MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ B8

IME I PREZIME: VESNA ŠARIĆ **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice
$$\begin{array}{c} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{array}$$
i obavezno <u>provjeri</u> rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 5}$ treba:
   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno:

4) $g(x)=\sqrt{x^2+4x+5}$

$x^2+4x+5 \geq 0$

$x^2+4x+5=0$

$1, 4, 5$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{-1\pm\sqrt{4^2-4\cdot1\cdot5}}{2\cdot1}$

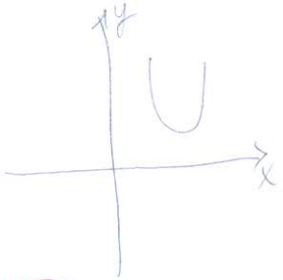
$x_{1,2}=\frac{-1\pm\sqrt{16-20}}{2}$

$x_{1,2}=\frac{-1\pm\sqrt{-4}}{2}$

$x_{1,2}=\frac{-1\pm 2i}{2}$

$\boxed{x_1=\frac{-1+2i}{2} \quad x_2=\frac{-1-2i}{2}}$

Nultočke ne sjeku graf funkcije.



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6) $\lim_{x\to-1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right)=\left(\frac{\sqrt{(-1)^2-8\cdot(-1)+7}-3+(-1)}{(-1)^2+4\cdot(-1)+3}\right)$

$=\frac{\sqrt{1+8+7}-3-1}{1-4+3}=\frac{\sqrt{16}-3-1}{1-4+3}=\frac{4-3-1}{1-4+3}=\boxed{\frac{0}{0}}$ neodređeni oblik

HOPITALOVO PRAVILO

$\lim_{x\to-1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right)'=\left[\frac{0}{0}\right]=\frac{\frac{1}{2\sqrt{x^2-8x+7}}\cdot(x^2-8x+7)'-(3)'+(x)'}{(x^2+4x+3)'}$

$=\frac{\frac{1}{2\sqrt{x^2-8x+7}}\cdot(2x-8)+1}{2x+4}=\frac{\frac{2x-8}{2\sqrt{x^2-8x+7}}+1}{2x+4}=\lim_{x\to(-1)}\frac{\frac{2\cdot(-1)-8}{2\sqrt{(-1)^2-8(-1)+7}}+1}{2\cdot(-1)+4}$

$=\frac{\frac{-2-8}{2\sqrt{1+8+7}}+1}{-2+4}=\frac{\frac{-10}{2\cdot4}+1}{-2+4}=\frac{\frac{-10}{8}+1}{2}=\frac{-\frac{1}{4}}{\frac{2}{1}}=\boxed{-\frac{1}{8}}$

PROVJERA:

VESNA SAMEC

②

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & -3 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$1r \cdot (-1) + 2r$
$1r \cdot (-4) + 3r$
$1r \cdot (-5) + 4r$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \sim$$

$1r \cdot (-5) + 2r$
$1r \cdot (-4) + 3r$
$1r \cdot (-1) + 4r$

$2r \cdot (-1) + 3r$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & -7 & 0 & -3 & 4 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \cdot (-1) \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim$$

$3r \cdot (-6) + 2r$

$2r \cdot (-2) + 3r$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 0 & -1 & -6/7 & 1/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$4r \cdot \frac{6}{7} + 3r$

$4r \cdot \left(-\frac{3}{7}\right) + 2r$

$4r \cdot (-1) + 1r$

$3r \cdot 2 + 1r$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 4/7 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/7 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \cdot (-1) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/7 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & 1 & 0 & 5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$2r + 1r$

$$x = \frac{3}{7}$$

$$y = -\frac{1}{7}$$

$$z = \frac{5}{7}$$

$$t = -1$$

$$\frac{3}{7} + \left(-\frac{1}{7}\right) + \frac{5}{7} + (-1) = 0$$

$$\frac{3}{7} - \frac{1}{7} + \frac{5}{7} - \frac{7}{7} = 0$$

$$0 = 0$$

$$x - y + 2z + t = 1$$

$$\frac{3}{7} - \left(-\frac{1}{7}\right) + 2 \cdot \frac{5}{7} + (-1) = 1$$

$$\frac{3}{7} + \frac{1}{7} + \frac{10}{7} - \frac{7}{7} = 1$$

$$5x + 4z + 2t = 3$$

$$5 \cdot \frac{3}{7} + 4 \cdot \frac{5}{7} + 2 \cdot (-1) = 3$$

$$\frac{15}{7} + \frac{20}{7} - 2 = 3$$

$$4x + y + 2z = 1$$

$$4 \cdot \frac{3}{7} + \left(-\frac{1}{7}\right) + 2 \cdot \frac{5}{7} = 1$$

$$\frac{7}{7} = 1$$

VESNA ŠARIĆ

1. $\left|\frac{z^2}{2}\right| = z+1+i$

$\frac{|z|^2}{4} = z+1+i$

$\left(\frac{\sqrt{x^2+y^2}}{4}\right)^2 = x+y+1+i$

$\frac{x^2+y^2}{4} = x+iy+1+i \;/\cdot 4$?

$x^2+y^2 = 4x+4y+4+4i$

$x^2+y^2 = 4x+4y+4$

$\boxed{0 = 4i}$

$4i = 0$

$i = \frac{0}{4}$

$\boxed{i = 0}$

$x^2+y^2 = 4x+4y$

$x^2-4x = 4y-y^2 \;/\sqrt{\ }$

$x-\sqrt{4x} = \sqrt{4y}-y$

$x-2x = 2y-y$

NEMA RJEŠENJA ?

~~3~~

③ $f'(x) = \frac{\sin(2x)}{x} =$ VESNA ŠARIĆ

$f'(x) = \frac{\cos 2x + \sin 2}{x} = \frac{(\cos 2x + \sin 2)' \cdot x - }{x^2}$

$\sin \cdot 2x$

$(\sin)' \cdot 2x + \sin \cdot (2x)'$

$= \cos \cdot 2x + \sin 2$

$\cos \cdot 2x$

$= \cos' \cdot 2x + \cos \cdot (2x)'$

$f'(x) = \frac{(\cos 2x + \sin 2) \cdot x - (\cos 2x + \sin 2) \cdot x'}{x^2}$

$= -\sin \cdot 2x + \cos 2$

$\sin 2 = \sin' \cdot 2 + \sin \cdot 2'$

$= \cos 2 + \sin$

$f'(x) = \frac{(-\sin 2x + \cos 2 + \cos 2 + \sin) - (\cos 2x + \sin 2)}{x^2}$

$f'(x) = \frac{-\sin 2x + \cos 2 + \cos 2 + \sin - \cos 2x - \sin 2}{x^2}$

$f'(x) = \frac{-\sin 2x + 2\cos 2 + \sin - \cos 2x - \sin 2}{x^2}$

$f'(\pi) = \frac{0 + (-0.83) + \sin - 1 - 0.909}{9.86}$

$f'(\pi) = \frac{-0.83 + \sin - 1.909}{9.86}$

$f'(\pi) = \frac{-2.739 \sin}{9.86}$ //

VESNA ŠARIĆ

3) $f(x) = \frac{\sin(2x)}{x}$

$(\sin \cdot 2x)'$

$((\sin x)' \cdot 2x + \sin \cdot (2x)'$

$(\cos \cdot 2x + \sin 2)$

$f(x) = \frac{\sin(2x)}{x}$

$f'(x) = \frac{(\sin(2x))' \cdot x - \sin(2x) \cdot x'}{(x)^2}$

$f'(x) = \frac{(\cos \cdot 2x + \sin 2) - \sin 2x}{x^2} \quad \frac{\cos 2x + \sin 2 - \sin 2x}{x^2}$

$f'(\pi) = \frac{\cos 2\pi + \sin 2 - \sin 2\pi}{(\pi)^2} = \frac{1 + 0.90 - 0}{9.86} = \frac{1.90}{9.86} = 0.19$

$f'(x) = \frac{(\cos 2x + \sin 2)}{x} = \frac{(\cos 2x + \sin 2)' \cdot x - (\cos 2x + \sin 2) \cdot x'}{x^2}$

$f'(x) = \frac{(-\sin 2 + \cos 2) \cdot x - \cos 2x + \sin 2}{x^2} = \frac{-x \sin 2 + \cos 2x - \cos 2x + \sin 2}{x^2}$

$f'(\pi) = \frac{-2\sin 2 + \cos(\pi) - \cos 2(\pi) + \sin}{(\pi)^2} = \frac{-2\sin 2 + (-1) - 1 + \sin}{9.86}$

$= \frac{-2\sin - 2 + \sin}{9.86} = \frac{-\sin - 2}{9.86} =$