

Popunite odmah!

IME I PREZIME: Marijan Štrk

BROJ INDEKSA:

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓
bodova
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1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,0)$, $B(2,3)$ i $C(4,2)$.

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2. Zadano je $f(x) = \frac{1}{\sqrt{x+1}}$. Odrediti $\int_{-1}^1 f(x) dx$. Skicirati graf funkcije f i površinu koja je određena integralom $\int_{-1}^1 f(x) dx$.

3. Grafički prikazati funkciju $f(x,y) = \frac{x^2}{y}$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$?

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4. Istražiti domenu i ekstreme funkcije $f(x,y) = x - y + \frac{1}{xy}$.

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5. Riješiti diferencijalnu jednačinu: $\sqrt[3]{x} y y' = 1 - x^2$

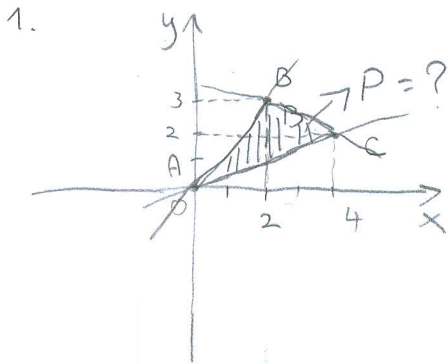
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6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednačine:

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$$y'' + 4y = 4, \quad y(0) = 0, \quad y'(0) = 2$$

GRUPNO 95



$$\vec{n}_{AB} \equiv (x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$$

$$\vec{n}_{AC} \equiv (4 - 0)(y - 0) = (2 - 0)(x - 0)$$

$$(2 - 0)(y - 0) = (3 - 0)(x - 0)$$

$$4y = 2x / :4$$

$$2y = 3x / :2$$

$$y = \frac{1}{2}x$$

$$y = \frac{3}{2}x$$

$$\vec{n}_{BC} \equiv (4 - 2)(y - 3) = \overset{-1}{(2 - 3)}(x - 2)$$

$$2y - 6 = -x + 2$$

$$\int -x + 4 dt = \int -x + 4 \int dt$$

$$2y = -x + 8 / :2$$

$$= -\frac{x^2}{2} + 4x$$

$$y = -\frac{1}{2}x + 4$$

$$P_1 = \int_0^2 \frac{3}{2}x - \frac{1}{2}x dt$$

$$P_2 = \int_2^4 -\frac{1}{2}x + 4 - \frac{1}{2}x dt$$

$$P_{\Delta ABC} = P_1 + P_2$$

$$= \int_0^2 x dt = \frac{x^2}{2} \Big|_0^2$$

$$= \int_2^4 -x + 4 dt$$

$$= 4 //$$

$$= 2$$

$$= -\frac{x^2}{2} + 4x \Big|_2^4$$

$$= 2$$

$$2. f(x) = \frac{1}{\sqrt{x+1}}$$

$$\int_{-1}^1 f(x) dx = ?$$

$$\int_{-1}^1 \frac{1}{\sqrt{x+1}} dx =$$

-kočka nuly. $x = -1$

$$= \lim_{t \rightarrow -1^+} \left(\int_t^1 \frac{1}{\sqrt{x+1}} dx \right)$$

$$= \lim_{t \rightarrow -1^+} \left(2\sqrt{x+1} \Big|_t^1 \right)$$

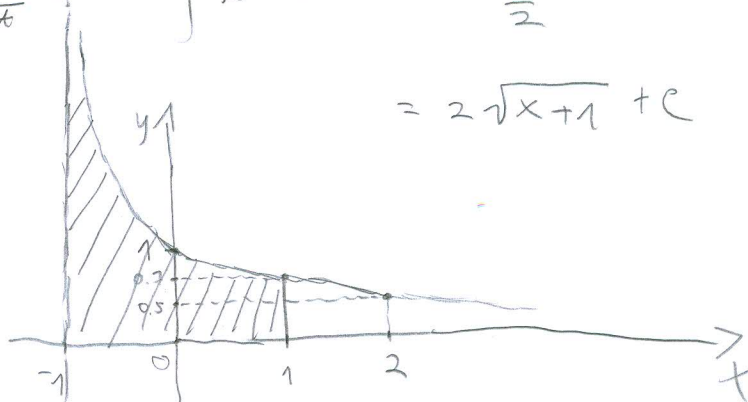
$$= 2\sqrt{2} - 0$$

$$= 2\sqrt{2} \approx 2.83 \quad \checkmark$$

$$\int \frac{1}{\sqrt{x+1}} dx = \left[\begin{array}{l} x+1 = t / \uparrow \\ dx = dt \end{array} \right]$$

$$\int \frac{1}{\sqrt{t}} dt = \int t^{-\frac{1}{2}} dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= 2\sqrt{x+1} + C$$



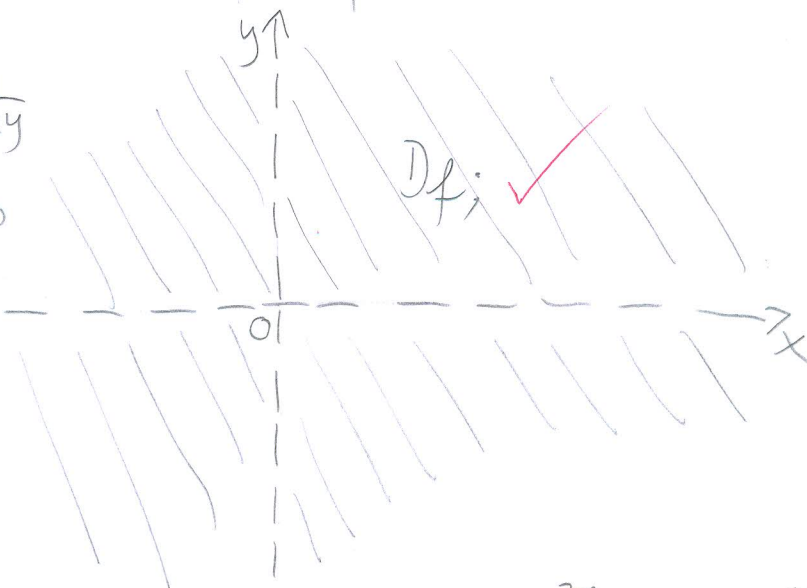
$$4. f(x,y) = x - y + \frac{1}{xy}$$

Df: mjest: $xy \neq 0$

$$x \neq 0$$

$$y \neq 0$$

$$Df: \mathbb{R}^2 \setminus \{0\}$$



$$\frac{\partial f}{\partial x} = 1 + \frac{1}{y} \cdot (x^{-1})'$$

$$= 1 + \frac{1}{y} \cdot (-1)x^{-2}$$

$$= 1 - \frac{1}{y} \cdot \frac{1}{x^2}$$

$$= 1 - \frac{1}{yx^2}$$

$$\frac{\partial f}{\partial y} = -1 + \frac{-1 \cdot x}{x^2 y^2}$$

$$= -1 - \frac{1}{xy^2}$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{-1 \cdot x \cdot 2y}{x^2 y^3}$$

$$= \frac{2}{xy^3}$$

$$\frac{\partial^2 f}{\partial y \partial x} = -\frac{-1 \cdot x}{y^2 x^2}$$

$$= \frac{1}{y^2 x^2} = \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{0 \cdot yx^2 - 1 \cdot 2yx}{y^2 x^3} = \frac{2}{yx^3}$$

mat. 4. rad.

Marijan Strk

$$1 - \frac{1}{yx^2} = 0 \rightarrow 1 - \frac{1}{y \cdot \left(-\frac{1}{y^2}\right)^2} = 0$$

$$-1 - \frac{1}{xy^2} = 0 \quad | \cdot xy^2$$

$$1 - \frac{1}{y \cdot \frac{1}{y^3}} = 0$$

$$-xy^2 - 1 = 0$$

$$-xy^2 = 1 \quad | : y^2$$

$$1 - \left(\frac{1}{\frac{1}{y^3}}\right) = 0$$

$$-x = \frac{1}{y^2} \quad | \cdot (-1)$$

$$x = -\frac{1}{y^2}$$

$$1 - y^3 = 0$$

$$x = -\frac{1}{1^2} \quad T_0(-1, 1)$$

$$y^3 = 1$$

$$y = 1$$

$$x = -1$$

$$\Delta = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}$$

$$\frac{\partial^2 f}{\partial x^2}(T_0) = -2 < 0 \quad \frac{\partial^2 f}{\partial x \partial y} = 1$$

$$\Delta = 4 - 1$$

$$\frac{\partial^2 f}{\partial y^2}(T_0) = -2$$

$$\Delta = 3 > 0$$

T_0 je maksimum funkcije. ✓

$$t = \sqrt[3]{x} = x^{\frac{1}{3}}$$

5. $\sqrt[3]{x} y y' = 1 - x^2 \quad | : \sqrt[3]{x}$

$$\int \frac{1-x^2}{\sqrt[3]{x}} dx = \left[\begin{array}{l} x = t \\ dx = 3t^2 dt \end{array} \right]$$

$$y y' = \frac{1-x^2}{\sqrt[3]{x}}$$

$$= \int \frac{1-t^6}{\sqrt[3]{t^3}} \cdot 3t^2 dt$$

$$y \cdot \frac{dy}{dx} = \frac{1-x^2}{\sqrt[3]{x}} \quad | \cdot dx$$

$$= \int \frac{3t^2 - 3t^8}{t} dt$$

$$y dy = \frac{1-x^2}{\sqrt[3]{x}} dx \quad | \int$$

$$\int y dy = \int \frac{1-x^2}{\sqrt[3]{x}} dx$$

$$= \int 3t - 3t^7 dt$$

$$\frac{y^2}{2} = \frac{3 \cdot x^{\frac{2}{3}}}{2} - \frac{3x^{\frac{8}{3}}}{8} \quad | \cdot 2 \quad \checkmark$$

$$= \int 3t dt - 3 \int t^7 dt$$

$$y^2 = 3x^{\frac{2}{3}} - \frac{3x^{\frac{8}{3}}}{4} + C$$

$$= 3 \cdot \frac{t^2}{2} - 3 \cdot \frac{t^8}{8}$$

$$y = \pm \sqrt{3x^{\frac{2}{3}} - \frac{3x^{\frac{8}{3}}}{4} + C}$$

$$= \frac{3 \cdot x^{\frac{2}{3}}}{2} - \frac{3 \cdot x^{\frac{8}{3}}}{8} + C$$

