MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ *A9*	POPUNJAVA NASTAVNIK
IME I PREZIME: IVAN RUBELJ **BROJ INDEKSA:** 17-2-0085-2011	Broj ↓ bodova
ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića	

1. Izračunati i obavezno provjeriti $\lim_{x\to 3}\left(\frac{\sqrt{6+x}-3}{x-3}\right)$. ~~6+2~~

2. Ispitati konvergenciju reda $\sum n\left(\sqrt{n+1}-\sqrt{n-1}\right)$. 7

3. Na osnovi ispitivanja tijeka funkcije skicirati graf: $f(x)=\frac{x+4}{x^2-2x-3}$. ~~20 (graf)~~ 5

4. Zapisati treću parcijalnu sumu razvoja funkcije $g(x)=e^{8x}$ u Taylorov red po potencijama od x. Taylorov red oko točke $x_0=0$ naziva se još i Maclaurinov red. 15

5. Odrediti domenu i asimptote funkcije $h(x)=\frac{2x+3}{x+\sqrt{x^2-x}}$. 0 ~~6+14~~ 10

6. Posebno izračunati rang, a posebno determinantu matrice $\mathbf{A}=\begin{bmatrix}0&8&0&0\\2&0&1&0\\-8&0&0&1\\0&8&0&-18\end{bmatrix}$. 8+7 10

7. Na sljedećem primjeru pokazati kako se nejednadžba može riješiti grafički, a kako analitički: $x-4\le\sqrt{x}$.
Provjeravaj gdje god možeš uvrštavanjem! ~~6+6+3~~

Ukupno: 28

1)

$$\lim_{x\to 3}\left(\frac{\sqrt{6+x}-3}{x-3}\right)=\lim_{x\to 3}\left(\frac{\sqrt{6+3}-3}{3-3}\right)=\left[\frac{0}{0}\right]$$ - neodređeni oblik

$$\lim_{x\to 3}\left(\frac{\sqrt{6+x}-3}{x-3}\right)' \overset{\text{NE TAKO}}{=} \lim_{x\to 3}\frac{(\sqrt{6+x})'\cdot(-3)-(\sqrt{6+x})\cdot(-3)'}{(x-3)^2}=\lim_{x\to 3}\frac{\frac{1}{2\sqrt{6+x}}\cdot(-3)-0}{(x-3)^2}=$$

$$\lim_{x\to 3}\frac{\frac{-3}{2\sqrt{6+x}}}{\frac{x^2-2\cdot x\cdot 3+3^2}{1}}=\lim_{x\to 3}\frac{-3}{(x^2-6x+9)\cdot 2\sqrt{6+x}}=\lim_{x\to 3}\frac{-3}{(2x^2-12x+18)\sqrt{6+x}}=$$

$$=\lim_{x\to 3}\frac{-3}{(2\cdot 3^2-12\cdot 3+18)\cdot\sqrt{6+3}}=\lim_{x\to 3}\frac{-3}{(18-36+18)\cdot 3}=\frac{-3}{54-108+54}=$$

$$=\frac{-3}{0}=-\infty$$

$\frac{-3}{0} \ne -\infty$

$$\boxed{\frac{-3}{0_+}=-\infty \quad \frac{-3}{0_-}=+\infty}$$

PROVJERENO U DIGITRONU. I ISPALO DOBRO.

ŠTO JE PROVJERENO, KOJE VRIJEDNOSTI?

$f(\_\_)=\square$

$f(\_\_)=\square$

7. $x-4 \le \sqrt{x}$

$f(x)$	-1	0	1
$x^2-9x+16$	26	16	8

$x-4 \le \sqrt{x} \quad /^2$

$(x-4)^2 \le x$

$x^2-2\cdot x\cdot 4+4^2 \le x$

$x^2-8x+16 \le x$

$x^2-8x+16-x \le 0$

$x^2-9x+16 \le 0$

$x^2-9x+16=0$

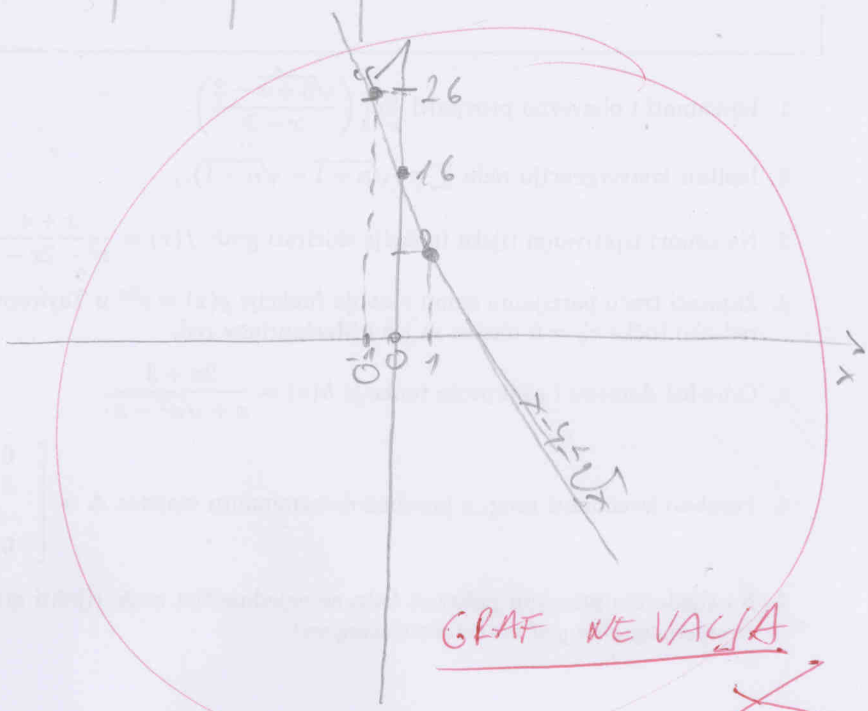
$a=1, b=-9, c=16$

$x_{1,2}=\frac{-b\mp\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{9\mp\sqrt{81-4\cdot 1\cdot 16}}{2}$

$x_{1,2}=\frac{9\mp\sqrt{17}}{2}$

$x_1=\frac{9-\sqrt{17}}{2}$ $\qquad x_2=\frac{9+\sqrt{17}}{2}$



GRAF NE VALJA

KOJA SU U OPĆE RJEŠENJA NEJEDNADŽBE?

PROVJERA:

$x-4 \le \sqrt{x}$

$\frac{9-\sqrt{17}}{2}-4 \le \sqrt{\frac{9-\sqrt{17}}{2}}$

$\frac{9-\sqrt{17}-8}{2} \le \sqrt{\frac{9-\sqrt{17}}{2}}$

$\frac{1-\sqrt{17}}{2} \le \sqrt{\frac{9-\sqrt{17}}{2}}$ ✓

UVRŠTENO U DIGITRON I DOBRO ISPALO

$x-4 \le \sqrt{x}$

$\frac{9+\sqrt{17}}{2}-4 \le \sqrt{\frac{9+\sqrt{17}}{2}}$

$\frac{9+\sqrt{17}-8}{2} \le \sqrt{\frac{9+\sqrt{17}}{2}}$

$\frac{1+\sqrt{17}}{2} \le \sqrt{\frac{9+\sqrt{17}}{2}}$ ✓

UVRŠTENO U DIGITRON I DOBRO ISPALO

ŠTO JE UVRŠTENO I KOLIKI JE REZULTAT?

NIJE PROVJERENO RJEŠENJE GORNJE JEDNADŽBE!!!

Ivan Rubelj MAT 1

3.

1. DOMENA

$f(x) = \frac{x+4}{x^2-2x-3}$

$x^2-2x-3 \neq 0$

$a=1, b=-2, c=-3$

$x_{1,2} = \frac{2 \mp \sqrt{4-4\cdot 1\cdot(-3)}}{2}$

$x_{1,2} = \frac{2 \mp 4}{2}$

$x_1 = \frac{2-4}{2}$ $\quad x_2 = \frac{2+4}{2}$

$x_1 = -1$ $\quad x_2 = 3$

$D(f) = \langle -\infty, -1\rangle \cup \langle 3, +\infty\rangle$ ✗

$\cup \langle -1, 3\rangle$

BODUJE SE SAMO SKICA GRAFA

2. NULTOČKE

$x+4=0$

$x=-4$

$T_1(-4,0)$ ✓

3. ASIMPTOTE

V.A.

$\lim_{x\to -1} \frac{x+4}{x^2-2x-3} = \lim_{x\to -1} \frac{-1+4}{(-1)^2-2\cdot(-1)-3} = \lim_{x\to -1} \frac{3}{1+2-3} = \frac{3}{0} = \infty^{\text{L.H.A.}}$

$\lim_{x\to 3} \frac{x+4}{x^2-2x-3} = \lim_{x\to 3} \frac{3+4}{3^2-2\cdot 3-3} = \lim_{x\to 3} \frac{7}{9-6-3} = \frac{7}{0} = \infty^{\text{D.H.A.}}$

-1 i 3 su V.A funkcije

KOLIKO JE $\lim_{x\to -1_-} f(x)$?

$\lim_{x\to 3_+} f(x) = ?$

H.A.

$\lim_{x\to \infty} \frac{x+4}{x^2-2x-3} \,/ :x^2 \,/ :x^2 = \lim_{x\to\infty} \frac{\frac{x}{x^2}+\frac{4}{x^2}}{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}} = 0$

$\lim_{x\to -\infty} \frac{x+4}{x^2-2x-3} = \left\{ \begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix} \right\} = \lim_{x\to\infty} \frac{-x+4}{(-x)^2-2\cdot(-x)-3} = \lim_{x\to\infty} \frac{-x+4}{x^2+2x-3} \,/ :x^2 \,/ :x^2 = \lim_{x\to\infty} \frac{-\frac{x}{x^2}+\frac{4}{x^2}}{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}$

$= 0$

0 je O.H.A.

kosu nemamo jer imamo horizontalnu

4. DERIVACIJE

$$f'(x)=\left(\frac{x+4}{x^2-2x-3}\right)'=\frac{(x+4)'\cdot(x^2-2x-3)-(x+4)\cdot(x^2-2x-3)'}{(x^2-2x-3)^2}=\frac{1(x^2-2x-3)-(x+4)\cdot(2x-2)}{(x^2-2x-3)^2}=$$

$$=\frac{x^2-2x-3-(2x^2-2x+8x-8)}{(x^2-2x-3)^2}=\frac{\cancel{x^2}-\cancel{2x}-3-2x^2+\cancel{2x}-8x+8}{(x^2-2x-3)^2}=\frac{-x^2-8x+5}{(x^2-2x-3)^2}$$

$$-x^2-8x+5=0$$

$$a=-1,\ b=-8,\ c=5$$

$$x_{1,2}=\frac{8\mp\sqrt{64-4\cdot(-1)\cdot5}}{-2}$$

$$x_{1,2}=\frac{8\mp\sqrt{84}}{-2}$$

$$x_1=\frac{8-\sqrt{84}}{-2}\qquad x_2=\frac{8+\sqrt{84}}{-2}$$

$$-(-9)^2-8\cdot(-9)+5$$

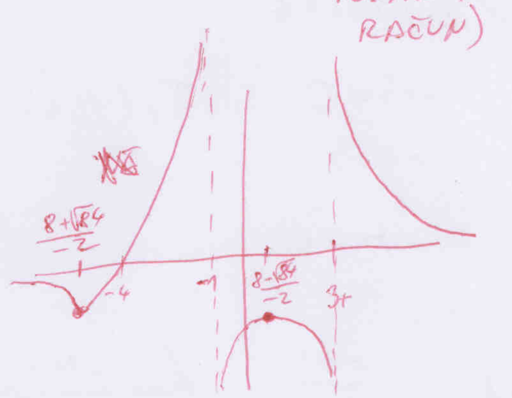
$$-81+72+5=-4$$

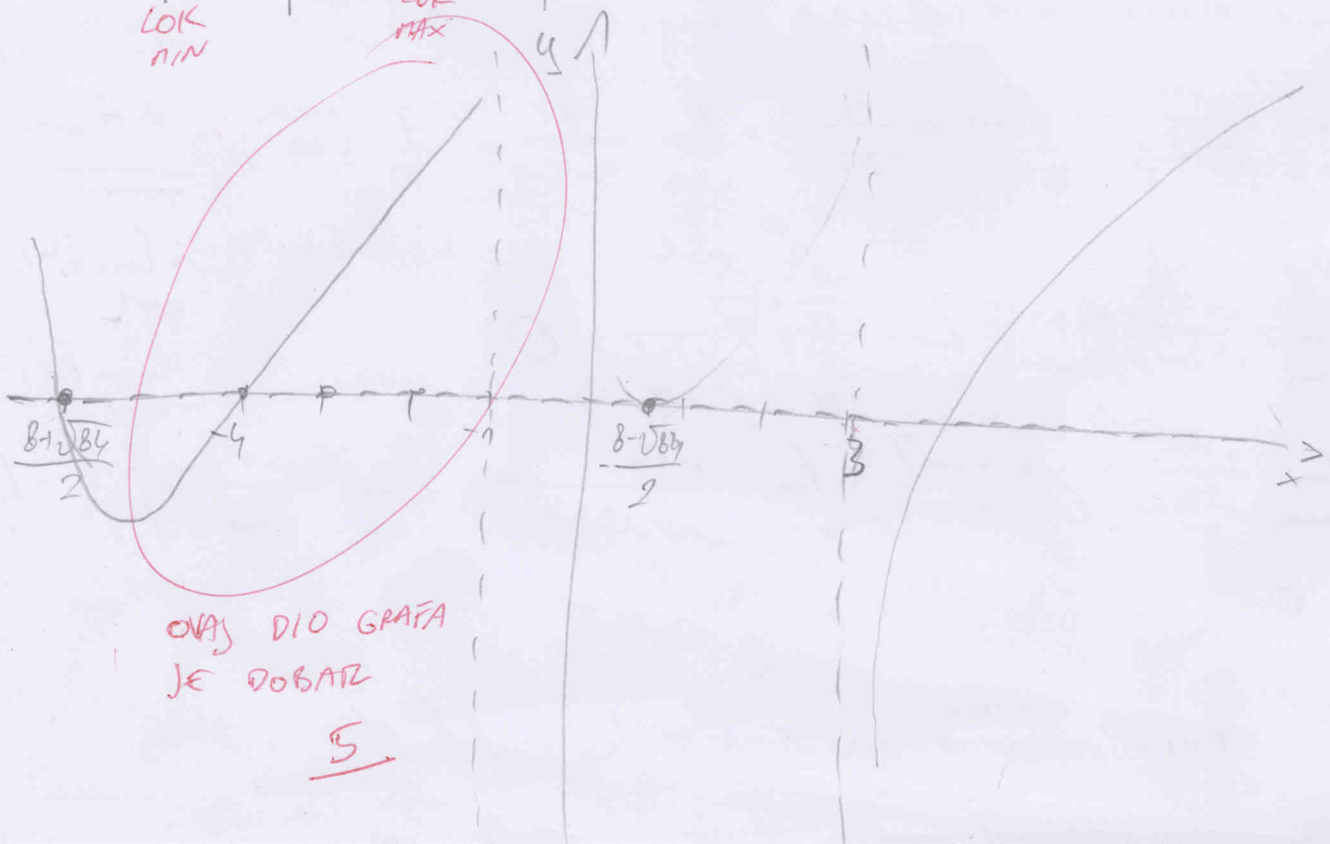
	$-\infty$	$\frac{8+\sqrt{84}}{-2}$	-4	$\frac{8+\sqrt{84}}{-2}$	$+\infty$
$f'(x)$	$-$	$+$	$+$	$+$ $-$	
$f(x)$	↘	↗	↗	↗ ↘	

LOK MIN

LOK MAX

GRAF TREBA IZGLEDATI OVAKO (AKO JE DOBAR VAŠ RAČUN)





OVAJ DIO GRAFA JE DOBAR

5

Ivan Rubdj MAT 1

6.

$$A=\begin{bmatrix} 0 & 8 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -8 & 0 & 0 & 1 \\ 0 & 8 & 0 & 18 \end{bmatrix} = \det A = (-1)^{2+3}\cdot 1 \begin{vmatrix} 0 & 8 & 0 \\ -8 & 0 & 1 \\ 0 & 8 & 18 \end{vmatrix} = \det A = (-1)\cdot(-1)^{2+1}$$

$$\cdot(-8)\begin{vmatrix} 8 & 0 \\ 8 & 18 \end{vmatrix} = \det A = (-8)\cdot\left[(8\cdot 18)+(8\cdot 0)\right] = \det A = -8\cdot 144 = -1152$$

a) $\det A \neq 0$ matrica ima inverz $\det A = -1152$ ✓

$$A=\begin{bmatrix} 0 & 8 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -8 & 0 & 0 & 1 \\ 0 & 8 & 0 & 18 \end{bmatrix} \sim \begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 8 & 0 & 0 \\ -8 & 0 & 0 & 1 \\ 0 & 8 & 0 & 18 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 8 & 0 & 0 \\ -8 & 0 & 0 & 1 \\ 0 & 8 & 0 & 18 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 8 & 0 & 18 \end{vmatrix} \sim$$

1R<->2R 1R:2 1R·8+3R 2R:8

$$\sim \begin{vmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 8 & 0 & 18 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 18 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 18 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 0 & -1/8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/4 \\ 0 & 0 & 0 & 18 \end{vmatrix} \sim$$

2R·(-8)+4R 3R:4 3R×(-1/2)+1R 4R:18

b) Rang matrice je 4 (broj redaka × broj stupaca)

$1\times 4 = 4$

POGREŠAN ARGUMENT !!

5. $f(x) = \dfrac{2x+3}{x+\sqrt{x^2-x}}$

1. DOMENA

$x+\sqrt{x^2-x} \neq 0$

$\sqrt{x^2-x} \neq -x \quad /^2$

$x^2-x \neq x^2$

$\cancel{x^2}-x-\cancel{x^2} \neq 0$

$-x \neq 0$

$x \neq 0$

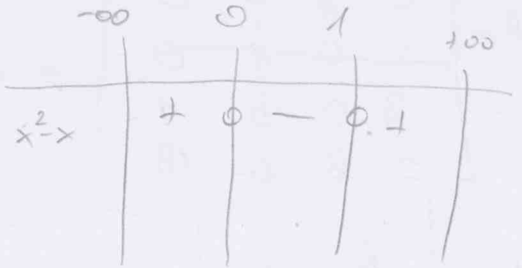
$x^2-x \geq 0$

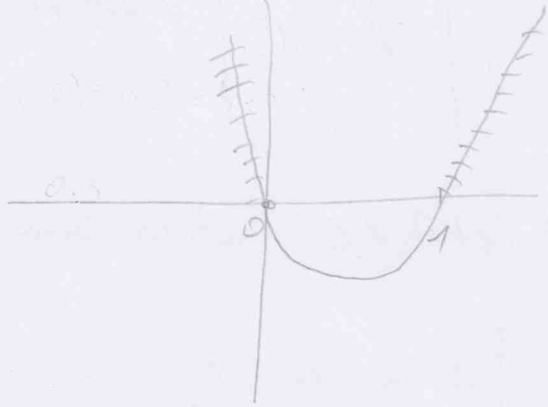
$x^2-x=0$

$x(x-1)=0$

$x=0 \quad x-1=0$

$x=1$





$D(f) = \langle -\infty, 0 \rangle \cup [1, +\infty\rangle$ ✓ 6

$x \in \langle -\infty, 0] \cup [1, +\infty\rangle$

1. V.A.

$\lim_{x\to 1} \dfrac{2x+3}{x+\sqrt{x^2-x}} = \lim_{x\to 1} \dfrac{2\cdot 1+3}{1+\sqrt{1-1}} = \lim_{x\to 1} \dfrac{5}{1} = 5$ NIJE V.A.

$\lim_{x\to 0} \dfrac{2\cdot 0+3}{0+\sqrt{0-0}} = \dfrac{3}{0} = \infty \quad 0 \notin D(f)$ 0 je V.A. ✓

2. H.A.

$\lim_{x\to \infty} \dfrac{2x+3}{x+\sqrt{x^2-x}} \; \begin{matrix} /:x \\ /:x \end{matrix} = \dfrac{\frac{2x}{x}+\frac{3}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}} = \dfrac{2}{2} = 1$ D.H.A. ✓

$\lim_{x\to -\infty} \dfrac{2x+3}{x+\sqrt{x^2-x}} = \left\{ \begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix} \right\} = \lim_{x\to +\infty} \dfrac{-2x+3}{-x+\sqrt{x^2+x}} \; \begin{matrix} /:x \\ /:x \end{matrix} = \dfrac{\frac{-2x}{x}+\frac{3}{x}}{\frac{-x}{x}+\sqrt{\frac{x^2}{x}+\frac{x}{x}}} = \dfrac{-2}{0} = \infty$

10

Lijeve nema.

Imamo horizontalnu, stoga nemamo kosu asimptotu. L.K.A.?