MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: STIPE VULIĆ BROJ INDEKSA: 57663-2009	Broj ↓ bodova

1. Neka je K krug radijusa $r=2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K}(2x+3)\,ds$.	20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(2,1)$. Izračunati $\iint_K (2x+3)\,dxdy$.	20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z\geq 1$.	15
4. Izračunati volumen paraboloida omeđenog plohama: $z=x^2+y^2$, $z=4$.	15
5. Zadana krivulja Γ s parametrizacijom $x=2\cos t$, $y=2\sin t$ i $z=t^2$, $t\in[-1,1]$. Još je zadano $f(x,y,z)=\sqrt{z}$. Izračunati: $\int_\Gamma f\,ds$.	~~15~~ 12
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:	15

$$y'''(t)-2y''(t)+y'(t)=t^2,\quad y(0)=0,\ y'(0)=0,\ y''(0)=1.$$

Ukupno: 82

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tablica integrala

$\int dx=x+C$	$\int\sin x\,dx=-\cos x+C$	$\int\frac{dx}{\cos^2 x}=\tan x+C$
$\int x^n\,dx=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int\cos x\,dx=\sin x+C$	$\int\frac{dx}{\sin^2 x}=-\cot x+C$
$\int\frac{dx}{x}=\ln\lvert x\rvert+C$	$\int\tan x\,dx=-\ln\lvert\cos x\rvert$	$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x\,dx=\frac{a^x}{\ln a}+C$	$\int\cot x\,dx=\ln\lvert\sin x\rvert$	$\int\frac{dx}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int\sqrt{x^2\pm a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int\sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$		
$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$	$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$

STIPE VULIĆ 57663-2009

(1.) KRUG $r=2$ $T(0,0)$ $\int_{\partial K}(2x+3)\,ds=?$

PARAMETRIZACIJA

$r(t)=\begin{pmatrix} 2\cos t \\ 2\sin t \end{pmatrix}$

$r'(t)=\begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix}$

$\|r'(t)\|=\sqrt{(-2\sin t)^2+(2\cos t)^2}$

$\|r'(t)\|=\sqrt{4\sin^2 t+4\cos^2 t}$

$\|r'(t)\|=\sqrt{4}=2$

$$=\int_0^{2\pi} 2(2\cos t)+3\cdot 2\,dt$$

$$=\int_0^{2\pi} 4\cos t+6\int_0^{2\pi} dt$$

$$=4\underbrace{\int_0^{2\pi}\cos t\,dt}_{0}+6\int_0^{2\pi} dt$$

$$=6\cdot 2\pi=12\pi$$

(2.) KRUG $r=1$ $T(2,1)$ $\iint(2x+3)\,dx\,dy \Rightarrow r\,dr\,d\varphi$

$x=r\cos\varphi+p$

$y=r\sin\varphi+q$

$x=r\cos\varphi+2$

$y=r\sin\varphi+1$

$$=\int_0^{2\pi}\int_0^{1} 2(r\cos\varphi+2)+3\;r\,dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^{1} 2r^2\cos\varphi+4+3\;r\,dr\,d\varphi$$

$$=\underbrace{\int_0^{2\pi}\int_0^{1} 2r^2\cos\varphi}_{0}+7\int_0^{2\pi}\int_0^{1} r\,dr\,d\varphi$$

$$=7\pi$$

3. $x^2+y^2+z^2=4$ ZA KOJI VRIJEDI $z \geq 1$.

$x^2+y^2+z^2=R^2$

$R^2=4 \;/\sqrt{}$

$R=\sqrt{4}$

$R=2$

$$\boxed{\begin{array}{l} \varphi \in (0, 2\pi) \\ z \in (1, 2) \\ r \in (0, \sqrt{4-z^2}) \end{array}}$$

$r^2+z^2=4$

$r^2=4-z^2 \;/\sqrt{}$

$r=\sqrt{4-z^2}$

$$V=\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dz\,dr\,d\varphi$$

$$V=2\pi\int_1^2 \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz$$

$$V=2\pi\int_1^2 \frac{(\cancel{\sqrt{4-z^2}})^{\cancel{2}}}{2}$$

$$V=\cancel{2}\pi\int_1^2 \frac{4-z^2}{\cancel{2}}$$

$$V=\pi\Big|_1^2 4z-\frac{z^3}{3}$$

$$V=\pi\left(4\cdot 2-\frac{2^3}{3}\right)-\left(4\cdot 1-\frac{1^3}{3}\right)$$

$$V=\pi\left(\frac{16}{3}\right)-\left(\frac{11}{3}\right)$$

$$V=\frac{5\pi}{3}$$

STIPE VULIĆ 57663-2009

4. VOLUMEN PARABOLOIDA

$z = x^2 + y^2 \quad , z = 4$

$R^2 = z \quad / \sqrt{\ }$

$R = \sqrt{z}$

$$\boxed{\begin{array}{l} r \in [0, \sqrt{z}) \\ \varphi \in [0, 2\pi) \\ z \in (0, 4) \end{array}}$$

$$V = \int_0^{2\pi} \int_0^{4} \int_0^{\sqrt{z}} r \, dz \, dr \, d\varphi$$

$$V = 2\pi \int_0^4 \frac{r^2}{2} \Big|_0^{\sqrt{z}} dz$$

$$V = 2\pi \int_0^4 \frac{(\sqrt{z})^2}{2}$$

$$V = 2\pi \int_0^4 \frac{z}{2}$$

$$V = \frac{1}{2} \cdot 2\pi \cdot \frac{z^2}{2} \Big|_0^4$$

$$V = \frac{1}{2} \cdot 2\pi \cdot \frac{4^2}{2}$$

$$V = \frac{1}{2} \cdot 2\pi \cdot 8$$

$$V = 8\pi$$ ✓

5.

$x = 2\cos t$

$y = 2\sin t$

$z = t^2$

PARAMETRIZACIJA

$$r(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t^2 \end{pmatrix}$$

$$r'(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 2t \end{pmatrix}$$

$$\int f \, ds = ?$$

$$= \int_{-1}^{1} \sqrt{t^2} \cdot \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (2t)^2}$$

12

$$= \int_{-1}^{1} \sqrt{t^2} \cdot \sqrt{4\sin^2 t + 4\cos^2 t + 4t}$$

$$= \int_{-1}^{1} \sqrt{t^{\cancel{2}}} \cdot 4 + 4t$$

$$= \int_{-1}^{1} t \cdot 4 + 4t$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: FRANE DUNAT **BROJ INDEKSA:** 17-2-0020-2010

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} (2x+3)\,\mathrm{d}s$. 20
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6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 70

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2\pm a^2}\right\| + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$

FRANC DUNAT

(1.) $r=2 \qquad T(0,0)$

$$\int_{\partial K}(2x+3)\,ds \qquad r=\begin{pmatrix} r\cos t \\ r\sin t\end{pmatrix}=\begin{pmatrix} 2\cos t \\ 2\sin t\end{pmatrix},\quad r'(t)=\begin{pmatrix} -2\sin t \\ 2\cos t\end{pmatrix}$$

$$\|r'(t)\|=\sqrt{(-2\sin t)^2+(2\cos t)^2}=\sqrt{4\sin^2 t+4\cos^2 t}=\sqrt{4(\sin^2 t+\cos^2 t)}=\sqrt{4\cdot 1}=\sqrt{4}=2$$

$$\int_{\partial K}(2x+3)\,ds=\int_0^{2\pi}2\left[2(2\cos t)+3\right]dt=\int_0^{2\pi}2\left[4\cos t+3\right]dt=\int_0^{2\pi}\left[8\cos t+6\right]dt$$

$$=\left[8\sin t+6t\right]\Big|_0^{2\pi}=\left[8\underset{0}{\sin 2\pi}+6\cdot 2\pi-(8\underset{0}{\sin 0}+\underset{0}{6\cdot 0})\right]=12\pi \checkmark$$

(2.) $r=1, \quad T(2,1)$

$$\iint_K(2x+3)\,dx\,dy \qquad \begin{aligned}x&=r\cos\varphi+2\\ y&=r\sin\varphi+1\\ dx\,dy&=r\,dr\,d\varphi\end{aligned} \qquad \begin{aligned}\varphi&\in[0,2\pi]\\ r&\in[0,1]\end{aligned}$$

$$\iint_K(2x+3)\,dx\,dy=\int_0^{2\pi}\int_0^1\left[2(r\cos\varphi+2)+3\right]r\,dr\,d\varphi=\int_0^{2\pi}\int_0^1\left[2r\cos\varphi+4+3\right]r\,dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^1\left[2r\cos\varphi+7\right]r\,dr\,d\varphi=\int_0^{2\pi}\int_0^1\left[2r^2\cos\varphi+7r\right]dr\,d\varphi=\int_0^{2\pi}\left[2\frac{r^3}{3}\cos\varphi+7\frac{r^2}{2}\right]\Big|_0^1 d\varphi$$

$$=\int_0^{2\pi}\left[\frac{2}{3}\cos\varphi+\frac{7}{2}\right]d\varphi=\left[\frac{2}{3}\sin\varphi+\frac{7}{2}\varphi\right]\Big|_0^{2\pi}=\left[\frac{2}{3}\underset{0}{\sin 2\pi}+\frac{7}{2}\cdot 2\pi-\left(\frac{2}{3}\underset{0}{\sin 0}+\underset{0}{\frac{7}{2}\cdot 0}\right)\right]$$

$$=7\pi \checkmark$$

③ $x^2+y^2+z^2=4$, $z\geq 1$

$R^2=4$

$R=2$

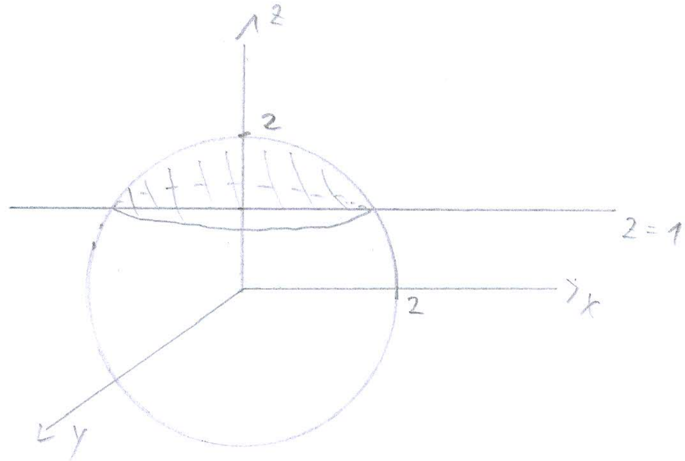
$\varphi\in[0,2\pi]$

$x^2+y^2+z^2=4$ $\qquad r\in[0,\sqrt{4-z^2}]$

$r^2+z^2=4$ $\qquad z\in[1,2]$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$



$$V=\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi=\int_0^{2\pi}\int_1^2 \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz\,d\varphi$$

$$=2\pi\int_1^2 \frac{(\sqrt{4-z^2})^2}{2}\,dz\,d\varphi=\pi\int_1^2(4-z^2)\,dz=\pi\left[4z-\frac{z^3}{3}\right]\Big|_1^2$$

$$=\pi\left[8-\frac{8}{3}-\left(4-\frac{1}{3}\right)\right]=\pi\left[4-\frac{8}{3}+\frac{1}{3}\right]=\pi\left[\frac{12-8+1}{3}\right]=\frac{5\pi}{3}$$ ✓

④ $z=x^2+y^2$, $z=4$

r	0	-1	1	-2	2
$z=r^2$	0	1	1	4	4

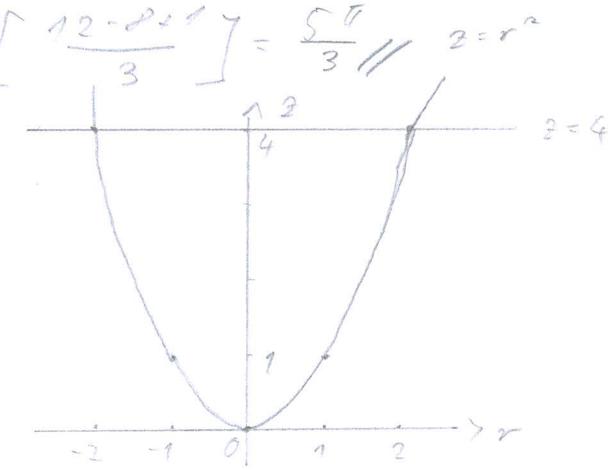
$z=r^2$

$r^2=z$ $\qquad \varphi\in[0,2\pi]$

$r^2=4$ $\qquad r\in[0,2]$

$r=\sqrt{4}$ $\qquad z\in[r^2,4]$

$r=2$



$$V=\int_0^{2\pi}\int_0^2\int_{r^2}^4 r\,dz\,dr\,d\varphi=2\pi\int_0^2 r(4-r^2)\,dr=2\pi\int_0^2(4r-r^3)\,dr$$

$$=2\pi\left[4\frac{r^2}{2}-\frac{r^4}{4}\right]\Big|_0^2=2\pi[8-4]=2\pi[4]=8\pi$$ ✓

FRANC ...

1) $y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0)=0, \; y'(0)=0, \; y''(0)=1$

$$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - 2\left[s^2Y(s) - sy(0) - y'(0)\right] + sY(s) - y(0) = \frac{2}{s^3}$$

$$s^3Y(s) - s^2\underbrace{y(0)}_{0} - s\underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{1} - 2s^2Y(s) + 2s\underbrace{y(0)}_{0} + 2\underbrace{y'(0)}_{0} + sY(s) - \underbrace{y(0)}_{=0} = \frac{2}{s^3}$$

$$s^3Y(s) - 1 - 2s^2Y(s) + sY(s) = \frac{2}{s^3}$$

$$s^3Y(s) - 2s^2Y(s) + sY(s) = \frac{2}{s^3} + 1$$

$$Y(s)\left(s^3 - 2s^2 + s\right) = \frac{s^3+2}{s^3}$$

$$Y(s) = \frac{\frac{s^3+2}{s^3}}{s^3-2s^2+s} = \frac{s^3+2}{s^3(s^3-2s^2+s)} = \frac{s^3+2}{s^4(s^2-2s+1)} = \frac{s^3+2}{s^4(s-1)^2}$$

$$= \frac{s^3+2}{s^4(s-1)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{(s-1)} + \frac{F}{(s-1)^2} \quad / \cdot s^4(s-1)^2$$

$$s^3+2 = As^3(s-1)^2 + Bs^2(s-1)^2 + Cs(s-1)^2 + D(s-1)^2 + Es^4(s-1) + Fs^4$$

$$s^3+2 = As^3(s^2-2s+1) + Bs^2(s^2-2s+1) + Cs(s^2-2s+1) + D(s^2-2s+1) + Es^5 - Es^4 + Fs^4$$

$$s^3+2 = As^5 - 2As^4 + As^3 + Bs^4 - 2Bs^3 + Bs^2 + Cs^3 - 2Cs^2 + Cs + Ds^2 - 2Ds + D + Es^5 - Es^4 + Fs^4$$

$$s^3+2 = (A+E)s^5 + (-2A+B-E+F)s^4 + (A-2B+C)s^3 + (B-2C+D)s^2 + (C-2D)s + D$$

$A+E=0$

$-2A+B-E+F=0$

$A-2B+C=1$

$B-2C=0$

$C-2D=0$

$\boxed{D=2}$

$C-2D=0$

$C-4=0$

$\boxed{C=4}$

$B-2C=0$

$B-8=0$

$\boxed{B=8}$

$A-2B+C=1$

$A-16+4=1$

$A=1+12$

$\boxed{A=13}$

$A=9$

$A+E=0$

$13+E=0$

$\boxed{E=-13}$

$-2\cdot 13 + 8 + 13 + F = 0$

$\boxed{F=5}$

$$Y(s) = 13\cdot\frac{1}{s} + 8\cdot\frac{1}{s^2} + 4\cdot\frac{1}{s^3} + 2\cdot\frac{1}{s^4} - 13\cdot\frac{1}{(s-1)} + 5\cdot\frac{1}{(s-1)^2}$$

$$Y(s) = 13\cdot\frac{1}{s} + 8\cdot\frac{1}{s^2} + \frac{4}{2}\cdot\frac{2!}{s^3} + \frac{2}{6}\cdot\frac{3!}{s^4} - 13\cdot\frac{1}{s-1} + 5\cdot\frac{1}{(s-1)^2}$$

$$y(t) = 13 + 8t + \frac{1}{2}t^2 + \frac{1}{3}t^3 - 13e^t + te^t$$

$y(0) = 13 + 8\cdot 0 + \frac{1}{2}\cdot 0^2 + \frac{1}{3}0^3 - 13e^0 + 0\cdot e^0 = 13 - 13 = 0$

$y'(0) = \left[8 + t + t^2 - 13e^t + e^t + te^t\right](t=0) = 8 - 13 + 1 = -4$

RJEŠENJE NE ODGOVARA

0

5.

$r'(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t^2 \end{pmatrix}, \quad \|r'(t)\| = \sqrt{(2\cos t)^2 + (2\sin t)^2 + (t^2)^2}$ $\quad f(x,y,z) = \sqrt{z}$

$t \in [-1,1]$

$= \sqrt{4\cos^2 t + 4\sin^2 t + t^4}$

$\int_\Gamma f\,ds = \int_{-1}^{1} f \cdot \|r'(t)\|\,dt$

$= \sqrt{4 + t^6} = \sqrt{4 + t^4}$

$r(t) = \begin{pmatrix} 2\sin t \\ -2\cos t \\ \frac{1}{3}t^3 \end{pmatrix}$

$\int_\Gamma f\,ds = \int_{-1}^{1} \sqrt{\frac{1}{3}t^3} \cdot \sqrt{4 + t^4}\,dt = \int_{-1}^{1}$

$\emptyset$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: LOVRE NIKITOVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 4$ za koji vrijedi $z \geq 1$. 15
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5. Zadana krivulja Γ s parametrizacijom $x = 2\cos t$, $y = 2\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,\mathrm{d}s$. ~~15~~
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~15~~

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 70

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

1. $r = 2$ $T(0,0)$ $\int (2x+3)\,ds$

$2k$ 12π

$x = 2\cos t$

$y = 2\sin t$

$r(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \end{pmatrix}$

$r'(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix}$

$\|r'(t)\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2}$

$= \sqrt{4\sin^2 t + 4\cos^2 t}$

$= \sqrt{4(\sin^2 t + \cos^2 t)}$

$= \sqrt{4} = 2$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi} (2 \cdot 2\cos t + 3) \cdot 2\,dt = \boxed{12\pi}$$

2. $r = 1$ $\quad T(2,1)$ $\quad \iint_K (2x+3)\,dx\,dy$ $\quad (s-1)^2$

$s^2 - 2s + 1$

$r \in [0,1]$
$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_0^1 [(2r\cos\varphi + 2) + 3]\, r\, dr\, d\varphi$$

$x = r\cos\varphi + 2$
$y = r\sin\varphi + 1$

$$\int_0^{2\pi}\int_0^1 [(2r\cos\varphi + 7)]\, r\, dr\, d\varphi$$

$$\int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 7r)\, dr\, d\varphi = \int_0^{2\pi} \left(2\cdot\frac{r^3}{3}\cos\varphi + 7\cdot\frac{r^2}{2}\right)\Big|_0^1 d\varphi$$

$$= \int_0^{2\pi}\left(2\cdot\frac{1}{3}\cos\varphi + 7\cdot\frac{1}{2}\right) d\varphi = \int_0^{2\pi}\left(\frac{2}{3}\cos\varphi + \frac{7}{2}\right) d\varphi$$

$$= \frac{2}{3}\sin\varphi + \frac{7}{2}\varphi\Big|_0^{2\pi} = \frac{2}{3}\sin 2\pi + \frac{7}{2}\cdot 2\pi = 0 + 7\pi = \boxed{7\pi}$$ ✓

6. $y'''(t) - 2y''(t) + y'(t) = t^2$ $\quad y(0) = 0 \quad y'(0) = 0 \quad y''(0) = 1$

$$s^3Y(s) - s^2\overset{=0}{y(0)} - s\overset{=0}{y'(0)} - \overset{=1}{y''(0)} - 2\cdot(s^2Y(s) - sy(0) - y'(0)) + sY(s) - y(0) = \frac{1}{s^3}$$

$$s^3Y(s) - s^2\overset{=0}{y(0)} - s\overset{=0}{y'(0)} - \overset{=1}{y''(0)} - 2s^2Y(s) + 2s\overset{=0}{y(0)} + 2\overset{=0}{y'(0)} + sY(s) - \overset{=0}{y(0)} = \frac{1}{s^3}$$

$$s^3Y(s) - 1 - 2s^2Y(s) + sY(s) = \frac{1}{s^3}$$

$$s^3Y(s) - 2s^2Y(s) + sY(s) = \frac{1}{s^3} + 1 = \frac{1+s^3}{s^3}$$

$$Y(s)(s^3 - 2s^2 + s) = \frac{s^3+1}{s^3}$$

$$\frac{\frac{s^3+1}{s^3}}{\frac{s^3-2s^2+s}{1}} = \frac{s^3+1}{(s^3-2s^2+s)s^3} = \frac{s^3+1}{s(s^2-2s+1)s^3}$$

$$\frac{s^3+1}{s^4(s-1)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{Es+F}{(s-1)^2} \Big/ s^4(s-1)^2$$

$s^3+1 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{Es+F}{(s-1)^2} \quad / \; s^4(s-1)^2$ LOVRE NIKITOVIĆ

$= As^3(s-1)^2 + Bs^2(s-1)^2 + Cs(s-1)^2 + D(s-1)^2 + (Es+F)s^4$

$= As^3(s^2-2s+1) + Bs^2(s^2-2s+1) + Cs(s^2-2s+1) + D(s^2-2s+1) + Es^5 + Fs^4$

$= \cancel{As^5} - 2As^4 + \cancel{As^3} + \cancel{Bs^4} - 2\cancel{Bs^3} + \cancel{Bs^2} + \cancel{Cs^3} - 2\cancel{Cs^2} + Cs + \cancel{D}s^2 - 2Ds + \cancel{D}$
$+ \cancel{Es^5} + Fs^4$

$= (A+E)s^5 + (-2A+B+F)s^4 + (A-2B+C)s^3 + (2B-2C+D)s^2$
$+ (C-2D)s + D$

$s^3+1 =$ $\boxed{D=1}$

$C-2=0$

$\boxed{C=2}$

$2B-4+1=0$

$2B=-5$

$\boxed{B=-\frac{5}{2}}$

$A-5-2=1$

$A=1+5+2$

$\boxed{A=8}$ ×

$A=9$

$8+E=0$

$\boxed{E=-8}$ ×

$-16-\frac{5}{2}+F=0$

$F=\frac{5}{2}+16$

$=\frac{5+32}{2}=\boxed{\frac{37}{2}}$

$\frac{8}{s} - \frac{\frac{5}{2}}{s^2} + \frac{2}{s^3} + \frac{1}{s^4} + \frac{-8s+\frac{37}{2}}{(s-1)^2}$

$= 8 - \frac{5}{2}t + \frac{1}{s^2} \ldots t^2 + t^3 \ldots \frac{s}{(s-1)^2} + \frac{37}{2}\cdot\frac{1}{(s-1)^2}$

$= 8 - \frac{5}{2}t + 2t^2 + t^3 \ldots e^{t} + \frac{37}{2} \ldots e^{t}$

4. $z = x^2 + y^2$ $\quad z = 4$

$x^2 + y^2 = r^2$

$r^2 = 4$

$r = \sqrt{4}$

$r \in [0, 2]$

$\varphi \in [0, 2\pi]$

$z = r^2$

$z \in [r^2, 4]$

$$\int_0^{2\pi}\int_0^2\int_{r^2}^4 r \, dz\, dr\, d\varphi = \int_0^{2\pi}\int_0^2 (4 - r^2)\, r \, dr\, d\varphi$$

$$= \int_0^{2\pi}\int_0^2 (4r - r^3)\, dr\, d\varphi = \int_0^{2\pi} \left(4 \cdot \frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^2 d\varphi$$

$$= \int_0^{2\pi} \left(2r^2 - \frac{r^4}{4}\right)\Big|_0^2 d\varphi = \int_0^{2\pi} \left(2 \cdot 2^2 - \frac{2^4}{4}\right)$$

$$= \int_0^{2\pi} (8 - 4)\, d\varphi = 4\varphi \Big|_0^{2\pi} = 4 \cdot 2\pi = \boxed{8\pi}$$ ✓

5. $x = 2\cos t \quad y = 2\sin t \quad z = t^2 \quad t \in [-1, 1]$

$$r(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t^2 \end{pmatrix} = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix}$$

$\varnothing$

$$\|r'(t)\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1^2}$$ ✗

$$= \sqrt{4\sin^2 t + 4\cos^2 t + 1}$$

$$= \sqrt{5}$$

$$\int_{-1}^{1} \sqrt{5}\, dt = \sqrt{5} \cdot 1 - \sqrt{5} \cdot (-1) = \sqrt{5} + \sqrt{5} = \boxed{5}$$

LOVRE NIKITOVIĆ

3. $x^2+y^2+z^2=4 \qquad z \geq 1$

$x^2+y^2+z^2=R$

$R^2=4$

$R=2$

$x^2+y^2+z^2=4$

$x^2+y^2=r^2$

$r^2+z^2=4$

$z=1$

$r^2+1=4$

$r^2=3$

$r=\sqrt{3}$

$x=r\cos\varphi$

$y=r\sin\varphi$

$dx\,dy\,dz=r\,dr\,d\varphi$

$\varphi\in[0,2\pi]$

$r\in[0,\sqrt{3}]$

$z\in[1,\sqrt{4-r^2}]$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r\,dz\,dr\,d\varphi = 2\pi\int_0^{\sqrt{3}} r\left(\sqrt{4-r^2}-1\right)dr$$

$$=2\pi\int_0^{\sqrt{3}}\left(r\sqrt{4-r^2}-r\right)dr = \begin{bmatrix} 4-r^2=t \\ -2r\,dr=dt \\ r\,dr=-\frac{1}{2}dt \end{bmatrix}$$

$$2\pi\int_{t_1}^{t_2}\sqrt{t}\cdot\left(-\frac{1}{2}dt\right)-2\pi\left(\frac{r^2}{2}\right)\Big|_0^{\sqrt{3}}$$

$$=2\pi\cdot\left(-\frac{1}{2}\right)\int_{t_1}^{t_2} t^{\frac{1}{2}}\,dt-2\pi\frac{(\sqrt{3})^2}{2}=-\pi\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\Big|_{t_1}^{t_2}-3\pi$$

$$=-\pi\cdot\frac{2}{3}\sqrt{(4-r^2)^3}\Big|_0^{\sqrt{3}}-3\pi=-\frac{2}{3}\pi\left(\sqrt{(4-\sqrt{3}^2)^3}-\sqrt{(4-0^2)^3}\right)-3\pi$$

$$=-\frac{2}{3}\pi(1-8)-3\pi=\frac{14\pi}{3}-3\pi=\frac{14\pi-9\pi}{3}=\boxed{\frac{5\pi}{3}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: BORIS PUDELKO **BROJ INDEKSA:** IA-1-0039-2010

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. ~~15~~
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 4$. ~~15~~ 12
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Ukupno: 67

Tablica Laplaceovih transformacija:

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1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

BORIS PUDELKO

1.) $\int\limits_{\partial K} (2x+3)\, ds \qquad r=2 \qquad T(0,0)$

$$\int\limits_{\partial K} f\, ds = \int_a^b (f\circ r)\cdot \|r'\|(t)\, dt$$

$$r = \begin{pmatrix} 2\cos t \\ 2\sin t \end{pmatrix} \qquad r' = \begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix}$$

$$\|r'\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2} = \sqrt{4(\sin^2 t + \cos^2 t)}$$

$$= \sqrt{4} = 2$$

$$= \int_0^{2\pi} \left(2(2\cos t)+3\right)\cdot 2\, dt = \int_0^{2\pi} (4\cos t + 3)\cdot 2\, dt$$

$$= \int_0^{2\pi} 8\cos t + 6\, dt = 8\underbrace{[\sin t]_0^{2\pi}}_{=0} + 6\,[t]_0^{2\pi} = 12\pi$$

2.) $\iint\limits_K (2x+3)\, dx\, dy \qquad r=1 \qquad T(2,1)$

$$\iint\limits_K (w\,|\,dr) = \iint\limits_D w(r)\cdot \vec{n} \qquad w = \begin{pmatrix} 0 \\ 0 \\ 2x+3 \end{pmatrix}$$

$$r = \begin{pmatrix} u \\ v \\ \sqrt{1-u^2-v^2} \end{pmatrix}$$ Parametrizacija

$$\vec{n} = \frac{dr}{du} \times \frac{dr}{dv}$$

$$\vec{n} = \begin{pmatrix} 1 \\ 0 \\ \frac{-u}{\sqrt{1-u^2-v^2}} \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{-v}{\sqrt{1-u^2-v^2}} \end{pmatrix} = \begin{pmatrix} \frac{u}{\sqrt{1-u^2-v^2}} \\ \frac{v}{\sqrt{1-u^2-v^2}} \\ 1 \end{pmatrix}$$

$1 \quad 0$

$0 \quad 1$

$$= \iint_D \begin{pmatrix} 0 \\ 0 \\ 2u+3 \end{pmatrix} \cdot \vec{n} \, du\,dv = \iint_D 2u+3 \, du\,dv$$

$u = r\cos\varphi + 2$

$v = r\sin\varphi + 1$

$$= \int_0^{2\pi}\int_0^1 \left(2(r\cos\varphi + 2) + 3\right) \cdot r \, dr\,d\varphi = \int_0^{2\pi}\int_0^1 (2r\cos\varphi + 7) \cdot r \, dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^1 2r^2\cos\varphi + 7r \, dr\,d\varphi = \int_0^{2\pi} 2\cos\varphi \left[\frac{r^3}{3}\right]_0^1 + 7\left[\frac{r^2}{2}\right]_0^1 d\varphi$$

$$= \frac{2}{3}\int_0^{2\pi}\cos\varphi \, d\varphi + \frac{7}{2}\int_0^{2\pi} d\varphi = \frac{2}{3}\underbrace{\left[\sin\varphi\right]_0^{2\pi}}_{=0} + \frac{7}{2}\left[\varphi\right]_0^{2\pi}$$

$$= \frac{7}{2} \cdot 2\pi = 7\pi \quad ✓$$

3) $x^2 + y^2 + z^2 = 4$ $\qquad z \geq 1$

$r^2 + z^2 = 16$ $\times$

$r = \sqrt{16 - z^2}$

$r = \sqrt{15}$

$z = \sqrt{16 - r^2}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{15}]$

$z \in [1, \sqrt{16 - r^2}]$

$$\underbrace{V = \int_0^{2\pi}\int_0^{\sqrt{15}}\int_1^{\sqrt{16-r^2}} 1 \cdot r \, dz\,dr\,d\varphi}$$ $\times$

BORIS PUDELKO

5.) $\int_\Gamma f\,ds \qquad x = 2\cos t \qquad y = 2\sin t \qquad z = t^2 \qquad t \in [-1,1]$

$$f(x,y,z) = \sqrt{z}$$

$$\int_\Gamma f\,ds = \int_a^b (f\circ r)\cdot \|r'\|(t)\,dt$$

$$r = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t^2 \end{pmatrix} \qquad r' = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 2t \end{pmatrix}$$

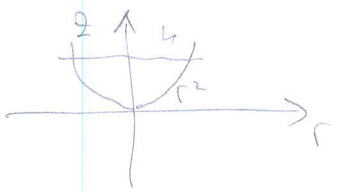
$$\|r'\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (2t)^2} = \sqrt{4\sin^2 t + 4\cos^2 t + 4t^2}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t) + 4t^2} = \sqrt{4 + 4t^2}$$

$$= \int_{-1}^{1} \sqrt{t^2}\cdot\sqrt{4+4t^2}\,dt = \int_{-1}^{1} t\sqrt{4+4t^2}\,dt = 0$$

t	-1	0	1
$f(t)$	$-2,8$	0	$2,8$

4.) $z = x^2 + y^2 \qquad z = 4$



$r^2 = z$

$r^2 = 4$

$r = 2$

$r \in [0,2]$

$z \in [r^2, 4]$

$\varphi \in [0, 2\pi]$

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$$V = \int_0^{2\pi}\int_0^2\int_{r^2}^4 1\cdot r\,dz\,dr\,d\varphi$$

6.) $y'''(t) - 2y''(t) + y'(t) = t^2$ $\qquad y(0)=0$

$y'(0)=0$

$y''(0)=1$

$$s^3 Y(s) - \cancel{s^2 \overset{0}{y(0)}} - \cancel{s \overset{0}{y'(0)}} - \underbrace{y''(0)}_{1} - 2\left(s^2 Y(s) - \cancel{s \overset{0}{y(0)}} - \cancel{\overset{0}{y'(0)}}\right)$$

$$+ s Y(s) - \cancel{\overset{0}{y(0)}} = \frac{2!}{s^{2+1}}$$

$$s^3 Y(s) - 1 - 2s^2 Y(s) + s Y(s) = \frac{2}{s^3}$$

$$s^3 Y(s) - 2s^2 Y(s) + s Y(s) = \frac{2}{s^3} + 1$$

$$Y(s)\left(s^3 - 2s^2 + s\right) = \frac{2+s^3}{s^3}$$

$$Y(s) = \frac{2+s^3}{s^3(s^3-2s^2+s)}$$

$$s\underbrace{(s^2-2s+1)}_{(s-1)^2}$$

$$\frac{s^3+2}{s^4(s-1)^2} = \frac{A}{s^4} + \frac{B}{s^3} + \frac{C}{s^2} + \frac{D}{s} + \frac{Es+F}{(s\pm1)^2}$$

$$\frac{E}{s-1} + \frac{F}{(s-1)^2}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: Luka Huljev BROJ INDEKSA: 58079	Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 4$ za koji vrijedi $z \geq 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 4$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 2\cos t$, $y = 2\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,\mathrm{d}s$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 55

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

① $r=2 \quad T(0,0)$ $\qquad \int_{\partial k} (2x+3)\, ds$

$x = r\cos t + 0 \Rightarrow x = 2\cos t$

$y = r\sin t + 0 \Rightarrow x = 2\sin t$

$r(t) = \begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$

$r'(t) = \begin{bmatrix} 2(-\sin t)\cdot 2 \\ 2\cos t \cdot 2 \end{bmatrix}$

$|r'(t)| = \sqrt{(2(-\sin t)\cdot 2)^2 + (2\cos t \cdot 2)^2}$

$|r'(t)| = \sqrt{\cancel{8\sin^2 t + 8\cos^2 t}}$

$|r'(t)| = \sqrt{\cancel{8(\sin^2 t + \cos^2 t)}}$

$= \sqrt{8}$

$\cancel{\int_0^{2\pi}\int_0^{2} (2(2\cos t)+3)\,2\,dt}$

$\cancel{\int_0^{2\pi}\int_0^{2} (8\cos t + 6)\,dt}$

$\cancel{\int_0^{2\pi} 8\int_0^{2} \cos t\,dt + 6\int_0^{2} dt}$

$\int_0^{2\pi} (2(2\cos t)+3)\,2\,dt$

$\int_0^{2\pi} (8\cos t + 6)\,dt$

$8\int_0^{2\pi} \cos t\,dt + 6\int_0^{2\pi} dt$

$2(-\sin t)$

$2\cos t$

$\sqrt{(2(-\sin t))^2 + (2\cos t)^2}$

$\sqrt{4\sin^2 t + 4\cos^2 t}$

$\sqrt{4}$

$\boxed{= 2\,dt}$

$\cancel{\sqrt{(4\sin^2 t\cdot 4) + (4\cos^2 t \cdot 4)}}$

$\cancel{= \sqrt{16\sin^2 t + 16\cos^2 t}}$

$\cancel{= \sqrt{16}}$

$\cancel{= 4\,dt}$

$8\left(\sin t \Big|_0^{2\pi}\right) + 6\left(t\Big|_0^{2\pi}\right)$

$8(\sin 2\pi - \sin 0) + 6(2\pi - 0)$

$\boxed{= 12\pi}$ ✓

(2.) $r=1 \quad T(2,1) \quad \iint (2x+3)\,dx\,dy$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$dx\,dy = r\,dr\,d\varphi$

$$\int_0^{2\pi}\int_0^1 \left(2(r\cos\varphi+2)+3\right) r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1 (2r\cos\varphi + 7)\, r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 7r)\,dr\,d\varphi$$

$$\int_0^{2\pi}\left(2\int_0^1 r^2\cos\varphi\,dr + 7\int_0^1 r\,dr\right) d\varphi$$

$$\int_0^{2\pi}\left(2\frac{r^3}{3}\cos\varphi\Big|_0^1 + 7\frac{r^2}{2}\Big|_0^1\right) d\varphi$$

$$\int_0^{2\pi}\left(\frac{2}{3}\cos\varphi + \frac{7}{2}\right) d\varphi$$

$$\frac{2}{3}\int_0^{2\pi}\cos\varphi\,d\varphi + \frac{7}{2}\int_0^{2\pi} d\varphi$$

$$\frac{2}{3}\left(\sin\varphi\Big|_0^{2\pi}\right) + \frac{7}{2}\left(\varphi\Big|_0^{2\pi}\right)$$

$$\frac{2}{3}(\sin 2\pi - \sin 0) + \frac{7}{\cancel{2}}(2\pi - 0)$$

$$= \frac{7}{\cancel{2}}\,2\pi = \boxed{7\pi}$$

3. $x^2+y^2+z^2=4 \quad z \geq 1$

Luka Hugjev

$r^2+z^2=4$

$r^2=4-z^2 \,/\sqrt{\ }$

$r=\sqrt{4-z^2}$

$r \in [0, \sqrt{4-z^2}]$

$z \in [1,2]$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi$$

$$\int_0^{2\pi}\int_1^{2}\left(\frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}}\right) dz\,d\varphi$$

$$\int_0^{2\pi}\int_1^{2}\left(\frac{4-z^2}{2}\right) dz\,d\varphi$$

$$\int_0^{2\pi}\frac{1}{2}\int_1^{2}(4-z^2)\,dz\,d\varphi$$

$$\int_0^{2\pi}\left(\frac{1}{2}\left(4\int_1^2 dz-\int_1^2 z^2\,dz\right)\right)d\varphi$$

$$\int_0^{2\pi}\left(\frac{1}{2}\left(4\left(z\Big|_1^2\right)-\left(\frac{z^3}{3}\Big|_1^2\right)\right)\right)d\varphi$$

$$\int_0^{2\pi}\left(\frac{1}{2}\left(4-\left(\frac{8}{3}-\frac{1}{3}\right)\right)\right)d\varphi$$

$r^2=\sqrt{4}$

$r=2$

$4-\frac{7}{3}$

$\frac{12}{3}-\frac{7}{3}=\frac{5}{3}$

$$\int_0^{2\pi}\left(\frac{1}{2}\left(\frac{5}{3}\right)\right)d\varphi$$

$$\int_0^{2\pi}\frac{5}{6}\,d\varphi$$

$$\frac{5}{6}\int_0^{2\pi} d\varphi$$

$$\frac{5}{6}\left(\varphi\Big|_0^{2\pi}\right)$$

$$\frac{5}{6}(2\pi-0)$$

$$\frac{5}{\cancel{6}}\,2\pi$$

$$=\boxed{\frac{5}{3}\pi}$$

✓

$$\int (8\cos t + 6)\,4\,dt$$

$$\int_0^{2\pi} (32\cos t + 24)\,dt$$

$$32\int_0^{2\pi} \cos t\,dt + 24\int_0^{2\pi} dt$$

$$32 \sin t \Big|_0^{2\pi} + 24\, t \Big|_0^{2\pi}$$

$$32(\cancel{\sin 2\pi})0 + 24(2\pi)$$

$$48\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: NINO MIKULANDRA **BROJ INDEKSA:** 57645	

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20 20
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Ukupno: 52

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
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$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

NINO MIKULANDRA 57645

2.) $\iint\limits_K (2x+3)\,dx\,dy$ $\quad T(2,1)\quad r=1$

$$(x-2)^2+(y-1)^2=1$$

$$x-2=r\cos\varphi \Rightarrow x=r\cos\varphi+2$$
$$y-1=r\sin\varphi \Rightarrow y=r\sin\varphi+1$$

$$\varphi\in[0,2\pi]$$
$$r\in[0,1]$$

$$\iint\limits_K (2x+3)\,dx\,dy=\int_0^{2\pi}\int_0^1 [2(r\cos\varphi+2)+3]\,r\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^1 (4r\cos\varphi+4+3)\,r\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^1 (4r\cos\varphi+7)\,r\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^1 (4r^2\cos\varphi+7r)\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(4\frac{r^3}{3}\cos\varphi+7r\right)dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(4\frac{r^3}{3}\cos\varphi+7\frac{r^2}{2}\right)\Bigg|_0^1 d\varphi=$$

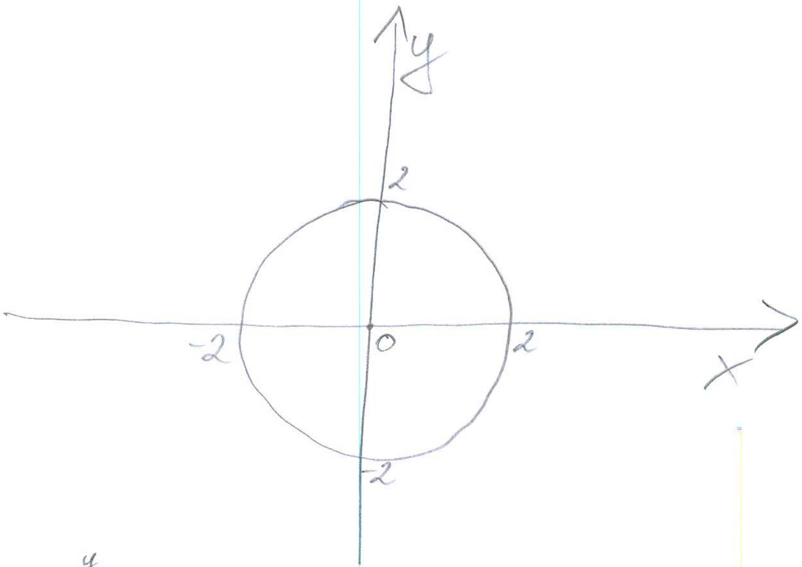
$$=\int_0^{2\pi}\left(\frac{4}{3}\cos\varphi+\frac{7}{2}\right)d\varphi=\frac{4}{3}\int_0^{2\pi}\cos\varphi\,d\varphi+\frac{7}{2}\int_0^{2\pi}d\varphi=$$

$$=\frac{4}{3}\sin\varphi\Bigg|_0^{2\pi}+\frac{7}{2}\varphi\Bigg|_0^{2\pi}=\frac{4}{3}(\overset{=0}{\sin 2\pi}-\overset{=0}{\sin 0})+\frac{7}{2}(2\pi-0)=$$

$$=\frac{7}{\cancel{2}}\cdot\cancel{2}\pi=7\pi$$

NINO MIKULANDRA 57645

1.) $\int\limits_{\partial K} (2x+3)\,ds \quad T(0,0) \quad \underline{r=2}$



$r(t) = (\overset{x}{r\cos t},\ \overset{y}{r\sin t})$

$r(t) = (2\cos t\ ,\ 2\sin t) \quad \underline{r=2}$

$r'(t) = (-2\sin t\ ,\ 2\cos t)$

$\|r'(t)\| = \sqrt{4\sin^2 t + 2\cos^2 t} =$

$= \sqrt{4(\sin^2 t + \cos^2 t)} =$

$= 2$

$(f\circ r)(t) = f[x(t), y(t)]$

$= \int\limits_0^{2\pi} (2\cdot 2\cos t + 3)\cdot 2\,dt$

$= \int\limits_0^{2\pi} 8\cos t\,dt + 6\int\limits_0^{2\pi} dt = 8\sin t\Big|_0^{2\pi} + 6t\Big|_0^{2\pi} =$

$= 8(\underset{0}{\sin 2\pi} - \underset{0}{\sin 0}) + 6(2\pi - 0) = 12\pi$

NINO MIKULANDRA 57645

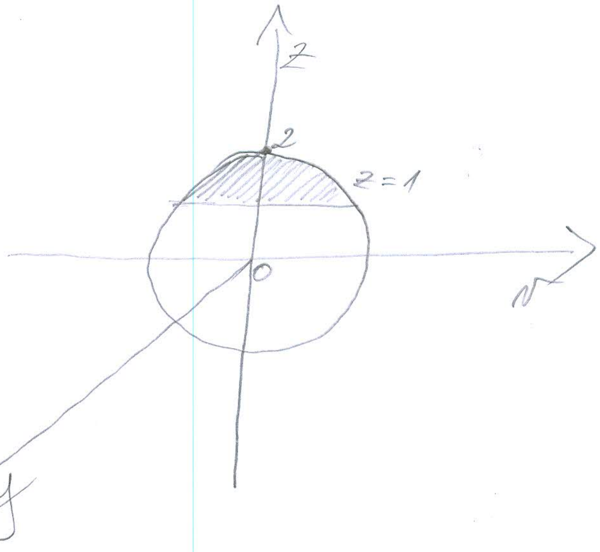
3.) $x^2+y^2+z^2=4 \quad \underline{z \geq 1}$

$\underbrace{x^2+y^2}+z^2=4$

$r^2+z^2=$

$r^2=4-z^2 \,/\sqrt{\ }$

$r=\pm\sqrt{4-z^2}$



$=\int_0^{2\pi} d\varphi \int_1^2 dz \int_0^{\sqrt{4-z^2}} r\,dr = \int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dr\,d\varphi\,dz$

$=\int_0^{2\pi} d\varphi \int_1^2 \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz\,d\varphi = \int_0^{2\pi}\int_1^2 \frac{4-z^2}{2}\,dz\,d\varphi$ $\quad \underline{12}$

$=\int_0^{2\pi}\int_1^2 \left(\frac{4}{2}-\frac{z^2}{2}\right) dz\,d\varphi = \int_0^{2\pi} \left(\frac{4}{2}z-\frac{z^3}{3}\right)\Big|_{z=1}^{z=2} d\varphi = \ldots$

$=$

NINO MIKULANDRA 57645

5.) $x = 2\cos t$, $y = 2\sin t$ i $z = t^2$ $t \in [-1, 1]$

$f(x,y,z)$ $\int f ds = ?$

$$r(t) = \begin{bmatrix} 2\cos t \\ 2\sin t \\ t^2 \end{bmatrix} \quad t \in [-1, 1]$$

$$r'(t) = \begin{bmatrix} -2\sin t \\ 2\cos t \\ 2 \end{bmatrix}$$

$$\|r'(t)\| = \sqrt{\underbrace{(-2\cos t)^2 + (2\cos t)^2}_{=1} + 2^2}$$

$$= \sqrt{1 + 4} = \sqrt{5}$$

$f(x,y,z) = \sqrt{z}$

$\int_{r} f ds = ?$

$$\int_{r} f ds = \int_a^b (f \circ r) \|r'(t)\| dt$$

$f \circ r(t) =$?

$\cancel{0}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. **IME I PREZIME:** BERNARDO IGOTCAR **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} (2x+3)\,\mathrm{d}s$. 20
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Ukupno: 50

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

BERNARDO KOTLAR

2. $r = 1$ $\quad T = (2,1)$ $\quad \iint_K (2x+3)\,dx\,dy$ $\quad r \in [0,1]$, $\varphi \in [0, 2\pi]$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$dx\,dy = r\,dr\,d\varphi$

$$\iint_K (2x+3)\,dx\,dy = \int_0^{2\pi}\int_0^1 (2r\cos\varphi + 4 + 3)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 7r)\,dr\,d\varphi$$

$$\int_0^{2\pi}\left(\cos\varphi \cdot 2\frac{r^3}{3} + 7\cdot\frac{r^2}{2}\right)\Big|_0^1 d\varphi = \int_0^{2\pi}\left(\frac{2}{3}\cos\varphi + \frac{7}{2}\right)d\varphi = \frac{2}{3}\sin\varphi + \frac{7}{2}\varphi\Big|_0^{2\pi}$$

$$\frac{2}{3}\underbrace{\sin 2\pi}_{0} + \frac{7}{2}2\pi - \left(\frac{2}{3}\underbrace{\sin 0}_{0} + \underbrace{\frac{7}{2}\cdot 0}_{0}\right) = 7\pi$$

---

3. $x^2 + y^2 + z^2 = R^2$

$x^2 + y^2 = r^2$ $\qquad r^2 + z^2 = 4$

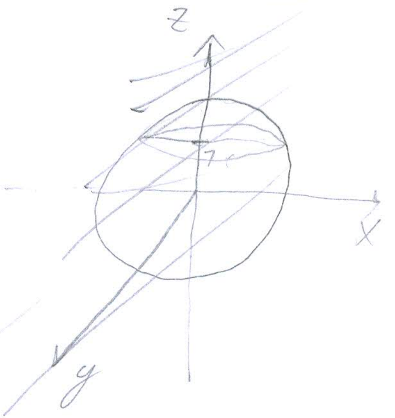
$r^2 + z^2 = 4$ $\qquad z^2 = 4 - r^2$

$z = 1$ $\qquad z = \sqrt{4 - r^2}$

$r^2 + 1 = 4$ $\qquad \varphi \in [0, 2\pi]$

$r^2 = 4 - 1$ $\qquad r \in [0, \sqrt{3}]$

$r = \sqrt{3}$ $\qquad z \in [1, \sqrt{4 - r^2}]$

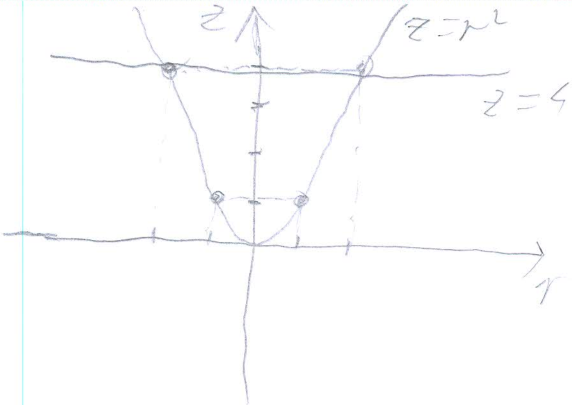


$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}} r\cdot(\sqrt{4-r^2})\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}} (r\sqrt{4-r^2})\,dr$$

$$\int_0^{2\pi} \frac{r^2}{2}\cdot\frac{1}{2}\left[r\sqrt{2^2 - r^2} + 2^2\arcsin\left(\frac{r}{2}\right)\right]\Big|_0^{\sqrt{3}} d\varphi$$

$$\int_0^{2\pi}\left(\frac{3}{4}\left[\sqrt{3}\cdot\sqrt{4-3} + 4\arcsin\left(\frac{\sqrt{3}}{2}\right)\right]\right)d\varphi = \int_0^{2\pi} \frac{5\sqrt{3}}{4}$$

(4.) $z = x^2+y^2 \quad z=4$



$x^2+y^2=r^2$

$z=r^2$

r	0	1	-1	2	-2
$z=r^2$	0	1	1	4	4

$z\in[r^2,4]$

$r^2=z$

$\varphi\in[0,2\pi]$

$z=4$

$r=\sqrt{4}$

$r\in[0,2]$

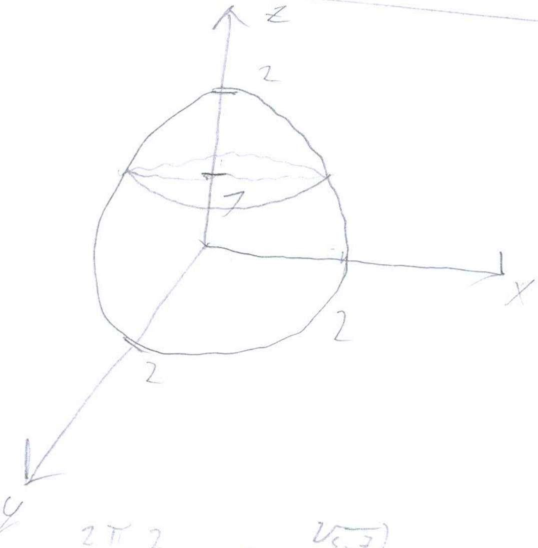
$r=2$

$$\int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} r\,dz\,dr\,d\varphi$$

$$2\pi\cdot\int_0^2 r\cdot(4-r^2)\,dr = 2\pi\cdot\int_0^2(4r-r^3)\,dr$$

$$2\pi\cdot\left(4\cdot\frac{r^2}{2}-\frac{r^4}{4}\right)\Big|_0^2 = 2\pi\cdot\left(8-\frac{16}{4}\right)=2\pi\cdot 4 = 8\pi$$

(3.) $x^2+y^2+z^2=4$ $\quad z\geq 1$



$x^2+y^2+z^2=R^2$

$z\in[1,2]$

$R^2=4$

$r\in[0,\sqrt{4-z^2}]$

$R=2$

$\varphi\in[0,2\pi]$

$x^2+y^2=r^2$

$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$

$$\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_1^{2}\frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}}dz\,d\varphi \rightarrow$$

$$\int_0^{2\pi}\int_1^{2}\frac{(\sqrt{4-z^2})^2}{2}\,dz\,d\varphi = 2\pi\cdot\int_1^2\frac{4}{2}-\frac{z^2}{2}\,dz = 2\pi\cdot\left(2z-\frac{1}{2}\frac{z^3}{3}\right)\Big|_1^2$$

BERNARDO KOTLAR

3. $2\pi \cdot \left(4 - \frac{8}{6} - \left(2 - \frac{1}{6}\right)\right)$

$= 2\pi \cdot \left(4 - \frac{8}{6} - 2 + \frac{1}{6}\right) = 2\pi \cdot \left(2 - \frac{7}{6}\right) = 2\pi \cdot \left(\frac{12}{6} - \frac{7}{6}\right)$

$= 2\pi \cdot \frac{5}{6} = \frac{10}{6}\pi = \frac{5}{3}\pi$ //

BERNARDO ICOTCAR

6. $y'''_{(t)} - 2y''(t) + y'(t) = t^2 \qquad y(0)=0 \;,\; y'(0)=0 \quad y''(0)=1$

$$s^3 Y_{(s)} - s^2 \underset{0}{y(0)} - s\underset{0}{y'(0)} - \underset{1}{y''(0)} - 2\cdot\left(s^2 Y_{(s)} - s\underset{0}{y(0)} - \underset{0}{y'(0)}\right) + sY_{(s)} - \underset{0}{y(0)} = \frac{2}{s^3}$$

$$Y_{(s)}\cdot(s^3-2s^2+s) - 1 = \frac{2}{s^3}$$

$$Y_{(s)} = \frac{2}{s^3\cdot(s^3-2s^2+s)} - 1 = \frac{2 - s^3\cdot(s^3-2s^2+s)}{s^3\cdot(s^3}$$

?

$\emptyset$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: IVAN GRZUNOV **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r=2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. ~~20~~
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. ~~20~~
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z\geq 1$. ~~15~~ 12
4. Izračunati volumen paraboloida omeđenog plohama: $z=x^2+y^2$, $z=4$. 15
5. Zadana krivulja Γ s parametrizacijom $x=2\cos t$, $y=2\sin t$ i $z=t^2$, $t\in[-1,1]$. Još je zadano $f(x,y,z)=\sqrt{z}$. Izračunati: $\int_\Gamma f\,\mathrm{d}s$. ~~15~~ 12
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t)-2y''(t)+y'(t)=t^2,\quad y(0)=0,\, y'(0)=0,\, y''(0)=1.$$

Ukupno: 39

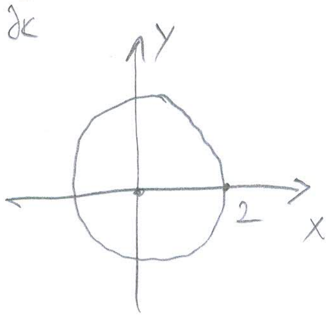
Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \sin x\,\mathrm{d}x=-\cos x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$
$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$
$\int \frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int \tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$		
$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$

1. $\int\limits_{\partial K} (2x+3)\, ds$



$x = r\cos t$

$y = r\sin t$

$r_{(t)} = r\cos t\ ,\ r\sin t$

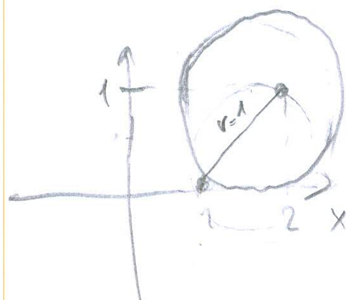
$r'_{(t)} = -r\sin t\ ,\ r\cos t$

$$\|r'_{(t)}\| = \sqrt{(-r\sin t)^2 + (r\cos t)^2}$$
$$= \sqrt{r^2 \sin^2 t + r^2\cos^2 t}$$
$$= \sqrt{r^2 (\sin^2 t + \cos^2 t)}$$
$$= \sqrt{r^2}$$
$$= r$$

$$\int_0^{2\pi} \big((2\cdot r\cos t + 3)\cdot r\big)\, dt = \int_0^{2\pi} 2r^2\cos t + 3r\, dt = 2\int_0^{2\pi} r^2\cos t\, dt + 3\int_0^{2\pi} r\, dt =$$

$$2r^2 \cdot \sin t\Big|_0^{2\pi} + 3rt\Big|_0^{2\pi} = \cancel{2r^2\cdot 0} - \cancel{2r^2\cdot 0} + 3r\cdot 2\pi - 0 = 3r\cdot 2\pi = 6r\pi$$

2. $\iint\limits_K (2x+3)\, dx\, dy$



$x = r\cos\varphi + 2$ ✓

$y = r\sin\varphi + 1$ ✓

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

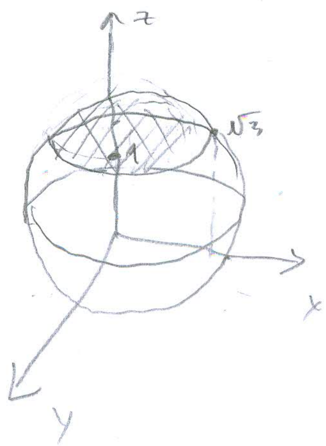
$$\int_0^{2\pi}\int_0^1 \big((2\cdot r\cdot\cos\varphi + 2) + 3)\cdot r\big)\, dr\, d\varphi = \int_0^{2\pi}\int_0^1 \big((2r\cos\varphi + 5)\cdot r\big)\, dr\, d\varphi =$$

$$\int_0^{2\pi}\int_0^1 2r^2\cos\varphi + 5r\, dr\, d\varphi = \int_0^{2\pi}\int_0^1 2r^2\cos\varphi\, dr\, d\varphi + \int_0^{2\pi}\int_0^1 5r\, dr\, d\varphi = \int_0^{2\pi}\Big(2\frac{r^3}{3}\cos\varphi\Big|_0^1\Big)\, d\varphi +$$

$$\int_0^{2\pi}\Big(5\frac{r^2}{2}\Big|_0^1\Big)\, d\varphi = \int_0^{2\pi} \frac{2}{3}\cos\varphi\, d\varphi + \int_0^{2\pi}\frac{5}{2}\, d\varphi = \frac{2}{3}\sin\varphi\Big|_0^{2\pi} + \frac{5}{2}\varphi\Big|_0^{2\pi} = \frac{5}{\cancel{2}}\cdot \cancel{2}\pi = 5\pi$$

IVAN GRZUNOV

3.



$x = r\cos\varphi$

$y = r\sin\varphi$

$(r\cos\varphi)^2 + (r\sin\varphi)^2 + 1^2 = 4$

$r^2\cos^2\varphi + r^2\sin^2\varphi + 1 = 4$

$r^2(\cos^2\varphi + \sin^2\varphi) + 1 = 4$

$r^2 = 4 - 1$

$r = \sqrt{3}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{3}]$

$z \in [1, \sqrt{4-r^2}]$

$(r\cos\varphi)^2 + (r\sin\varphi)^2 + z^2 = 4$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 4$

$r^2(\cos^2\varphi + \sin^2\varphi) + z^2 = 4$

$r^2 + z^2 = 4$

$z^2 = 4 - r^2$

$z = \sqrt{4-r^2}$

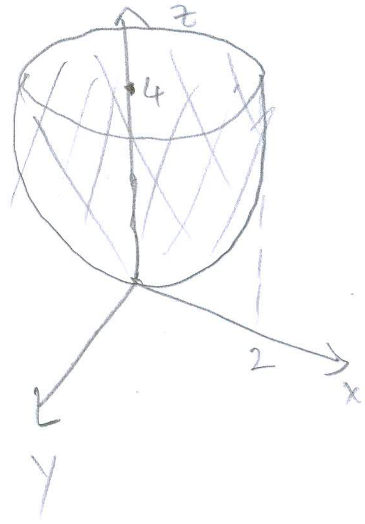
$$V = \int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r\,dz\,dr\,d\varphi =$$

$\underline{12}$

$$\int_1^{\sqrt{4-r^2}} r\,dz = r\cdot z\Big|_1^{\sqrt{4-r^2}} = r\cdot\sqrt{4-r^2} - \sqrt{4-r^2}$$

$$\int_0^{\sqrt{3}} r\sqrt{4-r^2}\,dr - \int_0^{\sqrt{3}} \sqrt{4-r^2}\,dr =$$

4.



$x = r\cos\varphi$

$y = r\sin\varphi$

$4 = (r\cos\varphi)^2 + (r\sin\varphi)^2$

$4 = r^2\cos^2\varphi + r^2\sin^2\varphi$

$4 = r^2(\cos^2\varphi + \sin^2\varphi)$

$4 = r^2$

$r = \sqrt{4} = 2$

$z = (r\cos\varphi)^2 + (r\sin\varphi)^2$

$z = r^2\cos^2\varphi + r^2\sin^2\varphi$

$z = r^2(\cos^2\varphi + \sin^2\varphi)$

$z = r^2$

$\varphi \in [0, 2\pi]$

$r \in [0, 2]$

$z \in [r^2, 4]$

$$\int_0^{2\pi}\int_0^2\int_{r^2}^4 r \, dz\, dr\, d\varphi =$$

$$\int_{r^2}^4 r\, dz = rz \Big|_{r^2}^4 = 4r - r^3$$

$$\int_0^2 4r\,dr - \int_0^2 r^3\, dr = 2r^2 \Big|_0^2 - \frac{r^4}{4}\Big|_0^2 = 8 - 4 = 4$$

$$\int_0^{2\pi} 4\, d\varphi = 4\varphi \Big|_0^{2\pi} = 4 \cdot 2\pi = 8\pi$$ ✓

5.

IVAN GRZUNOV

$r(t) = 2\cos t, 2\sin t, t^2$

$r'(t) = -2\sin t, 2\cos t, 2t$

$\|r'(t)\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (2t)^2}$

$= \sqrt{4\sin^2 t + 4\cos^2 t + 4t^2}$

$= \sqrt{4(\sin^2 t + \cos^2 t) + 4t^2}$

$= \sqrt{4 + 4t^2}$ ✓

$\int_{-1}^{1} \left|\sqrt{t^2} \cdot \sqrt{4+4t^2}\right| dt = \int_{-1}^{1} t \cdot \sqrt{4+4t^2}\, dt$ ✓

12

6. $y'''(t) - 2y''(t) + y'(t) = t^2 \qquad y(0)=0,\ y'(0)=0,\ y''(0)=1$

$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - (s^2Y(s) - sy(0) - y'(0)) + (sY(s) - y(0)) = \frac{2}{s^3}$

$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - s^2Y(s) + sy(0) + y'(0) + sY(s) - Y(0) = \frac{2}{s^3}$

$s^3Y(s) - 0 - 0 - 1 - s^2Y(s) + 0 + 0 + sY(s) - 0 = \frac{2}{s^3}$

$s^3Y(s) - 1 - s^2Y(s) + sY(s) = \frac{2}{s^3}$

$s^3Y(s) - s^2Y(s) + sY(s) = \frac{2}{s^3} + 1$

$Y(s)(s^3 - s^2 + s) = \frac{2}{s^3} + 1 \quad \Big| \cdot \frac{1}{(s^3 - s^2 + s)}$

$Y(s) = \dfrac{\frac{2}{s^3} + 1}{s^3 - s^2 + s} = \dfrac{2\frac{1}{s^3} + 1}{s(s^2 + s + 1)}$

$\dfrac{2\frac{1}{s^3} + 1}{s \cdot (s^3 + 1)} = \dfrac{A}{s} + \dfrac{Bs + C}{s^3 + 1} \quad \Big| \cdot s(s^3 + 1)$

$2\frac{1}{s^3} + 1 = A \cdot (s^3 + 1) + (Bs + C) \cdot s$

$= As^3 + A + Bs^2 + Cs$

$= s^3A + s^2B + sC + A$

$B = 0$

$C = 0$

$A = 1$

$Y(s) = \frac{1}{s} + \frac{0 \cdot s + 0}{s^3 + 1}$

$Y(s) = \frac{1}{s} \quad \Big/ \mathcal{L}^{-1}$

$Y(t) = 1$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. **IME I PREZIME:** LOVRE KOLEGA **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} (2x+3)\,ds$. 20 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint_K (2x+3)\,dxdy$. ~~20~~
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 4$ za koji vrijedi $z \geq 1$. ~~15~~ 12
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 4$. ~~15~~
5. Zadana krivulja Γ s parametrizacijom $x = 2\cos t$, $y = 2\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,ds$. ~~15~~
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~15~~

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 32

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int dx = x + C$	$\int \sin x\,dx = -\cos x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,dx = \sin x + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

LOVRÉ KOLEGA

1. $r=2$ $\quad T(0,0)$ $\quad x = 2\cos t$ $\quad t \in [0, 2\pi]$

$y = 2\sin t$

$$\int_{\partial K} (2x+3)\, ds$$

$r(t) = 2\cos t,\ 2\sin t$

$r'(t) = -2\sin t,\ 2\cos t$

$\|r'\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2}$

$\|r'\| = \sqrt{4\sin^2 t + 4\cos^2 t}$

$\|r'\| = \sqrt{4(\underbrace{\sin^2 t + \cos^2 t}_{1})}$

$\|r'\| = \sqrt{4} = 2$

$$\int_0^{2\pi} (4\cos t + 3)\cdot 2\, dt$$

$$\int_0^{2\pi} (8\cos t + 6)\, dt = 8\sin t\Big|_0^{2\pi} + 6t\Big|_0^{2\pi}$$

$$= 0 + 12\pi = 12\pi$$ ✓

2. $r=1$ $\quad \iint_K (2x+3)\, dx\, dy$

$T(2,1)$

$x = r\cos\varphi + 2$ $\quad \varphi \in [0, 2\pi]$

$y = r\sin\varphi + 1$ $\quad r \in [0, 1]$

$$\int_0^1\int_0^{2\pi} (2r\cos\varphi + 5)\, r\, d\varphi\, dr = \int_0^1\int_0^{2\pi} (2r^2\cos\varphi + 5r)\, d\varphi\, dr$$

$$= \int_0^1 2r^2\sin\varphi \Big|_0^{2\pi} + \int_0^1 5r\varphi\Big|_0^{2\pi} + \int_0^1 dr = \int_0^1 2r^2 + \int_0^1 10\pi r + \int_0^1 dr$$

$$= 2\frac{r^3}{3}\Big|_0^1 + 10\pi\frac{r^2}{2}\Big|_0^1 + r\Big|_0^1 = \frac{2}{3} + \frac{5}{2}\pi + 1$$

$\frac{2}{3} + 1 = \frac{2+3}{3} = \frac{5}{3}$

$$= \frac{5}{3} + \frac{5}{2}\pi$$

3. $x^2+y^2+z^2=4$ $\quad z \geq 1$ $\quad \varphi \in [0, 2\pi]$ $\quad 1{,}732$

$z=1$

$x^2+y^2+1=4$ $\quad T(0,0,0)$ $\quad r \in [0, \sqrt{3}]$

$x^2+y^2=3$ $\quad z \in [1, \sqrt{4-r^2}]$

$r=\sqrt{3}$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r\,dz\,dr\,d\varphi \quad ✓$$

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$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 4$

$r^2\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1} + z^2 = 4$

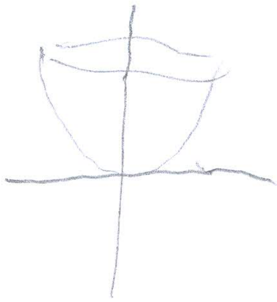
$z^2 = 4 - r^2$

$z = \sqrt{4-r^2}$

4. $z = x^2+y^2$ $\quad z=4$ $\quad \varphi \in [0, 2\pi]$

$x^2+y^2=4$ $\quad r \in [0, 2]$

$r=2$ $\quad z \in [4, r^2]$ ×



$z = x^2+y^2$

$z = r^2\cos^2\varphi + r^2\sin^2\varphi$

$z = r^2$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$\int_0^{2\pi}\int_0^{2}\int_4^{r^2} r\,dz\,dr\,d\varphi =$$ ×

∅

LOVRE KOLEGA

5. $x = 2\cos t \qquad t \in [-1, 1] \qquad (1^2)^2$

$y = 2\sin t$

$z = t^2 \qquad f(x,y,z) = \sqrt{z} \qquad 1^4$

$\int_\Gamma f\,ds$

$=$

$\|r'\| = \sqrt{(x')^2 + (y')^2 + (z')^2} = \sqrt{(2\cos t)^2 + (2\sin t)^2 + (1^2)^2}$

$\|r'\| = \sqrt{4\cos^2 t + 4\sin^2 t + 1}$

$\|r'\| = \sqrt{4(\underbrace{\cos^2 t + \sin^2 t}_{1}) + 1} = \sqrt{5}$

$\int_\Gamma f\,ds = \int_{-1}^{1} \sqrt{t^{\cancel{2}}} \cdot \sqrt{5}\,dt = \int_{-1}^{1} t\sqrt{5}\,dt = \sqrt{5}\,\frac{t^2}{2}\Big|_{-1}^{1}$

$= \sqrt{5}\,\frac{t^2}{2}\Big|_{-1}^{1} \cdot t\Big|_{-1}^{1} = 2\sqrt{5}$

6. $y'''(t) - 2y''(t) + y'(t) = t^2 \quad y(0)=0 \quad y'(0)=0 \quad y''(0)=1$

$$s^3 F_{(s)} - \cancel{s^2 f(0)} - \cancel{s f'(0)} - \underbrace{f''(0)}_{1} - 2\left(s^2 F_{(s)} - s f(0) - f'(0)\right)$$

$$+ s F_{(s)} - f(0) =$$

$$s^3 F_{(s)} - 1 - 2s^2 F_{(s)} + \cancel{2s f(0)} + \cancel{2 f'(0)} + s F_{(s)} - \cancel{f(0)}$$

$$s^3 F_{(s)} - 2s^2 F_{(s)} + s F_{(s)} = 1$$

$$F_s \left(s^3 - 2s^2 + s\right) = 1 \quad / : \left(s^3 - 2s^2 + s\right)$$

$$F_s = \frac{1}{s^3 - 2s^2 + s} = \frac{1}{s\left(s^2 - 2s + 1\right)} \qquad (s-1)^2$$

$$F_{(s)} = \frac{A}{s} + \frac{Bs + C}{s^2 - 2s + 1}$$

DALJE ... ?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: DINO KURIĆ **BROJ INDEKSA:** 56192-2008

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} (2x+3)\,\mathrm{d}s$. ~~20~~
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. ~~20~~
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 4$ za koji vrijedi $z \geq 1$. 15 12
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 4$. ~~15~~ 12
5. Zadana krivulja Γ s parametrizacijom $x = 2\cos t$, $y = 2\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,\mathrm{d}s$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 24

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

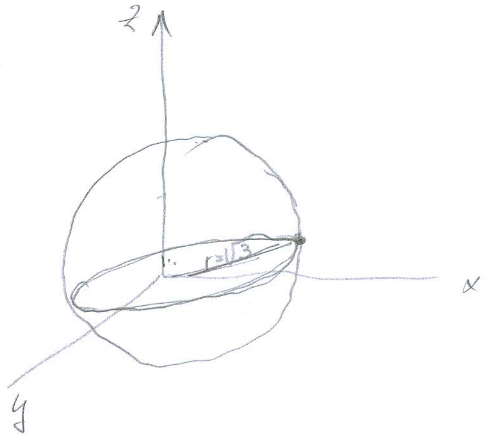
Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

③

DINO KURIĆ

$x^2 + y^2 + z^2 = 4 \quad z \geq 1$



$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{3}]$

$z \in [1, \sqrt{4-r^2}]$

$x^2 + y^2 + z^2 = 4$

$z = 1$

$x^2 + y^2 + 1^2 = 4$

$x^2 + y^2 = 4 - 1$

$x^2 + y^2 = 3$

$r = \sqrt{3}$

$(r\cos\varphi)^2 + (r\sin\varphi)^2 + z^2 = 4$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 4$

$r^2 + z^2 = 4$

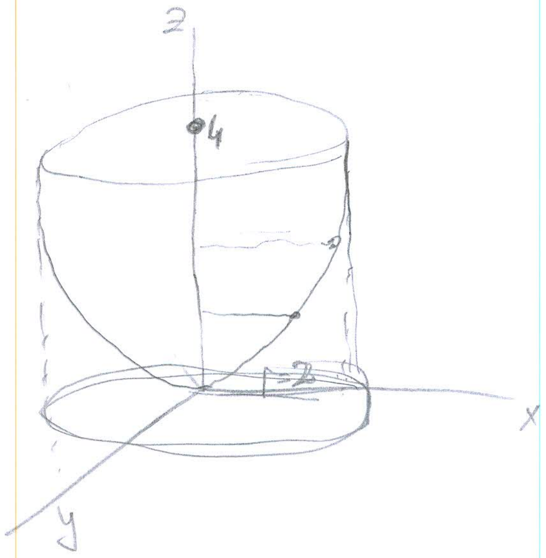
$z^2 = 4 - r^2 \quad / \sqrt{\ }$

$z = \sqrt{4-r^2}$

$V = \int_0^{2\pi} \int_0^{\sqrt{3}} \int_1^{\sqrt{4-r^2}} r \, dz \, dy \, dx =$ ✓

12

(4) $z = x^2 + y^2$, $z = 4$



$x^2 + y^2 = z$

$z = 4$

$x^2 + y^2 = 4$

$\varphi \in [0, 2\pi]$ $r = \sqrt{4}$

$r \in [0, 2]$ $\boxed{r = 2}$

$z \in [r^2, 4]$

$(r\cos\varphi)^2 + (r\sin\varphi)^2 = z$

$r^2 = z$

$z = r^2$

$$V = \int_0^{2\pi} \int_0^{2} \int_{r^2}^{4} r \, dz \, dr \, d\varphi =$$

✓

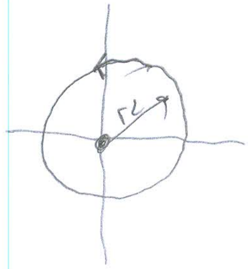
12

DINO KURIĆ

1. $r=2$ $\quad T(0,0)$ $\quad \int_{\partial K} (2x+3)\, ds$

$r(t) = (r\cos t + 0,\ r\sin t +)$ ×

$r'(t) = (-\sin t,\ \cos t)$ ×



$|r'(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2}$

$r(t) = (2\cos t, 2\sin t)$

$= \sqrt{\sin^2 t + \cos^2 t}$

$= 1$

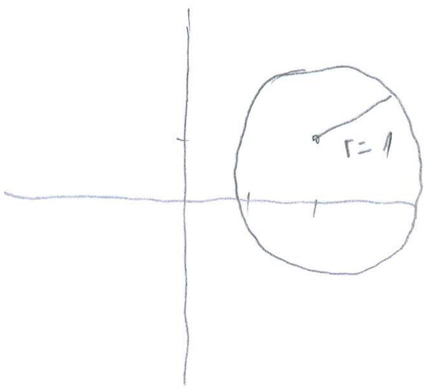
$\int_0^{2\pi} (2\cdot \cos t + 3)\, dt = 2\int_0^{2\pi} \cos t\, dt + \int_0^{2\pi} 3\, dt$ ×

$= 2\cdot \sin t \Big|_0^{2\pi} + 3t\Big|_0^{2\pi}$

$= \cancel{2\cdot \sin 2\pi} + 3\cdot 2\pi = 6\pi$ ×

---

2) K $\quad r=1$ $\quad T(2,1)$ $\quad \iint_K (2x+3)\, dx\, dy$



$x = r\cos\varphi + 2$ $\qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi + 1$ $\qquad r \in [0, 1]$

$\int_0^{2\pi} \left[\int_0^1 \left(2\cdot(r\cos\varphi + 1) + 3\right) r\, dr \right] d\varphi =$ ×

$\int_0^{2\pi} \left[\int_0^1 2r\cos\varphi + 2 + 3r\, dr \right] d\varphi$

$$\int_0^{2\pi}\left[\int_0^1 (2r\cos\varphi+5)\cdot r\,dr\right]d\varphi =$$

$$\int_0^{2\pi}\left[\int_0^1 2r^2\cdot\cos\varphi + \int_0^1 5r\,dr\right]d\varphi =$$

$$\int_0^{2\pi}\left[\cos\varphi\cdot\int_0^1 2r^2 + 5\int_0^1 r\,dr\right]d\varphi =$$

$$\int_0^{2\pi}\left[2\cos\varphi\int_0^1 r^2\,dr + 5\int_0^1 r\,dr\right]d\varphi$$

$$\int_0^{2\pi}\left[2\cos\varphi\,\frac{r^3}{3}\Big|_0^1 + 5\,\frac{r^2}{2}\Big|_0^1\right]d\varphi =$$

$$\int_0^{2\pi}\left[2\cos\varphi\,\frac{1}{3} + 5\cdot\frac{1}{2}\right]d\varphi$$

$$\int_0^{2\pi}\left[\frac{2}{3}\cdot\cos\varphi + \frac{5}{2}\right]d\varphi$$

$$\frac{2}{3}\int_0^{2\pi}\cos\varphi\,d\varphi + \frac{5}{2}\int_0^{2\pi}d\varphi$$

$$\frac{3}{2}\cdot\overset{0}{\sin 2\pi} + \frac{5}{2}\cdot 2\pi = \frac{\cancel{10}\,\pi}{\cancel{2}} = 5\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. **IME I PREZIME:** MATEJA MITROVIĆ **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2 + y^2$, $z = 4$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 2\cos t$, $y = 2\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,\mathrm{d}s$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = t^2, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 0

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\, f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\, f(t)$	$(-1)^n\, F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

Mateja Mitrović

$y'''(t) - 2y''(t) + y'(t) = t^2 \quad y(0)=0 \; y'(0)=0 \; y''(0)=1$

$= y'''(t) = s^3 F(s) - s^2 \underbrace{f(0)}_{0} - s \underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{1}$

$y'''(t) = s^3 F(s) - 1$

$= y''(t) = s^2 F(s) - s \underbrace{f(0)}_{0} - \underbrace{f'(0)}_{0}$

$y''(t) = s^2 F(s)$

$= y'(t) = s F(s) - \underbrace{f(0)}_{0} = s F(s)$

$s^3 F(s) - 1 - 2s^2 F(s) + s F(s) = \left(\frac{1}{s^2}\right)^2$

$s^3 F(s) - 1 - 2s^2 F(s) + s F(s) = \frac{1}{s^4}$

$(s^3 - 2s^2 + s) F(s) = \frac{1}{s^4} + 1$

$F(s) = \dfrac{s^3 - 2s^2 + s}{\frac{1}{s^4} + 1}$ 1

$1 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+s^4} \Big/ \cdot (s \cdot s^2 + s^3 + s + s^4)$

$1 = A \cdot (s^2 \cdot s^3 \cdot s + s^4) + B(s \cdot s^3 \cdot s + s^4) + D(s \cdot s^2 \cdot s^3)$

$1 = As^2 \, As^3 \, As + s^4 \,|+ Bs \, Bs^2 \, Bs + s^4 + Ds \, Ds^2 \, Ds^3$

0

1. $\int_0^0 \left(\frac{2x}{x} + \frac{3}{y}\right) ds + \int_0^2 \left(\frac{2x}{x} + \frac{3}{y}\right) ds = \frac{1}{2}\Big|_0^0 + \frac{1}{2}\Big|_0^2 = 0 + \left(\frac{1}{2}\right)\Big|^2 = 0 + 1 = 1$

2. $\int_0^1 (2x+3)\,dx\,dy + \int_1^2 (2x+3)\,dx\,dy = \frac{2x^2}{2} + 3y\Big|_0^1 + \frac{2x^2}{2} + 3y\Big|_1^2 =$

$= \left(\underbrace{\frac{2\cdot 1^2}{2} + 3\cdot 1}_{3}\right) - \left(\underbrace{\frac{2\cdot 0^2}{2} + 3\cdot 0}_{0}\right) + \left(\underbrace{\frac{2\cdot 2^2}{2} + 3\cdot 2}_{10}\right) - \left(\underbrace{\frac{2\cdot 1^2}{2} + 3\cdot 1}_{4}\right)$

$= 3 + 6 = 9$

4. $z = x^2 + y^2 \quad z = 4$

$x^2 + y^2 = r$

$4 = r$