MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

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POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

2. Izračunati dvostruki integral: $\iint_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 1 \,\mathrm{i}\, x \ge y\}$. 20

3. Izračunati $\int_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x+\mathrm{d}y)$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{8}{2},0)$, $B(8,\frac{8}{2})$ i $C(\frac{8}{2},\frac{8}{2})$. Izračunati dvostruki integral 15

$$\iint_X x^3 \mathrm{d}x\mathrm{d}y.$$

5. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint_{\partial K} 2\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2y''(t) + y'(t) = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = 1.$$

Ukupno: 55

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \ne -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

① $\iiint (2x+3)\,dx\,dy\,dz \qquad r=2$

Parametrizacija

$x = r\cos\varphi \qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi \qquad r \in [0, \sqrt{4-z^2}]$

$z = z \qquad z \in [-2, 2]$

$x^2+y^2+z^2 \leq 4$

$r^2+z^2 \leq 4$

$r \leq \sqrt{4-z^2}$

$$I = \int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}} (2r^2\cos\varphi + 3r)\,dr\,d\varphi\,dz = \int_{-2}^{2}\int_0^{\sqrt{4-z^2}} \left[2r^2\sin\varphi\Big|_0^{2\pi} + 3r\,\varphi\Big|_0^{2\pi}\right] dr\,dz$$

$$= 6\pi \int_{-2}^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz = \frac{6\pi}{2}\int_{-2}^{2}(4-z^2)\,dz$$

$$= 3\pi\left[4\cdot 2\Big|_{-2}^{2} - \frac{z^3}{3}\Big|_{-2}^{2}\right]$$

$$= 3\pi\left(16 - \frac{16}{3}\right) = 3\pi\,\frac{48-16}{3} = 32\pi$$

③ $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy)$

$$\begin{Bmatrix} x+y \\ x+y \end{Bmatrix} = -\operatorname{grad} f = -\begin{pmatrix} \partial_x f \\ \partial_y f \end{pmatrix}$$

$\partial_x f = -x-y \;/\int$

$f = \int(-x-y)\,dx = -\frac{x^2}{2} - xy + C(y)$

$\partial_y f = -x-y$

$0 - x + C'(y) = -x-y$

$C'(y) = -y \;/\int$

$C(y) = -\int y\,dy$

$C(y) = -\frac{y^2}{2} + C_2$

$f(x,y) = -\frac{x^2}{2} - xy - \frac{y^2}{2} + C_2$ ✓

$$I = \int_{(-1,2)}^{(2,3)} (x+y)dx + (x+y)dy = f(-1,2) - f(2,3)$$

$$= -\frac{1}{2} + 2 - \frac{4}{2} - \left(-\frac{4}{2} - 6 - \frac{9}{2}\right) = \underline{12}$$