MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. **IME I PREZIME:** DUJE KRALJEV **BROJ INDEKSA:** 17-2-0015-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$. 76 — 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. 77 — ~~20~~

3. Izračunati $\int\limits_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$ 14 — 10

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}x\mathrm{d}y$? 35 — 20

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = x^2 + y^2$ što leži iznad područja $D \ldots x^2 + y^2 \leq 4$. Nije potrebno računati površinu baze. — 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — ~~15~~

$$y'''(t) + 2y''(t) + y'(t) = t, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 40

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
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$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x + \sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

6) $y'''(t) + 2y''(t) + y'(t) = t \qquad y(0) = 2,\ y'(0) = 0,\ y''(0) = 1$

$$s^3 Y(s) - s^2 \underbrace{y(0)}_{2} - s \underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{1} + 2\left(s^2 Y(s) - s \underbrace{y(0)}_{2} - \underbrace{y'(0)}_{0}\right) + s Y(s) - \underbrace{y(0)}_{2} = \frac{1}{s^2}$$

$$s^3 Y(s) - 2s^2 - 1 + 2s^2 Y(s) - 4s + s Y(s) - 2 = \frac{1}{s^2}$$

$$s^3 Y(s) + 2s^2 Y(s) = \frac{1}{s^2} + 2 + 4s + 1 + 2s^2$$

$$Y(s)\left(s^3 + 2s^2\right) = \frac{1 + 3s^2 + 4s^3 + 2s^4}{s^2} \quad \Big/ :\left(s^3 + 2s^2\right)$$

$$Y(s) = \frac{\dfrac{1 + 3s^2 + 4s^3 + 2s^4}{s^2}}{s^3 + 2s^2} = \frac{2s^4 + 4s^3 + 3s^2 + 1}{\left(s^3 + 2s^2\right)s^2} = \frac{2s^4 + 4s^3 + 3s^2 + 1}{s^4(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{s+2} \quad \Big/ \cdot s^4(s+2)$$

$$2s^4 + 4s^3 + 3s^2 + 1 = A \underbrace{s^3(s+2)}_{s^4 + 2s^3} + B \underbrace{s^2(s+2)}_{s^3 + 2s^2} + C \underbrace{s(s+2)}_{s^2 + 2s} + D(s+2) + E s^4$$

$s = 0$

$1 = D \cdot 2$

$D = \frac{1}{2}$

$$2s^4 + 4s^3 + 3s^2 + 1 = \cancel{As^4} + \cancel{2As^3} + \cancel{Bs^3} + \cancel{2Bs^2} + \cancel{Cs^2} + \cancel{2Cs} + \cancel{Ds} + 2D + \cancel{Es^4}$$

$$2s^4 + 4s^3 + 3s^2 + 1 = (A+E)s^4 + (2A+B)s^3 + (2B+C)s^2 + (2C+D)s + 2D$$

$A + E = 2$

$2A + B = 4$

$2B + C = 3$

$2C + D = 0$

$2D = 1$

$D = \frac{1}{2}$

$2C + \frac{1}{2} = 0$

$2C = -\frac{1}{2}$

$C = -\frac{1}{4}$

$2B - \frac{1}{4} = 3$

$2B = 3 + \frac{1}{4}$

$2B = \frac{13}{4}$

$B = \frac{13}{8}$

$2A + B = 4$

$2A + \frac{13}{8} = 4$

$2A = 4 - \frac{13}{8}$

$2A = \frac{19}{8}$

$A = \frac{19}{16}$

$\frac{19}{16} + E = 2$

$E = 2 - \frac{19}{16}$

$E = \frac{13}{16}$

c)

$$Y(s) = \frac{13}{16}\frac{1}{s} + \frac{13}{8}\frac{1}{s^2} - \frac{1}{4}\frac{1}{s^3} + \frac{1}{2}\frac{1}{s^4} + \frac{13}{16}\cdot\frac{1}{s+2} \quad \leftarrow 6)$$

$$y(s) = \frac{13}{16} + \frac{13}{8}t - \frac{1}{4}\frac{t^2}{2} + \frac{1}{2}\frac{t^3}{3} + \frac{13}{16}e^{-2t} \quad \cancel{0}$$

PROVJERA:

$$y(0) = \frac{13}{16} + \frac{13}{16} = \frac{26}{16} \neq 2 \quad \times$$

1)

$$r(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \quad r'(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

$\varphi \in [0, 2\pi]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$\int_0^{2\pi} \cos t \sin t \cdot \underbrace{\sqrt{(-\sin t)^2 + (\cos t)^2}}_{1}\, dt$$

$$\int_0^{2\pi} \cos t \sin t \, dt = 0 \quad ✓$$

2)

$$r(t) = \begin{pmatrix} \cos t \\ -1 + \sin t \end{pmatrix}$$

$$w = \begin{pmatrix} 0 \\ 2x + 3 \end{pmatrix}$$

$$r'(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

$x = \cos t$

$y = \sin t - 1$

$$\int_0^{2\pi} \begin{pmatrix} 0 \\ 2\cos t + 3 \end{pmatrix} \cdot \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} dt$$

$$\int_0^{2\pi} (2\cos^2 t + 3\cos t)\, dt = 2\int_0^{2\pi} \cos^2 t \, dt + 3\int_0^{2\pi} \cos t \, dt$$

$$= 2\int_0^{2\pi} \frac{1 + \cos(2t)}{2} \, dt = 2\pi$$

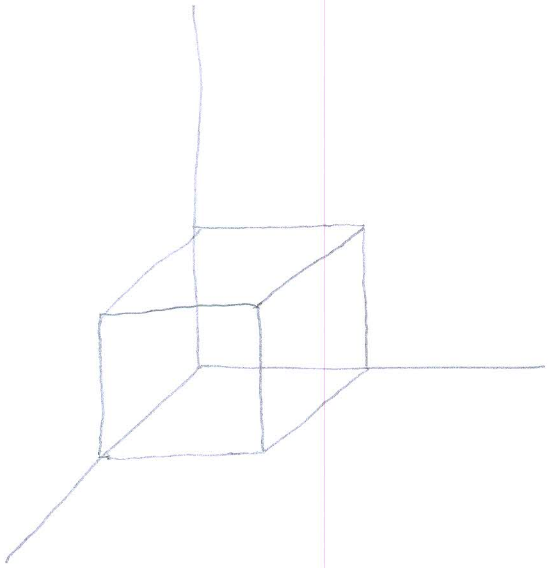
17-2-0015-2010 DUŠO KRALJEV

4)

$a = 2cm$

$K = T(0,0,0)$

$$\iint_{\partial K} w_x \, dy\, dz + w_y \, dx\, dz + w_z \, dx\, dz$$



$$w = \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} \quad w = \begin{pmatrix} 0 \\ 0 \\ 2x+3 \end{pmatrix}$$

$$\operatorname{div} w = \frac{\partial 0}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial (2x+3)}{\partial z} = 0 + 0 + 0 = 0$$

$$\iint_{\partial K} (2x+3)\, dx\, dy = \iint_{\partial K} (w \mid ds) = \iiint_K \operatorname{div} w \; dx\, dy\, dz$$

$$= \iiint_K 0 \, dx\, dy\, dz = 0 \quad ✓$$

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IME I PREZIME: Toma Medić **BROJ INDEKSA:** 17-2005 2-1010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy \, \mathrm{d}s$. 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\, \mathrm{d}y$. 20

3. Izračunati $\displaystyle\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$ 10

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x+3)\, \mathrm{d}x\mathrm{d}y$? 20

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = x^2 + y^2$ što leži iznad područja $D \ldots x^2 + y^2 \leq 4$. Nije potrebno računati površinu baze. 15

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Tablica integrala

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$\int \sqrt{a^2 - x^2}\, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

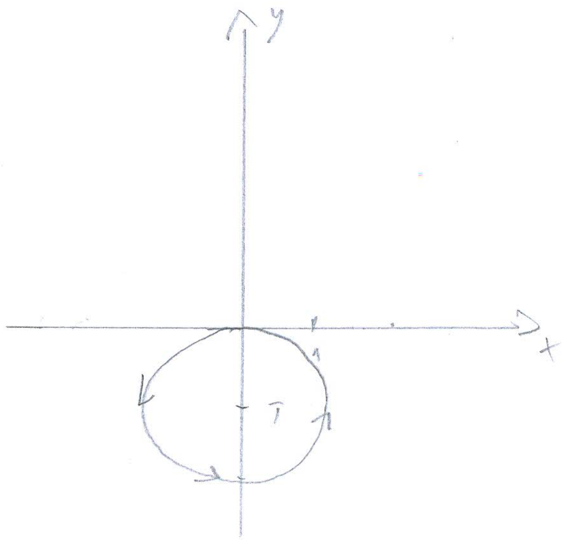
Tomo Međić

2.) $r = 1$

$T(0,-1)$

$$\oint_{\partial K} (2x+3)\,dy = ?$$

$\cancel{0}$



6) $y'''(t)+2y''(t)+y'(t)=t$ $y(0)=2,\ y'(0)=0\ \ y''(0)=1$

$$s^3Y(s)-s^2y(0)-sy'(0)-y''(0)+2(s^2Y(s)-sy(0)-y'(0))+sY(s)-y(0)=\frac{1}{s^2}$$

$$s^3Y(s)-2s^2-0-1+2s^2Y(s)-4s-0+sY(s)-2=\frac{1}{s^2}$$

$$s^3Y(s)+2s^2Y(s)+sY(s)-2s^2-4s-2=\frac{1}{s^2}$$

$$s^3Y(s)+2s^2Y(s)+sY(s)=2s^2+4s+2+\frac{1}{s^2}$$

$$(sY(s)(s^2+2s+1)=2s^2+4s+2+\frac{1}{s^2}$$

$\emptyset$

Toma Modić

1) $\iint_{\partial K} (2x+3)\,dx\,dy$

$\iint_{\partial K} w \cdot dS = \iiint_K w_x\,dy\,dz + w_y\,dx\,dz + w_z\,dx\,dy$

$w = \begin{pmatrix} 0 \\ 0 \\ 2x+3 \end{pmatrix}$, $\operatorname{div} w = \frac{\partial(0)}{\partial x} + \frac{\partial(0)}{\partial y} + \frac{\partial(2x+3)}{\partial z}$

$= 0 + 0 + 0$

$= 0$

$\iint_{\partial K} (2x+3)\,dx\,dy = \iiint_K \operatorname{div} w\,dx\,dy\,dz = \int_0^2\int_0^2\int_0^2 0\,dx\,dy\,dz = 0$ ✓

1.) $T(0,0)$ $r = 1$

$x = r\cos\varphi$
$y = r\sin\varphi$

$x = \cos\varphi$
$y = \sin\varphi$

$r(t) = r\begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$; $r'(t) = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}$

$\varphi \in [0, 2\pi]$
$r \in [0, 1]$

$\|r'\| = \sqrt{(-\sin\varphi)^2 + (\cos\varphi)^2}$
$\|r'\| = \sqrt{\sin^2\varphi + \cos^2\varphi}$
$\|r'\| = \sqrt{1}$ ✓
$\|r'\| = 1$

$\int_{\partial K} xy\,ds = \int_0^{2\pi}\int_0^1 \cos\varphi\sin\varphi\, r\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 r\cos\varphi\sin\varphi\,dr\,d\varphi$

$= \int_0^{2\pi} \frac{r^2}{2}\cos\varphi\sin\varphi \Big|_0^1 d\varphi = \int_0^{2\pi} \frac{1}{2}\cos\varphi\sin\varphi\,d\varphi$

$= \frac{1}{2}\int_0^{2\pi} \cos\varphi\sin\varphi\,d\varphi = \frac{1}{2}\left[\cos\varphi\sin\varphi\right]\Big|_0^{2\pi} = \frac{1}{2}\left[\cos 2\pi \cdot \sin 2\pi - (\cos 0 \cdot \sin 0)\right] =$

$= \frac{1}{2}\left[1 \cdot 0 - 1 \cdot 0\right] = \frac{1}{2} \cdot 0 = 0$

5) $z = x^2 + y^2$

$x^2 + y^2 \leq 4$

$r^2 = 4$

$r = 2$

$P(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$

$x^2 + y^2 = z$ $\qquad$ $x^2 + y^2 = z$

$2x\, \partial x = \partial z$ $\qquad$ $2y\, \partial y = \partial z$

$\frac{\partial z}{\partial x} = \frac{2x}{1}$ $\qquad$ $\frac{\partial z}{\partial y} = \frac{2y}{1}$

$\iint_D \sqrt{1 + (2x)^2 + (2y)^2}\, dx\, dy = \int_0^{2\pi}\int_0^{2} \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy$ X $\emptyset$

TOČNO RJEŠENJE

$$\int_0^{2\pi}\int_0^{2} \sqrt{1 + 4r^2\cos^2\varphi + 4r^2\sin^2\varphi}\, dr\, d\varphi$$

3.) $\int_{(3,2)}^{(5,5)} x\,dy + y\,dx$

$\int x\,dy = xy \qquad \int y\,dx = yx$

$\int_{(3,2)}^{(5,5)} xy + yx = 5\cdot5 + 5\cdot5 - (3\cdot2 + 3\cdot2)$ X $\emptyset$

$= 25 + 25 - 6 - 6$

$= 50 - 12$

$= 38$

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IME I PREZIME: [illegible] **BROJ INDEKSA:** [illegible]

POPUNJAVA NASTAVNIK — Broj ↓ bodova

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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

3. $\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = -xy + xy = ?$ ×

$= -3 \cdot 2 + 5 \cdot 5$

$25 - 6 = 19$

$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = -6 + 25$

$= \boxed{19}$

4. $a = 2$

$\iint (2x+3)\,dx\,dy$

$x \in [-1,1]$

$y \in [-1,1]$

$z \in [-1,1]$

$\operatorname{div} w = 0$

$f(x) = \int_{-1}^{1}\int_{-1}^{1} \operatorname{div} w \cdot dx\,dy\,dz = \boxed{0}$ ×

DVOSTRUKI INTEGRAL
JE POGREŠAN.

1. $r = 1 \quad T(0,0)$

$\int xy\,ds$

$x = r\cos t$

$y = r\sin t$

$\begin{bmatrix} r\cos t \\ r\sin t \\ 1 \end{bmatrix} \quad \begin{bmatrix} -1\sin t \\ 1\cos t \\ 1 \end{bmatrix}$

$\|r'\| = \sqrt{(-1\sin t)^2 + (1\cos t)^2 + 1^2}$

$= \sqrt{1\sin^2 t + 1\cos^2 t + 1}$

$= \sqrt{1(\underbrace{\sin^2 t + \cos^2 t}_{1}) + 1}$

$= \sqrt{2}$ ×

$\int_0^{2\pi} xy\sqrt{2}\,dt = \sqrt{2}\int_0^{2\pi} 1\cos t \sin t \; dt$

$= 2\sqrt{2}\pi$ ×

?

(2) $T(0,-1)$ $r=1$

$\int (2x+3)\,dy$ $\quad \int (2x+3)\,r\,dr\,d\varphi = \int_0^{2\pi} (2(r\cos\varphi)+3)\,r\,dr\,d\varphi$

$x = r\cos\varphi + 0$

$y = r\sin\varphi - 1$

$= \int_0^{2\pi} (2r\cos\varphi + 3)\,r\,dr\,d\varphi$

$= \underbrace{\int_0^{2\pi} 2r^2\cos\varphi\,dr\,d\varphi}_{I} + \underbrace{\int_0^{2\pi} 3r\,dr\,d\varphi}_{II}$

I

$2\int_0^{2\pi}\cos\varphi\,d\varphi \int_0^{2\pi} r^2\,dr = 2\int_0^{2\pi}\cos\varphi\,d\varphi\,\frac{r^2}{2}\Big|_0^{2\pi} = 2\int\cos\varphi\,d\varphi\,\frac{1}{2}(2\pi-0)^2 = 2\int\cos\varphi\,d\varphi\,\frac{1}{2}4\pi^2$

$= 4\pi\sin\varphi\Big|_0^{2\pi} = 4\pi(\sin 2\pi - \sin 0) = \boxed{0}$

II

$3\int_0^{2\pi} d\varphi + \int_0^{2\pi} r\,dr = 3\int_0^{2\pi} d\varphi + \frac{r^2}{2}\Big|_0^{2\pi} = 3\int_0^{2\pi} d\varphi + \frac{1}{2}(2\pi-0)^2 = 3\int_0^{2\pi} d\varphi + 2\pi = 6\pi(2\pi-0)$

$= 12\pi$

$I + II = 12\pi$

(1) $x^2+y^2=1$ [illegible]

$\begin{bmatrix} 1\cos t \\ 1\sin t \end{bmatrix} \quad \begin{bmatrix} -1\sin t \\ 1\cos t \end{bmatrix}$

$\int xy\,ds \qquad \|r'\| = \sqrt{(-1\sin t)^2+(1\cos t)^2}$

$x = r\cos t \qquad = \sqrt{1\sin^2 t + 1\cos^2 t}$

$y = r\sin t \qquad = \sqrt{1} = 1$

$$\int_0^{2\pi} xy\,ds = \int_0^{2\pi} xy\cdot 1\,dt$$

$$= \int_0^{2\pi} \cos t \sin t \cdot 1\,dt$$

$= 2\pi$ ✗

✓ 15

(2) $y'''(t)+2y''(t)+y'(t)=t \qquad y(0)=2 \quad y'=0 \quad y''(0)=1$

$s^3 f(s) - s^2 f(0) - s f'(0) - f''(0) + 2\left[s^2 f(s) - s f(0) - f'(0)\right] + s f(s) - f(0) = \frac{1}{s^2}$

$s^3 f(s) - s^2\cdot 2 - s\cdot 0 - 1 + 2\left[s^2 f(s) - s\cdot 2 - 0\right] + s f(s) - 2 = \frac{1}{s^2}$

$s^3 f(s) - 2s^2 - 1 + 2\left[s^2 f(s) - 2s\right] + s f(s) - 2 = \frac{1}{s^2}$

$s^3 f(s) - 2s^2 - 1 + 2s^2 f(s) - 4s + s f(s) - 2 = \frac{1}{s^2}$

$f(s)(s^3+2s^2+s) = \frac{1}{s^2} + 2 + 4s + 1 + 2s^2 \Rightarrow f(s)(s^3+2s^2+s) = \frac{1}{s^2} + 2s^2 + 4s + 3$

$f(s)(s^3+2s^2+s) = 1 + 2s^4 + 4s^3 + 3s^2$

$f(s) = \frac{2s^4+4s^3+3s^2+1}{s^3+2s^2+s} \Rightarrow \frac{2s^4+4s^3+3s^2+1}{s(s^2+2s+1)}$

$f(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2s+1} \Rightarrow A(s^2+2s+1) + (Bs+C)s =$

$= As^2 + 2As + A + Bs^2 + Cs$

$= s^2(A+B) + s(2A+C) + A$

$A+B=3 \Rightarrow 1+B=3 \Rightarrow B=2$

$2A+C=0 \Rightarrow 2+C=0$

$A=1 \qquad C=0$

0

$\frac{1}{s} + \frac{2s}{s^2+2s+1} = 1 + \frac{2}{s^2}\cdot\frac{s}{2s+1} = 1 + 2t\cos t$

✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: ROBERT SKOBLAR

BROJ INDEKSA: 52487

1. Neka je K krug radijusa $r=1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$. 20

2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. 20

3. Izračunati $\displaystyle\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$ 10

4. Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}x\mathrm{d}y$? 20

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z=x^2+y^2$ što leži iznad područja $D \ldots x^2+y^2 \leq 4$. Nije potrebno računati površinu baze. 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + 2y''(t) + y'(t) = t, \quad y(0)=2,\ y'(0)=0,\ y''(0)=1.$$

Ukupno: 0

Tablica Laplaceovih transformacija:

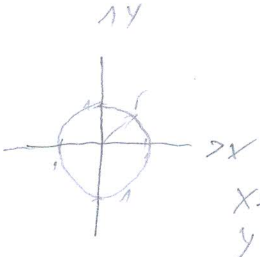
$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

ROBERT SKOBLAR 524 PT

1) Krug, $r=1$, $T(0,0)$ $\oint_{\partial K} xy\,ds = ?$



$r \in (0,1)$
$\varphi \in (0, 2\pi)$
$x = r\cdot\cos\varphi$
$y = r\cdot\sin\varphi$

$\cos\varphi\cdot\sin\varphi = \sin\frac{1}{2}\varphi$

$$\int_0^{2\pi}\int_0^{2}(r\cdot\cos\varphi\cdot r\cdot\sin\varphi)\, r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{2}(r^3\cos\varphi\sin\varphi)\,dr\,d\varphi$$

$$\int_0^{2\pi}\cos\varphi\sin\varphi\,d\varphi\int_0^{2} r^3\,dr$$

$$\frac{1}{2}\int_0^{2\pi}\sin\frac{1}{2}\varphi\cdot\frac{r^4}{4}\Big|_0^2$$

$$\frac{1}{2}\int_0^{2\pi}\sin\varphi\left(\frac{2^4}{8}\right)d\varphi$$

$$\frac{1}{2}(\sin\varphi)\cdot(2)\Big|_0^{2\pi}$$

$$\frac{1}{2}\cdot\sin 2\pi\cdot 2\cdot 2\pi$$

$= 0$

4) Kocka $a=2$ u ishodištu

$$\iint(2x+3)\,dx\,dy = ?$$

$$\iiint(2x+3)\,dx\,dy\,dz\cdot\mathrm{div}$$

$$\iiint \mathrm{div}\cdot(2x+3)\,dx\,dy\,dz$$

$= 0$

6) $y'''(t)+2y''(t)+y'(t)=t$

$y(0)=2$

$y'(0)=0$

$y''(0)=1$

$$s^3Y(s)-s^2y(0)-\overset{=0}{sy'(0)}-y''(0)+2s^2Y(s)-sy(0)-\overset{=0}{y'(0)}+sY(s)-y(0)=\frac{1}{s^2}$$

$$s^3Y_s-s^2\cdot 2-1+(2\cdot s^2Y(s)-s\cdot 2+sY(s)-2=\frac{1}{s^2}$$

$$s^3Y_s-2s^2-1+2s^2Y(s)-2s+sY(s)-2=\frac{1}{s^2}$$

$$s^3Y_s+2\cdot s^2Y(s)+sY(s)=\frac{1}{s^2}+2s^2+1+2s+2$$

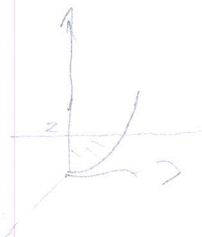
$$s^3Y_s+2s^2Y(s)+sY(s)=\frac{2s^2+2s+4}{s^2}$$

0

5) $z=x^2+y^2$ $x^2+y^2\le 4$

$z=r^2$ $r^2\le 4$

$r\le 2$



$\varphi\in(0,2\pi)$

$r\in(0,2)$

0

2) Krug $r=1$ $T(0,-1)$ $\int_{\partial K}(2x+3)\,dy$

$$\int_0^{2\pi}\int_0^1(2x+3)\,r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1(r\cdot 2\cos\varphi\, 3r)\,dr\,d\varphi$$

$$2\int_0^{2\pi}(\cos\varphi\cdot r\cdot 3r)\Big|_0^1$$

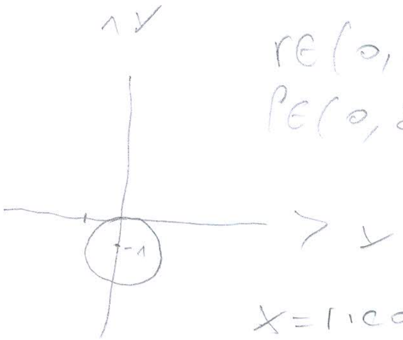
$$2\int_0^{2\pi}\cos\varphi\cdot 3\,d\varphi$$

$$6\cos\varphi\Big|_0^{2\pi}$$

$$6\cdot(\cos 2\pi-\cos 0)$$

$$6\cdot 0$$

$$=0$$



$r\in(0,1)$

$\varphi\in(0,2\pi)$

$x=1\cdot\cos\varphi+\overset{=0}{x_1}$

$y=1\cdot\sin\varphi+\overset{=-1}{y_1}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Frane Ženić BROJ INDEKSA: 57649 -2009	

1. Neka je K krug radijusa $r=1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$. 20

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$$y'''(t)+2y''(t)+y'(t)=t,\quad y(0)=2,\ y'(0)=0,\ y''(0)=1.$$

Ukupno: $\emptyset$

Tablica Laplaceovih transformacija:

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$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
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$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
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1. $y'''(t) + 2y''(t) + y'(t) = t$ $y(0)=2$ $y'(0)=0$ $y''(0)=1$ Prune Zemit

$\cancel{s^3 F(s) - s^2 f}$

$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + 2\cdot\left(s^2 Y(s) - s y(0) - y'(0)\right) + sY(s) - y(0) = t$

$y(0)=2$, $y'(0)=0$, $y''(0)=1$, $y(0)=2$, $y'(0)=1$, $y(0)=2$

$s^3 Y(s) - 2s^2 - 1 + 2\cdot\left(s^2 Y(s) - 2s - 1\right) + sY(s) - 2 = t$

$s^3 Y(s) - 2s^2 - 1 + 2s^2 Y(s) - 4 - 2s + sY(s) - 2\cancel{t} = t$

$s^3 Y(s) - 2s^2 - 1 + 2s^2 Y(s) - \cancel{6}\,7 + sY(s) - 2\cancel{t} = t$

$\cancel{s^3 Y(s) + 2s^2 Y(s) + sY(s) = 2s^2 + 1 + 6 + 2}$

$1, \sqrt{4\cdot 1\cdot 2}$

$0, \sqrt{3}$

$\cancel{s^3 Y(s) + 2s^2 Y(s) + sY(s) = 2s^2 + 9}$

$\cancel{Y(s)\cdot(s^3 + 2s^2 + s) = 2s^2 + 9}$

$\cancel{Y(s) = \frac{2s^2+9}{s^3+2s^2+s}}$

$\cancel{Y(s) = \frac{2s^2+9}{s\cdot(s^2+2s+1)}}$

$Y(s) = \frac{A}{s} + \frac{Bs^2 + Cs + D}{s^2+2s+1}$

$Y(s) = \cancel{\frac{A}{s} \cdot \frac{Bs^2+Cs}{(s+1)^2}}$ naz

$\cancel{2s^2+9} = \cancel{s\cdot(s+1)^2 = As^2 + 2As + A + Bs^3 + Cs^2}$

$\cancel{s^2+2s+1 = As^2 + Cs^2 + Bs^3 + A}$

$\cancel{B=0}$ $\cancel{1+C=1}$

$\cancel{A=1}$ $\cancel{C=0}$

$s^3 Y(s) - 2s^2 - 1 + 2s^2 Y(s) - 4 - 2s + sY(s) - 2 = t$

$s^3 Y(s) + 2s^2 Y(s) + sY(s) - 2s^2 - 7 - 2s = t$

$Y(s)\cdot(s^3 + 2s^2 + s) = 2s^2 + 7 + 2s$

$Y(s)(s^3 + 2s^2 + s) = 2s^2 + 2s + 7$

$Y(s) = \frac{2s^2 + 2s + 7}{s^3 + 2s^2 + s}$

$Y(s) = \frac{2s^2 + 2s + 7}{s\cdot(s^2+2s+1)}$

$Y(s) = \frac{2s^2 + 2s + 7}{s\cdot(s+1)^2}$

$(s+1)^2$

$(s+1)^2$

s^2+2s+1

$\cancel{s^3 + 2s^2 + s}$

$\cancel{(s^2+s)\cdot(s+2)}$

$\cancel{s^3 + 4s^2 + s^2 + 2s}$

2. $r=1$ $\quad T(0,-1)$

3 $\int x\,dy + y\,dx$

$\int x+y$

$(s+1)^2 = s^2+2s+1$

6. $Y(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+}{(s+1)^2}$ | $\cdot$ nuz

$(s^2+s)\cdot(s+1)^2 = As+A+As^2+2As+A+As+A+Bs+Bs^2+2Bs+B+Cs^2+Cs^2+C$

$(s^2+s)\cdot(s+1)^2 = 3As+3Bs+2Cs^2+3A+B+C$

$s^3+s^2+s^2+s$

$s^3+2s^2+s =$

$3A+3B=1$

$3A = 1-3B$

$A = \frac{1-3B}{3}$