

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **STIPE JULIC**

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1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0, 0)$. Izračunati $\int_{\partial K} (2x + 3) ds$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint_K (2x + 3) dx dy$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2 + y^2$, $z = 4$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 2 \cos t$, $y = 2 \sin t$ i $z = t^2$, $t \in [-1, 1]$. Još je zadano $f(x, y, z) = \sqrt{z}$. Izračunati: $\int_{\Gamma} f ds$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0, y'(0) = 0, y''(0) = 1.$$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

Ukupno:

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Tablica integrala

$\int dx = x + C$	$\int \sin x dx = -\cos x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \cos x dx = \sin x + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$		$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$		
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$

① KRUŽG $r=2$ $T(0,0)$ $\int_K (2x+3) ds = ?$

PARAMETRIZACIJA

$$r(t) = \begin{pmatrix} 2 \cos t \\ 2 \sin t \end{pmatrix}$$

$$r'(t) = \begin{pmatrix} -2 \sin t \\ 2 \cos t \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-2 \sin t)^2 + (2 \cos t)^2} = 2$$

$$\|r'(t)\| = \sqrt{4 \sin^2 t + 4 \cos^2 t}$$

$$\|r'(t)\| = \sqrt{4} = 2$$

$$= \int_0^{2\pi} 2(2 \cos t) + 3 \cdot 2 dt$$

$$= \int_0^{2\pi} 4 \cos t + 6 dt$$

$$= 4 \int_0^{2\pi} \cos t dt + 6 \int_0^{2\pi} dt$$

$$= 6 \cdot 2\pi = 12\pi, \checkmark$$

② KRUŽG $r=1$ $T(2,1)$ $\iint_S (2x+3) dx dy \Rightarrow r dr d\phi$

$$x = r \cos \phi + 2$$

$$y = r \sin \phi + 1$$

$$x = r \cos \phi + 2$$

$$y = r \sin \phi + 1$$

$$= \int_0^{2\pi} \int_0^1 2(r \cos \phi + 2) + 3 r dr d\phi$$

$$= \int_0^{2\pi} \int_0^1 2r^2 \cos \phi + 4 + 3r dr d\phi$$

$$= \int_0^{2\pi} \int_0^1 2r^2 \cos \phi + 7r dr d\phi$$

$$= 7\pi, \checkmark$$