

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

F4

NASTAVNIK

IME I PREZIME: JURE DUNDOVIĆ

BROJ INDEKSA:

Broj ↓
bodova

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 8 = 0$. *Prikaži ih u kompleksnoj ravnini!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\left(\frac{z_2}{z_1}\right)\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

~~10+5~~ 5

$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

$$3x_1 + x_2 - x_3 - x_4 = 2$$

$$9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$$

$$x_1 - x_2 - x_4 + 2x_5 = 1$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 6}{x^2 + 8}$

20(graf)

~~10~~

5. Odrediti i provjeriti:

(a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} =$

4+1

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $8 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadzbe uvrštavanjem, a također provjeri nekoliko rješenja nejednadzbe uvrštavanjem!*

15+5

Ukupno:

48

3) $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$
= 0 \mathbb{R}

$$x^2 + x - 2 = 0$$

$$x^2 + x - 2 = 0$$

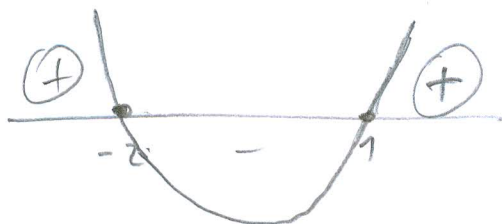
$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x_1 = \frac{-1+3}{2}$$

$$x_2 = \frac{-1-3}{2}$$

$$x_1 = 1$$

$$x_2 = -2$$



$$Df = \langle -\infty, -2 \rangle \cup [1, +\infty)$$

✓

$$4) f(x) = \frac{x^2 - 6}{x^2 + 8}$$

$$x^2 + 8 \neq 0$$

$$x^2 \neq -8 \cup$$

$$D_f = \mathbb{R}$$

$$x^2 - 6 = 0$$

$$x^2 = 6$$

$$x_1 = \sqrt{6}, x_2 = -\sqrt{6} \Rightarrow \text{MULTIPLICI}$$

$$f'(x) = \frac{2x \cdot (x^2 + 8) - (x^2 - 6) \cdot 2x}{(x^2 + 8)^2} = \frac{2x(x^2 + 8 - x^2 + 6)}{(x^2 + 8)^2} = \frac{2x \cdot 14}{(x^2 + 8)^2} = \frac{28x}{(x^2 + 8)^2}$$

$-\infty$	0	$+\infty$
$28x$	$-$	$+$
$28x$	$-$	$+$
$(x^2 + 8)^2$		

$$f''(x) = \frac{28 \cdot (x^2 + 8)^2 - 28x \cdot 2(x^2 + 8) \cdot 2x}{(x^2 + 8)^4} = \frac{28(x^2 + 8)(-3x^2 + 8)}{(x^2 + 8)^4}$$

$-\infty$	$-\sqrt{\frac{8}{3}}$	$\sqrt{\frac{8}{3}}$	$+\infty$
$-3x^2 + 8$	$-$	$+$	$-$
$f''(x)$	$-$	$+$	$-$

$-\infty$	$-\sqrt{\frac{8}{3}}$	0	$\sqrt{\frac{8}{3}}$	$+\infty$
$f'(x)$	$-$	$-$	$+$	$+$
$f''(x)$	$-$	$+$	$+$	$-$
$f(x)$	$)$	$($	$)$	$($

$$x^2 + 8 = 0$$

$$x^2 = -8$$

$$-3x^2 + 8 = 0$$

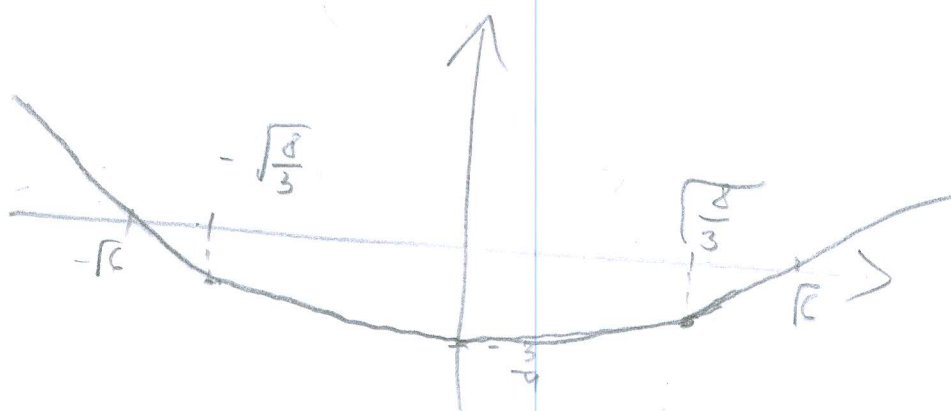
$$3x^2 = 8 \quad x_1 = \sqrt{\frac{8}{3}}$$

$$x^2 = \frac{8}{3} \quad x_2 = -\sqrt{\frac{8}{3}}$$

10

SAT GRAFA SE NE
VIDI ŠTO SE DOGAĐA
SASVIM LIJEVO I
DESNO?

H.A.



$$\begin{aligned} 2) \quad & x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2 \\ & 3x_1 + x_2 - x_3 - x_4 = 2 \\ & 9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5 \\ & x_1 - x_2 - x_4 + 2x_5 = 1 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{array} \right] \begin{array}{l} \\ \text{II} - 3\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 2 & 8 & -12 & -4 \\ 0 & -8 & 7 & 26 & -38 & -13 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{array} \right] \begin{array}{l} \\ \text{IV} - \text{IV} \\ \text{III} - 4\text{IV} \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 3 & 18 & -30 & -9 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 1 & 2 & -2 & -1 \\ 0 & 0 & 3 & 18 & -30 & -9 \\ 0 & 0 & 1 & 6 & -10 & -3 \end{array} \right] \begin{array}{l} \text{I} + \text{IV} \\ \\ \text{III} - 3\text{IV} \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 2 & 2 \\ 0 & -2 & 1 & 2 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 6 & -10 & -3 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 2 & 2 \\ 0 & -2 & 1 & 2 & -2 & -1 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \text{II} - \text{III} \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 2 & 2 \\ 0 & -2 & 0 & -4 & 8 & 2 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

5

Sustav ima beskonечно mnogo rjesenja!

$$\begin{aligned} x_1 &= a - 3b + c + 2e = 2 \\ x_2 &= -2b - 4d + 8e = 2 \\ x_3 &= c + 6d - 10e = -3 \\ x_4 &= a \end{aligned}$$

DA, ALI ISPOD NISU
RJESENJA
 $x_5 = ?$
PROVJERA?

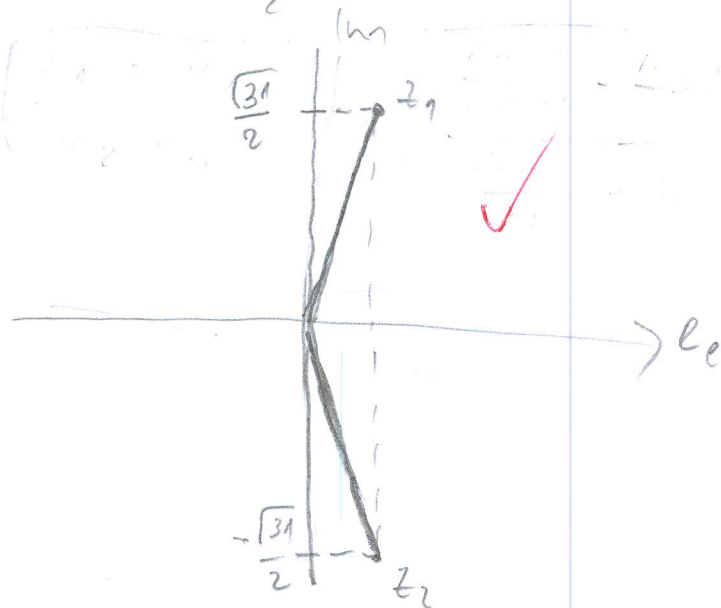
$$1) z^2 - z + 8 = 0$$

$$z_{1,2} = \frac{1 \pm \sqrt{1 - 32}}{2} =$$

$$z_{1,2} = \frac{1 \pm \sqrt{-31}}{2}$$

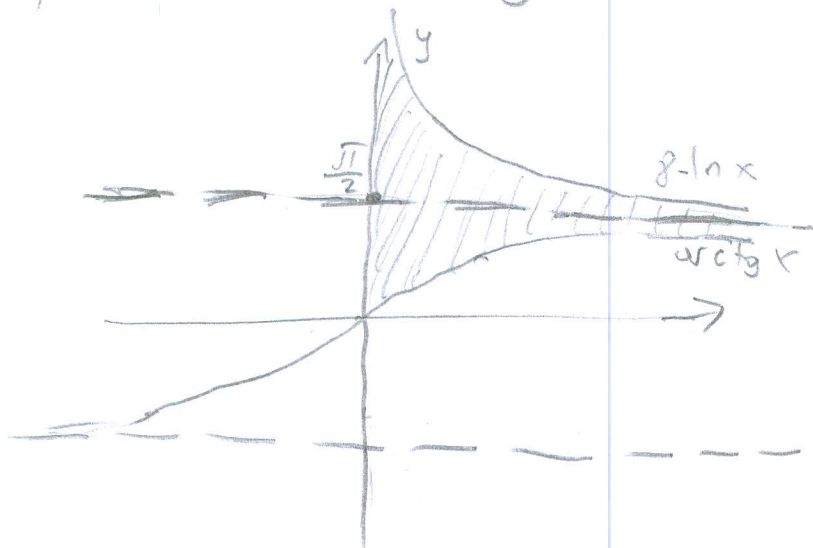
$$z_1 = \frac{1 - \sqrt{-31}}{2} \Rightarrow z_1 = \frac{1}{2} + \frac{\sqrt{31}}{2} i$$

$$z_2 = \frac{1 + \sqrt{-31}}{2} \Rightarrow z_2 = \frac{1}{2} - \frac{\sqrt{31}}{2} i$$



5

$$6) 8 - \ln x > \arctg x$$



PROVĚRA NEJEDNODUŠNĚ
UVRSTAVENÍ

$$x = 650$$

$$8 - \ln 650 \approx \arctg 650$$

$$1.52 \approx 1.57$$

$$x \in \langle 0, 650 \rangle$$

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

F4

NASTAVNIK

IME I PREZIME: **KRISTIAN MARTINović**

Broj INDEKSA: **17-2-0110-2011**

Broj ↓

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

bodova

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 8 = 0$. *Prikaži ih u kompleksnoj ravnini!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 4}\right)}$ i $\text{Im}\left(\left(\frac{z_2}{z_1}\right)\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

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$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

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15+5

Ukupno:

20

6. DRUGA DERIVACIJA

$$f'(x) = \frac{(22x)'(x^2+8)^2 - (22x)(x^2+8)'}{(x^2+8)^4}$$

$$f'(x) = \frac{22(x^2+8)^2 - (22x)2(x^2+8) \cdot 2x}{(x^2+8)^4}$$

$$f'(x) = \frac{22(x^2+8)^2 - 88x^2(x^2+8)}{(x^2+8)^4}$$

$$f'(x) = \frac{(x^2+8)[22(x^2+8) - 88x^2]}{(x^2+8)^4}$$

$$f'(x) = \frac{22x^2 + 176 - 88x^2}{(x^2+8)^3}$$

$$f'(x) = \frac{-66x^2 + 176}{(x^2+8)^3}$$

$f''(-2) = \ominus$ $f''(0) = \oplus$ $f''(2) = \ominus$

$$f''(x) = 0$$

$$-66x^2 + 176 = 0$$

$$-66x^2 = -176$$

$$x^2 = \frac{176}{66} = \frac{8}{3}$$

$$x = \pm \sqrt{\frac{8}{3}} = \pm \frac{2\sqrt{6}}{3}, T_i$$

(≈ 1.63)
(≈ -1.63)

	$-\infty$	$-\frac{2\sqrt{6}}{3}$	0	$\frac{2\sqrt{6}}{3}$	$+\infty$
$f''(x)$	-	0	+	0	-
$f(x)$	\cap		\cup		\cap

1) $f(x) = \frac{x^2+6}{x^2+8}$

1) $x^2+8 \neq 0$ Df: \mathbb{R}
 ~~$x^2 \neq -8$~~

2) $f(x) = 0$ NT $(\sqrt{6}, 0)$
 $x^2 - 6 = 0$ NT $(-\sqrt{6}, 0)$
 $x^2 = 6$
 $x = \pm \sqrt{6}$

$f(0) = \frac{0-6}{0+8} = -\frac{6}{8} = -\frac{3}{4}$

NT $(0, -\frac{3}{4})$

3) $f(-x) = \frac{(-x)^2-6}{(-x)^2+8} = \frac{x^2-6}{x^2+8} = \text{PARNA}$

4) VA. NEMA

H.A.

$\lim_{x \rightarrow \infty} \frac{x^2-6}{x^2+8} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{x^2-6}{x^2+8} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \rightarrow \infty} \frac{1 - \frac{6}{x^2}}{1 + \frac{8}{x^2}} = \frac{1}{1} = 1$

$\lim_{x \rightarrow \infty} \frac{x^2-6}{x^2+8} = \left[\begin{matrix} x \rightarrow -x \\ -\infty \rightarrow \infty \end{matrix} \right] = \lim_{x \rightarrow \infty} \frac{(-x)^2-6}{(-x)^2+8} = \lim_{x \rightarrow \infty} \frac{x^2-6}{x^2+8} \dots = 1$ } $y=1$
 O.H.A.

K.A. NEMA

5) $f'(x) = \frac{(x^2-6)'(x^2+8) - (x^2-6)(x^2+8)'}{(x^2+8)^2}$

$f'(x) = \frac{2x(x^2+8) - (x^2-6)2x}{(x^2+8)^2}$

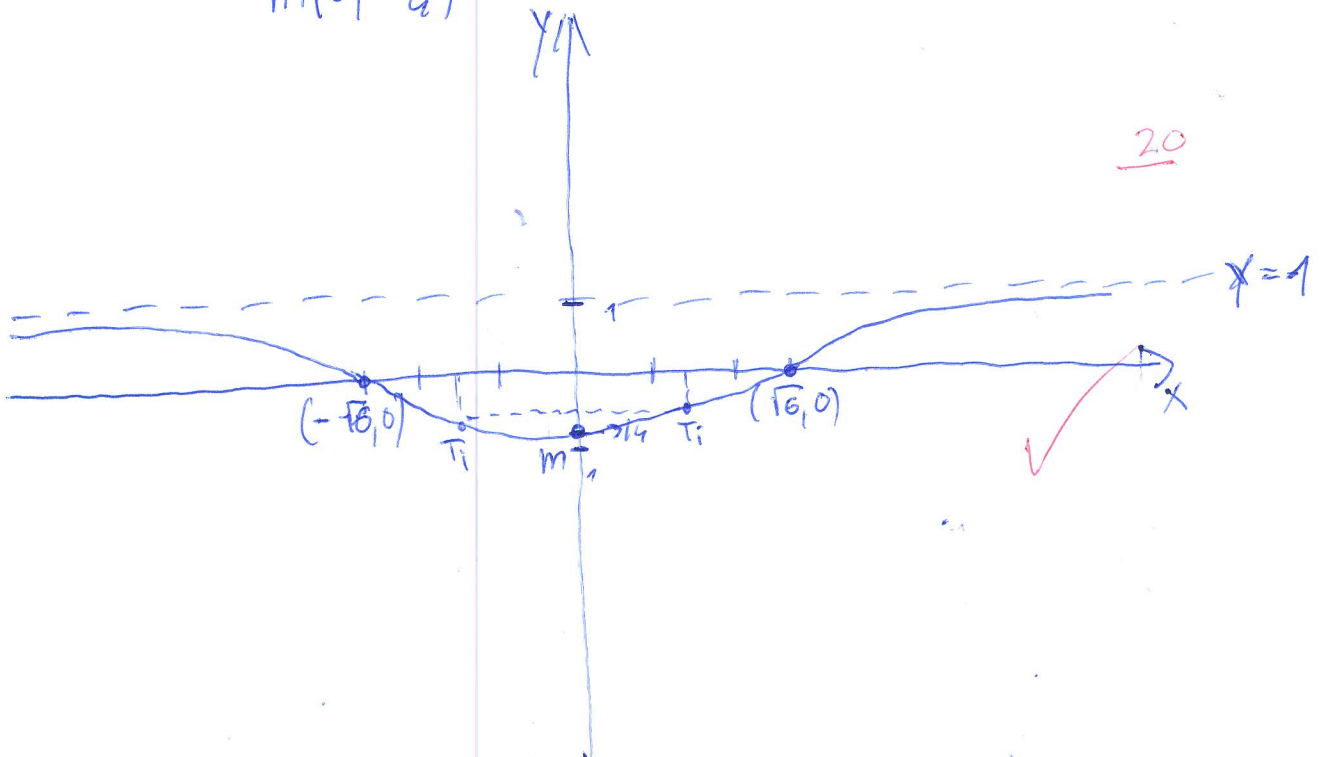
$f'(x) = \frac{2x^3 + 10x - (2x^3 - 12x)}{(x^2+8)^2} = \frac{2x^3 + 10x - 2x^3 + 12x}{(x^2+8)^2}$

$f'(x) = \frac{22x}{(x^2+8)^2}$

$f'(x) = 0$ $f(0) = -\frac{3}{4}$
 $22x = 0$ $ST(0, -\frac{3}{4})$
 $x = 0, \text{ S.T.}$ $m(0, -\frac{3}{4})$

	$-\infty$	0	$+\infty$
$f'(x)$	$-$	$+$	
$f(x)$	\searrow	\nearrow	
		m	

$f(-1) = -$
 $f(1) = +$



20

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

$(-3)R_1 + R_2$
 $(-9)R_1 + R_3$
 $(-1)R_1 + R_4$

$$\begin{bmatrix} -3 & -3 & 3 & 9 & -12 & -6 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 8 & -12 & -4 \cdot 1/2 \\ 0 & -1 & 1 & 4 & -6 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -1 & 1 & 4 & -6 & -2 \\ 0 & 8 & 7 & 26 & -38 & -13 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix}$$

$8R_2 + R_3$
 $(-2)R_2 + R_4$

$$\begin{bmatrix} -9 & -9 & 9 & 27 & -36 & -18 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 0 & 8 & 7 & 26 & -38 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -1 & 1 & 4 & -6 & -2 \\ 0 & 0 & 15 & 58 & 86 & -29 \\ 0 & 0 & -1 & -6 & 10 & 3 \end{bmatrix}$$

$R_3 + 15R_4$

$$\begin{bmatrix} -1 & -1 & 1 & 3 & -4 & -2 \\ 1 & -1 & 0 & -1 & 2 & 1 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -1 & 1 & 4 & -6 & -2 \\ 0 & 0 & 15 & 58 & 86 & -29 \\ 0 & 0 & 0 & -1 & 59/8 & 1/2 \end{bmatrix}$$

$(-3)R_4 + R_1$
 $4R_4 + R_2$
 $5R_4 + R_3$

$$\begin{bmatrix} 0 & -8 & 8 & 32 & -48 & -16 \\ 0 & 8 & 7 & 26 & -38 & -13 \\ 0 & 0 & 15 & 58 & 86 & -29 \\ 0 & 0 & 1 & 58/15 & 86/15 & -29 \end{bmatrix}$$

NEVALJA
 POGLEDATI DEUGI PAPIR

$$\begin{bmatrix} 0 & 2 & -2 & -8 & 12 & 4 \\ 0 & -2 & 1 & 2 & -2 & -1 \\ 0 & 0 & -1 & -6 & 16 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 15 & 58 & 86 & -29 \\ 0 & 0 & 15 & -90 & 150 & 45 \\ 0 & 0 & 0 & -32 & 236 & 16 \\ 0 & 0 & 0 & -16 & 118 & 8 \\ 0 & 0 & 0 & -8 & 59 & 4 \\ 0 & 0 & 0 & -1 & 59/8 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & -58 & 171/4 & 29 \\ 0 & 0 & 15 & 58 & 86 & -29 \\ 0 & 0 & 15 & 0 & 205/4 & 0 \\ 0 & 0 & 1 & 0 & 137/4 & 0 \end{bmatrix}$$

KRISTIAN MARTINOVIC

$$③ \quad g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$$

~~Handwritten scribbles~~

$$x^2 + x - 2 \geq 0$$

$$-1 \geq x^2 - x \geq 1$$

$$x^2 + x - 2 = 0$$

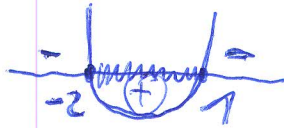
$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{9}}{2}$$

$$x_{1,2} = \frac{-1 \pm 3}{2}$$

$$x_1 = \frac{2}{2} = 1$$

$$x_2 = \frac{-4}{2} = -2$$



$$D_f = [-2, 1] \quad \times$$

④

$$5. a) \quad \lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} = \lim_{x \rightarrow -4} \frac{(-4)^2 - 10}{(-4)^2 + 8 \cdot (-4) + 16} = \lim_{x \rightarrow -4} \frac{16 - 10}{16 - 32 + 16} = \frac{6}{-32} = -\frac{3}{16}$$

$$b) \quad \lim_{n \rightarrow \infty} \left(\frac{n+3}{n}\right)^n = \lim_{n \rightarrow \infty} \left(\frac{\infty+3}{\infty}\right)^{\infty} = \left(\frac{\infty}{\infty}\right)^{\infty} = \infty \quad \times$$

a) PROUJERT

$$\frac{\left(\frac{3}{16}\right)^2 - 10}{\left(\frac{3}{16}\right)^2 + 8 \cdot \left(\frac{3}{16}\right) + 16} =$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 3 & 1 & -1 & -1 & 0 & | & 2 \\ 9 & 1 & -2 & -1 & -2 & | & 5 \\ 1 & -1 & 0 & -1 & 2 & | & 1 \end{bmatrix} \begin{array}{l} (-3)R_1 + R_2 \\ (-9)R_1 + R_3 \\ (-1)R_1 + R_4 \end{array}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & -1 & 1 & 4 & -6 & | & -2 \\ 0 & -8 & 7 & 26 & -38 & | & -13 \\ 0 & -2 & 1 & 2 & -2 & | & -1 \end{bmatrix} \begin{array}{l} R_2 + R_1 \\ (8)R_2 + R_3 \\ (2)R_2 + R_4 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & -1 & 1 & 4 & -6 & | & -2 \\ 0 & 0 & -1 & -6 & 10 & | & 3 \\ 0 & 0 & -1 & -6 & 10 & | & 3 \end{bmatrix} \begin{array}{l} R_3 + R_2 \\ (1)R_3 + R_2 \\ (1)R_3 + R_4 \\ (-1)R_3 + R_4 \end{array}$$

~~$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & -1 & 1 & 4 & -6 & | & -2 \end{bmatrix}$$~~

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & | & 0 \\ 0 & -1 & 0 & -2 & 4 & | & -1 \\ 0 & 0 & -1 & -6 & 10 & | & 3 \\ 0 & 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

SUSTAV NEMA RJEŠENJA



$$\begin{bmatrix} -3 & -3 & 3 & 9 & -12 & -6 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 8 & -12 & -4 \\ 0 & -1 & 1 & 4 & -6 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -9 & -9 & 9 & 27 & -36 & -18 \\ 3 & 1 & -2 & -1 & -2 & 5 \\ 0 & -8 & 7 & 26 & -38 & -13 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 1 & 3 & -4 & -2 \\ 1 & -1 & 0 & -1 & 2 & 1 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 & 4 & -6 & -2 \\ 1 & 1 & -1 & -3 & 4 & 2 \\ 1 & 0 & 0 & 1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 8 & -8 & -32 & 48 & 16 \\ 0 & -8 & 7 & 26 & -38 & -13 \\ 0 & 0 & -1 & -6 & 10 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -2 & -8 & 12 & 4 \\ 0 & -2 & 1 & 2 & -2 & -1 \\ 0 & 0 & -1 & -6 & 10 & 3 \end{bmatrix}$$

ovaj

$$\begin{bmatrix} 0 & 0 & -1 & -6 & 10 & 3 \\ 0 & -1 & 1 & 4 & -6 & -2 \\ 0 & -1 & 0 & -2 & 4 & -1 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & -1 & -6 & 10 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

KRISTIAN MARTINOVIC

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
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IME I PREZIME: ELENA BEG

BROJ INDEKSA: 3735

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

F4

1. Neka su z_1 i z_2 rješenja kvadratne jednadžbe $z^2 - z + 8 = 0$. Prikaži ih u kompleksnoj ravnini! Dalje izračunaj: $\operatorname{Re}\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\operatorname{Im}\left(\frac{z_2}{z_1}\right)$.

~~12+3~~

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

~~10+5~~

$$\begin{aligned} x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\ 3x_1 + x_2 - x_3 - x_4 &= 2 \\ 9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\ x_1 - x_2 - x_4 + 2x_5 &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$.

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4. Odrediti tok funkcije $f(x) = \frac{x^2 - 6}{x^2 + 8}$

~~20(graf)~~

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(a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} =$

~~4+1~~

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $8 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadžbe uvrštavanjem, a također provjeri nekoliko rješenja nejednadžbe uvrštavanjem!*

15+5

Ukupno:

15

1.

$$z^2 - z + 8 = 0$$

$$z = x + yi$$

$$z_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 8}}{2}$$

$$z_{1,2} = \frac{-1 \pm \sqrt{-31}}{2}$$

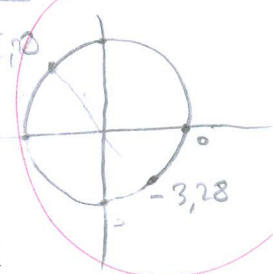
$$z_1 = \frac{-1 - \sqrt{-31}}{2}$$

$$z_2 = \frac{-1 + \sqrt{-31}}{2}$$

$$z_1 = \frac{-1 - \sqrt{31}i}{2} \quad z_2 = \frac{-1 + \sqrt{31}i}{2}$$

$$\approx -3,283i \quad \Downarrow \quad -\frac{1}{2} - \frac{\sqrt{31}}{2}i \approx 2,28i$$

$$\approx -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$



$$\operatorname{Re}\left(\frac{z_1 - z_2}{z_2 + 4}\right) = \frac{z_1 + z_2}{z_2 - 4} = \frac{-\frac{1}{2} + (-\frac{1}{2})}{-\frac{1}{2} - 4} = \frac{-1}{-\frac{9}{2}} = \frac{2}{9} //$$

$$\operatorname{Im}\left(\frac{z_2}{z_1}\right) = \frac{-\frac{\sqrt{31}}{2}}{\frac{\sqrt{31}}{2}} = \frac{-2\sqrt{31}}{2\sqrt{31}} = -1$$

4) $f(x) = \frac{x^2-6}{x^2+8}$

1) DOMENA $x^2+8 \neq 0$

$D_f \langle -\infty, +\infty \rangle$

$x^2 \neq -8$
 $x \neq \sqrt{-8}$

\rightarrow nemoguće.
 \rightarrow funkcija nema realnih nultočaka.

3) GLOBALNA SVOJSTVA

$f(x) = \frac{-x^2-6}{-x^2+8} = \frac{x^2-6}{x^2+8} = f(x)$

funkcija je parna, nije periodična, nije omeđena funkcija je omeđena od gore

2) ASIMPTOTE

V.A. $\lim_{x \rightarrow \pm\infty} \frac{x^2-6}{x^2+8} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \pm\infty} \frac{2x}{2x} = \lim_{x \rightarrow \pm\infty} 1 = 1$ $y=1$

HA \rightarrow ne postoji horizontalna a, je nema realnih nultočaka, * može nemati

5) $f'(x)$ i $f''(x)$

$f'(x) = \frac{(x^2-6)' \cdot (x^2+8) - (x^2-6)(x^2+8)'}{(x^2+8)^2} = \frac{2x(x^2+8) - (x^2-6)2x}{(x^2+8)^2}$
 $= \frac{2x^3+16x-2x^3+12x}{(x^2+8)^2} = \frac{16x+12x}{(x^2+8)^2} = \frac{28x}{(x^2+8)^2}$

$f''(x) = \frac{(28x)'(x^2+8)^2 - (28x)((x^2+8)^2)'}{(x^2+8)^4} = \frac{28(x^2+8)^2 - 28x(2(x^2+8) \cdot 2x)}{(x^2+8)^4}$
 $= \frac{28 - 28x(4x(x^2+8))}{(x^2+8)^2} = \frac{28 - 28x(4x^3 + 16x)}{(x^2+8)^2}$
 $= \frac{28 - 112x^4 - 448x^2}{(x^2+8)^2} = \frac{(-112x^4 - 448x^2 + 28)}{(x^2+8)^2}$
 $= \frac{28(-4x^4 - 16x^2 + 1)}{(x^2+8)^2}$

4) SPECIJA S KOORDINATNIM OVIŠTA =

$f(x) = \frac{x^2-6}{x^2+8}$ $x=0$ $f(0) = \frac{0-6}{0+8} = -\frac{6}{8}$

- funkcija sjeca x-os u točki $(0, -\frac{6}{8})$

6) KRITIČNE TOČKE

$D_f = D_{f'}$
 $28x = 0$
 $x = -28$

KRITIČNA TOČKA JE: $(-28, 0)$
 $f(-28) = \frac{(-28)^2-6}{(-28)^2+8} = \frac{389}{396} \approx 0,98$ rubna točka $(-28, 0,98)$

7) MONOTONOST

BORJE SE SA MO GRAF!

(2.)

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 12 \\ 9 & 1 & -2 & -1 & -2 & 15 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix} \begin{array}{l} \text{II} - 3\text{I} \\ \text{III} - 9\text{I} \\ \text{IV} - \text{I} \end{array} \sim \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 2 & 8 & -14 & -4 \\ 0 & -8 & 7 & 26 & -38 & -13 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \text{IV} + \text{II} \end{array} \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 2 & 8 & -14 & -4 \\ 0 & 0 & -1 & -6 & 10 & 3 \\ 0 & 0 & 3 & 10 & -14 & -5 \end{bmatrix} \begin{array}{l} \\ \\ \\ \text{IV} + 3\text{III} \end{array} \sim \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 2 & 8 & -14 & -4 \\ 0 & 0 & -1 & -6 & 10 & 3 \\ 0 & 0 & 0 & -8 & 16 & 14 \end{bmatrix} \begin{array}{l} \\ \\ \\ \text{I} + \text{II} \end{array} \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -2 & 2 & 8 & -14 & -4 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & -8 & 16 & 14 \end{bmatrix} \begin{array}{l} \\ \\ \\ \text{II} \cdot (-1) \end{array} \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & 2 & -2 & -8 & 14 & 4 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & -8 & 16 & 14 \end{bmatrix} \begin{array}{l} \\ \\ \\ \text{II} \cdot (-1) \end{array} \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -1 & 1 & 2 & -7 & -2 \\ 0 & 0 & 1 & 6 & -10 & -3 \\ 0 & 0 & 0 & -2 & 4 & 1 \end{bmatrix} \begin{array}{l} \\ \\ \\ \text{III} + \text{II} \end{array} \sim$$

\Rightarrow sistem nemo rjstanya!
(2 \neq 0)

(5.) a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} = \lim_{x \rightarrow -4} \frac{(-4)^2 - 10}{(-4)^2 + 8(-4) + 16} = \frac{6}{0} = +\infty$

$\frac{6}{0^+} = +\infty$
 $\frac{6}{0^-} = -\infty$

(3.) $g(x) = \sqrt{x^2 + x - 2} - \text{arctg}(x^2 - x)$
 $x^2 + x - 2 \geq 0$ $x^2 - x > 0$

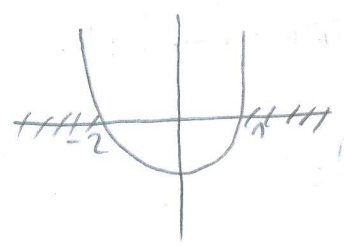
$D_f: (-\infty, -2) \cup (1, +\infty)$

$$x^2 + x - 2 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{-1 \pm \sqrt{9}}{2}$$

$$x_1 = \frac{-1 - 3}{2} = -\frac{4}{2} = -2$$

$$x_2 = \frac{-1 + 3}{2} = \frac{2}{2} = 1$$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA
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odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

F4

IME I PREZIME: TONI PERKOVIC

BROJ INDEKSA: 17-2-0134-2011

ZAOKRUŽITI AKO ŽELITE:

ustmeni kod prof. Uglešića

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 8 = 0$. Prikaži ih u kompleksnoj ravnini! Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\frac{z_2}{z_1}\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

$$3x_1 + x_2 - x_3 - x_4 = 2$$

$$9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$$

$$x_1 - x_2 - x_4 + 2x_5 = 1$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 6}{x^2 + 8}$

20(graf)

5. Odrediti i provjeriti:

(a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} =$

4+1

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $8 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. Provjeri približno rješenje jednadzbe uvrštavanjem, a također provjeri nekoliko rješenja nejednadzbe uvrštavanjem!

15+5

Ukupno:

115

③ $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$

$x^2 + x - 2 \geq 0$

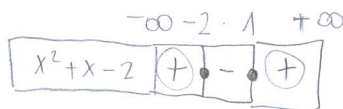
$a=1, b=1, c=-2$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2}$

$x_1 = 1$

$x_2 = -2$



$D(f) = (-\infty, -2] \cup [1, +\infty)$

④ $f(x) = \frac{x^2 - 6}{x^2 + 8}$
 $x^2 + 8 \neq 0$
 $x^2 \neq -8$

$D(f) = \mathbb{R}$

$f(-x) = \frac{(-x)^2 - 6}{(-x)^2 + 8} = \frac{x^2 - 6}{x^2 + 8} = f(x)$ = FUNKCIJA PARNA.
NIJE PERIODIČNA.

V.A. → NEMA

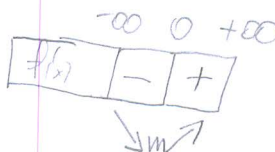
H.A. $\lim_{x \rightarrow \pm\infty} \frac{x^2 - 6}{x^2 + 8} = \left[\frac{\infty}{\infty}\right] = \lim_{x \rightarrow \pm\infty} \frac{-5}{9} = \boxed{H.A. \dots -0,55}$

K.A. → NEMA

$f'(x) = \frac{2x \cdot (x^2 + 8) - (x^2 - 6) \cdot 2x}{(x^2 + 8)^2} = \frac{2x^3 + 16x - 2x^3 - 12x}{(x^2 + 8)^2} = \frac{4x}{(x^2 + 8)^2}$

$4x = 0 \quad / : 4$

$x = 0$



$f'(0) = \frac{0}{8} = 0$

$\min(0, 0)$

GRAF →

$\frac{x^2 - 6}{x^2 + 8} = 0 \quad / \cdot x^2 + 8$

$x^2 - 6 = 0$
 $x^2 = 6 \quad / \sqrt{\quad}$

$x_1 = 2,45 \quad T_1 = (2,45, 0)$
 $x_2 = -2,45 \quad T_2 = (-2,45, 0)$

$f(0) = \frac{-6}{8} = -\frac{3}{4}$
 $S(0, -\frac{3}{4})$

$$f''(x) = \frac{4 \cdot (x^2+8)^2 - 4x \cdot 2(x^2+8) \cdot 2x}{(x^2+8)^3} = \frac{4 \cdot (x^2+8) - 4x \cdot 2 \cdot 2x}{(x^2+8)^3} = \frac{4x^2+32-16x^2}{(x^2+8)^3} = \frac{32-12x^2}{(x^2+8)^3}$$

$$32-12x^2=0$$

$$-12x^2=-32$$

$$12x^2=32$$

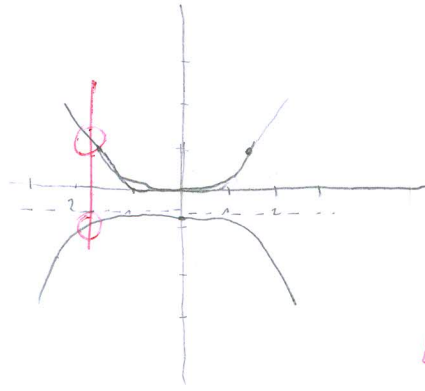
$$x^2=2.66/\text{m}$$

$$x_1=1.6 \quad \text{T.I. } (1.6, 1)$$

$$x_2=-1.6 \quad \text{T.I. } (-1.6, 1)$$

$$f(1.6)=1$$

	$-\infty$	-1.6	1.6	$+\infty$
$f(x)''$		-	+	-
		n	u	n



VERTIKALAN
PRAVAC NE
MOŽE SJEĆI
GRAF FUNKCIJE
U DVIJE TOČKE!!!

$$\textcircled{5.} \lim_{x \rightarrow -4} \frac{x^2-10}{x^2+8x+16} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow -4} \frac{1-10}{1+\frac{8}{x}+\frac{16}{x^2}} = \frac{-9}{1} = -9 \quad \text{X}$$

$$\textcircled{1.} z^2-2+8=0$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
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IME I PREZIME: **DOMAGOŠ GROZAS**

BROJ INDEKSA: **17-1-0056-2011**

ZAOKRUŽITI AKO ŽELITE:

ustmeni kod prof. Uglešića

F4

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 8 = 0$. *Prikaži ih u kompleksnoj ravnini!* Dalje izracunaj: $\overline{\left(\frac{z_1 - z_2}{z_2 + 4}\right)}$ i $\text{Im}\left(\left(\frac{z_2}{z_1}\right)\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

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3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 6}{x^2 + 8}$

20(graf)

5. Odrediti i provjeriti:

(a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} =$

4+1

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $8 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadzbe uvrštavanjem, a također provjeri nekoliko rješenja nejednadzbe uvrštavanjem!*

15+5

Ukupno:

20

④ $f(x) = \frac{x^2 - 6}{x^2 + 8}$

① DOMENA

$$x^2 + 8 \neq 0$$

$$x^2 = -8 \quad | \sqrt{}$$

$$x = \pm \sqrt{-8}$$

$$D(f) = \mathbb{R}$$

② NULTOČKE

$$x^2 - 6 = 0$$

$$x^2 = 6 \quad | \sqrt{}$$

$$x_1 = \sqrt{6}$$

$$x_2 = -\sqrt{6}$$

③ EKSPlicitni RAST

$$f'(x) = \frac{(x^2 - 6)'(x^2 + 8) - (x^2 - 6)(x^2 + 8)'}{(x^2 + 8)^2}$$

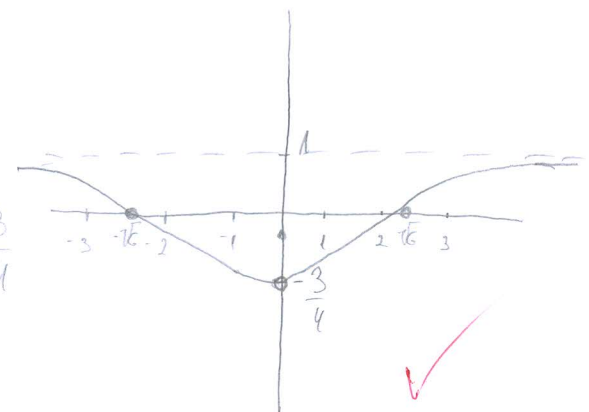
$$= \frac{2x(x^2 + 8) - (x^2 - 6)2x}{(x^2 + 8)^2}$$

$$= \frac{2x^3 + 16x - 2x^3 + 12x}{(x^2 + 8)^2}$$

$$= \frac{28x}{(x^2 + 8)^2}$$

$$x = 0$$

$$y = -\frac{6}{8} = -\frac{3}{4}$$



④ ASIMPTOTE

V.A. → NEMA

H.A. $\lim_{x \rightarrow \pm\infty} \frac{x^2 - 6}{x^2 + 8} = \lim_{x \rightarrow \pm\infty} \frac{\frac{x^2}{x^2} - \frac{6}{x^2}}{\frac{x^2}{x^2} + \frac{8}{x^2}} = \frac{1 - 0}{1 + 0} = 1$

$$\textcircled{3} f(x) = \sqrt{x^2+x-2} - \arctan(x^2-x)$$

$$\textcircled{1} x^2+x-2 \geq 0$$

$$x^2+x-2=0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-4 \cdot 1 \cdot (-2)}}{2}$$

$$= \frac{-1 \pm \sqrt{1+8}}{2}$$

$$= \frac{-1 \pm 3}{2}$$

$$x_1 \geq 1 \quad x_2 \geq -2$$

$$D(f) = [-2, 1] \cup [1, +\infty)$$

$$\textcircled{2} x^2-x=0$$

$$x(x-1)=0$$

$$x \neq 0$$

$$x-1 \neq 0$$

$$x \neq 1$$

$$D(f) = \mathbb{R} \setminus \{0, 1\}$$

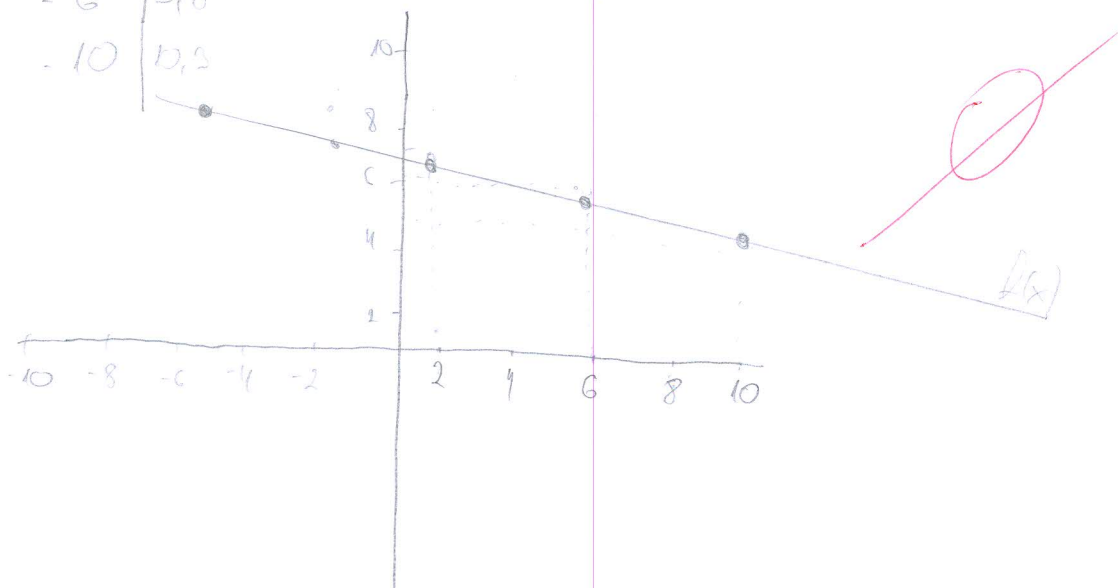
$$D(f) = [-2, 0) \cup (1, +\infty)$$

$$\textcircled{6} f(x) = 8 - \ln x$$

$$g(x) = \arctan x$$

x	f(x)
2	7,3
6	6,2
10	5,6
-2	8,4
-6	9,8
-10	10,2

x	g(x)
2	1,1
6	1,1
10	1,1



5) (a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} \stackrel{L'H}{=} \lim_{x \rightarrow -4} \frac{2x}{2x + 8} = \frac{2 \cdot (-4)}{2 \cdot (-4) + 8} = \frac{-8}{0} = -\infty$ ~~$-\infty$~~

(b) $\lim_{x \rightarrow \infty} \left(\frac{n+3}{n} \right)^n$

$\frac{-8}{0_+} = -\infty$
 $\frac{-8}{0_-} = +\infty$

2

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 12 \\ 3 & 1 & -1 & -1 & -1 & 12 \\ 9 & 1 & -2 & -1 & -2 & 15 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: **LUKA ŽIKUŠIĆ**

BROJ INDEKSA: **17-2-0013-2010**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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F4

1. Neka su z_1 i z_2 rjesenja kvadratne jednadžbe $z^2 - z + 8 = 0$. *Prikaži ih u kompleksnoj ravnini!* Dalje izračunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\operatorname{Im}\left(\left(\frac{z_2}{z_1}\right)\right)$.

12+3

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3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$.

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20(graf)

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4+1

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8+2

6. Grafički približno riješiti: $8 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadžbe uvrštavanjem, a također provjeri nekoliko rješenja nejednadžbe uvrštavanjem!*

15+5

Ukupno:



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: **DINO ĐOKOZA**

BROJ INDEKSA: **17-2-0214-12**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

F4

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 8 = 0$. *Prikaži ih u kompleksnoj ravnini!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\left(\frac{z_2}{z_1}\right)\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{aligned}x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\3x_1 + x_2 - x_3 - x_4 &= 2 \\9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\x_1 - x_2 - x_4 + 2x_5 &= 1\end{aligned}$$

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8+2

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15+5

Ukupno:



$$2. \quad x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

$$3x_1 + x_2 - x_3 - x_4 = 2$$

$$9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$$

$$x_1 - x_2 - x_4 + 2x_5 = 1$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 3 & 1 & -1 & -1 & 0 & | & 2 \\ 9 & 1 & -2 & -1 & -2 & | & 5 \\ 1 & -1 & 0 & -1 & 2 & | & 1 \end{bmatrix} \xrightarrow{-R_4} \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & -4 & -1 & 2 & -6 & | & -1 \\ 9 & 1 & -2 & -1 & -2 & | & 5 \\ 1 & -1 & 0 & -1 & 2 & | & 1 \end{bmatrix} \xrightarrow{-9R_4} \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & -4 & -1 & 2 & -6 & | & -1 \\ 0 & 10 & -2 & 8 & -20 & | & -4 \\ 1 & -1 & 0 & -1 & 2 & | & 1 \end{bmatrix} \xrightarrow{-R_1} \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & -4 & -1 & 2 & -6 & | & -1 \\ 0 & 10 & -2 & 8 & -20 & | & -4 \\ 0 & -2 & -1 & -4 & -2 & | & -1 \end{bmatrix} \xrightarrow{/:2} \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & -4 & -1 & 2 & -6 & | & -1 \\ 0 & 5 & -1 & 4 & -10 & | & -2 \\ 0 & -2 & -1 & -4 & -2 & | & -1 \end{bmatrix} \xrightarrow{-2R_4} \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & 0 & 1 & 10 & -2 & | & 1 \\ 0 & 5 & -1 & 4 & -10 & | & -2 \\ 0 & -2 & -1 & -4 & -2 & | & -1 \end{bmatrix} \xrightarrow{/:2R_4} \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & | & 2 \\ 0 & 0 & 1 & 10 & -2 & | & 1 \\ 0 & 1 & -3 & 0 & -14 & | & -4 \\ 0 & 2 & 1 & 4 & 2 & | & 1 \end{bmatrix} \xrightarrow{-R_3} \begin{bmatrix} 1 & 0 & 2 & -3 & 18 & | & 14 \\ 0 & 1 & -3 & 0 & -14 & | & -4 \\ 0 & 0 & 1 & 10 & -2 & | & 1 \\ 0 & 2 & 1 & 4 & 2 & | & 1 \end{bmatrix} \xrightarrow{-2R_2} \begin{bmatrix} 1 & 0 & 0 & -23 & 22 & | & 4 \\ 0 & 1 & -3 & 0 & -14 & | & -4 \\ 0 & 0 & 1 & 10 & -2 & | & 1 \\ 0 & 0 & 7 & 4 & 30 & | & 5 \end{bmatrix} \xrightarrow{+3R_3} \begin{bmatrix} 1 & 0 & 0 & -23 & 22 & | & 4 \\ 0 & 1 & 0 & 30 & -20 & | & -1 \\ 0 & 0 & 1 & 10 & -2 & | & 1 \\ 0 & 0 & 7 & 4 & 30 & | & 5 \end{bmatrix}$$

$$5. a) \lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} = \frac{(-4)^2 - 10}{(-4)^2 + 8(-4) + 16} = \frac{13}{16}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

F4

IME I PREZIME: *Ankovića Knežević*

BROJ INDEKSA: *57672*

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 8 = 0$. *Prikaži ih u kompleksnoj ravnini!* Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\left(\frac{z_2}{z_1}\right)\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{aligned}x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\3x_1 + x_2 - x_3 - x_4 &= 2 \\9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\x_1 - x_2 - x_4 + 2x_5 &= 1\end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(x^2 - x)$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 6}{x^2 + 8}$

20(graf)

5. Odrediti i provjeriti:

(a) $\lim_{x \rightarrow -4} \frac{x^2 - 10}{x^2 + 8x + 16} =$

4+1

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $8 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadzbe uvrštavanjem, a također provjeri nekoliko rješenja nejednadzbe uvrštavanjem!*

15+5

$$\begin{aligned}\left[\begin{array}{cccc|c} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & 1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{array} \right] & \sim & \left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 2 & 1 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 1 & 1 & -1 & -3 & 4 & 2 \end{array} \right] & \begin{array}{l} (-3)/(-9)/(+1) \\ \end{array} \\ \left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 2 & 1 \\ 0 & 4 & -1 & 2 & -6 & -4 \\ 0 & 10 & -2 & 8 & -20 & -4 \\ 0 & 2 & -1 & -2 & 2 & 1 \end{array} \right] & & \end{aligned}$$

Ukupno:

~~0~~

$$f(x) = \sqrt{x^2 + x + 2} - \arctan(x^2 - x)$$

$$(1) \sqrt{x^2 + x - 2} \leq 0$$

$$x^2 + x - 2 \leq 0$$

$$x \in \langle +\infty, -\infty \rangle \quad \times$$

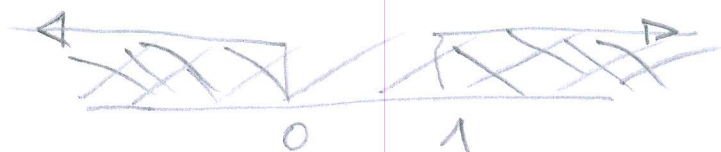
$$D(f) = \langle +\infty, 0 \rangle \cup \langle -2, -\infty \rangle$$

$$(2) x^2 - x > 0$$

$$x(x-1) > 0$$

	$x=0$		$x \cdot 1 = 0$	$x=1$
	$-\infty$	0	$\frac{1}{2}$	1
	$-\infty$	0	2	$+\infty$
x	-	0	+	+
$(x-1)$	-	-	0	+
	(+)	-	(+)	

$$x \in \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$$



$$4] f(x) = \frac{x^2 - 6}{x^2 + 8}$$

$$(1) \text{DOMENA}$$

$$x^2 + 8 \neq 0$$

$$x^2 + 8 = 0$$

$$x^2 = -8$$

NI JEDAN BROJ
KVADRIRAN NE DAJE
NEG, BR=0)

$$(2) \text{NULTOČKE}$$

$$x^2 - 6 = 0$$

$$x^2 = 6$$

$$x = \pm\sqrt{6}$$

$$D(f) = \mathbb{R}$$

3) EKSTREMI RAST I PAD

$$f'(x) = \frac{2x(x^2+8) - 2x(x^2-6)}{(x^2+8)^2} = \frac{\cancel{2x^3} + 16x - \cancel{2x^3} + 12x}{(x^2+8)^2}$$

$$f'(x) = \frac{28x}{(x^2+8)^2} \quad | \quad f(x) = 0$$

$$28x = 0 \quad | :28$$

$$x = 0$$

$$y = -\frac{6}{8}$$

	$-\infty$	-10	$1+\infty$
$f'(x)$	-	0	+
$f(x)$		MIN	

ASIMPTOTE

$$D(f) = \mathbb{R} \Rightarrow \text{NEMA V.A.}$$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2-6}{x^2+8} = \frac{\frac{x^2}{x^2} - \frac{6}{x}}{\frac{x^2}{x^2} + \frac{8}{x}} = \frac{1}{1} = 1 \rightarrow \text{H.A.}$$

