MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK
IME I PREZIME: Lovro Sorić BROJ INDEKSA: 57638-2009	Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. — 20

2. Izračunati dvostruki integral: $\iint\limits_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 1 \text{ i } x \ge y\}$. — 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x + \mathrm{d}y)$. — 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral — 15
$$\iint\limits_X x^3 \mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? — 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 15
$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 55

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

1. $\iiint (2x+3)\,dx\,dy\,dz \qquad r=2$

Parametrizacija

$x = r\cos\varphi \qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi \qquad r \in [0, \sqrt{4-z^2}]$

$z = 2 \qquad z \in [-2, 2]$

$x^2+y^2+z^2 \leq 4$

$r^2 + 2^2 \leq 4$

$r \leq \sqrt{4-z^2}$

$$I = \int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}} (2r^2\cos\varphi + 3r)\,dr\,d\varphi\,dz = \int_{-2}^{2}\int_0^{\sqrt{4-z^2}} \Big[\underbrace{2r^2\sin\varphi \Big|_0^{2\pi}}_{0} + 3r\,\varphi\Big|_0^{2\pi}\Big]\,dr\,dz$$

$$= 6\pi \int_{-2}^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz = \frac{6\pi}{2}\int_{-2}^{2}(4-2^2)\,dz$$

$$= 3\pi\left[4\cdot 2\Big|_{-2}^{2} - \frac{2^3}{3}\Big|_{-2}^{2}\right]$$

$$= 3\pi\left(16 - \frac{16}{3}\right) = 3\pi\,\frac{48-16}{3} = \underline{32\pi}$$

Lovro Sorić

3. $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy)$

$$\begin{Bmatrix} x+y \\ x+y \end{Bmatrix} = -\operatorname{grad} f = -\begin{pmatrix} \partial_x f \\ \partial_y f \end{pmatrix}$$

$$\partial_x f = -x-y \quad / \int$$

$$f = \int (-x-y)\,dx = -\frac{x^2}{2} - xy + C(y)$$

$$\partial_y f = -x-y$$

$$0 - \cancel{x} + C'(y) = -\cancel{x} - y$$

$$C'(y) = -y \quad / \int$$

$$C(y) = -\int y\,dy$$

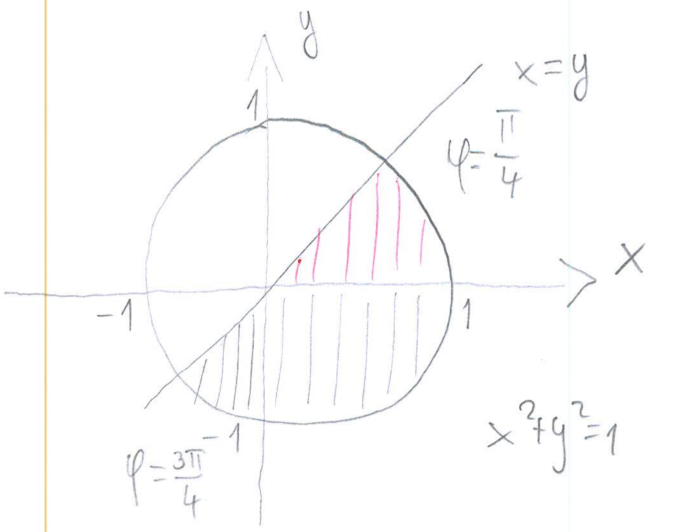
$$C(y) = -\frac{y^2}{2} + C_2$$

$$f(x,y) = -\frac{x^2}{2} - xy - \frac{y^2}{2} + C_2$$

$$I = \int_{(-1,2)}^{(2,3)} (x+y)dx + (x+y)dy = f(-1,2) - f(2,3)$$

$$= -\frac{1}{2} + 2 - \frac{4}{2} - \left(-\frac{4}{2} - 6 - \frac{9}{2}\right) = \underline{12}$$

② $\iint_S xy\,dx\,dy \qquad x^2+y^2 \leq 1$



Polarne koordinate

$x = r\cos\varphi$

$y = r\sin\varphi$

$r \in [0,1]$ ✓

$\varphi \in [-\frac{3\pi}{4}, \frac{\pi}{4}]$ ✓

$$\iint_S = xy\,dx\,dy = \int_0^1 \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} r\cos\varphi\, r\sin\varphi\, r\,d\varphi\,dr \checkmark$$

$$= \int_0^1 r^3 \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \cos\varphi \sin\varphi\, d\varphi = \frac{1}{4} \cdot \frac{\sin^2\frac{\pi}{4} - \sin^2(-\frac{3\pi}{4})}{2}$$

$$= \frac{1}{4} \cdot 0 = \underline{\underline{0}} \checkmark$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DOMAGOJ KNEŽEVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r=2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. ~~20~~ 15

2. Izračunati dvostruki integral: $\iint\limits_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq 1 \text{ i } x\geq y\}$. ~~20~~

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x+\mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral 15
$$\iint\limits_X x^3\mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r=1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~15~~
$$y'''(t)-2y''(t)+y'(t)=0,\quad y(0)=1,\, y'(0)=-1,\, y''(0)=1.$$

Ukupno: 45

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
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t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
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Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
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$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
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(1.) $r = 2$ $\iiint_K (2x+3) \, dx\,dy\,dz = ?$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$

$\varphi \in [0, 2\pi]$
$r \in [0, 2]$
$z \in [-\sqrt{4-r^2}, \sqrt{4-r^2}]$

$(x-0)^2 + (y-0)^2 + (z-0)^2 \leq 4$

$r^2(\cos^2\varphi + \sin^2\varphi) + z^2 \leq 4$

$r^2 + z^2 \leq 4$

$z^2 \leq 4 - r^2$

$I = \int_0^{2\pi}\int_0^2\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} (2r^2\cos\varphi + 3r) \, dz \, dr \, d\varphi =$

1.5

3. $\int_{(-1,2)}^{(2,3)} (x+y)\,dx\,dy$

$$w = \begin{bmatrix} -(x+y) \\ -x+y) \\ 0 \end{bmatrix} = -\operatorname{grad} f = \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -(x+y) \Rightarrow f = -\int (x+y)\,dx = -\frac{x^2}{2} - xy + C(y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial\left[-\frac{x^2}{2} - xy + C(y)\right]}{\partial y} = -x - y$$

$$\cancel{-x} + \frac{\partial C(y)}{\partial y} = \cancel{-x} - y$$

$$C(y) = -\int y\,dy = -\frac{y^2}{2} + C$$

$$f(x,y) = -\frac{x^2}{2} - xy - \frac{y^2}{2}$$

$$f(-1,2) - f(2,3) = -\frac{1}{2} \cancel{+2} \cancel{-2} - \left(-2 - 6 - \frac{9}{2}\right)$$

$$= -\frac{1}{2} + 8 + \frac{9}{2} = 12$$

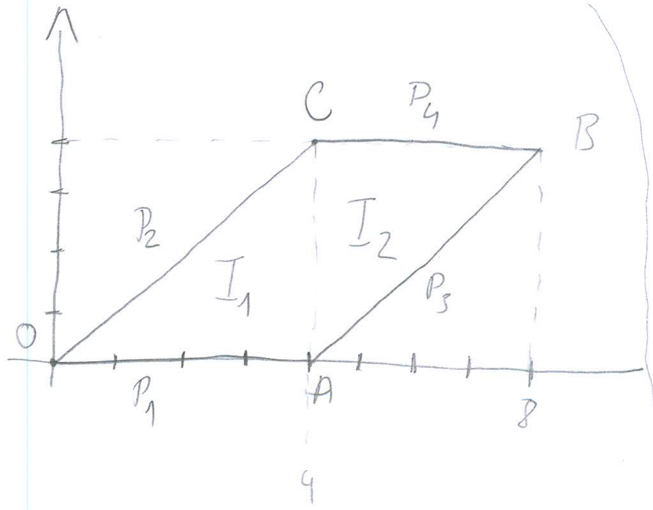
4. $O(0,0)$

$A(4,0)$

$B(8,4)$

$C(4,4)$

$$\iint\limits_X x^3\,dx\,dy$$



$P_1 \dots O(0,0)$, $A(4,0)$

$y=0$

$P_2 \dots O(0,0)$, $C(4,4)$

$y=\frac{4-0}{4-0}(x-0)$

$y=x$

P_3 $A(4,0)$, $B(8,4)$

$y=\frac{4-0}{8-4}(x-4)$

$y=x-4$

$P_4 \dots C(4,4)$, $B(8,4)$

$y-4=\frac{4-4}{8-4}(x-4)$

$y=4$

$I=I_1+I_2$

$$I_1=\int_0^4\int_{P_1}^{P_2} x^3\,dy\,dx=\int_0^4\int_0^x x^3\,dy\,dx=\int_0^4 x^3y\Big|_0^x dx=\int_0^4 x^4\,dx=\frac{x^5}{5}\Big|_0^4=\frac{1024}{5}$$

$$I_2=\int_4^8\int_{P_3}^{P_4} x^3\,dy\,dx=\int_4^8\int_{x-4}^4 x^3\,dy\,dx=\int_4^8 x^3y\Big|_{x-4}^4 dx=$$

$$=\int_4^8\left[x^3(4-x+4)\right]dx=\int_4^8(8x^3-x^4)\,dx=\left(2x^4-\frac{x^5}{5}\right)\Big|_4^8$$

$$I_2=\frac{6656}{5}$$

$$I=I_1+I_2=\frac{1024}{5}+\frac{6656}{5}=\boxed{1536}$$

6. $y'''(t) - 2y''(t) + y(t) = 0$

$y(0) = 1$

$y'(0) = -1$

$y''(0) = 1$

$-s^2 \qquad s \qquad -1 \qquad -2s^2Y(s) \qquad -s \qquad +1 \qquad -1$

$$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - 2\left(s^2Y(s) - sy(0) - y'(0)\right) + sY(s) - y(0) = 0$$

$$s^3Y(s) - s^2 + s - 1 - 2s^2Y(s) + 2s - 2 + sY(s) - 1 = 0$$

$$Y(s)\left(s^3 - 2s^2 + s\right) = s^2 - s + 1 - 2s + 2 + 1 =$$

$$Y(s) = \frac{s^2 - 3s + 4}{s(s^2 - 2s + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 - 2s + 1} \quad / \cdot s(s^2 - 2s + 1)$$

$$s^2 - 3s + 4 = As^2 - 2As + A + Bs^2 + Cs$$

$\boxed{A = 4}$ $\qquad -2A + C = -3$ $\qquad A + B = 1$

$-8 + C = -3$ $\qquad \boxed{B = -3}$

$C = -3 + 8$

$\boxed{C = 5}$

$$Y(s) = \frac{4}{s} - \frac{3s + 5}{s^2 - 2s + 1}$$

3. $\underline{r=1}$ $\quad \iint\limits_{\partial K} 2\,dS$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

$z \in [-\sqrt{1-r^2}, \sqrt{1-r^2}]$

$x^2+y^2+z^2 \le 1$

$r^2(\cos^2\varphi + \sin^2\varphi) + z^2 \le 1$

$r^2 + z^2 \le 1$

$z^2 \le 1 - r^2$

$$I = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2r \, dz \, dr \, d\varphi =$$

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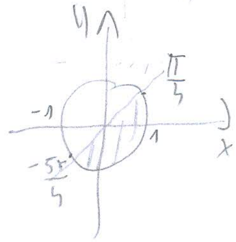
2. $\iint xy \, dx\,dy \qquad x^2+y^2=1 \text{ i } x=y$



$\varphi \in [-\frac{5\pi}{4}, \frac{\pi}{4}]$ X

$r \in [0, 1]$

$\emptyset$

X

$$I = \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \int_0^1 r^3 \cos\varphi \sin\varphi \, dr = \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \left(\frac{r^4}{4} \cos\varphi \sin\varphi\right)\Big|_0^1 d\varphi = \frac{1}{4} \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \cos\varphi \sin\varphi \, d\varphi$$

$$= \frac{1}{4} \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \frac{\sin 2\varphi}{2} d\varphi = \frac{1}{8} \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \sin 2\varphi \, d\varphi = \left| \begin{array}{l} 2\varphi = t \\ 2d\varphi = dt \\ d\varphi = \frac{dt}{2} \end{array} \right| = \frac{1}{16} \int_{-\frac{5\pi}{2}}^{\frac{\pi}{2}} \sin t \, dt$$

$$= -\frac{1}{16} \cos t \Big|_{-\frac{5\pi}{2}}^{\frac{\pi}{2}} = -\frac{1}{16} \left[\cos\frac{\pi}{2} - \cos\left(-\frac{5\pi}{2}\right)\right] = 0$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: ANTE BOTICA **BROJ INDEKSA:** 19-1-0017-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,dxdydz$. $r \in [0,2)$ 20

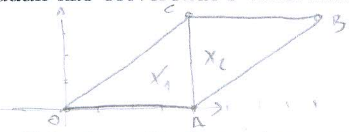
$x^2+y^2+z^2=R^2$ $y = r\sin\varphi$, $z=z$, $x = r\cos\varphi$ $\Rightarrow f(2r\cos\varphi+3)$ $\varphi \in [0,2\pi]$

2. Izračunati dvostruki integral: $\iint\limits_S xy\,dxdy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 1 \text{ i } x \ge y\}$. 20

$x^2+y^2 \le 1$; $x \ge y$

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(dx+dy)$. $= (x+y)dx + (x+y)dy \ldots \nabla f = \begin{bmatrix} -x-y \\ -x-y \end{bmatrix}$ 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral 15



OA ... $x=0$
OC ... $y=x$
BC ... $x=4$
AB ... $y-0=(4-4$

$$\iint\limits_X x^3 dxdy.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2dS$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 30

Tablica Laplaceovih transformacija:

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$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
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Tablica integrala

$\int dx = x + C$	$\int \sin x\,dx = -\cos x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \ne -1$	$\int \cos x\,dx = \sin x + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

ANTE BOTICA

1. K - kugla $r=2$ $S(0,0)$ $\iiint\limits_K (2x+3)\,dx\,dy\,dz$

$r \in [0,2]$

$\varphi \in [0,2\pi]$

$y = r\sin\varphi$

$x = r\cos\varphi$

$z = z$

kugla: $x^2+y^2+z^2=R^2$

$\underbrace{r^2\cos^2\varphi + r^2\sin^2\varphi}_{1} + z^2 = 4$

$r^2+z^2=4$

$z^2 = 4-r^2$

$z=\sqrt{4-r^2}$

za $r=2$

$z^2 = 4-2^2 = 4-4=0$

$z=0$

$f(x,y,z) : 2x+3$

$= 2r\cos\varphi + 3 \quad d\varphi\, dr\, dz$

$I = \int_0^{2\pi} d\varphi \int_0^{2} dr \int_0^{\sqrt{4-r^2}} (2r\cos\varphi+3)r\, dz$ ✓ ✗

$I = \int_0^{2\pi} d\varphi \int_0^{2} dr \int_0^{\sqrt{4-r^2}} 2r^2\cos\varphi + 3r\, dz$

$= \int_0^{2\pi} d\varphi \int_0^{2} (2r^2\cos\varphi\, z + 3rz)\Big|_0^{\sqrt{4-r^2}} dr = \int_0^{2\pi} d\varphi \int_0^{2} 2r^2\cos\varphi\sqrt{4-r^2} + 3r\sqrt{4-r^2}\, dr$

$= 2\int_0^{2\pi} d\varphi \int_0^{2} r^2\cos\varphi\sqrt{4-r^2}\, dr + 3\int_0^{2\pi} d\varphi \int_0^{2} r\sqrt{4-r^2}\, dr$

$= 2\int_0^{2\pi} \cos\varphi\, d\varphi \int_0^{2} r^2\sqrt{4-r^2}\, dr + 3\int_0^{2\pi} d\varphi \int_0^{2} r\sqrt{4-r^2}\, dr$

(5.) K – kugla: $r=1$ $\quad x^2+y^2+z^2=R^2$ $\quad x^2=r^2\cos^2\varphi$

$y^2=r^2\sin^2\varphi$

$z^2=x^2$

$$\iint_{\partial K} z\,dS$$

$\varnothing$

(3.) $$\int_{-1,2}^{2,3} (x+y)\,dx + (x+y)\,dy$$

$$\nabla_f = \begin{bmatrix} -x-y \\ -x-y \end{bmatrix} \begin{array}{l} f_1 \\ f_2 \end{array}$$

(1.)

$$\Rightarrow \int -x-y\,dx = -\int (x+y)\,dx = -\left(\frac{x^2}{2}+yx\right)+C(y)$$

$$= -\frac{x^2}{2} - yx + C(y)$$

$$\frac{\partial\left(-\frac{x^2}{2} - yx + C(y)\right)}{\partial y} = -x + C'(y)$$

$$-x + C'(y) = -x - y$$

$$C'(y) = -x + \cancel{x} - y = -y$$

(4.) $$\Rightarrow \int -y\,dy = -\frac{y^2}{2}$$

$$\Rightarrow -\frac{x^2}{2} - yx - \frac{y^2}{2}$$

$$f_1(\overset{x}{-1},\overset{y}{2}) - f_2(\overset{x}{2},\overset{y}{3})$$

$$\left[-\frac{(-1)^2}{2} - (-1)(2) - \frac{2^2}{2}\right] - \left[-\frac{2^2}{2} - (2\cdot 3) - \frac{3^2}{2}\right]$$

$$= \left[-\frac{1}{2} + \cancel{2} - \cancel{2}\right] - \left[-2 - 6 - \frac{9}{2}\right] = -\frac{1}{2} + \frac{25}{2} = 12$$

✓

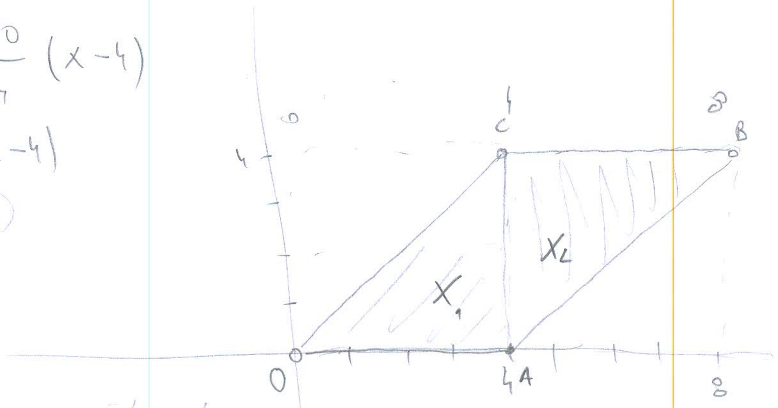
ANTE BOTICA

(4.) X četverokut: $O(0,0)$, $A(4,0)$, $B(8,4)$, $C(4,4)$

$y-0=\frac{4-0}{8-4}(x-4)$

$y=\frac{4}{4}(x-4)$

$y=x-4$



$p_{OA} \dots y=0$

$p_{OC} \dots y=x$

$p_{BC} \dots y=4$

$p_{AB} \dots y=x-4$

$p_{OA} \dots (y-0)=\frac{0-0}{4-0}(x-0)$

$y=0$

$p_{AB} \dots y-0=\frac{4-0}{8-4}(x-4)$

$y=x-4$

$$I=\iint_{X_1} x^3\,dx\,dy+\iint_{X_2} x^3\,dx\,dy$$

$$I=\int_0^4 x^3\,dx\int_0^x dy+\int_4^8 x^3\,dx\int_{x-4}^4 dy=\int_0^4 x^3\,dx\; y\Big|_0^x+\int_4^8 x^3\,dx\; y\Big|_{x-4}^4$$

$4-x+4=8-x$

$$=\int_0^4 x^3(x-0)\,dx+\int_4^8 x^3(4-(x-4))=\int_0^4 x^4\,dx+\int_4^8(8x^3-x^4)\,dx$$

$x^3(8-x)=8x^3-x^4$

$$=\frac{x^5}{5}\Big|_0^4+\left(\frac{\cancel{8}^{2}x^4}{\cancel{4}}\right)\Big|_4^8-\frac{x^5}{5}\Big|_4^8=$$

$$\left(\frac{4^5}{5}-\frac{0}{5}\right)+2x^4\Big|_4^8-\left[\frac{8^5}{5}-\frac{4^5}{5}\right]=\frac{1024}{5}+2\left(8^4-4^4\right)-\left(\frac{31744}{5}\right)$$

$$=\frac{1024}{5}+7680-\frac{31744}{5}=\underline{1536}$$

(6) $y'''(t) - 2y''(t) + y'(t) = 0 \qquad y(0)=1 \;;\; y'(0)=-1 \;,\; y''(0)=1$

$$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) - 2s^2F(s) + 2s f(0) + 2f'(0) + sF(s) - f(0) = 0$$

$$s^3F(s) - s^2 + s - 1 - 2s^2F(s) + 2s - 2 + sF(s) - 1 = 0$$

$$s^3F(s) - 2s^2F(s) + sF(s) = s^2 - 3s + 4$$

$$F(s)(s^3 - 2s^2 + s) = s^2 - 3s + 4$$

$$F(s) = \frac{s^2-3s+4}{s(s^2-2s+1)}$$

$s=0$; A

$s^2 + 2s + 1 = 0$; $s_{1,2} = \frac{2 \pm \sqrt{4-4\cdot 1\cdot 1}}{2\cdot 1} = \frac{2\pm\sqrt{0}}{2} = \frac{2}{2} = 1$

$(s-1)^2 = 0 \;/\sqrt{\ }$

$s-1=0$

$s=1$ B

RASTAV

$$\frac{A}{s} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\frac{s^2-3s+4}{s(s-1)^2} = \frac{A}{s} + \frac{B}{(s-1)^2} \;/\cdot s(s-1)^2$$

$$s^2-3s+4 = A(s-1)^2 + Bs$$

(1) za $s=0$

$$0 - 0 + 4 = A(\underbrace{0-1}_{(-1)^2=1})^2 + 0 \Rightarrow A=4$$

(2) za $s=1$

$$1-3+4 = A(\underbrace{1-1}_{0})^2 + B \Rightarrow B=2$$

$2 = B$

$$F(s) = \frac{4}{s} + \frac{2}{(s-1)^2} = 4\frac{1}{s} + 2\frac{1}{(s-1)^2}$$

$$= \frac{1}{(p-a)^n} \;;\; n=2,\; a=1$$

$$= \frac{e^{at}t^{n-1}}{(n-1)!} \Rightarrow \frac{e^t t}{\underbrace{(2-1)!}_{=1}} = 2e^t t$$

$$\boxed{y(t) = 4 + 2e^t \cdot t}$$

$1(s+1)$

$= s+1$

s^2-1

$\frac{1}{(s+1)} \Big| \frac{1}{(s+1)}$

s^2-2s+1

$(s^2+1)-2s$

$(s^2-1)-(2s+2)$

$(s-1)(s+1)-2(s+1)$

$(s+1)[(s-1)-2]$

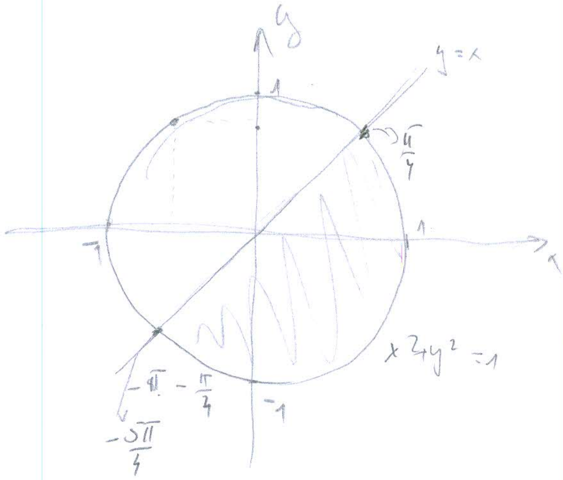
PROVJERA

$$y(0) = 4 + 2e^0\cdot 0 = 4 \neq 1$$

$\Rightarrow$ RJEŠENJE NIJE DOBRO.

AUTOBOTICA

(2.) $\iint\limits_S xy\,dx\,dy$ $\qquad S \in \quad x^2+y^2 \le 1, \quad x \geq y \qquad x^2+y^2=1$



$x^2+y^2=1$

$y=x$

$x^2+x^2=1$

$2x^2=1$

$x^2=\frac{1}{2}$

$x=\sqrt{\frac{1}{2}}$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r^2\cos^2\varphi + r^2\sin^2\varphi \le 1$

$r^2 \le 1$

$r \le \sqrt{1}$

$\boxed{r \le 1}$

$r \in [0,1]$

$\varphi \in \left[-\frac{5\pi}{4}; \frac{\pi}{4}\right]$

0

$$\int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_0^1 (r\sin\varphi\, r\cos\varphi)\, r\,dr = \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_0^1 [r(\sin\varphi\cos\varphi)]\, r\,dr = \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}}\int_0^1 r^2 \sin\varphi\cos\varphi\,dr$$

$\frac{1}{3} - \frac{0}{3} = \frac{1}{3}$

$$= \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} d\varphi\, \sin\varphi\cos\varphi\, \frac{r^3}{3}\Big|_0^1 = \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \frac{1}{3}\sin\varphi\cos\varphi\,d\varphi = \frac{1}{3}\int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} \sin\varphi\cos\varphi\,d\varphi$$

$\cos\varphi = t$

$\sin\varphi\,d\varphi = dt$

$\int t\,dt = \frac{t^2}{2}$

$= \frac{(\cos\varphi)^2}{2}$

$$= \frac{1}{3}\frac{(\cos\varphi)^2}{2}\Big|_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{3}\left[\left(\frac{\cos\frac{\pi}{4}}{2}\right)^2 - \left(\frac{\cos -\frac{5\pi}{4}}{2}\right)^2\right]$$

$$= \frac{1}{3}\left[\cancel{\frac{\frac{1}{2}}{2} - \frac{\frac{1}{2}}{2}}\right] = 0$$

0

(5) kugla: $x^2+y^2+z^2=r^2$

$r^2\cos^2 t + r^2\sin^2 t + z^2 = 1$

$r^2 + z^2 = 1$

$z^2 = 1 - r^2$

$z = \sqrt{1-r^2}$

$r \begin{bmatrix} r\cos t \\ r\sin t \\ z \end{bmatrix}$

$r' \begin{bmatrix} \cos t \\ \sin t \\ 1 \end{bmatrix}$

$$\|r'\| = \sqrt{\underbrace{\cos^2 t + \sin^2 t}_{1} + 1} = \underline{1}$$

$$\int_0^{2\pi}\int_0^1 2 \cdot 1 \cdot dt \, r dr = 2\int_0^{2\pi} dt \int_0^1 r dr = 2\int_0^{2\pi} dt \, \frac{r^2}{2}\Big|_0^1 = \cancel{2}\int_0^{2\pi} \frac{1}{\cancel{2}} dt = t\Big|_0^{2\pi} = \underline{\underline{2\pi}} \, ?$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: ANTE DUŠEVIĆ BROJ INDEKSA: 57641-2003	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

2. Izračunati dvostruki integral: $\iint\limits_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 1 \text{ i } x \ge y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x + \mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral 15 12

$$\iint\limits_X x^3 \mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\, y'(0) = -1,\, y''(0) = 1.$$

Ukupno: 27

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

③ $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy) = \int_{(-1,2)}^{(2,3)} (x+y)\,dx + \int_{(-1,2)}^{(2,3)} (x+y)\,dy$

$$w = \begin{bmatrix} x+y \\ x+y \end{bmatrix} = -\operatorname{grad} f(x,y) = \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -x-y \;/\; \int dx$$

$$f(x,y) = \int (-x-y)\,dx = \int -x\,dx - \int y\,dx = \boxed{-\frac{x^2}{2} - yx + c(y)}$$

$$\frac{\partial f}{\partial y} = -x-y \Rightarrow \frac{\partial\left(-\frac{x^2}{2} - yx + c(y)\right)}{\partial y} = -x-y$$

$$0 - \cancel{x} + \frac{\partial c(y)}{\partial y} = -\cancel{x} - y$$

$$\frac{\partial c(y)}{\partial y} = -y \;/\; \int dy$$

$$c(y) = -\frac{y^2}{2} + c$$

$$\boxed{f(x,y) = -\frac{x^2}{2} - yx - \frac{y^2}{2} + c}$$

$$I = f(-1,2) - f(2,3) = \left(-\frac{1}{2} \cancel{+2} \cancel{-2}\right) - \left(-2-6-\frac{9}{2}\right) = -\frac{1}{2} + 2 + 6 + \frac{9}{2} =$$

$$I = 8 + 4 = \underline{12}$$

✓

6. $y'''(t) - 2y''(t) + y'(t) = 0$; $y(0)=1$, $y'(0)=-1$, $y''(0)=1$

$$s^3x(s) - s^2\overset{1}{x(0)} - s\overset{-1}{x'(0)} - \overset{1}{x''(0)} - s^2x(s) - 2s\overset{1}{x(0)} - 2\overset{-1}{x'(0)} + sx(s) - \overset{1}{x(0)} = 0$$

$$s^3x(s) - s^2 + s - 1 - s^2x(s) - 2s + 2 + sx(s) - 1 = 0$$

$$s^3x(s) - s^2x(s) + s\,x(s) = s^2 + s + 1 + 2s - 2 + 1$$

$$x(s)(s^3 - s^2 + s) = s^2 + 3s \cancel{+1} \cancel{-1}$$

$$x(s)(s^3 - s^2 + s) = s^2 + 2s + s \;/ : s^3 - s^2 + s$$

$$x(s) = \frac{s^2+2s+s}{(s^3-s^2+s)} = \frac{s^2+2s+s}{s^3-(s(s+1))} =$$

$$\frac{s^2+2s+s}{s^3-(s(s+1))} = \frac{A}{s^3} + \frac{B}{s} + \frac{Cs+D}{s+1} \;/ : s^3-(s(s+1))$$

$$A(s(s+1)) + B(s^2(s+1)) + Cs+D(s^3-s)$$

$$s^2+2s+s = A(s^2+s) + B(s^3-s-1) + Cs+D(s^3-s)$$

$$s^2+2s+s = As^2 + As + Bs^3 - Bs - B + Cs^4 - Cs^2 + Ds^3 - Ds$$

$$s^2 + \overset{3s}{2s+s} = Cs^4 + s^3(B+D) + s^2(A-C) + s(A-B-D) - B$$

$0 = C$

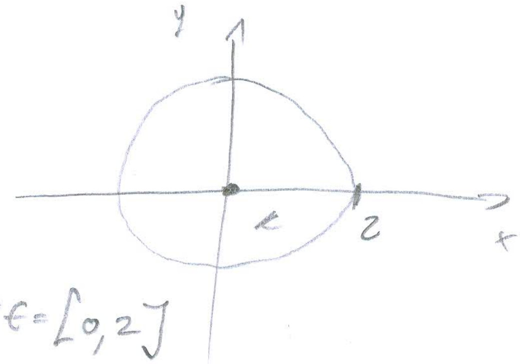
NA DRUGOM LISTU

1. $r = 2$

$\iiint (2x+3)\, dx\, dy\, dz$
K

$x = r\cos\varphi$
$y = r\sin\varphi$

$r^2 = 4$
$r = \pm 2$



$r \in [0,2]$
$\varphi \in [0,2\pi]$

$\int_0^{2\pi}\int_0^{2} (2 \cdot r\cos\varphi) + 3 \; r\,dr =$

6. NASTAVAK

$\boxed{0 = C}$

$0 = B + D \Rightarrow 0 = 0 + D \Rightarrow \boxed{D = 0}$

$1 = A - C \Rightarrow 1 = A - C \Rightarrow A = A + C = \underline{\underline{1}}$

$3 = A - B - D \Rightarrow$

$0 = -B$

$\boxed{B = 0}$

$$\frac{s^2 + 3s}{s^3 - (s(s+1))} = \frac{1}{s^3} + \frac{0}{s} + \frac{0s + 0}{s + 1}$$

$$X(s) = \frac{1}{s^3}$$

$X(s) =$?

(2.) $\iint_S xy\,dx\,dy$ $\quad S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1 ; x \geq y\}$

$y^2+x^2=1$

$y=x$

$x = r\cos\varphi$

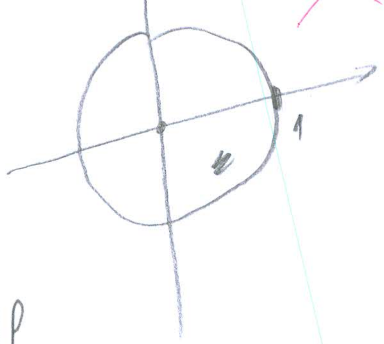
$y = r\sin\varphi$

$r \in [0,1]$

$\varphi \in \left[-\frac{5\pi}{4}, \frac{\pi}{4}\right]$

$\varnothing$

$= \int_{-\frac{5\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_0^1 r\cos\varphi \, r\sin\varphi \, r \, dr$



$x = r\cos\varphi$

$y = r\sin\varphi$

$\varphi \in [0, 2\pi]$

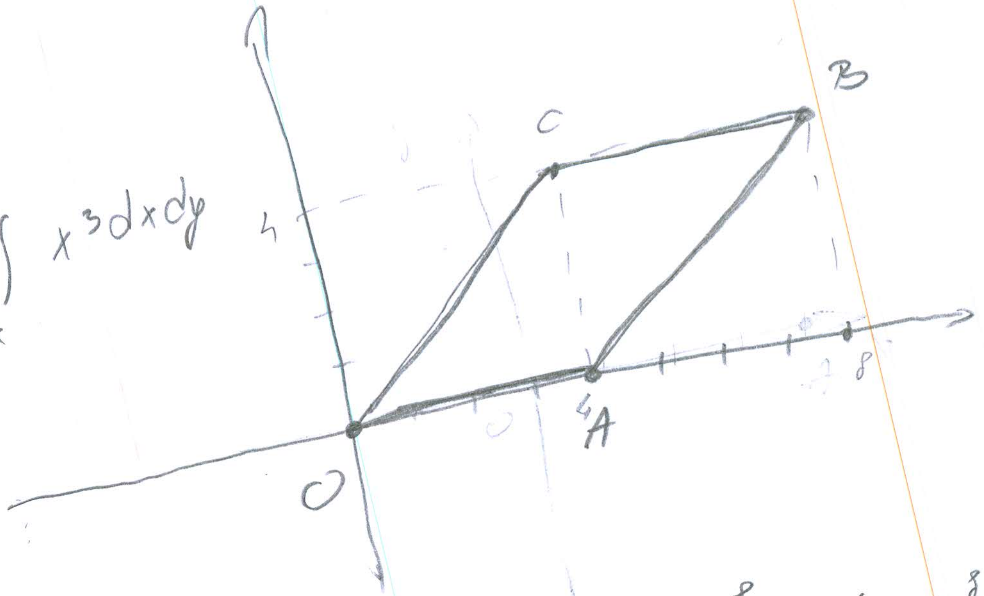
$r \in [0, 1]$

$$2\int_0^{2\pi}\int_0^1 r\,dr\,d\varphi$$

$$d\varphi = 2\cdot\int_0^{2\pi}\frac{1}{2}\,d\varphi$$

$$\frac{1}{2}\,\varphi\Big|_0^{2\pi} = \underline{\underline{2\pi}}$$

$$\iint_X x^3\,dx\,dy$$



$O(0, 0)$ $(0, 0)$

$A\left(\frac{8}{2}, 0\right)$ $(4, 0)$

$B\left(8, \frac{8}{2}\right)$ $(8, 4)$

$C\left(\frac{8}{2}, \frac{8}{2}\right)$ $(4, 4)$

$OC \Rightarrow y - y_1 = \frac{x_2 - x_1}{y_2 - y_1}(x - x_1)$

$$y - 0 = \frac{\frac{8}{2} - 0}{\frac{8}{2} - 0}(x - 0)$$

$y = 1\cdot x$

$\underline{\underline{y = x}}$

$CB \Rightarrow y - \frac{8}{2} = \frac{8 - \frac{8}{2}}{\frac{8}{2} - \frac{8}{2}}\left(x - \frac{8}{2}\right)$

$y - \frac{8}{2} = 4(x - 4)$

$y - 4 = 4x - 8$

$y = 4x - 8 + 4 = \underline{\underline{4x - 4}}$

$O(\overset{x_1}{0}, \overset{y_1}{0})$

$A(\overset{x_2}{4}, \overset{y_2}{0})$

$B(\overset{x_2}{8}, \overset{y_2}{4})$

$C(4, 4)$

$OA \Rightarrow \quad y - y_1 = \frac{x_2 - x_1}{y_2 - y_1}(x - x_1)$

$$y - 0 = \frac{4-0}{0-0}(x-0)$$

$$y = 4x$$

$$\int_0^4 dy \int_y^{y+4} x^3\,dx = \int_0^4 \left[\frac{x^4}{4}\right]_y^{y+4} dy$$

$$= \frac{1}{4}\int_0^4 (y+4)^4 - y^4\,dy$$

$AB \Rightarrow \quad y - 0 = \frac{4}{4}(x-4)$

$y = x - 4$

$x = y + 4$

$$= \int_0^4 x^3(\cancel{y}+4-\cancel{y})\,dy = \int_0^4 4x^3\,dy = 4\frac{x^4}{4}\Big|_0^4 =$$

$$= 4 \cdot \frac{4^4}{4} = 4 \cdot \frac{256}{4} = 256$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Andrea Savić **BROJ INDEKSA:** A-1-0017-2do	

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. ~~20~~ 15

2. Izračunati dvostruki integral: $\iint\limits_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1 \text{ i } x \geq y\}$. ~~20~~

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x + \mathrm{d}y)$. ~~15~~

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$$\iint\limits_X x^3\,\mathrm{d}x\mathrm{d}y.$$

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t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
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Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
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$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

1. $r=2 \quad T(0,0)$

$$\iiint_K (2x+3)\,dx\,dy\,dz$$

$x = r\cos\varphi \qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi \qquad z \in [-2, 2]$

$z = z \qquad r \in [0, \sqrt{4-z^2}]$

1,5

$$\int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}} (2r\cos\varphi+3)\,r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}} (2r^2\cos\varphi+3r)\,dr\,dz\,d\varphi =$$

DO KRAJA SE OVA TRICA "IZGUBILA"

$$= 2\int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}} r^2\cos\varphi\,dr\,dz\,d\varphi + 3\int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}} 3r\,dr\,dz\,d\varphi$$

$$= \int_0^{2\pi}\int_{-2}^{2} \cos\varphi\,dz\,d\varphi \int_0^{\sqrt{4-z^2}} r^2\,dr + \int_0^{2\pi}\int_{-2}^{2} \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz\,d\varphi$$

$$= \int_0^{2\pi}\int_{-2}^{2} \cos\varphi\,dz\,d\varphi \; \frac{r^3}{3}\Big|_0^{\sqrt{4-z^2}} + \int_0^{2\pi}\int_{-2}^{2} \left(\frac{4-z^2}{2} - 0\right) dz\,d\varphi$$

$$\int_0^{2\pi}\int_{-2}^{2} \frac{(\sqrt{4-z^2})^3}{3}\cos\varphi\,dz\,d\varphi + 2\int_0^{2\pi}\int_{-2}^{2} dz\,d\varphi - \frac{1}{2}\int_0^{2\pi}\int_{-2}^{2} z^2\,dz\,d\varphi$$

$$\int_0^{2\pi} (0-0)\cos\varphi\,d\varphi + 8\int_0^{2\pi} d\varphi - \frac{1}{2}\int_0^{2} \frac{z^3}{3}\Big|_{-2}^{2} d\varphi = 16\pi - \frac{8}{3}\int_0^{2\pi} d\varphi$$

$\underbrace{\cos 2\pi - \cos 0}_{0}$

$\frac{8}{3} + \frac{8}{3} = \frac{16}{3}$

$= 16 - \frac{8}{3}2\pi = \frac{32}{3}\pi$

3. $\int_{-(1,2)}^{(2,3)} (x+y)(dx+dy) = f(2,3) - f(-1,2) = $?

$w_x = x+y$

$w_y = x+y$

$w = -\text{grad} f = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}\right)$

$-\frac{\partial f}{\partial x} = x+y$

$\frac{\partial f}{\partial x} = -x-y$

$-\frac{\partial f}{\partial y} = x+y$

$\frac{\partial f}{\partial y} = -x-y$

$f = \int(-x-y)\partial x$

$f = -\frac{x^2}{2} - xy + C$

$f = \int(-x-y)\partial y$

$f = -xy - C'$

$f = -\frac{x^2}{2} - 2xy + \frac{1}{6}y^3$ × $\emptyset$

$\Rightarrow C' = \frac{y^2}{2}$

$C = \frac{1}{2}\frac{y^3}{3}$

5. $\iint_{\partial K} 2\,dS$

OVO JE ZA CILINDAR

$r = \begin{pmatrix} \cos u \\ \sin u \\ v \end{pmatrix}$ $\quad \frac{\partial r}{\partial u} = \begin{pmatrix} -\sin u \\ \cos u \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{\partial r}{\partial v}$

$r = 1$

$T(0,0)$

$\emptyset$

$= \begin{pmatrix} \cos u \\ \sin u \\ 0 \end{pmatrix}$

$\|\vec{n}\| = \sqrt{\underbrace{\cos^2 u + \sin^2 u}_{1}} = 1$

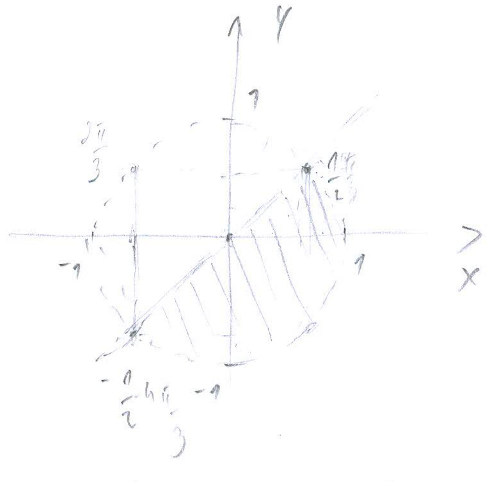
$u = [0, 2\pi]$

$v = [0, 1]$

$\int_0^{2\pi}\int_0^1 2\,dv\,du = 2\int_0^{2\pi} 1\,du = 2 \cdot 2\pi = \underline{\underline{4\pi}}$

Andrea Šarić

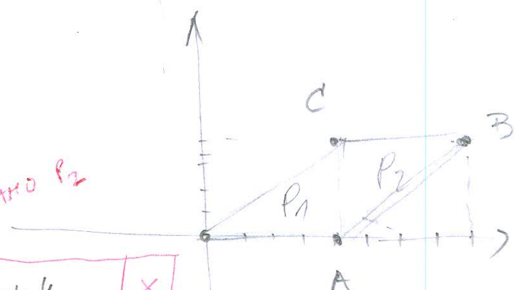
2. $\iint_S xy\,dx\,dy$



?

$\cancel{0}$

4. O



TREBA $x=4$ AKO SE ŽELI SAMO P_2

$$I_2 = \int_0^4 \int_{x=4}^{x=y-4} x^3\,dx\,dy = \int_0^4 \frac{x^4}{4}\Big|_{0}^{y-4} dy = \int_0^4 \frac{(y-4)^4}{4} - \frac{y^4}{4}\,dy$$

$$= \frac{1}{4}\frac{(y-4)^5}{5}\Big|_{0}^{4} - \frac{1}{4}\frac{y^5}{5}\Big|_0^4 = 0 + 51,2 - 51,2 = 0$$

$$I_1 = \int_0^4\int_{x=y}^{4} x^3\,dx\,dy = \int_0^4 \frac{x^4}{4}\Big|_y^4 dy$$ ✓

OVO JE SAMO DIO. DRUGI DIO NIJE DOBAR.

$\cancel{0}$

$$= \int_0^4 \left(\frac{256}{4} - \frac{y^4}{4}\right) dy$$

$$64\int_0^4 dy - \frac{1}{4}\int_0^4 y^4\,dy = 256 - \frac{1}{4}\left(\frac{y^5}{5}\right)\Big|_0^4$$

$$= 256 - \frac{1}{4}\cdot\frac{1024}{5} = 256 - \frac{256}{5} = \underline{307,2}$$

$\overline{OA}$ $y=0$

$\overline{AB}$ $(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$

$(y-0)(8-4)=(4-0)(x-4)$

$4y=4x-16$

$y=x-4$

BC $(y-4)(4-8)=(4-4)(x-x_1)$

$-4y+16=0$

$y=4$

$\overline{CO}$ $(y-4)(0-4)=(0-4)(x-4)$

$-4y+16=-4x+16$

$y=x$

AC ⇒ $x=4$ $y-4=u$ $dy=du$ $u^4\,du$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: IVAN ŠIKIĆ

BROJ INDEKSA: 17-1-0014-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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$$\iint\limits_X x^3\mathrm{d}x\mathrm{d}y.$$

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$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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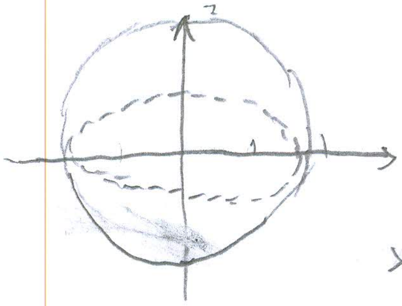
1. $K, r=2$, $\iiint (2x+3)\,dx\,dy\,dz$

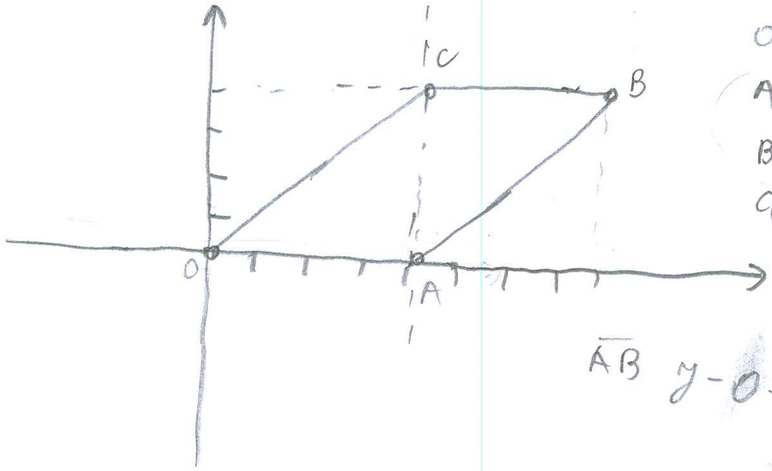


$$\int_0^{2\pi}\int_0^{2}\int_{r\cos\varphi}^{r\sin\varphi} 2r\cos\varphi+3 \; r\,dr\,d\varphi$$

$x = r\cos\varphi$

$y = r\sin\varphi$

4.



$O(0,0)$
$A(4,0)$
$B(8,4)$
$C(4,4)$

$\iint x^3\,dx\,dy$

$\overline{AB}$ $y-0 = \frac{4}{4}(x-4)$

$y = x-4$

$CB =$

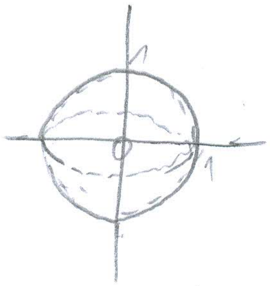
$y = 4$

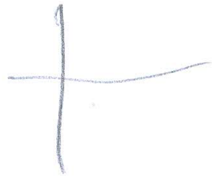
$$\int_0^{x}\int_{x-4}^{4} x^3\,dx\,dy = \int_0^{x} 64-0$$

$= 64$

IVAN ŠIKIĆ 17-1-0014-2010

5.



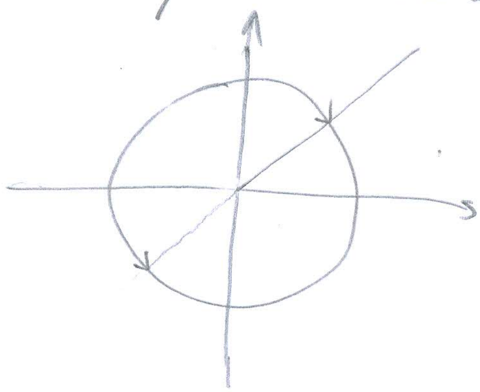


$$\iint_{\partial K} 2\, ds$$

$$\int_0^{2\pi}\int_0^1 2\, ds = 0$$

3. $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy) = \int_{(-1,2)}^{(2,3)} x\,dx + y\,dx + x\,dy + y\,dy = (3+1) = 4$

2. $\iint xy\,dx\,dy$ $\quad S = S(x,y) \in R^2 : x^2+y^2 \le 1 \quad x \ge y$



$|r| \le 1$

$$\int_0^{2\pi}\int_{-1}^{1} xy\,dx\,dy =$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: JOSIP KALEBIĆ **BROJ INDEKSA:** 56776-2008

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

⑥ $y'''(t) - 2y''(t) + y'(t) = 0$

$y(0)=1$
$y'(0)=-1$
$y''(0)=1$

$$s^3 X(s) - s^2 y(0) - s y'(0) - y''(0) - 2\cdot\left(s^2 X(s) - s\cdot y(0) - y'(0)\right) + s X(s) - y(0) = 0$$

$$s^3\cdot X(s) - s^2 + s - 1 - 2\left(s^2 X(s) - 1 + 1\right) + s X(s) - 1 = 0$$

$$s^3\cdot X(s) - s^2 + s - 1 - 2\cdot s^2\cdot X(s) + s\cdot X(s) - 1 = 0$$

$$s^3\cdot X(s) - 2\cdot s^2\cdot X(s) + s\cdot X(s) = s^2 - s + 2$$

$$X(s)\left(s^3 - 2s^2 + s\right) = s^2 - s + 2$$

$$X(s) = \frac{s^2 - s + 2}{s(s^2 - 2s + 1)} = \frac{s^2 - s + 2}{s\cdot(s-1)(s-1)}$$

$$X(s) = \frac{A}{s} + \frac{B}{(s-1)} + \frac{C}{(s-1)}$$

$$\frac{A(s-1)(s-1) + B(s\cdot(s-1)) + C(s\cdot(s-1))}{s\cdot(s-1)(s-1)}$$

$$\frac{A(s^2 - 2s + 1) + B(s^2 - s) + C(s^2 - s)}{s\cdot(s-1)(s-1)} =$$

$$\frac{As^2 - 2As + A + Bs^2 - Bs + Cs^2 - Cs}{s\cdot(s-1)(s-1)}$$

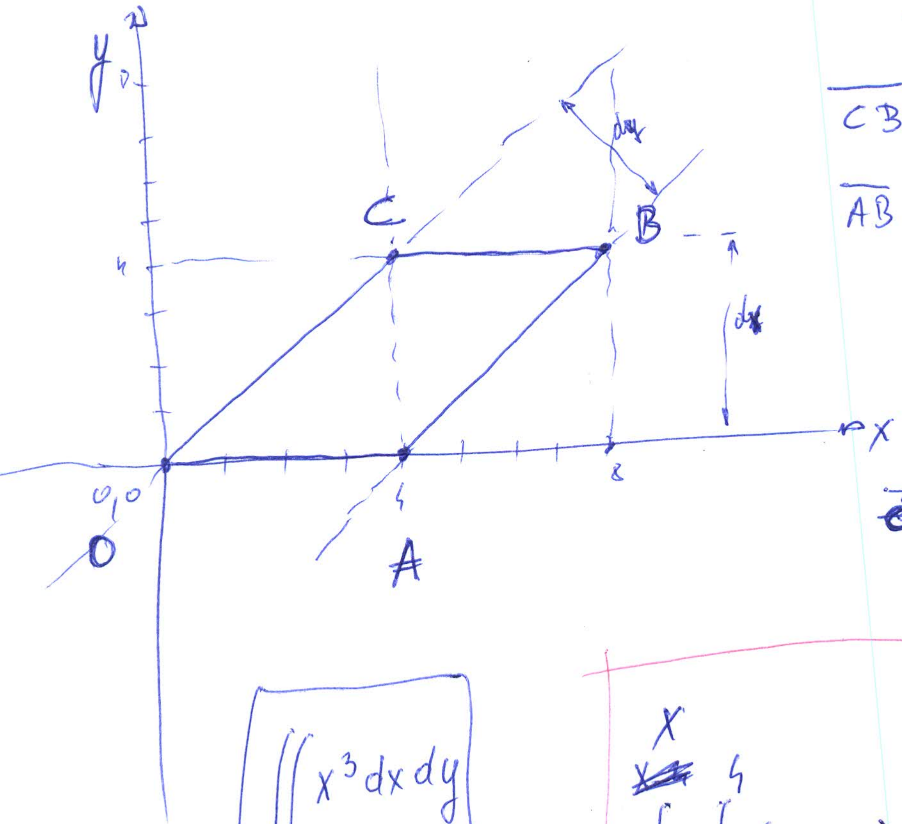
$$\frac{s^2(A+B+C) - s(2A+B+C) + A}{s\cdot(s-1)\cdot(s-1)}$$

$A+B+C=1$
$2A+B+C=1$
$A=2$

$\to$ $B+C = 1-2$
$B+C = 1-4$

$(\phi, \phi)\ ,\ A\left(\frac{8}{2}, 0\right)\ ,\ B\left(8, \frac{}{2}\right)\ ,\ C\left(\frac{}{2}, \frac{}{2}\right)$

0,0 , 4,0 8,4 , 4,4



$\overline{OA} \Rightarrow (y-0) = \frac{0-0}{4-0}(x-0)$

$\Rightarrow y = \phi$

$\overline{CB} \Rightarrow y = 4$

$\overline{AB} \Rightarrow (y-0) = \frac{4-0}{8-4}(x-4)$

$y = \frac{4}{4}(x-4)$

$y = x-4$

$\overline{OC} \Rightarrow (y-0) = \frac{4-0}{4-0}(x-0)$

$y = 1(x-0)$

$y = x$

$$\boxed{\iint_X x^3\,dx\,dy}$$

$$\int_{x-4}^{x}\int_{0}^{4}\left(x^3\,dx\right)dy$$

$$\int_{x-4}^{x}\left(\frac{x^4}{4}\Big|_0^4\right)dy = \int_{x-4}^{x}\left(\frac{4^4}{4}-\frac{0^4}{4}\right)dy = \int_{x-4}^{x}64\,dy$$

$$= \left(64y\Big|_{x-4}^{x}\right) = 64\cdot x - 64(x-4)$$

$$\cancel{64x} - \cancel{64x} + 256$$

$$= 256$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DANIJEL KAPOVIĆ **BROJ INDEKSA:** 52590-2005

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,dxdydz$. 20

2. Izračunati dvostruki integral: $\iint\limits_S xy\,dxdy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1 \text{ i } x \ge y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(dx+dy)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral $\cancel{15}$
$$\iint\limits_X x^3\,dxdy.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 2dS$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $\cancel{15}$
$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: Ø

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int dx = x + C$	$\int \sin x\,dx = -\cos x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \ne -1$	$\int \cos x\,dx = \sin x + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$		$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

6. $y'''(t) - 2y''(t) + y'(t) = 0 \qquad y(0) = 1 \quad y'(0) = -1 \quad y''(0) = 1$

$$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - 2\left(s^2Y(s) - sy(0) - y'(0)\right) + sY(s) - y(0) = 0$$

$$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - 2s^2Y(s) + 2sy(0) + 2y'(0) + sY(s) - y(0) = 0$$

$$s^3Y(s) - s^2 + s - 1 - 2s^2Y(s) + 2s - 2 + sY(s) - 1 = 0$$

$$s^3Y(s) - 2s^2Y(s) + sY(s) - s^2 + 3s - 4 = 0$$

$$s^3Y(s) - 2s^2Y(s) + sY(s) = s^2 - 3s + 4$$

$$Y(s)(s^3 - 2s^2 + s) = s^2 - 3s + 4$$

$$Y(s)\, s(s^2 - 2s + 1) = s^2 - 3s + 4$$

$$Y(s) = \frac{s^2 - 3s + 4}{s(s^2 - 2s + 1)} = \frac{s^2 - 3s + 4}{s(s-1)^2}$$

$$s^2 - 3s + 4 = \frac{A}{s} + \frac{B}{(s-1)^2} + \frac{C}{s-1} \quad \checkmark$$

$$s^2 - 3s + 4 = A(s-1)^2 + Bs(s-1) + Cs(s-1)^2$$

$$s^2 - 3s + 4 = A(s^2 - 2s + 1) + Bs(s-1) + Cs(s^2 - 2s + 1)$$

$$s^2 - 3s + 4 = As^2 - 2As + A + Bs^2 - Bs + Cs^3 - 2Cs^2 + Cs$$

$0 = C$

$1 = A + B - 2C \longrightarrow 1 = 4 + B - 0$

$B = 1 - 4$

$B = -3$

$-3 = -2A - B + C$

$4 = A$

DALJE ... ?

0

$$4 \cdot \frac{1}{s} - 3\,\frac{1}{(s-1)^2}$$

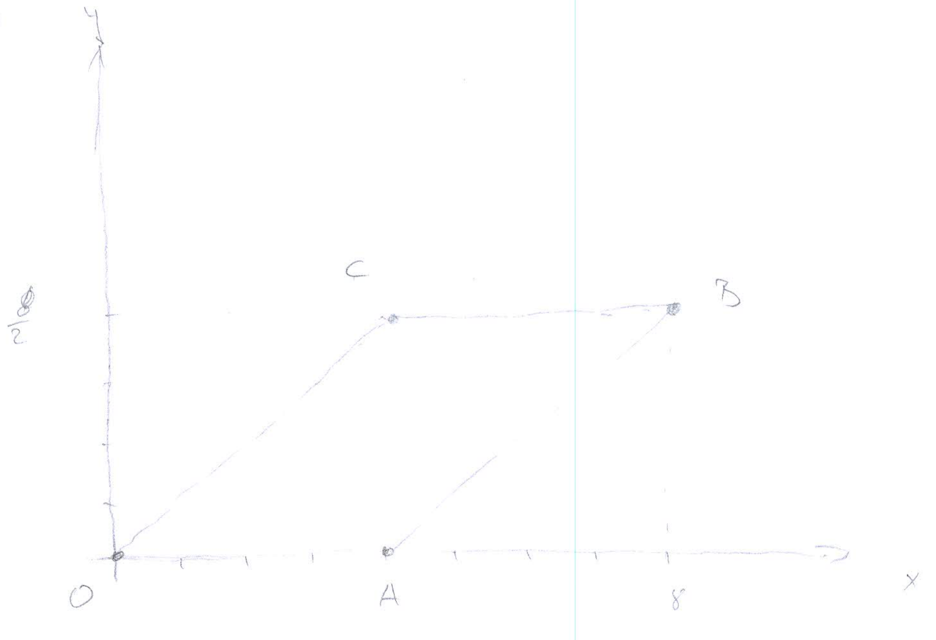
$$4 \cdot 1 - 3 \cdot e^t$$

$$4 - 3e^t$$

DANIJEL LABOVIĆ

(4) $O(0,0)$, $A(\frac{8}{2}, 0)$, $B(8, \frac{8}{2})$ $C(\frac{8}{2}, \frac{8}{2})$

$\iint y^3 \, dx\, dy$



$\overline{OC}$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{0 - \frac{8}{2}}{\frac{8}{2} - 0}(x - 0)$ X

$y = \frac{-\frac{8}{2}}{\frac{8}{2}} x$

$y = -x$

$x = -y$ X

$\overline{AB}$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{\frac{8}{2} - 0}{8 - \frac{8}{2}}\left(x - \frac{8}{2}\right)$

$y = \frac{\frac{8}{2}}{\frac{8}{2}}\left(x - \frac{8}{2}\right)$

$y = x - \frac{8}{2}$

$x = y + \frac{8}{2}$

$$\int_0^{\frac{8}{2}} \int_{-y}^{y+\frac{8}{2}} x^3 \, dx\, dy = \int_0^{\frac{8}{2}} \frac{x^4}{4}\Bigg|_{-y}^{y+\frac{8}{2}} dy = \int_0^{\frac{8}{2}} \left(\frac{(y+\frac{8}{2})^4}{4} - \frac{(-y)^4}{4} \right) dy$$

X

$\cancel{0}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: IVAN LOVIĆ BROJ INDEKSA: 37104	

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4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral 15

$$\iint\limits_X x^3 \mathrm{d}x\mathrm{d}y.$$

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Ukupno: $\cancel{0}$

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Tablica integrala

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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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$$6.)\ y'''(t) - 2y''(t) + y'(t) = 0 \quad , \quad \overset{f(0)}{y(0)} = 1 \ , \ \overset{f'(0)}{y'(0)} = -1 \ , \ \overset{f''(0)}{y''(0)} = 1$$

$$s^3F(s) - s^2f(0) - s\,f'(0) - f''(0) - \left(2 \cdot s^2F(s) - s\,f(0) - f'(0)\right) +$$
$$\left(s\,F(s) - f'(0)\right)$$

$$s^3F(s) - s^2 + s - 1 - 2\left(s^2F_s - s + 1\right) + sF(s) + 1$$
$$s^3F(s) - s^2 + s - 1 - 2s^2F_s + 2s - 2 + sF(s) + 1$$
$$s^3F(s) - 2s^2F(s) + sF(s) - s^2 + 3s - 2$$
$$F_s\left(s^3 - 2s^2 + s\right) - s^2 + 3s - 2$$