Popuniti odmah!

Ime i prezime: MARTIN SEDMAK **Broj indeksa:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

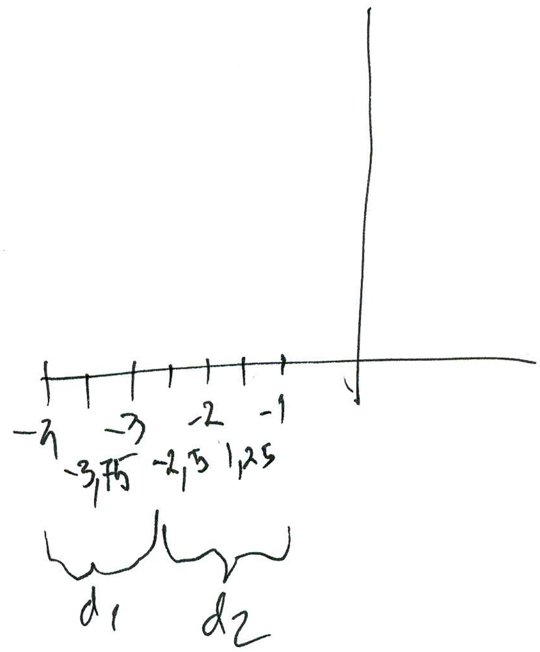
	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x dx$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x dx$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	20 2
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

37/80

1. $\int_{-4}^{-1} x^2 e^x dx = \int x^2 e^x dx =$

$= 1{,}36$

2. $\int_{-4}^{-1} x^2 e^x dx$



i	x_i	y
0	-4	-0,293050
1	-3,75	-4,25717
2	-2,5	
3	-1,25	-1,31336
4	-1	

i	x_i	y
0	-4	-0,293050
1	-3,75	-0,330712
2	-2,5	-0,51303
3	-1,25	-0,44766
4	-1	-0,36788

$$S = \frac{d}{6}(f_0 + 4f_1 + f_2)$$

$$= \frac{1,5}{6}(0{,}29305 + 1{,}322848 + 0{,}513$$

$S_1 = 0{,}532232$

$S_2 = 0{,}6678275$

$\sum_{i=1}^{2} S \approx 1{,}2000$

4. $f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$Df(x,y) = ($

$x \neq 0$

$y \neq 0$

$Df(x,y) = \langle -\infty, 0 \rangle \cup \langle 0, \infty \rangle$ $\{(y,x) \in R^2 : x \neq 0 \wedge y \neq 0\}$

$\frac{\partial}{\partial x} f(x,y) = 2x - \frac{2}{x^2 y}$ ✓

$\frac{\partial}{\partial y} f(x,y) = 2y - \frac{2}{y^2 x}$ ✓

$2x - \frac{2}{x^2 y} = 0$

$2x = \frac{2}{x^2 y} \;/ \cdot x^2 y$

$2x \; x^2 y = 2 \;/ :2$

$x \cdot x^2 y = 0$ ✗

$x = 0$

$2y - \frac{2}{x^2 y} = 0$

$2y = \cancel{0} + \frac{2}{x^2 y}$

$2y \cdot xy^2 = 2$

$y \cdot yx^3 = 0$ ✗

$y = 0$

FUNKCIJA NEMA EKSTREME

✗

MARTIN SEDMAK

6. $y'' - 4y' + 4y = 4$

$$Y(x) = Y_p(x) + Y_H(x)$$

$Y_H(x) = ?$

$\lambda^2 - 4\lambda + 4 = 0$

$$\frac{+4 \pm \sqrt{\cancel{16 \pm 16}\ 16 - 16}}{2} = \frac{4}{2} = 2$$

$$Y_H(x) = C_1 e^{2x} + C_2 x e^{2x}$$

$Y_p(x) = ?$

$$Y_p(x) = 1$$

$$Y(x) = C_1 e^{2x} + C_2 x e^{2x} + 1$$

Popuniti odmah!

IME I PREZIME: MARIN MATEK **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	20
~~5. Riješiti: $y' + 2xy = x - 3$.~~	20
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

3. $A(2,2,4)$

$B(0,-1,3)$

$C(-1,0,2)$

$AB(\overset{l_1}{-2}, \overset{m_1}{-3}, \overset{n_1}{-1})$ ✓

$AC(\overset{l_2}{-3}, \overset{m_2}{-2}, \overset{n_2}{-2})$ ✓

$$\cos\varphi = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\left(\sqrt{l_1^2 + m_1^2 + n_1^2}\right) \cdot \left(\sqrt{l_2^2 + m_2^2 + n_2^2}\right)}$$

$+6+6+2$

$$\cos\varphi = \frac{14}{\sqrt{19} \cdot \sqrt{17}}$$ ✓

$4+9+4$ $4+9+1$

$$\varphi = \cos^{-1}\left(\frac{14}{\sqrt{323}}\right)$$

$\varphi = 0{,}67775^\circ$

∅

4. $f(x,y)$ $\quad D: \mathbb{R}$

$x^2+y^2+\frac{2}{xy}$

$\frac{\partial f}{\partial x} = 2x + \cancel{\frac{2}{y}}$ $\quad \frac{\partial^2 f}{\partial x^2} = 2$

$\frac{\partial f}{\partial y} = 2y + \cancel{\frac{2}{x}}$ $\quad \frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial f}{\partial x \partial y} = 2$

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial x \partial y} \\ \frac{\partial f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} = 4-4=0$$ NEMA EKSTREMA

STAC. TOČKE

$\frac{\partial f}{\partial x} = 2x + \frac{2}{y}$ $\quad \frac{\partial f}{\partial y} = 2y + \frac{2}{x}$

$2x + \frac{2}{y} = 0$ $\quad 2\cdot(y) + \frac{2}{y} = 0$

$2x = \frac{2}{y} \; /:2$ $\quad 2y + \frac{2}{y} = 0 \; / y$

$x = y$ $\quad 2y^2+2=0$

$y^2 = -1$

$y = N/P$

6. $y'' - 4y' + 4y = 0$

$\Downarrow \quad \Downarrow \quad \Downarrow$

$r^2 - 4r - 4 = 0$

$r_{1,2} = \frac{4 \pm \sqrt{16+16}}{2}$

$r_1 = \frac{4 \pm 4\sqrt{2}}{2} = 2 + 2\sqrt{2}$

$r_2 = \quad = 2 - 2\sqrt{2}$

$y_H =$

1. $\int_{-4}^{-1} x^2 e^x dx = \left[\begin{matrix} e^x = u / & dv = x^2 dx / \int \\ e^x dx = du & v = \frac{x^3}{3} \end{matrix} \right.$ $\qquad \int x^2 dx = \frac{x^3}{3} + C$ //

MARIN MATEK

$= u \cdot v - \int v \cdot du$ $\qquad \left| \begin{matrix} 2x = u / & dv = e^x dx / \int \\ 2 dx = du & v = e^x \end{matrix} \right]$

$= x^2 \cdot e^x - \int e^x \cdot 2x \, dx$

$= x^2 \cdot e^x - (2x \cdot e^x - 2 \int e^x \cdot dx$ $\qquad \left[\begin{matrix} x^2 = u / & e^x dx = dv / \int \\ 2x \, dx = du & e^x = v \end{matrix} \right]$

$= x^2 \cdot e^x - (2x \cdot e^x - 2 e^x)$

$= x^2 \cdot e^x - 2x e^x + 2 e^x$

$= x^2 \Big|_{-4}^{-1} \cdot e^x \Big|_{-4}^{-1} - 2x \Big|_{-4}^{-1} \cdot e^x \Big|_{-4}^{-1} + 2 e^x \Big|_{-4}^{-1}$

$= ((-1)^2 - (-4)^2) \cdot (e^{-1} - e^{-4}) - (-2 - 8) \cdot (e^{-1} - e^{-4}) + \frac{1}{2} (e^{-1} - e^{-4})$

$= (-15) \cdot (0,3495638) + 10 \cdot (0,3495638) + 2 (0,3495638)$

$= -5,243457 + 3,495638 + 0,699127$

$= -1,04869395$ //

Popuniti odmah!

IME I PREZIME: Ivan Kovačević **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	20
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

0

4) $f(x,y) = x^2 + y^2 + \frac{2}{x\cdot y}$ D(f) $\{(x,y): x \neq 0, y \neq 0\}$ ✓

$\frac{a \cdot b' - a' \cdot b}{a^2}$

$\frac{2\cdot(xy)' - (2)' \cdot xy}{2}$

$\frac{2x - xy}{2}$

$\frac{df}{dx} = 2x + \frac{2x}{2}$ ✗ $\frac{df}{dy} = 2y + \frac{2y}{2}$ ✗

$\frac{d^2f}{dx^2} = 2 + \frac{2}{2}$ $\frac{d^2f}{dy^2} = 2 + \frac{2}{2}$ $\frac{d^2f}{dxdy} = 0$

$z_0 = f(-1,-1) = (-1)^2 + (-1)^2 + \frac{2}{-1(-1)}$

$z_0 = 1 + 1 + 2$

$z_0 = 4$

$x_0 =$

$2x + \frac{2x}{x} = 0$

$2x + 2 = 0$

$2x = -2 \quad /:2$

$x = -1$

$y_0 =$

$2y + 2 = 0$

$2y = -2 \quad /:2$

$y = -1$

$\Delta \begin{vmatrix} -3 & 0 \\ 0 & -3 \end{vmatrix} = -3 \cdot -3 = 9$

$\Delta > 0 \rightarrow$ lokalni extrem

$\frac{df}{dx} = 2\cdot -1 + -1 = -3 \rightarrow$ Točka $(-1,-1,4)$ je lokalni minimum

$$y' + 2xy = x - 3$$

$$y' = \frac{dy}{dx}$$

$$\frac{dy}{dx} + 2xy = x - 3 \quad / : x$$

$$2y = \frac{x-3}{x} - \frac{dy}{dx}$$

Popuniti odmah!

IME I PREZIME: Matija Miočić **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \dfrac{2}{xy}$.	20
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

Ø

3. $A(2,2,4)$

$B(0,-1,3)$

$C(-1,0,2)$

$\overline{AB} = \vec{i}(2-0) - \vec{j}(2+1) + \vec{k}(4-3)$

$\overline{AB} = 2\vec{i} - 3\vec{j} + \vec{k}$

$\overline{AC} = \vec{i}(2+1) - \vec{j}(2-0) + \vec{k}(4-2)$

$\overline{AC} = 3\vec{i} - 2\vec{j} + 2\vec{k}$

Popuniti odmah!

IME I PREZIME: MATE MITROVIĆ **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	~~20~~
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

$\frac{50}{80}$

(1.) $\int_{-4}^{-1} x^2 e^x dx = \begin{bmatrix} u = x^2 /' & e^x dx = dv /\int \\ du = 2x\,dx & \int e^x dx = \int dv \\ & e^x = v \end{bmatrix} = x^2 \cdot e^x - \int e^x \cdot 2x\,dx$ *

$= x^2 \cdot e^x - (e^x \cdot 2x - \int e^x \cdot 2 dx) = x^2 \cdot e^x - e^x \cdot 2x + 2e^x + C$

$* \int e^x \cdot 2x\,dx = \begin{bmatrix} u = 2x /' & e^x dx = dv /\int \\ du = 2dx & e^x = v \end{bmatrix} = e^x \cdot 2x - \int e^x \cdot 2 dx$

$\int_{-4}^{-1} x^2 e^x dx = \left[e^x(x^2 - 2x + 2)\right]_{-4}^{-1} = e^{-1}((-1)^2 - 2\cdot(-1) + 2) - e^{-4}((-4)^2 - 2\cdot(-4) + 4) - 2)$

$= e^{-1} \cdot 5 - e^{-4} \cdot 26 = \underline{1{,}36}$ ✓

(4.) $f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$\frac{\partial f}{\partial x} = 2x + \frac{2}{y}$ ✗ $\quad \frac{\partial f}{\partial y} = 2y + \frac{2}{x}$ ✗

$\frac{\partial^2 f}{\partial x^2} = 2 \quad \frac{\partial^2 f}{\partial y^2} = 2$

$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(T_0) & \frac{\partial^2 f}{\partial x \partial y}(T_0) \\ \frac{\partial^2 f}{\partial x \partial y}(T_0) & \frac{\partial^2 f}{\partial y^2}(T_0) \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} = 4 - 4 = 0$

$\Delta = 0$ Nema odluke o ekstremu

$2x + \frac{2}{y} = 0 \;/\cdot y \qquad 2y + \frac{2}{x} = 0$

$2xy + 2 = 0 \qquad -\frac{2}{x} + \frac{2}{x} = 0$

$2xy = -2 \;/: x$

$2y = -\frac{2}{x}$

$y = 0$

$T(0,0)$

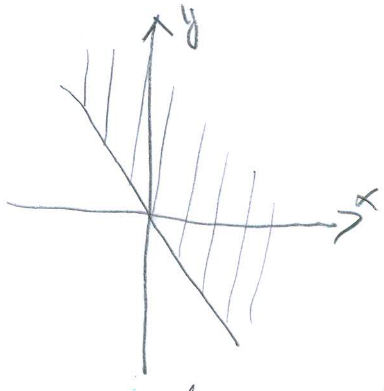
Domena:

$x^2 + y^2 > 0$

$\frac{2}{xy}$

$x\,y \neq 0$

$y > -x$



$\frac{\partial^2 f}{\partial x \partial y} = 2$

Domena je skup svih točaka između pravca neuključujući i pravac ✗

⑤ $y' + 2xy = x - 3$

$y' + 2xy = 0$

$\frac{dy}{dx} = -2xy \quad / \cdot dx$

$dy = -2xy\,dx \quad / : y$

$\int \frac{dy}{y} = -2\int x\,dx$

$\ln|y| = -x^2 + c \quad / e^{\square}$

$e^{\ln|y|} = e^{-x^2 + c}$

$y = c \cdot e^{-x^2}$

$y = U(x) \cdot e^{-x^2}$

$y' = U'(x) \cdot e^{-x^2} + U(x) \cdot (-2x e^{-x^2})$

$U'(x) \cdot e^{-x^2} \cancel{+ U(x) \cdot -2x e^{-x^2} + 2x \cdot (U(x) \cdot e^{-x^2})} = x - 3$

$U'(x) \cdot e^{-x^2} = x - 3$

$U'(x) = \frac{x-3}{e^{-x^2}} \Rightarrow U(x) = \int \frac{x-3}{e^{-x^2}}\,dx$

$$\boxed{y = c - \frac{7}{2} \cdot \frac{1}{x^2+1} \cdot e^{x^2+1} \cdot e^{-x^2}}$$

$\int \frac{x-3}{e^{-x^2}}\,dx = \int \frac{-3+x}{e^{-x^2}}\,dx = \left[\begin{matrix} -x^2 = t \ / ' \\ -2x\,dx = dt \\ x\,dx = -\frac{1}{2}dt \end{matrix}\right] = \int \frac{-3 + \frac{1}{2}dt}{e^t} = \int \frac{-\frac{7}{2}dt}{e^t}$

$= -\frac{7}{2}\int \frac{dt}{e^t} = -\frac{7}{2}\int e^{t+t}\,dt = -\frac{7}{2} \cdot \frac{e^{x^2+1}}{x^2+1} + C = -\frac{7}{2} \cdot \frac{1}{x^2+1} \cdot e^{x^2+1} + C$

⑥. $y'' - 4y' + 4y = 4$

$y'' - 4y' + 4y = 0$

$r^2 - 4r + 4 = 0$

$r_{1,2} = \frac{4 \pm \sqrt{16-16}}{2}$

$r_1 = \frac{4}{2} = 2$

$\underline{y_H(x) = C_1 e^{2x} + C_2 x e^{2x}}$ ✓

$y = e^{\alpha x}(P_m(x)\cos(\beta x) + Q_m(x)\sin(\beta x))$

$\alpha = 0 \qquad \alpha \pm i\beta = 0$

$\beta = 0 \qquad k = 0$

$\left.\begin{matrix} P_m = 4 & m = 0 \\ Q_m = 0 & m = 0 \end{matrix}\right\} N = 0$

$y_P = x^k e^{\alpha x}(S_N(x)\cos(\beta x) + T_N(x)\sin(\beta x))$

$y_P = C$

$y'_P = 0$

$y''_P = 0$

$y_P(x) = 1$

$y'' - 4y' + 4y = 4$

$0 - 4 \cdot 0 + 4 \cdot C = 4$

$4 \cdot C = 4$

$C = \frac{4}{4} = 1$

$y(x) = y_P(x) + y_H(x)$

$$\boxed{y(x) = 1 + C_1 e^{2x} + C_2 x e^{2x}}$$ ✓

, MATE MITROVIĆ

(5.) Provjera

$y' = u'(x) \cdot e^{-x^2} + u(x) \cdot (-2x e^{-x^2})$

$y' = \cancel{\frac{x-3}{e^{-x^2}}} \cdot \cancel{e^{-x^2}} + (C - \frac{7}{2} \cdot \frac{1}{x^2+1} \cdot e^{x^2+1} \cdot (-2x e^{-x^2})$

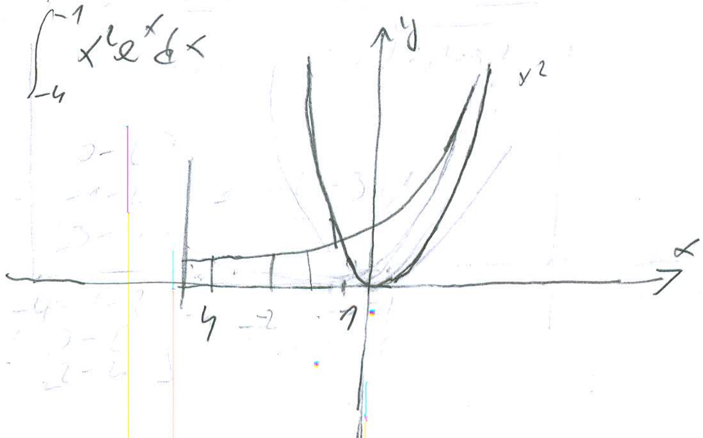
$y' = \cancel{x-3} + \underbrace{(C - \frac{7}{2} \cdot \frac{1}{x^2+1} \cdot e^{x^2+1}}\cdot (-2x e^{-x^2}) = -2x \cdot (\underbrace{(C - \frac{7}{2} \cdot \frac{1}{x^2+1} \cdot e^{x^2+1}} \cdot e^{-x^2}) + \cancel{x-3}$

$0 = 0$

$y' = -2x \cdot y + x - 3$

$y' = -2x \cdot (C - \frac{7}{2} \cdot \frac{1}{x^2+1} \cdot e^{x^2+1} \cdot e^{-x^2}) + x - 3$

(2.) $\int_{-4}^{-1} x^2 e^x \, dx$



$d_1 = 1$; $n = 4$

i	0	1	2	3	4	5	6	7
x_i	0	4	3,5	3	2.5	2	1.5	1
y_i	0	0,29	0,27					

i	0	1	2	3	4
x_i	0	4	3	2	1
y_i	0	0,29	0,44	0,54	0,37

$y_i = f(x_i)$

$T = \frac{d}{2}(f_0 + f_2)$

$T_1 = \frac{1}{2}(0 + 0,29) = 0,145$

$T_2 = \frac{1}{2}(0,29 + 0,44) = 0,36$

$T_3 = \frac{1}{2}(0,44 + 0,54) = 0,49$

$T_4 = \frac{1}{2}(0,54 + 0,36) = 0,45$

$T = \sum_1^4 T_i = 0,145 + 0,365 + 0,49 + 0,45$

$= 1,45$

Popuniti odmah!
Ime i prezime: TIN LOBOREC **Broj indeksa:** 17-2-0188-2012

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x \mathrm{d}x$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	~~20~~
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	~~20~~

$\frac{30}{80}$

1.

$$\int_{-4}^{-1} x^2 e^x \, dx = \left[\begin{matrix} u = x^2 \,/\, ' & dv = e^x dx \,/\int \\ du = 2x\,dx & v = e^x \end{matrix} \right]$$

$$= x^2 \cdot e^x - \int 2x \cdot e^x \, dx$$

$u \cdot v - \int v \cdot du$

$$= x^2 \cdot e^x - 2 \int x \cdot e^x \, dx =$$

$$* \int x \cdot e^x \, dx = \left[\begin{matrix} u = x \,|\, ' & dv = e^x dx \,/\int \\ du = dx & v = e^x \end{matrix} \right]$$

$$* \int x \cdot e^x \, dx = x \cdot e^x - \int e^x \, dx = x \cdot e^x - e^x$$

$$\int_{-4}^{-1} x^2 e^x \, dx = x^2 \cdot e^x - 2\left(xe^x - e^x\right)$$

$$= \left[x^2 \cdot e^x - 2\left(xe^x - e^x\right) \right]_{-4}^{-1}$$

$$= \left[0{,}367879441 - 2\left(-0{,}367879441 - 0{,}367879441\right)\right]$$

$$- \left[0{,}29305022 - 2\left(-0{,}073262555 - 0{,}018315638\right)\right]$$

$$= 1{,}839397 - 0{,}446206596 = 1{,}393190404$$

⑤ $y' + 2xy = x - 3$ $\Rightarrow$ LINEARNA ODJ. 1. REDA

$y' + 2xy = 0$

$\frac{dy}{dx} = -2xy \quad / \cdot dx$

$dy = (-2xy)\, dx \quad / : (y)$

$\int \frac{dy}{y} = -2\int x\, dx$

$\ln|y| = -\cancel{2}\frac{x^2}{\cancel{2}} + C$

$\ln y = -x^2 + C \quad / e^{\circ}$

$y = e^{-x^2 + C} = e^{-x^2} \cdot e^{C}$ $\rightarrow C \Rightarrow$ konstanta

$y = e^{-x^2} \cdot C \Rightarrow y(x) = e^{-x^2} \cdot C(x)$

$y'(x) = e^{-x^2} \cdot (-x^2)' \cdot C(x) + e^{-x^2} \cdot C'(x)$

$y'(x) = -2x e^{-x^2} \cdot C(x) + e^{-x^2} \cdot C'(x)$

$$\frac{-2xy}{y} = -2x$$

$-x^2$

$y = e^{-x^2} \cdot C \Rightarrow y(x) = e^{-x^2} \cdot C(x)$

$y'(x) = e^{-x^2} \cdot (-x^2)' \cdot C(x) + e^{-x^2} \cdot C'(x)$

$y'(x) = e^{-x^2} \cdot (-2x) \cdot C(x) + e^{-x^2} \cdot C'(x)$

$-2x e^{-x^2} \cdot C(x) + e^{-x^2} \cdot C'(x) + 2(x e^{-x^2} \cdot C(x)) = x - 3$

$-2x e^{-x^2} \cdot C(x) + e^{-x^2} \cdot C'(x) + 2x e^{-x^2} \cdot C(x) = x - 3$

$e^{-x^2} \cdot C'(x) = x - 3 \quad / : \ln$

$-x^2 \cdot C'(x) = \ln|x-3| \Rightarrow C'(x) = \frac{\ln|x-3|}{(-x)^2} \quad / \int$

INTEGRIRANO NA DRUGOM LISTU

$C(x) = \int x^2 \cdot \ln|x-3|\, dx$

$C(x) = (x-3) + 3\ln|x-3| + C_1$

KONAČNO RJEŠENJE:

$y(x) = e^{-x^2} \cdot (2x \cdot \ln|x-3| - 2((x-3) + 3\ln|x-3| + C_1$

TIN LOBOREC

$$C'(x) = \frac{\ln|x-3|}{-x^2} \quad / \int$$

$$C(x) = \int x^2 \ln|x-3| \, dx$$

$$C(x) = \left[\begin{array}{l} u = \ln|x-3| \; / \; ' \\ du = \frac{1}{x-3} \end{array} \quad \begin{array}{l} dv = x^2 \, dx \; / \int \\ \underline{\underline{v = 2x}} \end{array} \right.$$

$$\Rightarrow 2x \cdot \ln|x-3| - 2\int x \cdot \frac{1}{x-3}$$

$$= 2x \cdot \ln|x-3| - 2\int \frac{x}{(x-3)} \, dx$$

$$\int \frac{x}{x-3} = \left[\begin{array}{l} x-3 = t \\ dx = dt \end{array} \right] = \int \frac{t+3}{t} \, dt$$

$$\Rightarrow \underline{\underline{x = t+3}}$$

$$= \int \frac{t}{t} \, dt + \int \frac{3}{t} \, dt$$

$$= t + 3\ln|t| = |x-3| + 3\ln|x-3|$$

$$\Rightarrow C(x) = 2x \cdot \ln|x-3| - 2\big((x-3) + 3\ln|x-3|\big)$$

6. $y'' - 4y' + 4y = 4$

$r^2 - 4r + 4 = 0$

$r_{1,2} = \frac{4 \pm \sqrt{16-16}}{2}$

$r_{1,2} = \frac{4 \pm 0}{2} \Rightarrow r_1 = r_2 = 2$ $\Rightarrow$ samo 1 realno rješenje

$y_0 \Rightarrow$ homogeno y $(y_h) \Rightarrow$ $y_h = C_1 e^{2x} + C_2 x e^{2x}$

$y_p = e^0 (A) = A$

$y_p' = \cancel{A}$

$y_p'' = \cancel{A}$

$A - 4A + 4A = 4$ $\boxed{A = 4}$ $y_p = 4$

Konačno rješenje

$y = C_1 e^{2x} + C_2 x e^{2x} + 4\cancel{x}$

PROVJERA ?

2. NUMERIČKA INTEGRACIJA $\int_{-4}^{-1} x^2 e^x \, dx = ?$

$b = -1$ $h = \frac{b-a}{m}$
$a = -4$

$y = h\left(\frac{y_0 + y_m}{2} + y_1 + y_2 + \dots y_{m-1}\right)$

$h = \frac{-4+1}{m} = \frac{-3}{m} = \frac{-3}{6} = -\frac{1}{2}$

$y_i = f(x_i)$
$x_i = x_0 + ih$

$x_0 = a \rightarrow x_0 = -4$
$x_m = b$ $x_m = -1$

$m = 6$

$y_0 = f(x_0) = f(-4) = 0{,}29305$

$y_m = f(x_m) = f(-1) = 0{,}367879$

$x_i = x_0 + ih$
$x_i = -4 - \frac{1}{2} i$

$x_1 = -4 + \frac{1}{2} = -3{,}5$
$x_2 = -4 + 1 = -3$
$x_3 = -4 + \frac{3}{2} = -2{,}5$
$x_4 = -4 + \frac{4}{2} = -2$
$x_5 = -4 + \frac{5}{2} = -1{,}5$
$x_6 = -4 + 3 = -1$

$y \approx \frac{1}{2}\left(\frac{0{,}29305 + 0{,}367879}{2} + y_1 + y_2 + y_3 + y_4 + y_5\right)$

$y_1 = 0{,}369918$
$y_2 = 0{,}4480836$
$y_3 = 0{,}513031$
$y_4 = 0{,}54134132$
$y_5 = 0{,}502043$
$y_6 = 0{,}367879$

$y \approx \frac{1}{2}(0{,}3304645 + (y_1 + y_2 + y_3 + y_4 + y_5))$

$y \approx 1{,}35243811 6$

(4) $f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$\frac{\partial f}{\partial x} = 2x + \frac{2}{y}$

$\frac{\partial f}{\partial y} = 2y + \frac{2}{x}$

$\frac{\partial^2 f}{\partial^2 x} = 2$, $\frac{\partial^2 f}{\partial x \partial y} = -2y - 2x$

$\frac{\partial^2 f}{\partial^2 y} = 2$

$D = \begin{vmatrix} 2 & -2y-2x \\ -2y-2x & 2 \end{vmatrix}$

$2x + \frac{2}{y} = 0$

$2x = -\frac{2}{y} \quad \Big| :2$

$x = \frac{-\frac{2}{y}}{2}$

$x = -\frac{1}{y}$

$2y + \frac{2}{x} = 0$

$2y + \frac{2}{-\frac{1}{y}} = 0$

$2y + \frac{2}{x} = 0$

$2y + \frac{\frac{2}{1}}{-\frac{1}{y}} = 0$

$2y - 2y = 0$

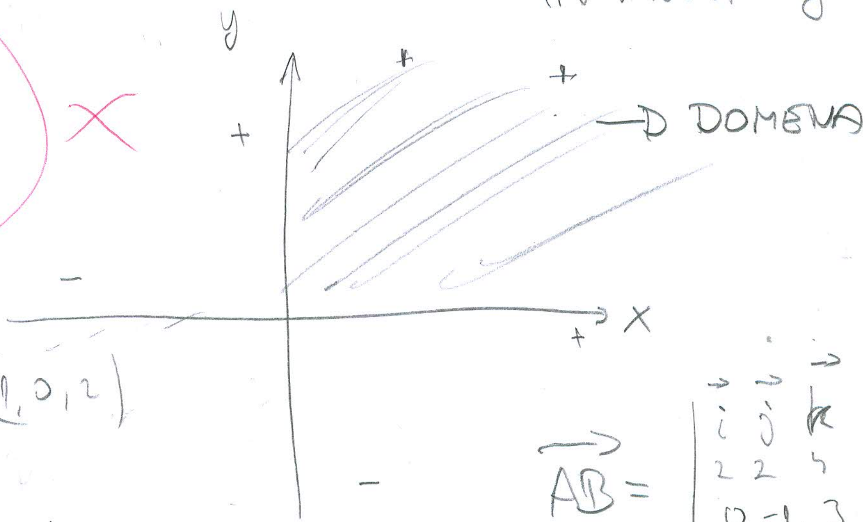
Nema stacionarnih točaka

DOMENA ⇒ $x^2 + y^2 + \frac{2}{xy}$?

x^2 i y^2 su uvijek veći od nule jer svaki broj na kvadrat daje broj > 0

$xy > 0$

$x > 0$ $\quad$ $y > 0$



3. $A(2,2,4)$ $B(0,-1,3)$ $C(-1,0,2)$

$\sphericalangle(AB ; AC) = ?$

$\cos\alpha = \frac{|196 + 100 + 16|}{\sqrt{196 + 100 + 16}} = \frac{312}{\sqrt{312}}$

$\cos\alpha =$

$\vec{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 4 \\ 0 & -1 & 3 \end{vmatrix}$

$\vec{AB} = (10)\vec{i} + (2)\vec{j} - 2\vec{k}$

$\vec{a} = \vec{AB} = 10\vec{i} + 2\vec{j} - 2\vec{k}$

$\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 4 \\ -1 & 0 & 2 \end{vmatrix}$

$\vec{AC} = (4)\vec{i} + (8)\vec{j} + (2)\vec{k}$

$\vec{b} = \vec{AC} = 4\vec{i} + 8\vec{j} + 2\vec{k}$

Popuniti odmah!
Ime i prezime: LUKA MILIN **Broj indeksa:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x dx$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x dx$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	20 2
~~5. Riješiti: $y' + 2xy = x - 3$.~~ $f(x)$ $g(x)$	20
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

17/80

1. $$\int_{-4}^{-1} x^2 e^x \, dx =$$

$u = x^2$
$du = 2x\,dx$
$dv = e^x dx$
$v = \int e^x dx$
$v = e^x$

$u = 2x$
$du = 2$
$dv = e^x$
$v = \int e^x$
$v = e^x$

$$= x^2 e^x \Big|_{-4}^{-1} - \int_{-4}^{-1} 2x e^x \, dx =$$

$$= x^2 \cdot e^x \Big|_{-4}^{-1} - 2xe^x \Big|_{-4}^{-1} - 2\int_{-4}^{-1} e^x dx = x^2 \cdot e^x - 2xe^x - 2e^x \Big|_{-4}^{-1}$$

$$= 1 \cdot e^{-1} + 2 \cdot e^{-1} - 2e^{-1} - (16 \cdot e^{-4} + 8e^{-4} - 2e^{-4})$$

$$= e^{-1} + 2e^{-1} - 2e^{-1} - 16e^{-4} - 8e^{-4} + 2e^{-4}$$

$$= e^{-1} - 16e^{-4} - 8e^{-4} + 2e^{-4}$$

2. $$\int_{-4}^{-1} x^2 e^x \, dx$$

$-\frac{7}{2}$ -3 $-\frac{5}{2}$ -2 $-\frac{3}{2}$ -1

-4 -1

$$h = \frac{b-a}{2n} = \frac{-1+4}{6} = \frac{3}{6} = \frac{1}{2}$$

k	0	1	2	3	4	5	6
x_i	-4	$-\frac{7}{2}$	-3	$-\frac{5}{2}$	-2	$-\frac{3}{2}$	-1
y_i	0,2931	0,3699	0,4481	0,5130	0,5413	0,5020	0,3679
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Nuhca Milui

$f(x) = x^2 e^x$

$f(-4) = (-4)^2 \cdot e^{-4} = 0,2931$ y_0

$f(-\frac{7}{2}) = \left(-\frac{7}{2}\right)^2 \cdot e^{-\frac{7}{2}} = 0,3699$ y_1

$f(-3) = (-3)^2 \cdot e^{-3} = 0,4481$ y_2

$f(-\frac{5}{2}) = \left(-\frac{5}{2}\right)^2 \cdot e^{-\frac{5}{2}} = 0,5130$ y_3

$f(-2) = (-2)^2 \cdot e^{-2} = 0,5413$ y_4

$f(-\frac{3}{2}) = \left(-\frac{3}{2}\right)^2 \cdot e^{-\frac{3}{2}} = 0,5020$ y_5

$f(-1) = (-1)^2 \cdot e^{-1} = 0,3679$ $y_6 = y_n$

$$I = \frac{h}{3} \cdot \left[y_0 + y_n + 4 \cdot (y_1 + y_3 + y_5) + 2 \cdot (y_2 + y_4) \right]$$

$$I = \frac{1}{6} \cdot \left[0,2931 + 0,3679 + 4 \cdot (0,3699 + 0,5130 + 0,5020) + 2 \cdot (0,4481 + 0,5413) \right]$$

$$= 1,363233333$$ ✓

1) $\int_{-4}^{-1} x^2 e^x \, dx$

$u = e^x$
$du = e^x$
$dv = x^2$
$v = \int x^2$
$v = \frac{x^3}{3}$

Luka Mlen

$$= \frac{x^3}{3} \cdot e^x - \frac{1}{3} \int_{-4}^{-1} x^3 e^x = \frac{x^3}{3} \cdot e^x - \frac{1}{3} \cdot \frac{x^4}{4} e^x \Big|_{-4}^{-1}$$

$$= -\frac{1}{3} e^{-1} - \frac{1}{3} \cdot \frac{1}{4} e^{-1} - \left(\frac{-64}{3} e^{-4} \right.$$

$$- \frac{1}{3} \cdot 64 e^{-4}$$

$$= -\frac{1}{3} e^{-1} - \frac{1}{12} e^{-1} + \frac{64}{3} e^{-4} + \frac{64}{3} e^{-4}$$

$$= 0{,}6281841588$$

Lulea Milen

5) $y' + 2xy = x - 3$

$P = e^{-\int f(x)dx} \cdot \left[\int e^{\int f(x)dx} \cdot g(x)dx + C\right]$

$f(x) = 2x$

$g(x) = x - 3$

$\int (x-3)dx = \frac{x^2}{2} - 3x$

$\int f(x)dx = \int 2x\,dx = \cancel{2} \cdot \frac{x^2}{\cancel{2}} = x^2$

$P = e^{-x^2} \cdot \left[\int e^{x^2} \cdot (x-3)dx + C\right] = e^{-x^2} \cdot \left[2e^x \cdot \int (x-3)dx + C\right]$ $\left| \begin{array}{l} x-3=t \\ dx=dt \end{array} \right|$

$= e^{-2x} \cdot \left[2e^x \cdot \left(\int t\,dt\right) + C\right]$

$= e^{-2x} \cdot \left[2e^x \cdot \frac{t^2}{2} + C\right]$

$= e^{-2x} \cdot 2e^x \cdot \frac{(x-3)^2}{2} + C = e^{-2x} \cdot \frac{2e^x \cdot (x-3)^2}{2} + C$

6) $y'' - 4y' + 4y = 4$

$r^2 - 4r + 4 = 4$ ✗

$r^2 - 4r + 4 - 4 = 0$

$r^2 - 4r = 0$

$r(r-4) = 0$

$r_1 = 0$

$r - 4 = 0$

$r_2 = 4$ ~~? DALJE~~

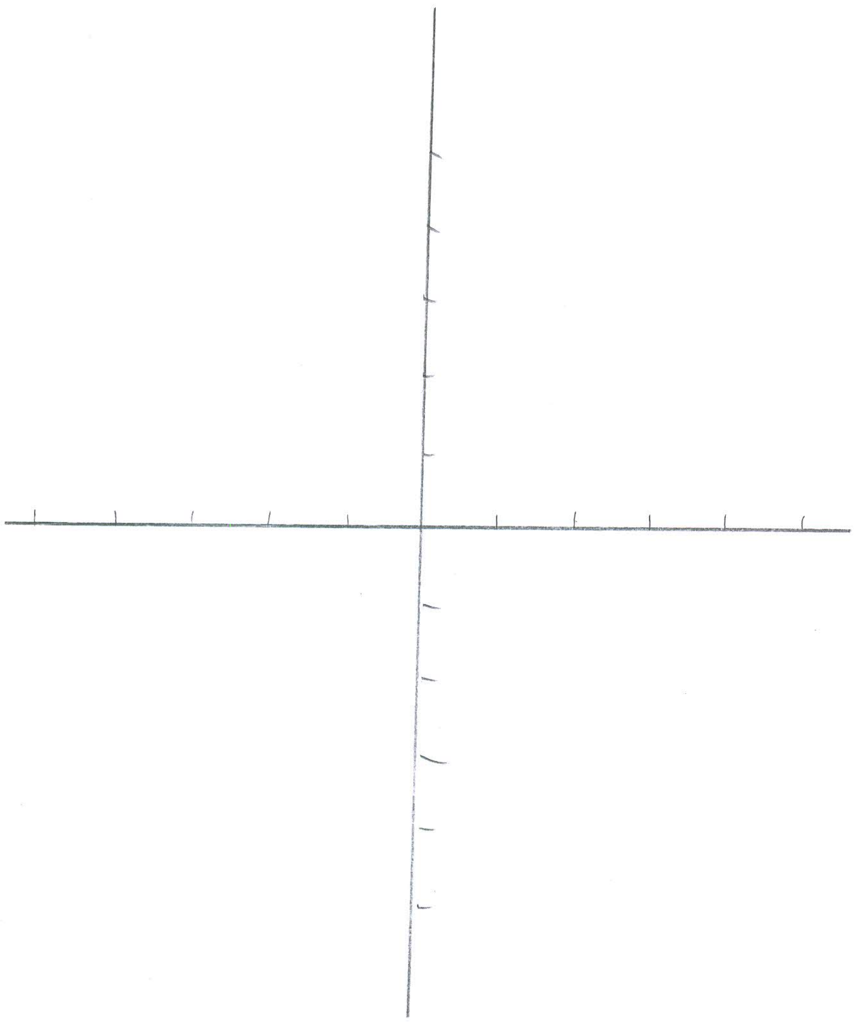
$y_H = C_1 \cdot e^{r_1 x} + C_2 \cdot e^{r_2 x}$ ✗

$y_H = C_1 \cdot e^0 + C_2 \cdot e^{4x}$ ✗

$y_H = C_1 + C_2 \cdot e^{4x}$ ✗

3) $A(2,2,4)$, $B(0,-1,3)$, $C(-1,0,2)$

Luka Milun



4) $f(x,y) = x^2+y^2+\frac{2}{xy}$

$\frac{\partial f}{\partial x} = 2x + \frac{2}{y^2}$ $\quad \frac{\partial^2 f}{\partial x \partial y} = y$

$\frac{\partial f}{\partial y} = 2y + \frac{2}{x^2}$

$D: \; xy \neq 0$

$\frac{\partial^2 f}{\partial^2 x} = 2$

$\frac{\partial^2 f}{\partial^2 y} = 2$

$Df = \{(x,y) \in \mathbb{R}^2 \mid xy \neq 0\}$

Popuniti odmah!

IME I PREZIME: JOSIP FEŠTINI **BROJ INDEKSA:**

MATEMATIKA 2: ZAVRŠNI KOLOKVIJ Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x dx$.	15
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x dx$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	~~20~~ 17
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

$\frac{47}{80}$

(2) $\int_{-4}^{-1} x^2 e^x dx$

$-4 \quad -3.5 \quad -3 \quad -2.5 \quad -2 \quad -1.5 \quad -1$

$n=6$

i	x_i	y_i
0	-4	0.293
1	-3.5	0.3699
2	-3	0.448
3	-2.5	0.513
4	-2	0.5413
5	-1.5	0.502
6	-1	0.3678

$P_1 = \frac{1}{6}(0.293 + 4\cdot(0.3699) + 0.448)$

$P_1 = 0.3701$

$P_2 = \frac{1}{6}(0.448 + 4(0.513) + 0.5413)$

$P_2 = 0.5067$

$P_3 = \frac{1}{6}(0.5413 + 4(0.502) + 0.3678)$

$P_3 = 0.4861$

$P_u \approx 1.3629$ ✓

(4) $f(x,y) = x^2 + y^2 + \frac{2}{xy}$

EKSTREMI

$\frac{\partial f}{\partial x} = 2x + \left(\frac{2}{x}\cdot\frac{1}{y}\right)'$

$= 2x - \frac{2}{x^2 y}$ ✓

$\frac{\partial f}{\partial y} = 2y - \frac{2}{xy^2}$ ✓

$\left(\frac{2}{x}\right)' = (2\cdot x^{-1})' = -2x^{-2} = -\frac{2}{x^2}$

$\left(\frac{2}{y}\right)' = (2\cdot y^{-1})' = \frac{-2}{y^2}$

$2x - \frac{2}{x^2 y} = 0 \; / \cdot x^2 y$

$2x^3 y - 2 = 0$

$2x^3 y = 2$

$x^3 = \frac{2}{2y}$

$x^3 = \frac{1}{y}$

$2y - \frac{2}{xy^2} = 0 \; / \cdot xy^2$

$2y^3 x - 2 = 0$

$2y^3 \cdot \frac{1}{y} - 2 = 0$

$2y^2 - 2 = 0$

$2y^2 = 2$

$y^2 = \frac{2}{2} \Rightarrow \boxed{y_1 = 1}$ $\boxed{y_2 = -1}$

$2x^3 = 2$

$x^3 = \frac{2}{2} = 1$

$\boxed{x_1 = 1}$

$2x^3 \cdot (-1) = 2$

$2x^3 = -2$

$x^3 = -\frac{2}{2}$

$\boxed{x_2 = -1}$

$T_1(1,1)$ ✓

$T_2(-1,-1)$ ✓

stacionarne točke

$$\frac{\partial^2 f}{\partial^2 x} = 2 - \left(\frac{2}{x^2 y}\right)'$$

$$= 2 + \frac{4}{x^3 y} = 6 \Rightarrow T_1 \quad T_2 = 6$$

$$\frac{\partial^2 f}{\partial^2 y} = 2 + \frac{4}{x y^3} \quad T_1 = 6, \; T_2 = 6$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{2}{x^2 y^2} = T_1 = 2 \quad T_2 = 2$$

$T_1(1,1) \to$ je minimum

$$\Delta_{T_1} = \begin{vmatrix} 6 & 2 \\ 2 & 6 \end{vmatrix} = 36 - 4 = 32 > 0$$

$$\Delta T_2 = \begin{vmatrix} 6 & 2 \\ 2 & 6 \end{vmatrix} = 36 - 4 = 32 > 0$$

$T_2(-1,1)$ je lokalen minimum.

$$\left(-\frac{2}{x^2}\right)' = \left(-2 \cdot x^{-2}\right)' = -2 \cdot (-2) \cdot x^{-3} = 4 x^{-3} = \frac{4}{x^{+3}}$$

$$2x - \frac{2}{x^2 y}$$

$$\frac{2}{x^2 y} = \frac{1}{x^2} \cdot \left(-\frac{2}{y}\right)'$$

$$= \left(-\frac{2}{y}\right)' = -2 \cdot (-1) y^{-2} = \frac{2}{x^2 y} = \frac{2}{y^2}$$

---

1) $\int_{-4}^{-1} x^2 e^x \, dx =$

$x^2 = u$, $2x\,dx = du$, $e^x = v$

$x^2 \cdot e^x - \int e^x 2x\,dx$ $\quad x = u$, $dx = du$, $e^x = v$

$x^2 \cdot e^x - 2\int e^x x\,dx$

$x^2 \cdot e^x - 2 \cdot (x \cdot e^x - \int e^x dx)$

$x^2 \cdot e^x - 2 \cdot (x \cdot e^x - e^x)$

$= x^2 \cdot e^x - 2xe^x + 2e^x + C$

$= \int_{-4}^{-1} x^2 e^x - 2xe^x + 2e^x$

$= \left[1 \cdot e^{-1} - [2 \cdot (-1) \cdot e^{-1}] + 2e^{-1}\right]$

$- \left[(-4)^2 \cdot e^{-4} - (2 \cdot (-4)) \cdot e^{-4} + 2e^{-4}\right)$

$= \left[e^{-1} - [-2e^{-1}] + 2e^{-1}\right] - \left[(16 \cdot e^{-4}) + 8e^{-4} + 2e^{-4}\right]$

$= 1.83939 - 0.4762$

$= 1.36319$

(3) JOSIP FESTINI

$A(2,2,4)$ $B(0,-1,3)$ $C(-1,0,2)$

$AB(-2,-3,-1)$ $AC(-3,-2,-2)$

$\measuredangle AB, AC = \cos\varphi \left(\dfrac{l_1 \cdot l_2 + m_1 \cdot m_2}{\sqrt{l_1 l_2 + m_1 m_2}} \right.$

?

(4) DOMENA

$f(x,y) = x^2 + y^2 + \dfrac{2}{xy}$

UVJET

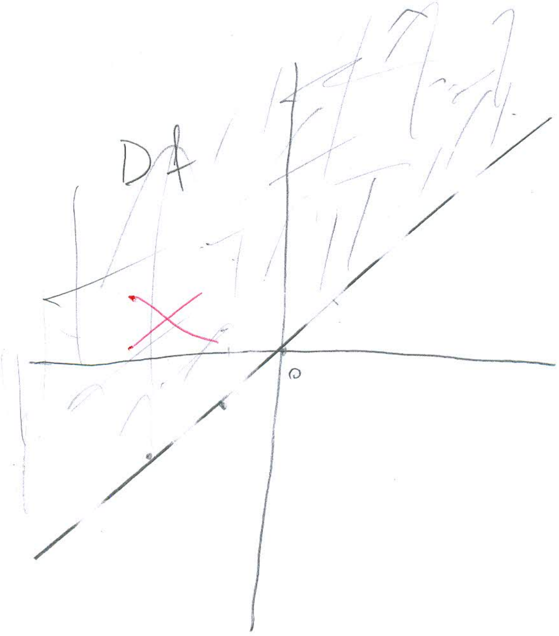
$xy > 0$

$y > x$

y	-1	0	-2
x	-1	0	-2

$T(1,1)$

$\frac{1}{1}$



JOSIP FEŠTINI

Popuniti odmah!
Ime i prezime: STIPE BRKLJAČA **Broj indeksa:**

MATEMATIKA 2: <u>ZAVRŠNI KOLOKVIJ</u> Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Odrediti integracijom (analitički): $\int_{-4}^{-1} x^2 e^x dx$.	~~15~~
2. Odrediti numeričkom integracijom: $\int_{-4}^{-1} x^2 e^x dx$.	15
3. Zadane su točke $A(2,2,4)$, $B(0,-1,3)$ i $C(-1,0,2)$. Koliki je kut između pravaca AB i AC?	10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y^2 + \frac{2}{xy}$.	~~20~~
~~5. Riješiti: $y' + 2xy = x - 3$.~~	~~20~~
6. Riješiti: $y'' - 4y' + 4y = 4$.	20

$\frac{45}{80}$

B-1

3) $A(2,2,4)$
$B(0,-1,3)$
$C(-1,0,2)$

$AB(\underset{l_1}{-2}, \underset{m_1}{-3}, \underset{n_1}{-1})$
$AC(\underset{l_2}{-3}, \underset{m_2}{-2}, \underset{n_2}{-2})$

$$\cos\varphi = \frac{l_1 \cdot l_2 + m_1 \cdot m_2 + n_1 \cdot n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}$$

$$\cos\varphi = \frac{(-2)\cdot(-3) + (-3)(-2) + (-1)\cdot(-2)}{\sqrt{(-2)^2 + (-3)^2 + (-1)^2} \cdot \sqrt{(-3)^2 + (-2)^2 + (-2)^2}}$$

$$\cos\varphi = \frac{6+6+2}{\sqrt{4+9+1}\ \sqrt{9+4+4}} = \frac{14}{\sqrt{14}\sqrt{17}}$$

$\varphi = 0.433547$

2) $\int_{-4_a}^{-1_b} x^2 e^x dx$

$k = \frac{b-a}{6}$

$$k = \frac{-1-(-4)}{6} = \frac{1}{2}$$

$$P = \frac{k}{3}\left(y_0 + y_m + 4(y_1 + y_3 + y_{n-1}) + 2(y_2 + y_4)\right)$$

$-4 \quad -\frac{7}{2} \quad -3 \quad -\frac{5}{2} \quad -2 \quad -\frac{3}{2} \quad -1$

$y_0(-4) = 0.2931$
$y_1(-\frac{7}{2}) = 0.3699$
$y_2(-3) = 0.4481$
$y_3(-\frac{5}{2}) = 0.5130$
$y_4(-2) = 0.5413$
$y_5(-\frac{3}{2}) = 0.5020$
$y_6(-1) = 0.3679$

$$= \frac{\frac{1}{2}}{3}\left(0.2931 + 0.3679 + 4(0.3699 + 0.5130 + 0.5020)\right.$$
$$\left. + 2(0.4481 + 0.5413)\right)$$

$$= 1.36323333 = 1.36.$$

1) $\int_{-4}^{-1} x^2 e^x \, dx$

$\left[\begin{matrix} x^2 = u' & e^x = dv \\ 2x\,dx = du & e^x = v \end{matrix}\right]$

$dx = \frac{du}{2x}$

$u \cdot v - \int v \cdot du$

$x^2 \cdot e^x - \int e^x \, 2x \, dx$

$x^2 \cdot e^x - 2 \int x e^x \, dx$

$x^2 \cdot e^x - 2(x \cdot e^x - e^x)$

$\left[\begin{matrix} u = x' & e^x = dv \\ du = dx & v = e^x \end{matrix}\right]$

$x^2 \cdot e^x - 2x e^x + 2e^x$

$\int_{-4}^{-1} x^2 \cdot e^x - 2x e^x + 2e^x + C$

$= x^2 - e^x \Big|_{-1}^{-4} - 2(x \cdot e^x - e^x) \Big|_{-1}^{-4}$

$x \cdot e^x - \int e^x dx$

$x \cdot e^x - e^x dx$

$= ((16 - e^{-4}) - (1 - e^{-1}))$

$- 2((-4 \cdot e^{-4} - e^{-4}) - (-1 \cdot e^{-1} - e^{-1}))$

$= 15.34 \quad -1.288$

$= 14.06$

$x^2 \cdot e^x - 2(x e^x - e^x dx)$

$x^2 \cdot e^x - 2x e^x + 2e^x dx$

$(16 - 1) \cdot e^5 - 2(5) e^5 + 2e^5$

STIPE BRKLJAČA

4) $f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$\frac{2}{xy} \Big|' =$

$\frac{df}{dx} = 2x - \frac{2}{yx^2}$ ✓

$\frac{df}{dy} = 2y + \frac{2}{\cancel{x}}$ ✗

$T_1(0,0) =$

$\frac{d^2f}{dx} = 2 + \frac{4y}{x^3}$

>0

$\frac{d^2f}{dy} = 2$

$\Delta = \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} = 4 - 4 = 0$

nema ekstre

T(

$2x = \frac{2}{yx^2}$

$2yx^3 = 2$

$yx^3 = 1$

$-x \cdot x^3 =$

$x^2 = 1 \quad x = \pm 1$

$T_1(0,0)$

$T_2(1,-1)$

$T_3(-1,1)$

$2y = -\frac{2}{x} \,/ :2$

$y = -\cancel{x}$

$y = 0$

$T_2(1,-1) \quad \Delta = \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} =$

$-4 - 4 = -8$

$-8 < 0$

$\frac{d^2f}{dx} < 0$

sedlasta

$T_3(-1,1)$

$\Delta = \begin{vmatrix} -2 & 2 \\ 2 & 2 \end{vmatrix} = -4-4 = -8 < 0$

sedlasta

6) $y'' - 4y' + 4y = 4$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda_{12} = \frac{4 \pm \sqrt{16-16}}{2}$

$\lambda_{12} = \frac{4}{2} = 2$

$\lambda_1 = 2 \quad \lambda_2 = 2$

$y_0 = C_1 e^{2x} + x C_2 e^{2x}$

$m = a_0$

$m' = 0$

$m'' = 0$

$0 - 0 + 4a_0 = 4 \,/ :4$

$a_0 = 1$

$y_0 = y + m$

$y_0 = y + m$

$m = a_0$

$y = C_1 e^{2x} + x C_2 e^{2x} + \cancel{A}$

$y = C_1 e^{2x} + x C_2 e^{2x} + 1$ ✓

$0 - 0 + 4a_0 = 4$

$4a_0 = 4 \,/ :4$

$a_0 = 1$

$5)\ y' + 2xy = x-3$

$y' + 2xy = 0$

$y' = -2xy$

$\frac{dy}{dx} = -2xy \ / \cdot dx$

$dy = -2xy\,dx \ / y$

$\int \frac{dy}{y} = -2\int x\,dx$

$\ln|y| = -\frac{2x^2}{8} + c \ / e^{\nabla}$

$\boxed{y = c \cdot e^{-x^2}}$

$y = c(x) \cdot e^{x^2}$

$y' = c'(x) \cdot e^{x^2} - 2x\,c(x) \cdot e^{x^2}$

$c'(x) \cdot e^{x^2} \cancel{- 2x\,c(x) \cdot e^{x^2}} \cancel{+ 2x\,c(x) \cdot e^{x^2}} = x-3$

$c'(x) \cdot e^{x^2} = x-3 \ /'$

$c'(x) = \frac{x-3}{e^{x^2}}$

$c(x) = \frac{3}{2xe^{x^2}}$

$y = \frac{3}{2xe^{x^2}} \cdot e^{x^2} + C$

$y = \frac{3}{2}x + C$

$= e^{-\int p(x)dx}\left[\int Q(x) \cdot e^{\int p(x)dx} dx + C\right]$

$\int p(x) = 2\int x\,dx$

$= \frac{2x^2}{2} = x^2$

$\int (x-3) \cdot e^{x^2} dx + C$

$\frac{x^2}{2} - 3\int e^{x^2}$

$\frac{x^2}{2} - 6e^{x^2} + C$

$y = e^{-x^2}\left(\frac{x^2}{2} - 6e^{x^2} + C\right)$