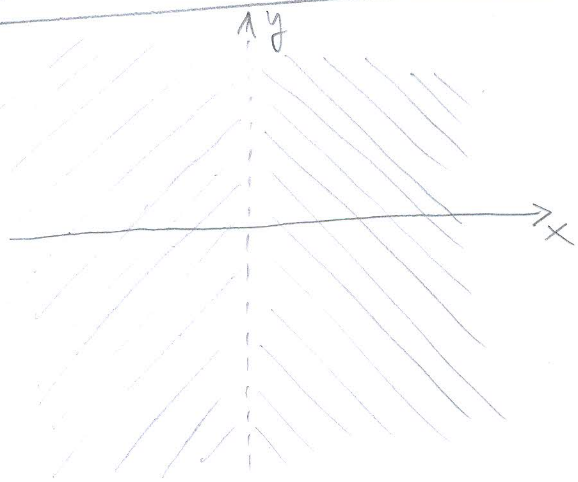
$$= \lim_{\varepsilon \to 0} \int_{\varepsilon}^{2} \frac{dt}{\sqrt{t}} = \lim_{\varepsilon \to 0} \int_{\varepsilon}^{2} t^{-\frac{1}{2}} dt = \lim_{\varepsilon \to 0} \left. \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right|_{\varepsilon}^{2} = \lim_{\varepsilon \to 0} \left. 2\sqrt{t} \right|_{\varepsilon}^{2} = \lim_{\varepsilon \to 0} (2\sqrt{2} - 2\sqrt{\varepsilon}) =$$

$$= 2\sqrt{2} - 2\sqrt{0} = 2\sqrt{2}$$

---

3. $f(x, y) = \frac{x^2}{y}$

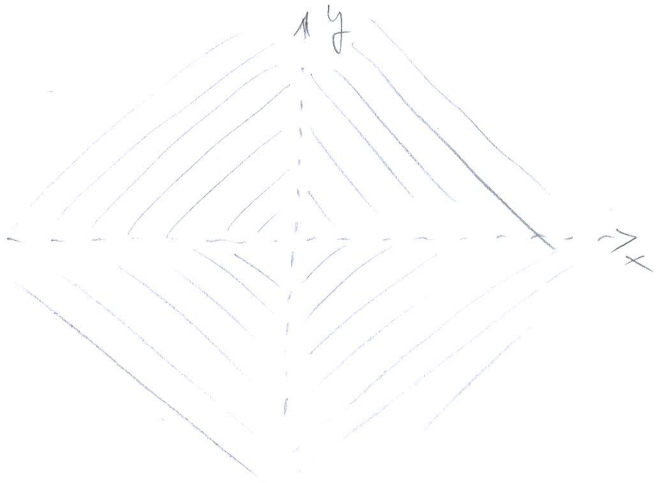
$y \neq 0$



4. $f(x,y) = x - y + \frac{1}{xy}$

$xy \neq 0$

$x \neq 0 \quad y \neq 0$ ✓ $\underline{2}$



5.

ROKO KRALJEV

$$\sqrt[3]{x}\, y\, y' = 1 - x^2$$

$$y' y = \frac{1-x^2}{\sqrt[3]{x}}$$

$$\frac{dy}{dx} y = \frac{1-x^2}{\sqrt[3]{x}} \Big/ \cdot dx$$

$$\int y\, dy = \int \frac{1-x^2}{\sqrt[3]{x}} dx$$

$$\frac{y^2}{2} = \int \frac{1}{\sqrt[3]{x}} dx - \int \frac{x^2}{\sqrt[3]{x}}$$

$$\frac{y^2}{2} = \int x^{-\frac{1}{3}} dx - \int x^{2-\frac{1}{3}} dx$$

$$\frac{y^2}{2} = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{8}{3}}}{\frac{8}{3}}$$

$$\frac{y^2}{2} = \frac{3\sqrt[3]{x^2}}{2} - \frac{3\sqrt[3]{x^8}}{8} \Big/ \cdot 2$$

$$y^2 = 3\sqrt[3]{x^2} - \frac{3\sqrt[3]{x^8}}{4} + C$$

$$y_1 = \sqrt{3 \cdot \sqrt[3]{x^2} - \frac{3 \cdot \sqrt[3]{x^8}}{4} + C}$$

$$y_2 = -\sqrt{3 \cdot \sqrt[3]{x^2} - \frac{3 \cdot \sqrt[3]{x^8}}{4} + C}$$

Popuniti odmah!
Ime i prezime: LUKA STIPIĆ **Broj indeksa:**

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25

4. a) $f(x,y) = x - y + \frac{1}{xy}$

$x - y = 0$ $\qquad x \cdot y \neq 0$

$x = y$

DOMENA CIJELI R^2 OSIM TOČKI O

$D_f: R^2 \setminus \{0\}$

b) $f(x,y) = x - y + \frac{1}{x \cdot y}$

$z_x = 1 + \left(\frac{0 \cdot xy - 1 \cdot (1 \cdot y + x \cdot 0)}{(xy)^2}\right) = 1 - \frac{y}{x^2y^2} = 1 - \frac{1}{x^2 y}$

$z_y = -1 - \frac{1}{xy^2}$

$z_{xx} = -\frac{0 \cdot x^2 y - 1 \cdot (2x \cdot y + x^2 \cdot 0)}{(x^2y)^2} = \frac{2xy}{x^4y^2} = \frac{2}{x^3 y}$

$z_{yy} = -\frac{0 \cdot (xy^2) - 1 \cdot (0 \cdot y^2 + x \cdot 2y)}{(xy^2)^2} = \frac{2xy}{x^2y^4} = \frac{2}{xy^3}$

$z_{xy} = -\frac{0 \cdot (x^2y) - 1 \cdot (0 \cdot y + x^2 \cdot 1)}{(x^2y)^2} = \frac{x^2}{x^4y^2} = \frac{1}{x^2y^2}$

$z_{yx} = -\frac{0 \cdot (xy^2) - 1 \cdot (1 \cdot y^2 + x \cdot 0)}{(xy^2)^2} = \frac{y^2}{x^2y^4} = \frac{1}{x^2y^2}$

6

4) NUŽAN UVJET

$z_x = 0$
$z_y = 0$

$z_x = 1 - \frac{1}{x^2 y} = 0$

$-\frac{1}{x^2y} = -1 \mid \cdot(-1)$

$\frac{1}{x^2y} = 1 \mid \cdot x^2 y$

$1 = x^2 y \mid \cdot \frac{1}{x^2}$

$y = \frac{1}{x^2}$

$y = \frac{1}{(-1)^2} = 1$

$z_y = -1 - \frac{1}{xy^2} = 0$

$-\frac{1}{xy^2} = 1 \mid \cdot xy^2$

$-1 = xy^2$

$-1 = x \cdot \frac{1}{x^2}$

$-1 = \frac{1}{x} \mid \cdot x$

$-x = 1$

$x = -1$

$T(-1, 1)$ ✓

DOVOLJAN UVJET

$z_{xx} \neq 0$

$z_{xx} = \frac{2}{x^3 y} = \frac{2}{(-1)^3 \cdot 1} = \frac{2}{-1} = -2 < 0$ MAKSIMUM

$\Delta = \begin{vmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 4 - 1 = 3 > 0$ IMAMO EKSTREM U TOČKI $(-1, 1)$ MAKSIMUM ✓

6) $y'' + 4y = 4 \longrightarrow m = 0$

$\lambda^2 + 4 = 0$

$\lambda^2 = -4 \mid \sqrt{\ }$

$\lambda_{1,2} = \pm 2i$

$a + bi = z \quad a = 0 \quad b_1 = 2 \quad b_2 = -2$

$y_0 = e^{0x} \cdot (C_1 \cos b_1 x + C_2 \sin bx)$

$y_0 = 1 \cdot (2 C_1 \cos x - 2 C_2 \sin x)$

$y = y_0 + y_p$

$y = 2C_1 \cos x - 2C_2 \sin x + 1$ ✗

$y(0) = 0 \qquad y'(0) = 2$

$0 = 2 \cdot C_1 \cos 0 - 2 \cdot C_2 \sin 0 + 1$

$0 = 2C_1 + 1$

$2C_1 = -1$

$C_1 = -\frac{1}{2}$

$y_p = a_0 \mid ''$

$y_p' = 0$

$y_p'' = 0$

$0 + 4a_0 = 4$

$4 \cdot a_0 = 4$

$a_0 = 1$

$y' = -2C_1 \sin x - 2C_2 \cos x$

$2 = -2 \cdot \frac{1}{2} \sin 0 - 2 \cdot C_2 \cos x$

$C_2 = -1$

$y = -\cos x + 2 \sin x + 1$ ✗

5) $\sqrt[3]{x}\, y\, y' = 1 - x^2 \quad | : \sqrt[3]{x}$

$y\, y' = \frac{1-x^2}{\sqrt[3]{x}}$

$y \frac{dy}{dx} = \frac{1-x^2}{\sqrt[3]{x}}$

$\int y\, dy = \int \frac{1-x^2}{\sqrt[3]{x}} dx$

$\frac{y^2}{2} = 3\sqrt[3]{x} + \ln|x| + C$

$\int y\, dy = \frac{y^2}{2}$

$\int \frac{1-x^2}{\sqrt[3]{x}} dx = \int \frac{1}{\sqrt[3]{x}} dx - \int \frac{\cancel{x}^2}{\sqrt[3]{x}} dx$

$= \int x^{-\frac{1}{3}} dx = 3x^{\frac{1}{3}}$

$\int \frac{1}{x} = \ln|x|$

2) $\int_{-1}^{1} \frac{1}{\sqrt{x+1}} dx = 2\sqrt{x+1}\Big|_{-1}^{1} = 2\cdot\sqrt{1+1} - 2\cdot\sqrt{-1+1} = 2\sqrt{2} - 0 = 2\sqrt{2} = 2{,}828427125$

$\int \frac{1}{\sqrt{x+1}} dx = \left| \begin{matrix} x+1=t \\ dx=dt \end{matrix} \right| = \int \frac{1}{\sqrt{t}} dt = \int t^{-\frac{1}{2}} dt = \int \frac{t^{\frac{1}{2}}}{\frac{1}{2}} dt = 2\sqrt{t}$

$= 2\sqrt{x+1}$

Popuniti odmah!

IME I PREZIME: Antun Žanetić **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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4. Istražiti domenu i ekstreme funkcije $f(x,y) = x - y + \dfrac{1}{xy}$. ~~20~~ 2
5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\, y\, y' = 1 - x^2$ 15
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe: ~~20~~

$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

10

6. $y'' + 4y = 4$

$\boxed{y = e^{kx}}$

$a = 1$
$b = 0$
$c = 4$

$y'' + 4y = 0$

$k^2 + 4 = 0$

$k^2 = -4$

$k_{1,2} = \sqrt{-4}$

$k_1 = 2i$

$k_2 = -2i$

$f(x) = 4$

$y_p = 4A \quad / \cdot x$

$\boxed{y_p = 4Ax}$

$y_p' = 4A$

$y_p'' = 0$

$0 + 4Ax = 4x$

$4Ax = 4x \quad / : 4x$

$\boxed{A = 1}$

$\boxed{y_p = 4x}$

$y_1 = e^{2ix}$

$y_2 = e^{-2ix}$

$y_H = C_1 \cdot e^{2ix} + C_2 \cdot e^{-2ix}$

Eulerova transformacija

$\boxed{e^{bix} = \cos bx - i \sin bx}$

$y_H = C_1 \cdot (\cos 2x - i \sin 2x) + C_2 \cdot (\cos(-2x) - i \sin(-2x))$

$y_H = C_1 \cdot \cos 2x - C_1 \cdot i \sin 2x + C_2 \cdot \cos(-2x) - C_2 \cdot i \sin(-2x)$

$y_H = C_1 \cdot \cos 2x - C_1 \cdot \cancel{i \sin 2x} + C_2 \cos 2x + C_2 \cdot \cancel{i \sin 2x}$

$y_H = C_1(\cos 2x - 1) + C_2(\cos 2x + 1)$

$y = y_H + y_p$

$y = C_1(\cos 2x - 1) + C_2(\cos 2x + 1) + 4x$

$y(0) = 0$

$C_1(\cos 0 - 1) + C_2(\cos 0 + 1) + 4 \cdot 0 = 0$

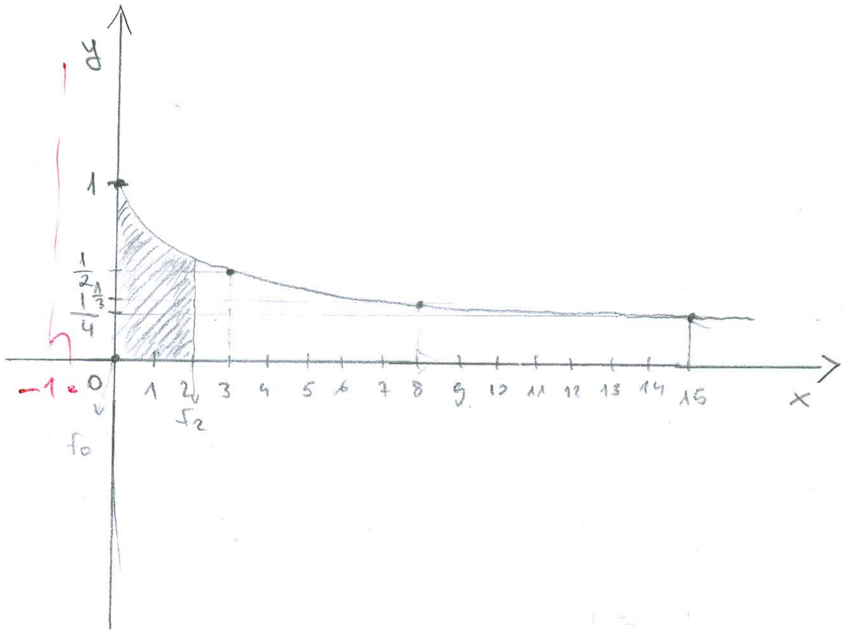
2. $f(x)=\frac{1}{\sqrt{x+1}}$

$$\int_{-1}^{1}\frac{1}{\sqrt{x+1}}dx = \left| \begin{matrix} x+1=t^2 \quad /' \\ dx = 2t\,dt \end{matrix}\right| = \int_0^{\sqrt{2}}\frac{2t\,dt}{\cancel{\sqrt{t^2}}} = \int_0^{\sqrt{2}}\frac{2\cancel{t}\,dt}{\cancel{t}} = 2\int_0^{\sqrt{2}}dt =$$

x	-1	1
t	0	2

$$= 2\cdot t\Big|_0^2 = (2\cdot 2)-(2\cdot 0) = 4$$

x	0	3	8	15
f(x)	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$



4. $f(x,y)=x-y+\frac{1}{xy}$

$\Downarrow$

$x-y+\frac{1}{x}\cdot\frac{1}{y}$

$x\cdot y\neq 0$

$D(f)=\{(x,y): x\cdot y\neq 0\}$ ✓

$$\Delta = \begin{vmatrix} \text{poz.b.} & 0 \\ 0 & \text{poz.br.}\end{vmatrix} = \text{poz. br.} > 0$$

minimum

EKSTREMI

$\frac{\partial f}{\partial x} = 1+(-x^{-2})$ ✗

$\frac{\partial f}{\partial y} = -1+(-y^{-2})$ ✗

$\frac{\partial^2 f}{\partial x^2} = 2x^{-3} = \frac{2}{x^3}$

$\frac{\partial^2 f}{\partial y^2} = 2y^{-3} = \frac{2}{y^3}$

$\frac{\partial^2 f}{\partial x\partial y} = 0$

Anton Žanetić

3. $f(x,y) = \frac{x^2}{y}$

DOMENA

$y \neq 0$

$D(f) = \{(x,y) : y \neq 0\}$ ✓

$\lim_{(x,y)\to(0,0)} \frac{x^2}{y}$ = NE POSTOJI ZAŠTO ?

$= \lim_{x\to 0}\left[\lim_{y\to 0} \frac{x^2}{y}\right]$ = NE POSTOJI

$\lim_{y\to 0}\left[\lim_{x\to 0} \frac{x^2}{y}\right] = \lim_{y\to 0} \frac{0}{y} = \lim_{y\to 0} 0 = 0$

$\lim_{x\to 0}\left[\lim_{y\to 0} \frac{x^2}{y}\right] \neq \lim_{y\to 0}\left[\lim_{x\to 0} \frac{x^2}{y}\right]$

LIMES NE POSTOJI JER NIJE ZADOVOLJENA OVAKVA JEDNAKOST. DA ✓

Popuniti odmah!
IME I PREZIME: PETAR PERICA **BROJ INDEKSA:**
MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

$\emptyset$

1. $A(0,0)$, $B(2,3)$, $C(4,2)$

$AB\ (y-0)(2-0) = (3-0)(x-0) = 2y = 3x \,|:2 \Rightarrow y = \frac{3x}{2}$

$BC\ (y-3)(4-2) = (2-3)(x-2) = 4y - 2y - 12 + 6 = 2x - 4 - 3x + 6$

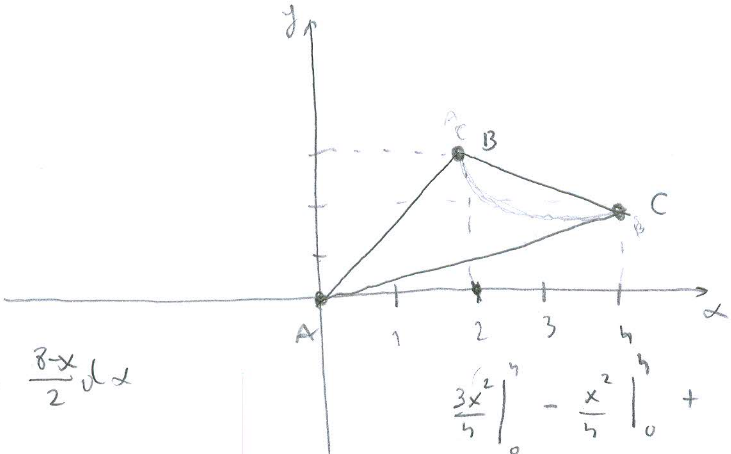
$2y - 6 = -x + 2$

$2y = -x + 8 \,|:2 \Rightarrow y = \frac{8-x}{2}$

$AC\ (y-0)(4-0) = (2-0)(x-0) = 4y = 2x \Rightarrow y = \frac{2x}{4_2} = \frac{x}{2}$

$[0,4]$ AB i AC

$[2,4]$ AB i BC



$\int_0^4 \frac{3x}{2} - \frac{x}{2}\,dx + \int_2^4 \frac{3x}{2} - \frac{8-x}{2}\,dx$

$\frac{3x^2}{4}\Big|_0^4 - \frac{x^2}{4}\Big|_0^4 + \frac{3x^2}{4}\Big|_2^4 - 4x\Big|_2^4 - \frac{x^2}{4}\Big|_2^4$

$12 - 4 + 12 - 8 - 3 = \boxed{9}$

$P_\triangle = 9$

$\int \frac{3x}{2} = \frac{3}{2}\int x$

$\boxed{\frac{3x^2}{4}}$

$\int \frac{x}{2} = \frac{1}{2}\int x$

$\boxed{\frac{x^2}{4}}$

$\int \frac{8-x}{2}$

$\int \frac{8}{2} - \int \frac{x}{2}$

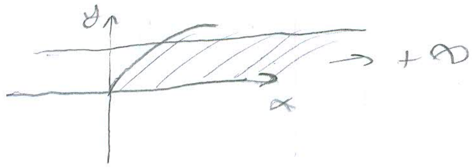
$4 \frac{8x}{2_1} = 4x - \frac{x^2}{4}$

2. $\int \frac{1}{\sqrt{x+1}} = \begin{bmatrix} x+1=t \\ dt = dx \end{bmatrix} = \int \frac{1}{\sqrt{t}}\,dt = \int \frac{1}{t^{\frac{1}{2}}}\,dt = \int t^{-\frac{1}{2}}\,dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} = 2t^{\frac{1}{2}} + C = 2\sqrt{t} + C =$

$= 2\sqrt{x+1} + C$

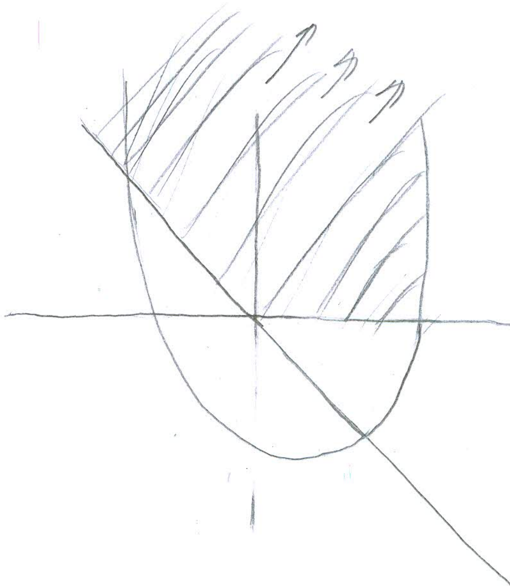
$\int_{-1}^{1} \frac{1}{\sqrt{x+1}}$ $\boxed{\begin{matrix} x+1 \neq 0 \\ x \neq -1 \end{matrix}}$

$= \lim_{x \to -1} \int_{x}^{1} \frac{1}{\sqrt{x+1}} = \lim_{x \to -1} \frac{1}{\sqrt{-1+1}} = \frac{1}{0} = +\infty$



3. $f(x,y) = \frac{x^2}{y}$

$y \neq 0$



D. f.

4. $f(x,y) = x - y + \frac{1}{xy}$

$\frac{\partial f}{\partial x} = 1 + \frac{-1\cdot y}{(xy)^2} = 1 - \frac{y}{(xy)^2} \Rightarrow \frac{y}{(xy)^2} = 1 \Rightarrow y = (xy)^2$

$y = x^2 \cdot y^2$

$\frac{y}{y^2} = x^2$

$\frac{1}{y} = x^2$

$1 = x^2 y$

$\boxed{y = \frac{1}{x^2}}$

$\boxed{x = -\frac{1}{y^2}}$

$\frac{\partial f}{\partial y} = -1 - \frac{x}{(xy)^2} \Rightarrow \frac{x}{(xy)^2} = -1 \Rightarrow x = -(xy)^2$

$y = 0, \quad x = 0$ $\quad T_0(0,0)$ - STACIONARNE TOČKE

$\frac{\partial^2 f}{\partial x^2} = -\frac{y \cdot 2\cancel{x}y^2 \cdot y}{(xy)^4} = \frac{y 2 y}{(xy)^2} = 0$

5. $\sqrt[3]{x}\, y y' = 1 - x^2$

$y y' = \frac{1-x^2}{\sqrt[3]{x}}$

$y \frac{dy}{dx} = \frac{1-x^2}{\sqrt[3]{x}} \quad / \cdot dx$

$\int y\, dy = \int \frac{1-x^2}{\sqrt[3]{x}}\, dx$

$\boxed{\frac{y^2}{2} = \frac{3 \cdot x^{\frac{2}{3}}}{2} - x}$ OPĆE RJEŠENJE D.J.

$\int \frac{1-x^2}{\sqrt[3]{x}} \overset{*}{=} \int \frac{1}{\sqrt[3]{x}} - \int \frac{x^2}{\sqrt[3]{x}} = \frac{3 \cdot x^{\frac{2}{3}}}{2} - x$

$* \int \frac{1}{x^{\frac{1}{3}}} = \int x^{-\frac{1}{3}} = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} = \boxed{\frac{3 \cdot x^{\frac{2}{3}}}{2}}$ ✓

$\int \frac{x^2}{\sqrt[3]{x}} = \left[\begin{array}{l} x = t^3 \\ dx = 3t^2 dt \end{array}\right] = \int \frac{(t^3)^2}{\sqrt[3]{t^3}} \cdot 3\cancel{t^2}\, dt = 3\int \frac{t^3}{t} = 3\int t^2$

$= 3 \cdot \frac{t^3}{3} = \boxed{t^3}$

$\boxed{= x}$

PETAR PERICA

6. $y'' + 4y = 4$

$4 = e^{\alpha x}\left(P_m(x)\cos(\beta x) + Q_n(x)\sin(\beta x)\right)$?

$\alpha = 0$

$\beta = 0$

$Q_m = 0$

$P_m = 0$

Popuniti odmah!

IME I PREZIME: JOSIP PREDOVAN **BROJ INDEKSA:**

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