MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} xy\,\mathrm{d}s$. 16 — 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\partial\widehat{K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\partial\widehat{K}} (2x+3)\,\mathrm{d}y$. 17 — 20

3. Izračunati $\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$ 14 — 10

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial\widehat{K}} (2x+3)\,\mathrm{d}x\mathrm{d}y$? 35 — 20

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = x^2+y^2$ što leži iznad područja $D \ldots x^2+y^2 \leq 4$. Nije potrebno računati površinu baze. — 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 15

$$y'''(t) + 2y''(t) + y'(t) = t, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 40

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$		$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$		
$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$

6) $y'''(t) + 2y''(t) + y'(t) = t \qquad y(0)=2,\ y'(0)=0,\ y''(0)=1$

$$s^3Y(s) - s^2\underbrace{y(0)}_{2} - s\underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{1} + 2\left(s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{0}\right) + sY(s) - \underbrace{y(0)}_{2} = \frac{1}{s^2}$$

$$s^3Y(s) - 2s^2 - 1 + 2s^2Y(s) - 4s + sY(s) - 2 = \frac{1}{s^2}$$

$$s^3Y(s) + 2s^2Y(s) = \frac{1}{s^2} + 2 + 4s + 1 + 2s^2$$

$$Y(s)\left(s^3 + 2s^2\right) = \frac{1 + 3s^2 + 4s^3 + 2s^4}{s^2} \qquad / : \left(s^3 + 2s^2\right)$$

$$Y(s) = \frac{\frac{1 + 3s^2 + 4s^3 + 2s^4}{s^2}}{s^3 + 2s^2} = \frac{2s^4 + 4s^3 + 3s^2 + 1}{\left(s^3 + 2s^2\right)s^2} = \frac{2s^4 + 4s^3 + 3s^2 + 1}{s^4(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{s+2} \qquad / \cdot s^4(s+2)$$

$$2s^4 + 4s^3 + 3s^2 + 1 = As^3(s+2) + Bs^2(s+2) + Cs(s+2) + D(s+2) + Es^4$$

$$\left(s^4 + 2s^3\right) \qquad \left(s^3 + 2s^2\right) \qquad \left(s^2 + 2s\right)$$

$s = 0$

$1 = D \cdot 2$

$D = \frac{1}{2}$

$$2s^4 + 4s^3 + 3s^2 + 1 = As^4 + 2As^3 + Bs^3 + 2Bs^2 + Cs^2 + 2Cs + Ds + 2D + Es^4$$

$$2s^4 + 4s^3 + 3s^2 + 1 = (A+E)s^4 + (2A+B)s^3 + (2B+C)s^2 + (2C+D)s + 2D$$

$A + E = 2$

$2A + B = 4$

$2B + C = 3$

$2C + D = 0$

$2D = 1$

$D = \frac{1}{2}$

$2C + \frac{1}{2} = 0$

$2C = -\frac{1}{2}$

$C = -\frac{1}{4}$

$2B - \frac{1}{4} = 3$

$2B = 3 + \frac{1}{4}$

$2B = \frac{13}{4}$

$B = \frac{13}{8}$

$2A + B = 4$

$2A + \frac{13}{8} = 4$

$2A = 4 - \frac{13}{8}$

$2A = \frac{19}{8}$

$A = \frac{19}{16}$

$\frac{19}{16} + E = 2$

$E = 2 - \frac{19}{16}$

$E = \frac{13}{16}$