MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ B8

IME I PREZIME: VESNA ŠARIĆ **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice
$$\begin{array}{c} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{array}$$
i obavezno <u>provjeri</u> rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 5}$ treba:
   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno:

4) $g(x)=\sqrt{x^2+4x+5}$

$x^2+4x+5 \geq 0$

$x^2+4x+5=0$

$1, 4, 5$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{-1\pm\sqrt{4^2-4\cdot1\cdot5}}{2\cdot1}$

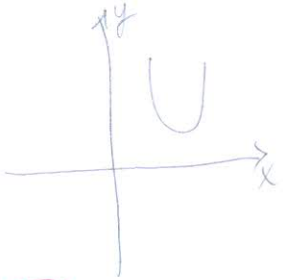
$x_{1,2}=\frac{-1\pm\sqrt{16-20}}{2}$

$x_{1,2}=\frac{-1\pm\sqrt{-4}}{2}$

$x_{1,2}=\frac{-1\pm 2i}{2}$

$\boxed{x_1=\frac{-1+2i}{2} \quad x_2=\frac{-1-2i}{2}}$

Nultočke ne sjeku graf funkcije.



?

6) $\lim_{x\to-1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right)=\left(\frac{\sqrt{(-1)^2-8\cdot(-1)+7}-3+(-1)}{(-1)^2+4\cdot(-1)+3}\right)$

$=\frac{\sqrt{1+8+7}-3-1}{1-4+3}=\frac{\sqrt{16}-3-1}{1-4+3}=\frac{4-3-1}{1-4+3}=\boxed{\frac{0}{0}}$ neodređeni oblik

HOPITALOVO PRAVILO

$\lim_{x\to-1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right)'=\left[\frac{0}{0}\right]=\frac{\frac{1}{2\sqrt{x^2-8x+7}}\cdot(x^2-8x+7)'-(3)'+(x)'}{(x^2+4x+3)'}$

$=\frac{\frac{1}{2\sqrt{x^2-8x+7}}\cdot(2x-8)+1}{2x+4}=\frac{\frac{2x-8}{2\sqrt{x^2-8x+7}}+1}{2x+4}=\lim_{x\to(-1)}\frac{\frac{2\cdot(-1)-8}{2\sqrt{(-1)^2-8(-1)+7}}+1}{2\cdot(-1)+4}$

$=\frac{\frac{-2-8}{2\sqrt{1+8+7}}+1}{-2+4}=\frac{\frac{-10}{2\cdot4}+1}{-2+4}=\frac{\frac{-10}{8}+1}{2}=\frac{-\frac{1}{4}}{\frac{2}{1}}=\boxed{-\frac{1}{8}}$

PROVJERA:

VESNA SAMEC

②

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & -3 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$1r \cdot (-1) + 2r$
$1r \cdot (-4) + 3r$
$1r \cdot (-5) + 4r$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \sim$$

$1r \cdot (-5) + 2r$
$1r \cdot (-4) + 3r$
$1r \cdot (-1) + 4r$

$2r \cdot (-1) + 3r$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & -7 & 0 & -3 & 4 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \cdot (-1) \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim$$

$3r \cdot (-6) + 2r$

$2r \cdot (-2) + 3r$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 0 & -1 & -6/7 & 1/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$4r \cdot \frac{6}{7} + 3r$

$4r \cdot \left(-\frac{3}{7}\right) + 2r$

$4r \cdot (-1) + 1r$

$3r \cdot 2 + 1r$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 4/7 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/7 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & -1 & 0 & -5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \cdot (-1) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/7 \\ 0 & 1 & 0 & 0 & -1/7 \\ 0 & 0 & 1 & 0 & 5/7 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$2r + 1r$

$$x = \frac{3}{7}$$

$$y = -\frac{1}{7}$$

$$z = \frac{5}{7}$$

$$t = -1$$

$$\frac{3}{7} + \left(-\frac{1}{7}\right) + \frac{5}{7} + (-1) = 0$$

$$\frac{3}{7} - \frac{1}{7} + \frac{5}{7} - \frac{7}{7} = 0$$

$$0 = 0$$

$$x - y + 2z + t = 1$$

$$\frac{3}{7} - \left(-\frac{1}{7}\right) + 2 \cdot \frac{5}{7} + (-1) = 1$$

$$\frac{3}{7} + \frac{1}{7} + \frac{10}{7} - \frac{7}{7} = 1$$

$$5x + 4z + 2t = 3$$

$$5 \cdot \frac{3}{7} + 4 \cdot \frac{5}{7} + 2 \cdot (-1) = 3$$

$$\frac{15}{7} + \frac{20}{7} - 2 = 3$$

$$4x + y + 2z = 1$$

$$4 \cdot \frac{3}{7} + \left(-\frac{1}{7}\right) + 2 \cdot \frac{5}{7} = 1$$

$$\frac{7}{7} = 1$$

VESNA ŠARIĆ

1. $\left|\frac{z^2}{2}\right| = z+1+i$

$\frac{|z|^2}{4} = z+1+i$

$\left(\frac{\sqrt{x^2+y^2}}{4}\right)^2 = x+y+1+i$

$\frac{x^2+y^2}{4} = x+iy+1+i \;/\cdot 4$?

$x^2+y^2 = 4x+4y+4+4i$

$x^2+y^2 = 4x+4y+4$

$\boxed{0 = 4i}$

$4i = 0$

$i = \frac{0}{4}$

$\boxed{i = 0}$

$x^2+y^2 = 4x+4y$

$x^2-4x = 4y-y^2 \;/\sqrt{\ }$

$x-\sqrt{4x} = \sqrt{4y}-y$

$x-2x = 2y-y$

NEMA RJEŠENJA ?

~~3~~

③ $f'(x) = \frac{\sin(2x)}{x} =$ VESNA ŠARIĆ

$f'(x) = \frac{\cos 2x + \sin 2}{x} = \frac{(\cos 2x + \sin 2)' \cdot x - }{x^2}$

$\sin \cdot 2x$

$(\sin)' \cdot 2x + \sin \cdot (2x)'$

$= \cos \cdot 2x + \sin 2$

$\cos \cdot 2x$

$= \cos' \cdot 2x + \cos \cdot (2x)'$

$f'(x) = \frac{(\cos 2x + \sin 2) \cdot x - (\cos 2x + \sin 2) \cdot x'}{x^2}$

$= -\sin \cdot 2x + \cos 2$

$\sin 2 = \sin' \cdot 2 + \sin \cdot 2'$

$= \cos 2 + \sin$

$f'(x) = \frac{(-\sin 2x + \cos 2 + \cos 2 + \sin) - (\cos 2x + \sin 2)}{x^2}$

$f'(x) = \frac{-\sin 2x + \cos 2 + \cos 2 + \sin - \cos 2x - \sin 2}{x^2}$

$f'(x) = \frac{-\sin 2x + 2\cos 2 + \sin - \cos 2x - \sin 2}{x^2}$

$f'(\pi) = \frac{0 + (-0.83) + \sin - 1 - 0.909}{9.86}$

$f'(\pi) = \frac{-0.83 + \sin - 1.909}{9.86}$

$f'(\pi) = \frac{-2.739 \sin}{9.86}$ //

VESNA ŠARIĆ

3) $f(x) = \frac{\sin(2x)}{x}$

$(\sin \cdot 2x)'$

$((\sin x)' \cdot 2x + \sin \cdot (2x)'$

$(\cos \cdot 2x + \sin 2)$

$f(x) = \frac{\sin(2x)}{x}$

$f'(x) = \frac{(\sin(2x))' \cdot x - \sin(2x) \cdot x'}{(x)^2}$

$f'(x) = \frac{(\cos \cdot 2x + \sin 2) - \sin 2x}{x^2} \quad \frac{\cos 2x + \sin 2 - \sin 2x}{x^2}$

$f'(\pi) = \frac{\cos 2\pi + \sin 2 - \sin 2\pi}{(\pi)^2} = \frac{1 + 0.90 - 0}{9.86} = \frac{1.90}{9.86} = 0.19$

$f'(x) = \frac{(\cos 2x + \sin 2)}{x} = \frac{(\cos 2x + \sin 2)' \cdot x - (\cos 2x + \sin 2) \cdot x'}{x^2}$

$f'(x) = \frac{(-\sin 2 + \cos 2) \cdot x - \cos 2x + \sin 2}{x^2} = \frac{-x \sin 2 + \cos 2x - \cos 2x + \sin 2}{x^2}$

$f'(\pi) = \frac{-2\sin 2 + \cos(\pi) - \cos 2(\pi) + \sin}{(\pi)^2} = \frac{-2\sin 2 + (-1) - 1 + \sin}{9.86}$

$= \frac{-2\sin - 2 + \sin}{9.86} = \frac{-\sin - 2}{9.86} =$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B8

IME I PREZIME: ANTON RAŠIĆ **BROJ INDEKSA:** 17-2-0054-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

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5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno: 15

(2)
$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 5R_1 \\ R_3 - 4R_1 \\ R_4 - R_1 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \begin{array}{l} \\ \\ \\ 5R_4 - 2R_2 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 7 & 6 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & 7 & 6 & 3 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right] \begin{array}{l} \\ \\ 1:5 \\ 1:7 \\ 1:(-1) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 6/7 & 3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 + R_2 \\ \\ \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 6/7 & 3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 - \frac{4}{5}R_3 \\ \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2/7 & 9/35 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 6/7 & 3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \overset{R_1 + \frac{2}{7}R_4}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 19/35 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 6/7 & 3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \overset{R_2 + \frac{6}{5}R_3}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 19/35 \\ 0 & 1 & 0 & 3/7 & 4/35 \\ 0 & 0 & 1 & 6/7 & 3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \overset{R_2 - \frac{3}{7}R_4}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 19/35 \\ 0 & 1 & 0 & 0 & -11/35 \\ 0 & 0 & 1 & 6/7 & 3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \overset{R_3 - \frac{6}{7}R_4}{}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 19/35 \\ 0 & 1 & 0 & 0 & -11/35 \\ 0 & 0 & 1 & 0 & -3/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \qquad \begin{Bmatrix} x \\ y \\ z \\ t \end{Bmatrix} = \begin{Bmatrix} 19/35 \\ -11/35 \\ -3/7 \\ 1 \end{Bmatrix}$$

PROVJERA:

$5x + 4z + 2t = 3$

$5 \cdot \frac{19}{35} - 4 \cdot \frac{3}{7} + 2 = 3$

$\frac{95}{35} - \frac{12}{7} + 2 = 3$

$\frac{19}{7} - \frac{12}{7} + \frac{2}{1} = 3$

$\frac{19 - 12 + 14}{7} = 3$

$\frac{21}{7} = 3$

$3 = 3$ //

$x - y + 2z + t = 1$

$\frac{19}{35} + \frac{11}{35} - 2 \cdot \frac{3}{7} + 1 = 1$

$\frac{30}{35} - \frac{6}{7} + \frac{1}{1} = 1$

$\frac{6}{7} - \frac{6}{7} + \frac{1}{1} = 1$

$\frac{1}{1} = 1$

$1 = 1$ //

$4x + y + 2z = 1$

$4 \cdot \frac{19}{35} - \frac{11}{35} - 2 \cdot \frac{3}{7} = 1$

$\frac{76}{35} - \frac{11}{35} - \frac{6}{7} = 1$

$\frac{76 - 11 - 30}{35} = 1$

$\frac{35}{35} = 1$

$1 = 1$ //

$x + y + z + t = 0$

$\frac{19}{35} - \frac{11}{35} - \frac{3}{7} + 1 = 0$

V.A. NEMA.

H.A. NEMA

K.A. NEMA

④ a) $g(x) = \sqrt{x^2+4x+5}$

$x^2+4x+5 \geq 0$

$x_{1/2} = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{4}\,i}{2}$

$x_1 = \frac{-4+\sqrt{4}\,i}{2} \quad x_2 = \frac{-4-\sqrt{4}\,i}{2}$

$Dg: \mathbb{R}$ ✓

⑥ $\lim_{x\to -1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right) = \lim_{x\to -1}\frac{\sqrt{(-1)^2-8(-1)+7}-3+(-1)}{(-1)^2+4(-1)+3} =$

$= \frac{\sqrt{1+8+7}-3-1}{1-4+3} = \frac{\sqrt{15}-4}{0} = \infty$ ✗

③ $f(x) = \frac{\sin(2x)}{x} = \frac{(\sin(2x))'\cdot x - \sin(2x)\cdot x'}{x^2}$

$= \frac{\cos(2x)\cdot(2x)'\cdot x - \sin(2x)\cdot 1}{x^2}$

$= \frac{2\cos(2x)\cdot x - \sin(2x)}{x^2} = \frac{2x\cos(2x)-\sin(2x)}{x^2}$ ✓

$= \frac{2\pi \overset{=1}{\cos(2\pi)} - \overset{=0}{\sin(2\pi)}}{\pi^2} = \frac{6{,}28\cdot \overset{=1}{0{,}9939} - \overset{=0}{0{,}05478}}{9{,}8596}$

$= 0{,}63$ // ✓

⑤ $h(x) = \frac{x^2+8}{x-4}$

$x-4 \neq 0$

$x \neq 4$

Dj: $\mathbb{R}\setminus\{4\}$ ✓

$x^2+8=0$

$x^2=-8 \mid \sqrt{}$

$x_1 = +\sqrt{8}\,i$

$x_2 = -\sqrt{8}\,i$

V.A.

$\lim_{x\to 4} \frac{x^2+8}{x-4} = \frac{(4)^2+8}{4-4} = \infty$

$y=4$

GRAF?

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① $\left|\frac{z}{2}\right|^2 = z+1+i$

$\left|\frac{z}{2}\right| = \sqrt{z+1+i}$ ✗

$\left|\frac{z}{2}\right| = \frac{\sqrt{z+1+i}}{1} \cdot \frac{\sqrt{z+1-i}}{1}$

$\left|\frac{z}{2}\right| = (z+1+i)(z+1-i)$

$\left|\frac{z}{2}\right| = z^2+z-zi+z+1-i+iz+i-i^2$

$\left|\frac{z}{2}\right| = z^2+2z+1-i^2$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B8

IME I PREZIME: Duje Mitrović **BROJ INDEKSA:** 17-2-0205-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{matrix} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{matrix}$ i obavezno provjeri rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 5}$ treba:
   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno: 20

(2)

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \begin{matrix} II-I \\ III-4I \\ IV-5I \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] II-III \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \begin{matrix} I-II \\ III+3II \\ IV+5II \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 14 & 17 & 3 \end{array}\right] IV-III \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 7 & 9 & 2 \end{array}\right] IV-II$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{matrix} I+3IV \\ II-4IV \\ III-8IV \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] (:7) \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] II-3III$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{matrix} x = 1 \\ y = -1 \\ z = -1 \\ t = 1 \end{matrix}$$

PROVJERA:

$5x + 4z + 2t = 3$
$5 - 4 + 2 = 3$
$3 = 3$ //

$4x + y + 2z = 1$
$4 - 1 - 2 = 1$ ✓
$1 = 1$ //

$x - y + 2z + t = 1$
$1 + 1 - 2 + 1 = 1$
$1 = 1$ //

$x + y + z + t = 0$
$1 - 1 - 1 + 1 = 0$
$0 = 0$ //

③ $f(x) = \frac{\sin(2x)}{x}$ $f'(\pi) = ?$

$f'(x) = \frac{\cos(2x)\cdot x - \sin(2x)\cdot 1}{x^2}$ ×

$f'(x) = \frac{x\cos(2x) - \sin 2x}{x^2}$

$f'(\pi) = \frac{\pi\cos(2\pi) - \sin 2\pi}{\pi^2}$

$f'(\pi) = \frac{3{,}14159 - 0}{9{,}86960}$

$f'(\pi) \approx 0{,}305309$

④ $g(x) = \sqrt{x^2+4x+5}$

a) $D: x^2+4x+5 \geq 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16-20}}{2}$

$x_{1,2} = \frac{-4 \pm \sqrt{-4}}{2}$

$Dg(x) = \mathbb{R}$ // ✓

b) $g'(x) = (\sqrt{x^2+4x+5})'$

$= (x^1 + 4x^{\frac{1}{2}} + \sqrt{5})'$ ×

$= 1 + 2x^{-\frac{1}{2}} + 5^{\frac{1}{2}}\ln 5$

$y = \sqrt{5}$

$T(0, \sqrt{5})$

$1 + 2x^{-\frac{1}{2}} + 5^{\frac{1}{2}}\ln 5 = 0$

$2x^{-\frac{1}{2}}$

$2\sqrt{x} = 1$

$\sqrt{x} = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$

c) $\lim_{x\to-\infty} \frac{f(x)}{x} = \frac{\sqrt{x^2+4x+5}}{x} = \frac{1}{1} = 1$ //

$\lim_{x\to-\infty} = \sqrt{x^2+4x+5} - x \cdot \frac{\sqrt{x^2+4x+5}+x}{\sqrt{x^2+4x+5}+x} =$

$= \frac{x^2+4x+5-x^2}{\sqrt{x^2-4x+5}+x} = \frac{4}{2} = 2$ //

$y = x + 2$

- Nima horizontalne
- Nima vertikalne

Nasa asimptota funkcije.

DUJE MITROVIĆ

⑤ $h(x) = \frac{x^2+8}{x-4}$

1) Domena

U: $x-4 \neq 0$

$x \neq 4$

$D_f = \mathbb{R} \setminus \{4\}$

2) V.A

$\lim_{x \to 4^-} \frac{x^2+8}{x-4} = \frac{24}{-0} = -\infty$

$\lim_{x \to 4^+} \frac{x^2+8}{x-4} = \frac{24}{+0} = +\infty$

H.A

$\lim_{x \to \infty} \frac{x^2+8}{x-4} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{1}{0} = 1$ $\boxed{y=1}$ OBOSTRANA ASIMPTOTA

$\lim_{x \to -\infty} \frac{x^2+8}{x-4} \begin{Bmatrix} x \to -x \\ -\infty \to \infty \end{Bmatrix} \frac{x^2+8}{-x-4} = \frac{1}{0} = 1$

Nema kose asimptote!

3) Globalne svojstva!

- Nije periodična
- Ni parna ni neparna

4) Izvodi:

$h'(x) = \frac{2x(x-4) - (x^2+8)}{(x-4)^2}$

$h'(x) = \frac{x^2-8x-8}{(x-4)^2}$

$h''(x) = \frac{(2x-8)(x-4)^2 - (x^2-8x-8) \cdot 2}{(x-4)^4}$

GRAF?

⑥ $\lim_{x \to -1} \frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3} \overset{L'H}{=} \frac{1-4x^{-\frac{1}{2}}+7^{\frac{1}{2}}\ln 7+1}{2x+2x^{\frac{1}{2}}}$

$= \frac{1+4^{-\frac{1}{2}}+7^{\frac{1}{2}}\ln 7+1}{-2-2} =$

⑥ $\lim_{x \to -1} \frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3} \cdot \frac{1:x^2}{1:x^2} = \frac{0}{1} = 0$

PROVERA:

$\frac{\sqrt{(-1)^2+8+7}-3-1}{(-1)^2-4+3} = \frac{4-3-1}{0}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B8

IME I PREZIME: Rikardo Radovčić **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{matrix} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{matrix}$ i obavezno provjeri rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 5}$ treba:
   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno: 29

② $5x + 4z + 2t = 3$

$x - y + 2z + t = 1$

$4x + y + 2z = 1$

$x + y + z + t = 0$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - 4R_1 \\ R_4 - 5R_1 \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & -2 & -3 & -4 & 1 \\ 0 & -1 & -5 & -3 & 3 \end{array}\right] \begin{array}{l} R_1 - R_2 \\ \\ R_3 + 2R_2 \\ R_4 + R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 1 & -1 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & -7 & -4 & 3 \\ 0 & 0 & -7 & -3 & 4 \end{array}\right] R_4 - R_3 \sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 1 & -1 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & -7 & -4 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 - R_4 \\ \\ R_3 + 4R_4 \\ \\ \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & -2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & -7 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] /\cdot(-7) \sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & -2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 - 3R_3 \\ R_2 + 2R_3 \\ \\ \\ \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$x = 1$

$y = -1$

$z = -1$

$t = 1$

$x = 1$

$y = -1$

$z = -1$

$t = 1$

*Provjera:

$5x + 4z + 2t = 3$

$5(1) + 0(-1) + 4(-1) + 2 \cdot (1) = 3$

$5 - 0 - 4 + 2 = 3$

$5 - 4 + 2 = 3$

$\boxed{3 = 3}$

$x - y + 2z + t = 1$

$1 - (-1) + 2(-1) + 1 = 1$

$1 + 1 - 2 + 1 = 1$

$\boxed{1 = 1}$

$4x + y + 2z = 1$

$4 \cdot 1 + (-1) + 2(-1) = 1$

$4 - 1 - 2 = 1$

$\boxed{1 = 1}$

$x + y + z + t = 0$

$1 + (-1) + (-1) + 1 = 0$

$1 - 1 - 1 + 1 = 0$

$\boxed{0 = 0}$

Ricardo Radovčić

(5) $h(x) = \frac{x^2+8}{x-4}$

① $x^2 + 8 \neq 0$

$x^2 + 8 = 0$

$x^2 = -8$

$D(h) = \mathbb{R}$ ✗

② NULTOČKE

$x - 4 = 0$

$x = \pm 4$

$f'(x) = \frac{2x(x-4) - 2x(x^2+8)}{(x-4)^2}$

$= \frac{2x^2 - 8x - 2x^3 - 16x}{(x-4)^2}$

$= \frac{x(2-2x) - 24x}{(x-4)^2}$ $\quad f'(x) = 0$

$x = 0$

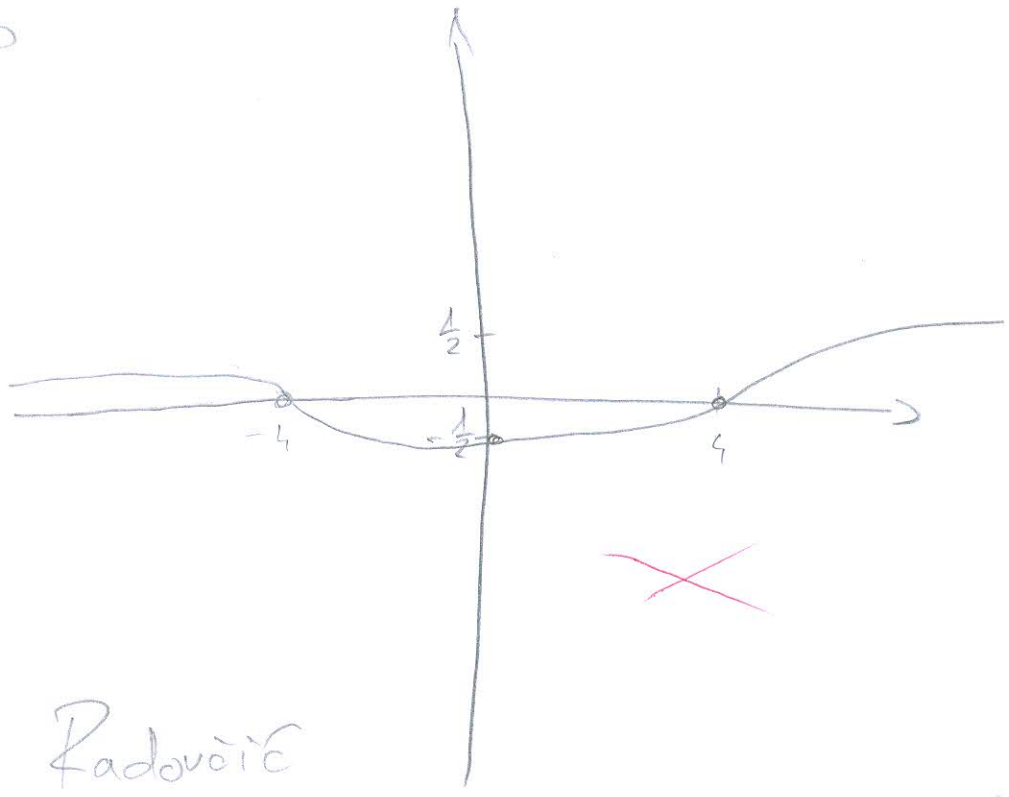
Asimptote

$D(h) = \mathbb{R} \rightarrow$ Nema V.A.

$y = \frac{4}{8} = -\frac{1}{2}$

$\lim_{x \to \pm\infty} \frac{x^2+8}{x-4} = \frac{\frac{x^2}{x^2} - \frac{8}{x^2}}{\frac{x}{x^2} - \frac{4}{x^2}} = \frac{1}{0} = 0$ = D.H.A.

	$-\infty$		0		$+\infty$
$f'(x)$		−	0	+	
$f(x)$		↘	MIN.	↗	



✗

Rikardo Radovčić

④ $g(x) = \sqrt{x^2+4x+5}$

Globalna svojstva
- funkcija je neomeđena
- $g(-x) = \sqrt{x^2+4x+5}$
  ↳ funkcija nije ni parna ni neparna
↳ nije periodična

$x^2+4x+5 \geq 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16+4\cdot 1\cdot 5}}{2}$

$x_{1,2} = \frac{-4 \pm \sqrt{16+20}}{2}$

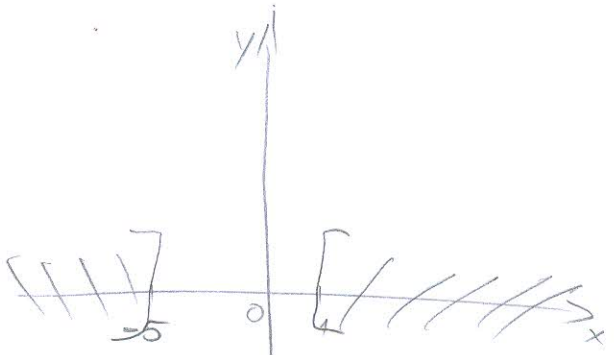
$x_{1,2} = \frac{-4 \pm \sqrt{36}}{2}$

$x_{1,2} = \frac{-4 \pm 6}{2}$

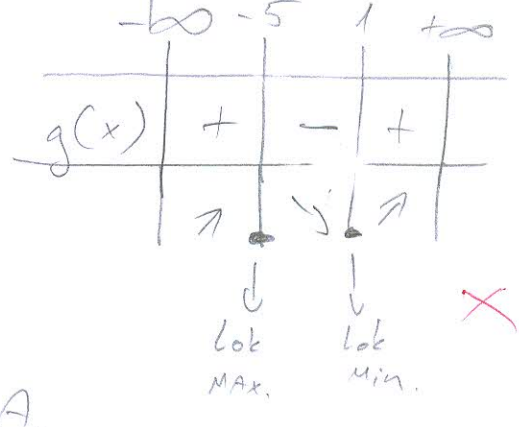
$x_1 = \frac{-4+6}{2}$ $\quad x_2 = \frac{-4-6}{2}$

$x_1 = \frac{2}{2}$ $\quad x_2 = \frac{-10}{2}$

$x_1 = 1$ $\quad x_2 = -5$



$x \in \langle -\infty, -5] \cup [1, +\infty \rangle$ ✗



✗

1° V.A.

$\lim_{x \to 1^-} \sqrt{x^2+4x+5} = 0^-$

$\lim_{x \to -5^+} \sqrt{x^2+4x+5} = 0^+$

} Nema V.A.

Nema H.A.

K.A.

$$\lim_{x\to+\infty}\frac{\sqrt{x^2+4x+5}}{x}\ \overset{/:x^2}{=}\ \lim_{x\to+\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{5}{x^2}}}{\frac{x}{x}}=1$$

$$\lim_{x\to+\infty}\left[g(x)-kx\right]=\lim_{x\to+\infty}\sqrt{x^2+4x+5}-x=$$

$$=\lim_{x\to+\infty}\sqrt{x^2+4x+5}-x\cdot\frac{\sqrt{x^2+4x+5}+x}{\sqrt{x^2+4x+5}+x}=$$

$$=\lim_{x\to+\infty}\frac{x^2+4x+5-x^2}{\sqrt{x^2+4x+5}+x}=\lim_{x\to+\infty}\frac{4x+5}{\sqrt{x^2+4x+5}+x}\ \overset{/:x}{=}\ 1$$

D.K.A., $y=x+1$

$$k=\lim_{x\to-\infty}\frac{\sqrt{x^2+4x+5}}{x}=\lim_{x\to+\infty}\frac{\sqrt{x^2-4x+5}}{-x}\ \overset{/:x}{=}\ \frac{1}{-1}=-1$$

$$l=\lim_{x\to-\infty}\sqrt{x^2+4x+5}+x=\lim_{x\to+\infty}\sqrt{x^2+4x+5}-x=$$

$$=\lim_{x\to+\infty}\sqrt{x^2+4x+5}-x\cdot\frac{\sqrt{x^2+4x+5}+x}{\sqrt{x^2+4x+5}+x}=$$

$$=\lim_{x\to+\infty}\frac{x^2-4x+5-x^2}{\sqrt{x^2-4x+5}+x}=\lim_{x\to+\infty}\frac{-4x+5}{\sqrt{x^2-4x+5}+x}\ \overset{/:x}{=}\ -1$$

$$y=kx+l\qquad \boxed{y=-x-1}$$

Lijeva kosa asimptota ✓

Rikardo Radovčić

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ B8

IME I PREZIME: MIHOVIL PEDISIĆ

BROJ INDEKSA: 17-2-0253-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{matrix} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{matrix}$ i obavezno provjeri rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 5}$ treba:
   (a) ispitati domenu 5
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5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno: 15

3. $f(x) = \frac{\sin(2x)}{x}$

$f'(x) = \frac{(\sin(2x))' \cdot x - \sin(2x) \cdot x'}{x^2} \cdot (2x)'$

$f'(x) = \frac{\cos(2x) \cdot x - \sin(2x)}{x^2} \cdot 2$

$f'(x) = \frac{x\cos(2x) - \sin(2x)}{x^2} \cdot 2$

$f'(\pi) = \frac{\pi\cos(2\pi) - \sin(2\pi)}{\pi^2} \cdot 2 = 0.6366$

2. $5x+4z+2t=3$
$x-y+2z+t=1$
$4x+y+2z=1$
$x+y+z+t=0$

$$\begin{bmatrix} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 2 & 2 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 2 & 2 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 5 & -6 & -8 & -7 \\ 0 & 2 & -1 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & -4 & -6 & -3 \\ 0 & 2 & -1 & -1 & -2 \end{bmatrix} \sim$$

$3r-3r$ | $2r-1r$, $3r-1r\cdot 4$, $4r-1r$ | $2r\cdot(-1)$, $3r-4r\cdot 2$ | $2r \leftrightarrow 4r$, $2r \leftrightarrow 3r$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 & 2 \\ 0 & 1 & -4 & -6 & -3 \\ 0 & 2 & -1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 2 & 2 \\ 0 & 1 & -4 & -6 & -3 \\ 0 & 0 & 7 & 11 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 3 \\ 0 & 0 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim$$

$3r-2r\cdot 2$ | $3r-4r\cdot 11$, $2r+4r\cdot 6$, $1r-4r\cdot 2$ | $3r:7$ | $2r+3r\cdot 4$, $1r-3r\cdot 2$ | $1r+2r$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$x=1$
$y=-1$
$z=-1$
$t=1$

$5x+4z+2t=3$
$5-4+2=3$
$3=3$

$x-y+2z+t=1$
$1+1-2+1=1$
$1=1$

$4x+y+2z=1$
$4-1-2=1$
$1=1$

$x+y+z+t=0$
$1-1-1+1=0$
$0=0$

✓

4. $g(x)=\sqrt{x^2+4x+5}$

a) $x^2+4x+5 \geq 0$

$a=1$, $\cup$

$x_{1,2}=\frac{-4\pm\sqrt{16-20}}{2}$

$D=\mathbb{R}$ ✓

c) asimptote

H.A. $\lim\limits_{x\to\infty} \sqrt{x^2+4x+5} \,/\, x = \sqrt{1+\frac{4}{x}+\frac{5}{x^2}} = \sqrt{1}=1$ $\boxed{V.A=1}$ ✗

Funkcija nema kose asimptote jer ima obe horizontalne asimptote za $x\to\infty$ i $x\to-\infty$, nema ni vertikalne jer je definirana svuda od minusa beskonačno do plus beskonačno. ✗

b) $g(x)=\sqrt{x^2+4x+5}$

$g'(x)=\frac{1}{2\sqrt{x^2+4x+5}}\cdot(x^2+4x+5)'$

$g'(x)=\frac{1}{2\sqrt{x^2+4x+5}}\cdot(2x+4)=\frac{2x+4}{2\sqrt{x^2+4x+5}}$

EKST.

$\frac{2x+4}{2\sqrt{x^2+4x+5}}\cdot\frac{\sqrt{x^2+4x+5}}{\sqrt{x^2+4x+5}}=\frac{(2x+4)\cdot\sqrt{x^2+4x+5}}{2x^2+8x+10}$?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ B8

IME I PREZIME: Franko Bakarić **BROJ INDEKSA:** 0269070613

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

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   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno: 15

2. $5x + 4z + 2t = 3$
$x - y + 2z + t = 1$
$4x + y + 2z = 1$
$x + y + z + t = 0$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \begin{matrix} \\ \text{II} - \text{I} \\ \text{III} - 4\,\text{I} \\ \text{IV} - 5\,\text{I} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \begin{matrix} \\ \\ \text{III} + 3\,\text{II} \\ \text{IV} + 5\,\text{II} \end{matrix} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{5}{2} & -4 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{array}\right] \cdot\left(-\frac{2}{5}\right) \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{5} & 5 \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{array}\right] \cdot\left(-\frac{2}{7}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{5} & 5 \\ 0 & 0 & 1 & \frac{21}{2} & -7 \end{array}\right] \begin{matrix} \\ \\ \\ \text{IV} - \text{III} \end{matrix} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{5} & 5 \\ 0 & 0 & 0 & \frac{13}{5} & -12 \end{array}\right] \cdot\frac{5}{13} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{5} & 5 \\ 0 & 0 & 0 & 1 & -\frac{60}{13} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \overset{\text{I}}{\sim} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \text{II}-5\text{I} \\ \text{III}-4\text{I} \\ \text{IV}-\text{I} \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \text{III}-\text{II}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \overset{\text{IV}}{\sim} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \text{I}-\text{IV} \\ \text{III}+4\text{IV} \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 0 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 5 & -6 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \text{III}-2\text{II}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 0 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 1 & -4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \overset{\text{II}}{\sim} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 0 \\ 0 & 1 & -4 & 0 & 3 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \text{I}+\text{II} \\ \text{III}-2\text{II} \end{array} \left[\begin{array}{cccc|c} 1 & 0 & -7 & 0 & 3 \\ 0 & 1 & -4 & 0 & 3 \\ 0 & 0 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] :7$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -7 & 0 & 3 \\ 0 & 1 & -4 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \text{I}+7\text{III} \\ \text{II}+4\text{III} \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$x + 4bz + t = 0$$

$$1 - 1 - 1 + 1 = 0$$

$$0 = 0 \;//$$

$$1 \neq -1 - 1 + 1 = 0$$

$$0 \neq 0$$

$$0 = 0$$

6. $\lim_{x \to -1} \dfrac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} = \dfrac{\sqrt{(-1)^2 + 8 + 7} - 3 - 1}{1 - 4 + 3} = \dfrac{\sqrt{16} - 4}{4 - 4} = \dfrac{4 - 4}{0} = \dfrac{0}{0}$

$$\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} = \frac{\sqrt{\frac{x^2}{x^4} - \frac{8x}{x^4} + \frac{7}{x^4}} - \frac{3}{x^2} + \frac{x}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} + \frac{3}{x^2}} = \frac{\sqrt{\frac{1}{x^2} - \frac{8}{x^3} + \frac{7}{x^4}} - \frac{3}{x^2} + \frac{1}{x}}{1 + \frac{4}{x} + \frac{3}{x^2}} =$$

$$\frac{x^{2\left(+\frac{1}{2}\right)} - 8x^{\left(+\frac{1}{2}\right)} + 7^{\frac{1}{2}} - 3 + x}{x^2 + 4x + 3} = \left(\frac{x - 8x^{\frac{1}{2}} + 7^{\frac{1}{2}} - 3 + x}{x^2 + 4x + 3}\right)'$$

$$\frac{1 - 4 + 1}{2 \cdot 4} = -\frac{2}{6} = -\frac{1}{3}$$

3. $f(x) = \dfrac{\sin 2x}{x}$

$$f'(x) = \left(\frac{\sin 2x}{x}\right)' = 2 \cdot (\sin x)' \cdot \left(\frac{1}{x}\right)' = 2 \cdot (\cos x) \cdot -\frac{1}{x^2}$$

$$= -\frac{2\cos 2x}{x^2}$$

$$= -0.318$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ B8

IME I PREZIME: ŠIME-BORNA MAGAŠ **BROJ INDEKSA:** 17-2-0108-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{matrix} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{matrix}$ i obavezno provjeri rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 5}$ treba:
   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 8}{x - 4}$ 20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno:

(2) $5x+4z+2t=3$
$x-y+2z+t=1$
$4x+y+2z=1$
$x+y+z+t=0$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] :5 \sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \cdot(-1) \cdot(-4) \cdot(-1)$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & -1 & -\frac{16}{5} & \frac{3}{5} & \frac{2}{5} \\ 0 & 1 & \frac{4}{5} & -\frac{8}{5} & -\frac{7}{5} \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{16}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 1 & \frac{4}{5} & -\frac{8}{5} & -\frac{7}{5} \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & \frac{2}{5} \end{array}\right] \cdot(-1) \cdot(-1)$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{16}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & -\frac{12}{5} & -1 & -\frac{9}{5} \\ 0 & 0 & -3 & \frac{4}{5} & 0 \end{array}\right] :\left(-\frac{12}{5}\right) \sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{16}{5} & -\frac{3}{5} & \frac{2}{5} \\ 0 & 0 & 1 & \frac{5}{12} & \frac{3}{4} \\ 0 & 0 & -3 & \frac{4}{5} & 0 \end{array}\right] \cdot$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{15} & -\frac{1}{5} \\ 0 & 1 & 0 & -\frac{4}{3} & -\frac{12}{5} \\ 0 & 0 & 1 & \frac{5}{12} & \frac{3}{4} \\ 0 & 0 & 0 & 6 & -\frac{3}{4} \end{array}\right] :(6) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{15} & -\frac{1}{5} \\ 0 & 1 & 0 & -\frac{4}{3} & -\frac{12}{5} \\ 0 & 0 & 1 & \frac{5}{12} & \frac{3}{4} \\ 0 & 0 & 0 & 1 & -\frac{3}{24} \end{array}\right] \cdot\left(-\frac{1}{15}\right) \cdot\left(\frac{4}{3}\right) \cdot\left(-\frac{5}{12}\right)$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{1079}{72} \\ 0 & 1 & 0 & 0 & \frac{5}{18} \\ 0 & 0 & 1 & 0 & \frac{241}{288} \\ 0 & 0 & 0 & 1 & -\frac{5}{24} \end{array}\right]$$