MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: MARTIN ŠOŠA **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja. 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 6A - 8B + 6C + 8D &= 8 \\ -8A + 6B - 8C - 6D &= 6 \\ -A - B + C + 8D &= 0 \\ 6A + 6B + 6C - 6D &= -6 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 2} - x$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+2}{x^2-4}$. 15(graf)
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^2)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4+x} + \sqrt{4-x}$. Koji su globalni ekstremi? 15

Ukupno: 20

2.

$$\begin{aligned} 6A - 8B + 6C + 8D &= 8 \\ -8A + 6B - 8C - 6D &= 6 \\ -A - B + C + 8D &= 0 \\ 6A + 6B + 6C - 6D &= -6 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 6 & -8 & 6 & 8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] : 6 \sim$$

$$\left[\begin{array}{cccc|c} 1 & -8/6 & 1 & 8/6 & 8/6 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \cdot \sim$$

$R1 \cdot 8 + R2$
$R1 + R3$
$R1 \cdot (-6) + R4$

$$\left[\begin{array}{cccc|c} 1 & -8/6 & 1 & 8/6 & 8/6 \\ 0 & -14/3 & 0 & 14/3 & 30/3 \\ 0 & -7/3 & 2 & 28/3 & 8/6 \\ 0 & 14 & 0 & -14 & -14 \end{array}\right] \cdot \left(-\frac{3}{14}\right) \sim$$

$$\left[\begin{array}{cccc|c} 1 & -8/6 & 1 & 8/6 & 8/6 \\ 0 & 1 & 0 & -1 & -25/7 \\ 0 & -7/3 & 2 & 28/3 & 8/6 \\ 0 & 14 & 0 & -14 & -14 \end{array}\right] \sim$$

$R2 \cdot \frac{8}{6} + R1$
$R2 \cdot \frac{7}{3} + R3$
$R2 \cdot (14) + R4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & -24/7 \\ 0 & 1 & 0 & -1 & -25/7 \\ 0 & 0 & 2 & 7 & -7 \\ 0 & 0 & 0 & 0 & 36 \end{array}\right] \sim$$

$: 2$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & -24/7 \\ 0 & 1 & 0 & -1 & -25/7 \\ 0 & 0 & 1 & 7/2 & -7/2 \\ 0 & 0 & 0 & 0 & 36 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7/2 & 1/14 \\ 0 & 1 & 0 & -1 & -25/7 \\ 0 & 0 & 1 & 7/2 & -7/2 \\ 0 & 0 & 0 & 0 & 36 \end{array}\right] \sim$$

$R_3 \cdot (-1) + R_1$

Sustav nema RJEŠENJA ✓

⑥ $f(x) = \sqrt{4+x} + \sqrt{4-x}$

GLOBALNI EKSTREMI $[-4, 0) \cup (0, +4]$ ✗

---

⑤ $g(x) = \sqrt{x^2+4x+2} - x$

$D\langle -\infty, -3.732] \cup [-0.268, +\infty)$ ✓

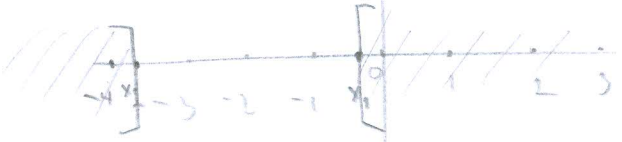
$x^2 + 4x + 2 \geq 0$

$x_{1,2} = \frac{-b \pm \sqrt{ab+4c}}{2a} = \frac{-4 \pm \sqrt{4+4\cdot 2}}{2}$

$x_1 = \frac{-4+\sqrt{12}}{2} = -0.268$

$x_{1,2} = \frac{-4 \pm \sqrt{12}}{2}$ =

$x_2 = \frac{-4-\sqrt{12}}{2} = -3.732$



MARTIN ŠOŠA

④ $f(x) = \frac{x+2}{x^2-4}$

$x+2 \neq 0$

$x^2-4 \neq 0$

⑤ $h(x) = \arctan(x^2)$

$h'(x) = \frac{1}{1+(x^2)^2} \cdot 2x$

$h'(x) = \frac{1}{1+x^4} \cdot 2x$ ✓

$h''(x) = \frac{1' \cdot (1+x^4) - 1 \cdot (1+x^4)'}{(1+x^4)^2} \cdot (2x)'$ ✗

$= \frac{-1 \cdot 4x^3}{1+x^6} \cdot 2$

$h''(x) = \frac{-4x^3}{1+x^6} \cdot \frac{2}{1} = \frac{-8x^3}{1+x^6}$ //

funkcija je neparna! ✗

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: MARKO PARANCIN **BROJ INDEKSA:** 17-1-0062-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja. 12+3
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$$\begin{aligned} 6A - 8B + 6C + 8D &= 8 \\ -8A + 6B - 8C - 6D &= 6 \\ -A - B + C + 8D &= 0 \\ 6A + 6B + 6C - 6D &= -6 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 2} - x$. 5+15 7
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+2}{x^2-4}$. 15(graf)
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^2)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4+x} + \sqrt{4-x}$. Koji su globalni ekstremi? 15

Ukupno: 22

2.

$$\left[\begin{array}{cccc|c} 6 & -8 & 6 & 8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \div 6 \sim \left[\begin{array}{cccc|c} 6 & -8 & 6 & 8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 1 & 1 & 1 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & -8 & 6 & 8 & 8 \end{array}\right] \cdot 8 \cdot (+6)$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 14 & 0 & -14 & -2 \\ 0 & 0 & 2 & 7 & -1 \\ 0 & -14 & 0 & 14 & 14 \end{array}\right] \div (-14) \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 2 & 7 & -1 \\ 0 & 14 & 0 & -14 & -2 \end{array}\right] \cdot 1 \cdot (-1) \cdot 1 \cdot (-14) \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 2 & 7 & -1 \\ 0 & 0 & 0 & 0 & 12 \end{array}\right]$$

SUSTAV NEMA RJEŠENJA

$0A + 0B + 0C + 0D \neq 12$

③ Domena funkcije

$g(x) = \sqrt{x^2 + 4x + 2} - x$

$x^2 + 4x + 2 \geq 0$

$a = 1$, $b = 4$, $c = 2$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$x_{1,2} = \frac{-4 \pm \sqrt{2\sqrt{2}}}{2} =$$

$x_1 = -4 + \frac{2\sqrt{2}}{2} = -0{,}58$

$x_2 = -4 - \frac{2\sqrt{2}}{2} = -3{,}41$

$D(g) = \mathbb{R} \setminus \{-3{,}41;\ -0{,}58\}$ ✗

Asimptote : $g(x) = \sqrt{x^2+4x+2} - x$

V.A

$\lim_{x \to 3{,}41} = \sqrt{-3{,}41^2 + 4\cdot(3{,}41) + 2} + 3{,}41$ NEMA . V. A

H.A.

$$\lim_{x\to\infty} = \sqrt{x^2+4x+2} - x = \lim_{x\to\infty} = \sqrt{x^2+4x+2} - x \cdot \frac{\sqrt{x^2+4x+2} - x}{\sqrt{x^2+4x+2} - x}$$

$$\lim_{x\to\infty} = \frac{x^2+4x+2-x^2}{\sqrt{x^2+4x+2} - x} = \frac{4x+2}{\sqrt{x^2+4x+2} - x} \quad /:x \quad /:x$$

$$= \lim_{x\to\infty} = \frac{4}{\sqrt{1} + 1} = \frac{4}{2} = 2$$

NEMA K.A.

H.A.

OVO SE SAMO D.H.A. KOJE JE L.H.A.

(4) $A(x) = \dfrac{x+2}{x^2-4}$

$D(A) = \langle -\infty, -4) \cup \langle 4, +\infty \rangle$

BORIJE SE SAMO SKICA GRAFA

$1^\circ\ x^2 - 4 \neq 0$

$x = -4$

$x = 4$

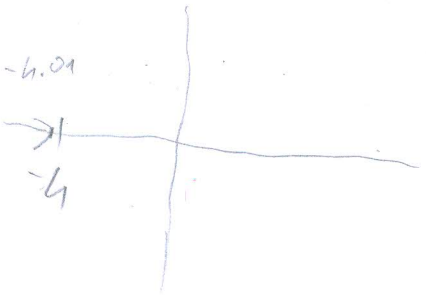
1. Asimptota V.A

$$\lim_{x\to 4^+} = \frac{x+2 \ /:x^2}{x^2-4 \ /:x^2} = \frac{\frac{x}{x^2} + \frac{2}{x^2}}{1^+ \frac{x^2}{x^2} - \frac{4}{x^2}} = +\infty$$

$$\lim_{x\to 4^-} = \frac{x-2 \ /:x^2}{x^2-4 \ /:x^2} = \frac{\frac{x}{x^2} - \frac{2}{x^2}}{1 \cdot \frac{x^2}{x^2} - \frac{4}{x^2}} = -\infty$$

$$\lim_{x\to -4^+} = \frac{x-2 \ /:x^2}{x^2-4 \ /:x^2} = \frac{\frac{x}{x^2} - \frac{2}{x^2}}{1 \cdot \frac{x^2}{x^2} - \frac{4}{x^2}} = +\infty$$

$$\lim_{x \to -4^-} = \frac{x-2 \mid :x^2}{x^2-4 \mid :x^2} = \frac{\frac{x}{x^2} - \frac{2}{x^2}}{1 \frac{x^2}{x^2} - \frac{4}{x^2}} = -\infty$$



⑤ $h(x) = \operatorname{arctan}(x^2)$

DERIV.

$h'(x) \frac{1}{x^2+1} \cdot (x^2)' \cdot$ ×

$$f'(x) = \frac{1}{x^2+1} \cdot \frac{2x}{1}$$

$$f'(x) = \frac{2x}{x^2} = \frac{2}{x} = 2x \text{ //}$$

$$f''(x) = (2x)'$$

$$f''(x) = 2 \text{ //}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ G3

IME I PREZIME: IVAN BEKAM **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 13

1. $\frac{|z|}{z+2i} = 3i$

$|z| = \sqrt{x^2+y^2}$

$\frac{z}{z+2i} = 3i \quad / \cdot z+2i$

$z = 3i(z+2i)$

$z = 3zi + 6i^2$

$z = 3zi + 6$

$z - 3zi = 6$

$z(1-3i) = 6 \quad / :(1-3i)$

$z = \frac{6}{1-3i}$

$z = \frac{6}{1-3i} \cdot \frac{1+3i}{1+3i}$

$z = \frac{6+18i}{(1)^2 - (3i)^2}$

$z = \frac{6+18i}{1+9} = \frac{6}{10} + \frac{18}{10}i$

3) $g(x)=\sqrt{x^2+4x+2}-x$

$\sqrt{x^2+4x+2}\geq 0$

$x^2+4x+2=0$

$x_{1,2}=\frac{-4\pm\sqrt{16-4\cdot1\cdot2}}{2}$

$x_{1,2}=\frac{-4\pm\sqrt{8}}{2}$

$x_1=\frac{-4+\sqrt{8}}{2}=\frac{-4+2\sqrt{2}}{2}=-2+\sqrt{2}$

$x_2=\frac{-4-\sqrt{8}}{2}=\frac{-4-2\sqrt{2}}{2}=-2-\sqrt{2}$

$-2-\sqrt{2}\approx-3,41$ $\quad$ $-2+\sqrt{2}\approx-0,59$

$x_1=-4$: $\sqrt{(-4)^2-16+2}\geq 0$, $\sqrt{16-16+2}\geq 0$, $\sqrt{+2}\geq 0$ ✓

$x_2=-2$: $\sqrt{(-2)^2-8+2}\geq 0$, $\sqrt{4-8+2}\geq 0$, $\sqrt{-2}\geq 0$ ✗

$x_3=0$: $\sqrt{(0)^2-0+2}\geq 0$, $\sqrt{2}\geq 0$ ✓

$D_f\ \langle-\infty,2-\sqrt{2}]\cup[-2+\sqrt{2},+\infty\rangle$ ✓

H. A.

$\lim_{x\to+\infty}\sqrt{x^2+4x+2}-x\cdot\frac{\sqrt{x^2+4x+2}+x}{\sqrt{x^2+4x+2}+x}=\lim_{x\to\infty}\frac{x^2+4x+2-x^2\,/:x^2}{\sqrt{x^2+4x+2}+x\,/:x^2}=\lim_{x\to\infty}\frac{1+\frac{4}{x}+2-1}{\sqrt{\frac{1}{x^2}+\frac{4}{x^3}+\frac{2}{x^4}}+\frac{1}{x}}=$

$=\frac{1+\frac{4}{+\infty}+2-1}{\sqrt{\frac{1}{\infty}+\frac{4}{\infty}+\frac{2}{\infty}}+\frac{1}{\infty}}=\frac{1+0+2-1}{\sqrt{0}+0}=\frac{2}{0}=+\infty$ ✗

$\lim_{x\to-\infty}\sqrt{x^2+4x+2}-x=\lim_{x\to\infty}\sqrt{x^2+4x+2}-x\cdot\frac{\sqrt{x^2+4x+2}+x}{\sqrt{x^2+4x+2}+x}=$

$\lim_{x\to-\infty}=\frac{x^2+4x+2-x^2\,/:x^2}{\sqrt{x^2+4x+2}+x\,/:x^2}=\frac{1+\frac{4}{x}+2-1}{\sqrt{\frac{1}{x^2}+\frac{4}{x^3}+\frac{2}{x^4}}+\frac{1}{x}}=\frac{1+\frac{4}{-\infty}+2-1}{\sqrt{\frac{1}{-\infty}+\frac{4}{-\infty}+\frac{2}{-\infty}}+\frac{1}{-\infty}}=\frac{1+0+2-1}{\sqrt{0}}=\frac{2}{0}=-\infty$

$y=kx+l$

$k=\lim_{x\to\infty}\frac{f(x)}{x}$

$k=\lim_{x\to\infty}\frac{\sqrt{x^2+4x+2}-x\,/:x}{x\,/:x}=$

$\lim_{x\to\infty}\frac{\sqrt{1+\frac{4}{x}+\frac{2}{x^2}}-1}{1}=$

$\lim_{x\to\infty}\frac{\sqrt{1+\frac{4}{x}+\frac{2}{x^2}}-1}{1}=$

$\frac{\sqrt{1+0+0}-1}{1}=\frac{1-1}{1}=0$

$l=\lim_{x\to\infty}[f(x)-k(x)]$

$l=\lim_{x\to\infty}\sqrt{x^2+4x+2}-x-0x$

V. A.

$\lim_{x\to-2+\sqrt{2}}=$

5. $h(x)=\operatorname{arctg}(x^2)$

$D_f\ \langle-\infty,+\infty\rangle$ ✓

$h(-x)=\operatorname{arctg}((-x)^2)$

$h(-x)=\operatorname{arctg}(x^2)\Rightarrow$ parna funkcija ✓

$h'(x)=\frac{1}{1+2x^2}$ ✗

$h''(x)=\frac{1}{1+2x^2}'=\frac{1'\cdot(1+2x^2)-1\cdot(1+2x^2)'}{(1+2x^2)^2}=\frac{1\cdot(0+4x)}{(1+2x^2)^2}=\frac{4x}{(1+2x^2)^2}$

$$\left(\begin{array}{cccc|c} 6 & -8 & 6 & -8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right) \begin{matrix} \text{IV}-\text{I} \\ \text{II}+8\text{III} \\ \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|c} 6 & -8 & 6 & -8 & 8 \\ 0 & 14 & -16 & -70 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 0 & -2 & 0 & -14 & -14 \end{array}\right) \begin{matrix} \text{I}+6\text{III} \\ \\ \\ \\ \end{matrix}$$

$$\left(\begin{array}{cccc|c} 0 & -14 & 12 & 56 & 8 \\ 0 & 14 & -16 & -70 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 0 & -2 & 0 & -14 & -14 \end{array}\right) \begin{matrix} \text{I}+\text{III} \\ \\ \\ \\ \end{matrix} \sim$$

$$\left(\begin{array}{cccc|c} -1 & -1 & 1 & 8 & 0 \\ 0 & -2 & 0 & -14 & -14 \\ 0 & -14 & 12 & 56 & 8 \\ 0 & 14 & -16 & -70 & 6 \end{array}\right) \begin{matrix} \\ :2 \\ :2 \\ :2 \end{matrix} \sim \left(\begin{array}{cccc|c} -1 & -1 & 1 & 8 & 0 \\ 0 & -1 & 0 & -7 & -7 \\ 0 & -7 & 6 & 28 & 4 \\ 0 & 7 & -8 & -35 & 0 \end{array}\right) \begin{matrix} \text{I}-\text{II} \\ \\ \\ \\ \end{matrix}$$

$$\left(\begin{array}{cccc|c} 0 & 1 & -5 & 7 & 7 \\ -1 & 0 & -7 & -7 & \\ -7 & 6 & 28 & 4 & \\ 7 & -8 & -35 & 0 & \end{array}\right) \begin{matrix} \text{III}-7\text{II} \\ \text{IV}+7\text{II} \\ \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|c} -1 & 0 & 1 & 15 & 7 \\ 0 & -1 & 0 & -7 & -7 \\ 0 & 0 & 6 & 77 & 53 \\ 0 & 0 & -8 & -84 & -49 \end{array}\right) =$$

$$\left(\begin{array}{cccc|c} -1 & 0 & 1 & 15 & 7 \\ 0 & -1 & 0 & -7 & -7 \\ 0 & 0 & 6 & 77 & 53 \\ 0 & 0 & -8 & -84 & -49 \end{array}\right) \text{III}+\text{IV} \quad \left(\begin{array}{cccc|c} -1 & 0 & 1 & 15 & 7 \\ 0 & -1 & 0 & -7 & -7 \\ 0 & 0 & -2 & -7 & 4 \\ 0 & 0 & -8 & -84 & -49 \end{array}\right) \begin{matrix} \\ \\ :3 \\ \\ \end{matrix}$$

$$\left(\begin{array}{cccc|c} -2 & 0 & 2 & 30 & 14 \\ 0 & -2 & 0 & -14 & -14 \\ 0 & 0 & -2 & -7 & 4 \\ 0 & 0 & -8 & -84 & -49 \end{array}\right) \begin{matrix} \text{I}+\text{III} \\ \\ \\ \\ \end{matrix} \sim$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{1315}{112} \\ 0 & 1 & 0 & 0 & \frac{315}{56} \\ 0 & 0 & 1 & 0 & \frac{301}{112} \\ 0 & 0 & 0 & 1 & \frac{11}{56} \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -2 & 0 & 0 & 23 & 28 \\ 0 & -2 & 0 & -14 & -14 \\ 0 & 0 & -2 & -7 & 4 \\ 0 & 0 & -8 & -84 & -49 \end{array}\right) \text{IV}-4\text{III} \sim$$

$$\left(\begin{array}{cccc|c} -2 & 0 & 0 & 23 & 28 \\ 0 & -2 & 0 & -14 & -14 \\ 0 & 0 & -2 & -7 & 4 \\ 0 & 0 & 0 & 56 & 11 \end{array}\right) \begin{matrix} :-2 \\ :-2 \\ :-2 \\ :56 \end{matrix} \sim$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{23}{2} & 14 \\ 0 & 1 & 0 & 7 & 7 \\ 0 & 0 & 1 & \frac{7}{2} & -2 \\ 0 & 0 & 0 & 1 & \frac{11}{56} \end{array}\right) \begin{matrix} \text{II}-\frac{23}{2}\text{IV} \\ \text{II}-7\text{IV} \\ \text{III}-\frac{7}{2}\text{IV} \\ \\ \end{matrix}$$

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 1315/112 \\ 315/56 \\ 301/112 \\ 11/56 \end{pmatrix}$$

$$14 - \frac{23}{2} \cdot \frac{11}{56} = \frac{253}{112} = 14 \qquad 7 - \left(7 \cdot \frac{11}{56}\right) = 7 - \frac{77}{56} = \frac{315}{56}$$

$$14 - \left(\frac{23}{2} \cdot \frac{11}{56}\right) \qquad \frac{11}{56} \cdot \frac{7}{2} = \frac{77}{112} =$$

$$14 - \left(\frac{253}{112}\right) \qquad -2 - \frac{77}{112} = \frac{-224-77}{112}$$

$$\frac{1568-253}{112} = \frac{1315}{112}$$

$$6A - 8B + 6C + 8D = 8$$

$$6 \cdot \frac{1315}{112} - 8 \cdot \frac{315}{56} + 6 \cdot \frac{301}{112} + 8 \cdot \frac{11}{56} = 8$$

$$\frac{7890}{112} - 45 + \frac{1806}{112} + \frac{88}{56} = 8$$

$$\frac{7890 - 5040 + 1806 + 176}{112} = 8 \quad ?$$

$$\left(\begin{array}{cccc|c} 6 & -8 & +6 & +8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & +8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right) \overset{\text{I}\leftrightarrow\text{III}}{\sim} \left(\begin{array}{cccc|c} -1 & -1 & 1 & 8 & 0 \\ -8 & 6 & -8 & -6 & 6 \\ 6 & -8 & 6 & 8 & 8 \\ 6 & 6 & 6 & -6 & 6 \end{array}\right) \begin{array}{l} \text{I}:(-1) \\ \\ \\ \text{IV}:6 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 8 & 0 \\ -8 & 6 & -8 & -6 & 6 \\ 6 & -8 & 6 & 8 & 8 \\ 1 & 1 & 1 & -1 & 1 \end{array}\right) \begin{array}{l} \text{II}+8\text{I} \\ \\ \text{III}-6\text{IV} \\ \\ \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 8 & 0 \\ 0 & 14 & 0 & 58 & 6 \\ 0 & -14 & 0 & 14 & 2 \\ 1 & 1 & 1 & -1 & 1 \end{array}\right) \begin{array}{l} \\ \\ \\ \text{II}\leftrightarrow\text{IV} \end{array} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 8 & 0 \\ 1 & 1 & 1 & -1 & 1 \\ 0 & -14 & 0 & 14 & 2 \\ 0 & 14 & 0 & 58 & 6 \end{array}\right) \begin{array}{l} \text{I}-\text{IV} \\ \\ \\ \\ \end{array} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 8 & 0 \\ 1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 72 & 8 \\ 0 & 14 & 0 & 52 & 6 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 6 & -8 & +6 & +8 & 8 \\ -8 & 6 & 8 & -6 & 6 \\ -1 & -1 & 1 & +8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right) \begin{array}{l} :2 \\ :2 \\ :(-1) \\ :6 \end{array} \sim \left(\begin{array}{cccc|c} 3 & -4 & 3 & 4 & 4 \\ -4 & 3 & 4 & -3 & 3 \\ 1 & 1 & 1 & -8 & 0 \\ 1 & 1 & 1 & -1 & -1 \end{array}\right) \begin{array}{l} \text{I}-2\text{IV} \\ \text{II}+4\text{IV} \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & -6 & 1 & 6 & 5 \\ 0 & 7 & 8 & -7 & -1 \\ 1 & 1 & 1 & -8 & 0 \\ 1 & 1 & 1 & -1 & -1 \end{array}\right) \begin{array}{l} \text{III}-\text{I} \\ \text{IV}-\text{I} \\ \\ \\ \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & -6 & 1 & 6 & 6 \\ 0 & 7 & 8 & -7 & -1 \\ 0 & 7 & 0 & -14 & -6 \\ 0 & 7 & 0 & -7 & -7 \end{array}\right) \begin{array}{l} \text{I}+\text{IV} \\ \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 7 & 8 & -7 & -1 \\ 0 & 7 & 0 & -14 & -6 \\ 0 & 7 & 0 & -7 & -7 \end{array}\right) \begin{array}{l} \\ \\ \\ :7 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -1 \\ 0 & 7 & 8 & -7 & -1 \\ 0 & 7 & 0 & -14 & -6 \\ 0 & 1 & 0 & -1 & -1 \end{array}\right) \begin{array}{l} \text{I}-\text{IV} \\ \\ \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 7 & 8 & -7 & -1 \\ 0 & 7 & 0 & -14 & -6 \\ 0 & 1 & 0 & -1 & -1 \end{array}\right) \begin{array}{l} \\ \\ \\ \\ \text{I}-\text{III} \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & 7 & 5 \\ 0 & 7 & 0 & -14 & -6 \\ 0 & 1 & 0 & -1 & -1 \end{array}\right) \begin{array}{l} \\ \\ \\ \text{II}\leftrightarrow\text{IV} \end{array} \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 7 & 0 & -14 & -6 \\ 0 & 0 & 8 & 7 & 5 \end{array}\right) \begin{array}{l} \\ \\ \text{III}-7\text{II} \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & -7 & 1 \\ 0 & 0 & 8 & 7 & 5 \end{array}\right)$$

IVAN BEZIĆ

6. $f(x) = \sqrt{4+x} + \sqrt{4-x}$

$[-4, 4]$

Funkcija je omeđena (sadrži domenu)

$f(-x) = -f(x)$ je neparna

$f(-x) = \sqrt{4-x} + \sqrt{4+x}$

	-4		4
f'(x)	−		+
f''(x)	∩		∪

Ø

4.) $f(x) = \frac{x+2}{x^2-1}$

Domena

$x^2 - 1 \neq 0$

$x^2 \neq 1$

$D_f \ \langle -\infty, -1] \cup \langle -1, 1\rangle \cup \langle 1, +\infty\rangle$

2. ASIMPTOTE

V.A

$\lim_{x \to 1^-} \frac{x^2+1}{x^2-1} = \frac{2}{0^+} = +\infty$

$\lim_{x \to -1^+} \frac{x^2+1}{x^2-1} = \frac{2}{0^-} = -\infty$

SKICA GRAFA ?

H.A

$\lim_{x \to +\infty} \frac{x^2+1}{x^2-1} = \frac{\infty}{\infty} = 1$

$\lim_{x \to -\infty} \frac{x^2+1}{x^2-1} = -1$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: MATE ĆOSIĆ **BROJ INDEKSA:** 55924

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 6A - 8B + 6C + 8D &= 8 \\ -8A + 6B - 8C - 6D &= 6 \\ -A - B + C + 8D &= 0 \\ 6A + 6B + 6C - 6D &= -6 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 2} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+2}{x^2-4}$. 15(graf)

5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^2)$. 2+4+6+8

6. Zadana je funkcija $f(x) = \sqrt{4+x} + \sqrt{4-x}$. Koji su globalni ekstremi? 15

Ukupno: 45

$$\left[\begin{array}{cccc|c} 6 & -8 & 6 & 8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \sim \left[\begin{array}{cccc|c} -1 & -1 & 1 & 8 & 0 \\ -8 & 6 & -8 & -6 & 6 \\ 6 & -8 & 6 & 8 & 8 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \cdot(-1)$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -8 & 0 \\ -8 & 6 & -8 & -6 & 6 \\ 6 & -8 & 6 & 8 & 8 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \begin{array}{l} \cdot 8 \\ \\ -6 \\ -6 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -8 & 0 \\ 0 & 14 & -16 & -70 & 6 \\ 0 & -14 & 12 & 56 & 8 \\ 0 & 0 & 12 & 42 & -6 \end{array}\right] \cdot\frac{1}{14}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -8 & 0 \\ 0 & 1 & -\frac{8}{7} & -5 & \frac{3}{7} \\ 0 & -14 & 12 & 56 & 8 \\ 0 & 0 & 12 & 42 & -6 \end{array}\right] \begin{array}{l} \\ -1 \\ \\ \end{array} \cdot 14 \sim \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{7} & -3 & -\frac{3}{7} \\ 0 & 1 & -\frac{8}{7} & -5 & \frac{3}{7} \\ 0 & 0 & -16 & -14 & 14 \\ 0 & 0 & 12 & 42 & -6 \end{array}\right] \cdot -\frac{1}{16}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{7} & -3 & -\frac{3}{7} \\ 0 & 1 & -\frac{8}{7} & -5 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{7}{8} & -\frac{7}{8} \\ 0 & 0 & 12 & 42 & -6 \end{array}\right] \begin{array}{l} \\ \\ \\ -12 \end{array} \cdot -\frac{1}{7} \quad \frac{8}{7}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{25}{8} & -\frac{17}{56} \\ 0 & 1 & 0 & -4 & -\frac{4}{7} \\ 0 & 0 & 1 & \frac{7}{8} & -\frac{7}{8} \\ 0 & 0 & 0 & \frac{63}{2} & \frac{9}{2} \end{array}\right] \cdot\frac{2}{63}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{25}{8} & -\frac{17}{56} \\ 0 & 1 & 0 & -4 & -\frac{4}{7} \\ 0 & 0 & 1 & \frac{7}{8} & -\frac{7}{8} \\ 0 & 0 & 0 & 1 & \frac{1}{7} \end{array}\right] \cdot \frac{25}{8} \quad \cdot 4 \quad \cdot\left(-\frac{7}{8}\right)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{1}{7} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & \frac{1}{7} \end{array}\right] \quad \left[\begin{array}{c} A \\ B \\ C \\ D \end{array}\right] = \left[\begin{array}{c} \frac{1}{7} \\ 0 \\ -1 \\ \frac{1}{7} \end{array}\right]$$

$6 \cdot \frac{1}{7} - 8 \cdot 0 + 6 \cdot (-1) + 8 \cdot \frac{1}{7} = 8$

$-8 \cdot \frac{1}{7} + 6 \cdot 0 - 8 \cdot (-1) - 6 \cdot \frac{1}{7} = 6$

$-\frac{1}{7} - 0 + (-1) + 8 \cdot \frac{1}{7} = 0$

$6 \cdot \frac{1}{7} + 6 \cdot 0 + 6 \cdot (-1) - 6 \cdot \frac{1}{7} = -6$

NEMOJTE TO VIŠE RADITI !!!

3. $g(x) = \sqrt{x^2+4x+2} - x$

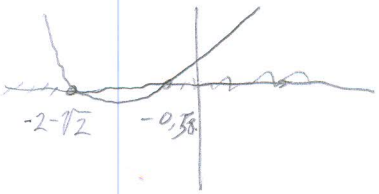
$x^2+4x+2 \geq 0 \quad a>0$

$x_{1,2} = \frac{-4 \pm \sqrt{4^2-4\cdot 1\cdot 2}}{2}$

$x_{1,2} = \frac{-4 \pm \sqrt{8}}{2}$

$x_1 = -2+\sqrt{2}$

$x_2 = -2-\sqrt{2}$



$Dg: (-\infty, -2-\sqrt{2}] \cup [-2+\sqrt{2}, +\infty)$ ✓

Asimptote

$\lim_{x \to -2+\sqrt{2}} \sqrt{x^2+4x+2} - x = \text{nema}$ V.A.

i/ili A. $\lim_{x\to\infty} \sqrt{x^2+4x+2} - x \cdot \frac{\sqrt{x^2+4x+2}+x}{\sqrt{x^2+4x+2}+x} = \frac{x^2+4x+2-x^2}{\sqrt{x^2+4x+2}-x}$

$\lim_{x\to\infty} \frac{4x+2}{\sqrt{x^2+4x+2}-x} \;/x \;/x = \frac{4}{\sqrt{0+4+0}-1} = \frac{4}{1} = 4$ ✗

$y = 4$

$\lim_{x\to-\infty} = \frac{-4x+2}{\sqrt{x^2-4x+2}+x} = \frac{-4}{\infty} \;\text{nema}$

5)

$f(x) = \arctan(x^2)$

$Df = \mathbb{R}$ ✓

$f'(x) = \frac{1}{1+(x^2)^2} \cdot 2x =$ ✓

$f'(x) = \frac{2x}{1+x^4}$ ✓

$f''(x) = \frac{2\cdot(1+x^4) - (2x\cdot 4x^3)}{(1+x^4)^2}$ ✓

$f''(x) = \frac{2\cdot(1+x^4) - 8x^4}{(1+x^4)^2} = \frac{2+2x^4-8x^4}{(1+x^4)^2}$

$f''(x) = \frac{2-6x^4}{(1+x^4)^2}$ //

MAIE ČÍSLO

1) $\frac{|z|}{z+2i} = 3i$

$\frac{\sqrt{x^2+y^2}}{x+yi+2i} = 3i$

$\sqrt{x^2+y^2} = 3i(x+yi+2i)$

$\sqrt{x^2+y^2} = 3xi + 3yi^2 + 6i^2$

$\sqrt{x^2+y^2} = 3xi - 3y - 6$

$0 = -3y - 6$

$6 = -3y$

$-2 = y$

$y = -2$ ✓

$\sqrt{x^2+y^2} = -3y - 6$

$-3x = 0$

$\sqrt{x^2+y^2} = 3x \quad /^2$

$x^2 + y^2 = 9x^2$

$4 = 9x^2 - x^2$

$8x^2 = 4 \quad /:2$

$x^2 = \frac{1}{2} \quad /\sqrt{\ }$

$x = \pm\frac{\sqrt{2}}{2}$

$x_1 = \frac{\sqrt{2}}{2}$

$x_2 = -\frac{\sqrt{2}}{2}$

$z = \frac{\sqrt{2}}{2} - 2i$

ZKOUŠKA

$\frac{\sqrt{-\left(\frac{\sqrt{2}}{2}\right)^2 + (-2)^2}}{-\frac{\sqrt{2}}{2} - 2 + 2} = 3$

$\frac{\frac{3\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 3$

$-3 = 3$

$\frac{\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + (-2)^2}}{\frac{\sqrt{2}}{2} - 2 + 2} = 3i$

$\frac{\frac{3\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 3$

$3 = 3$

4) $f(x) = \frac{x+2}{x^2-4}$ Domena $x^2-4 \neq 0$ $D_f: \mathbb{R} \setminus \{-2, 2\}$

$x^2 \neq 4$

$x \neq \pm 2$

$$f'(x) = \frac{1\cdot(x^2-4)-(x+2)\cdot 2x}{(x^2-4)^2} = \frac{x^2-4-2x^2-4x}{(x^2-4)^2} = \frac{-x^2-4x-4}{(x^2-4)^2}$$

N. T. $-x^2-4x-4=0$

$$x_{1,2} = \frac{4 \pm \sqrt{4^2-4\cdot(-1)\cdot(-4)}}{2\cdot(-1)} = \frac{4 \pm 0}{-2} =$$

$x = -2$

$-4 \quad -8 \quad -4$

$\quad\quad 10 \quad\quad 4$

$$f''(x) = \frac{-2x-4(x^2-4)^2-(-x^2-4x-4)\cdot(2x^2-4)\cdot 2x}{(x^2-4)^4}$$

$f(-2) = 16 > 0$ min $T_{MIN}(-2, 0)$

Sim P.

V. A. NEMA

N. A. $\lim_{x\to\infty} \frac{x+2}{x^2-4} \;/x^2 = \frac{0+0}{1-0} = \infty$ NEMA HA

SKICA GRAFA

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ G3

IME I PREZIME: KLARA POSTRUŽIN **BROJ INDEKSA:** 17 -1-0117-1012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja. 12+3
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$$\begin{aligned} 6A - 8B + 6C + 8D &= 8 \\ -8A + 6B - 8C - 6D &= 6 \\ -A - B + C + 8D &= 0 \\ 6A + 6B + 6C - 6D &= -6 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2+4x+2} - x$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+2}{x^2-4}$. 15(graf)
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^2)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4+x} + \sqrt{4-x}$. Koji su globalni ekstremi? 15

Ukupno:

(3.) $g(x) = \sqrt{x^2+4x+2} - x$

$x^2+4x+2 \geq 0$

$x^2+4x+2 = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16-8}}{2} = \frac{-4 \pm \sqrt{8}}{2}$

$\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

$x_1 = \frac{-4+2\sqrt{2}}{2}$

$x_2 = \frac{-4-2\sqrt{2}}{2}$

Dg. $\left\langle -\infty, \frac{-4-2\sqrt{2}}{2} \right] \cup \left[\frac{-4-2\sqrt{2}}{2}, \frac{-4+2\sqrt{2}}{2} \right] \cup \left[\frac{-4+2\sqrt{2}}{2}, +\infty \right\rangle$ (2)

NE ZNATE RIJEŠAVATI NEJEDNADŽBE! NAUČITE!

(5.) $h(x) = \arctan(x^2)$

Dg: $\mathbb{R}$ ✓

$h(-x) = -\arctan(-x)^2$

$h(-x) = -\arctan x^2$

niti parna niti neparna ✗

$(\arctan x)' = \frac{1}{1+x^2}$

$h'(x) = \frac{1}{1+x^2} \cdot (x^2)'$ ✗

$h'(x) = \frac{1}{(1+x^2)^2} \cdot 2x$

$h'(x) = \frac{2x}{(1+x^2)^2}$

① $\frac{|z|}{z+2i} = 3i \;/\cdot(z+2i)$

$|z| = 3i(z+2i)$

$\sqrt{x^2+y^2} = 3i(z+2i)\;/^2$

$x^2+y^2 = (3i)^2(z+2i)^2$

$x^2+y^2 = 9i^2(z+2i)^2$

$x^2+y^2 = -9(z+2i)^2$ ✗

$z = x+yi$

$|z| = \sqrt{x^2+y^2}$

$(z+2i)^2 =$

$z^2+2\cdot z\cdot 2i+(2i)^2 =$

$z^2+4zi+4i^2$

x^2+y^2+4

$(2i)^2 = 4i^2 = -4$

$(z+2i)^2 =$

$z^2-2\cdot z\cdot 2i+(2i)^2 =$

$z^2-4zi-4$

$(x+yi)^2 = x^2+2xyi-y$

$x^2+2xyi-y-4i(x+yi)-4$

$x^2+2xyi-y-4xi-4yi^2-4$

$x^2+2xyi-y-4xi+4y-4$

x^2

$\frac{|z|}{z+2i} = 3i$

$\frac{\sqrt{x^2+y^2}}{x+yi+2i} = 3i\;/\cdot(x+yi+2i)$

$\sqrt{x^2+y^2} = 3xi+3yi^2+6i^2$

$5xi$

4. $f(x)=\frac{x+2}{x^2-4}$

DOMENA

$D_f: \mathbb{R}\setminus\{-2,2\}$

$x^2-4\neq 0$

$x^2\neq 4$

$x\neq\pm 2$

ASIMPTOTE

$\lim_{x\to -2}\frac{x+2}{x^2-4}=\frac{0}{0}$

$\lim_{x\to 2}\frac{x+2}{x^2-4}=\frac{4}{0^+}=+\infty$

$x=2$ je vertikalna asimptota

$\lim_{x\to+\infty}\frac{x+2}{x^2-4}\ /:x$

$\lim_{x\to-\infty}\frac{-x+2}{x^2-4}$

nema horizontalne asimptote

$y=kx+l$

$k=\lim_{x\to\pm\infty}\frac{f(x)}{x}$

$k=\lim_{x\to+\infty}\frac{\frac{x+2}{x^2-4}}{x}=\lim_{x\to+\infty}\frac{x+2}{x^3-4x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to+\infty}\frac{x+2\ /:x^3}{x^3-4x\ /:x^3}=0$

k ne smije biti nula

- nema kose asimptote

G.S.

- nije periodična jer nije trigonometrijska

$f(-x)=\frac{-x+2}{(-x)^2-4}=\frac{-x+2}{x^2-4}$

- niti parna niti neparna

DERIVACIJE

$f'(x)=\frac{(x+2)'\cdot(x^2-4)-(x+2)\cdot(x^2-4)'}{(x^2-4)^2}$

$f'(x)=\frac{x^2-4-2x(x+2)}{(x^2-4)^2}=\frac{x^2-4-2x^2-4x}{(x^2-4)^2}$

$f'(x)=\frac{-x^2-4x-4}{(x^2-4)^2}$

$f''(x)=\frac{(-x^2-4x-4)'\cdot(x^2-4)^2-(x^2-4x-4)\cdot[(x^2-4)^2]'}{(x^2-4)^4}$

$\frac{-x^2-4x-4}{(x^2-4)^2}=0$

$-x^2-4x-4=0\ /:(-1)$

$x^2+4x+4=0$

$x_{1,2}=\frac{-4\pm\sqrt{16-16}}{2}$

$x=-\frac{4}{2\cdot 1}$

$x=2$

GRAF?

⑥ $f(x)=\sqrt{4+x}+\sqrt{4-x}$

$4+x\geq 0$ $\quad 4-x\geq 0$ $\quad Dg:\langle -4,4\rangle$

$x\geq -4$ $\quad -x\geq -4$

$x\leq 4$ $\quad \sqrt{4-x}^2$

$$f'(x)=\frac{1}{2\sqrt{4+x}}\cdot(4+x)'+\frac{1}{2\sqrt{4-x}}\cdot(4-x)'$$

$$f'(x)=\frac{1}{2\sqrt{4+x}}+\frac{1}{2\sqrt{4-x}}$$

$$f'(x)=\frac{2\sqrt{4-x}+2\sqrt{4+x}}{(2\sqrt{4+x})(2\sqrt{4-x})}$$

$f'(x)>0$ min

$f'(x)<0$ Max

$$\frac{2\sqrt{4-x}+2\sqrt{4+x}}{(2\sqrt{4+x})(2\sqrt{4-x})}=0 \quad /\cdot(2\sqrt{4+x})(2\sqrt{4-x})$$

$2\sqrt{4-x}+2\sqrt{4+x}=0 \quad /^2$

$(2\sqrt{4-x})^2+(2\sqrt{4+x})^2=0$

$4(4-x)+4(4+x)=0$

$16-4x+16+4x=0$

$\not\in$

-funkcija nema globalnih ekstrema

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ G3

IME I PREZIME: Tibor Rak **BROJ INDEKSA:** 17-1-0060-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja. 12+3
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4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+2}{x^2-4}$. 15(graf) 5
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^2)$. 2+4+6+8
6. Zadana je funkcija $f(x) = \sqrt{4+x} + \sqrt{4-x}$. Koji su globalni ekstremi? 15

Ukupno: 5

1. $\frac{|z|}{z+2i} = 3i$

$\frac{|x+yi|}{z+2i} = 3i \mid \cdot (z+2i)$

$\sqrt{x^2+y^2} = 3i(z+2i) \mid^2$

$x^2+y^2 = 9i(z+2i)^2$

$x^2+y^2 = 9i \cdot z^2 + 12zi + 4i^2$

$x^2+y^2 = 9i \cdot (x+yi)^2 + 12zi - 4$

$x^2+y^2 = 9i \cdot x^2 + 2xyi - 16$

Re: $x^2 = x^2 - 16$ Im: $y^2 = 9i + 2xyi$

3. $g(x) = \sqrt{x^2+4x+2} - x$

1) $x^2+4x+2 \geq 0$

$x^2+4x \geq 2 \mid$

$x+2x \geq \sqrt{2}$

$3x \geq \sqrt{2} \mid :3$

$x \geq \frac{\sqrt{2}}{3}$ ✗

1) $x \neq 0$

$D(f) = \langle -\infty, \frac{\sqrt{2}}{3} \rangle \cup \langle \frac{\sqrt{2}}{3}, \infty \rangle$

V.A.

5. $h(x)=\arctan(x^2)$

$$h'(x)=\frac{1}{1+x^2}\cdot(x^2)'=\frac{1}{1+x^2}\cdot 2x=\frac{2x}{1+x^2}$$

$$h''(x)=\frac{(2x)'(1+x^2)-2x\,(1+x^2)'}{(1+x^2)^2}=\frac{2(1+x^2)-2x\cdot 2x}{(1+x^2)^2}=\frac{2+2x^2-4x}{(1+x^2)^2}$$

$h=\arctan(x^2)$

$x^2\neq 0$ ×

$D(h)=\mathbb{R}$

6

$k=\lim_{x\to\infty}\frac{f(x)}{x}$

$l=\lim_{x\to\infty}[f(x)-k\cdot x]$

4. $f(x)=\frac{x+2}{x^2-4}$

1. $x^2-4\neq 0 \mid \sqrt{\ }$

$x-2=0$ $\quad D(f)=\mathbb{R}\setminus\{2\}$

$x=2$

2. V.A. $\lim_{x\to 2^-}\frac{x+2}{x^2-4}=\frac{4}{0^-}=-\infty$

$\lim_{x\to 2^+}\frac{x+2}{x^2-4}=\frac{4}{0^+}=+\infty$

H.A. $\lim_{x\to\infty}\frac{x+2\mid :x^2}{x^2-4\mid :x^2}=\frac{\frac{x}{x^2}+\frac{2}{x^2}}{1-\frac{4}{x^2}}=\frac{0}{1}=0$

3. $f'(x)=\frac{(x+2)'(x^2-4)-(x+2)(x^2-4)'}{(x^2-4)^2}$

$f'(x)=\frac{x^2-4-(x+2)2x}{(x^2-4)^2}$

$f'(x)=\frac{x^2-4-2x^2-4x}{(x^2-4)^2}$

$f'(x)=\frac{-x^2-4x-4}{(x^2-4)^2}$

V.A. $y=kx+l$

$k=\lim_{x\to\infty}\frac{\frac{x+2}{x^2-4}}{x}=\lim_{x\to\infty}\frac{x+2\mid :x^3}{x^3-4x\mid :x^3}=\lim_{x\to\infty}\frac{\frac{x}{x^3}+\frac{2}{x^3}}{\frac{x^3}{x^3}-\frac{4x}{x^3}}=\frac{0}{1}=0$

$l=\lim_{x\to\infty}\left[\frac{x+2}{x^2-4}-0\cdot x\right]$

$=\lim_{x\to\infty}\left[\frac{x+2}{x^2-4}-0\right]=\lim_{x\to\infty}\frac{x+2\mid :x^2}{x^2-4\mid :x^2}=$

$=\lim_{x\to\infty}\frac{\frac{x}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}-\frac{4}{x^2}}=\frac{0}{1}=0$

$y=0\cdot x+l$

$y=0$

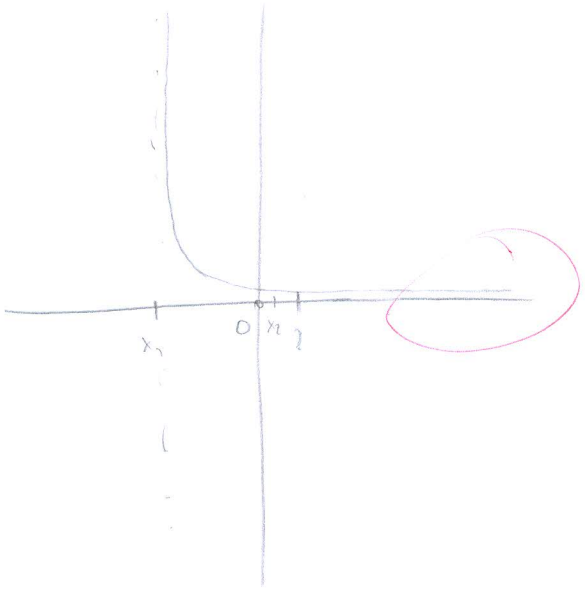
$-x^2-4x-4$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{4\pm\sqrt{16+16}}{-2}=\frac{4\pm\sqrt{32}}{-2}$

$x_1=-4{,}8$ $\quad x_2=0{,}82$

4.

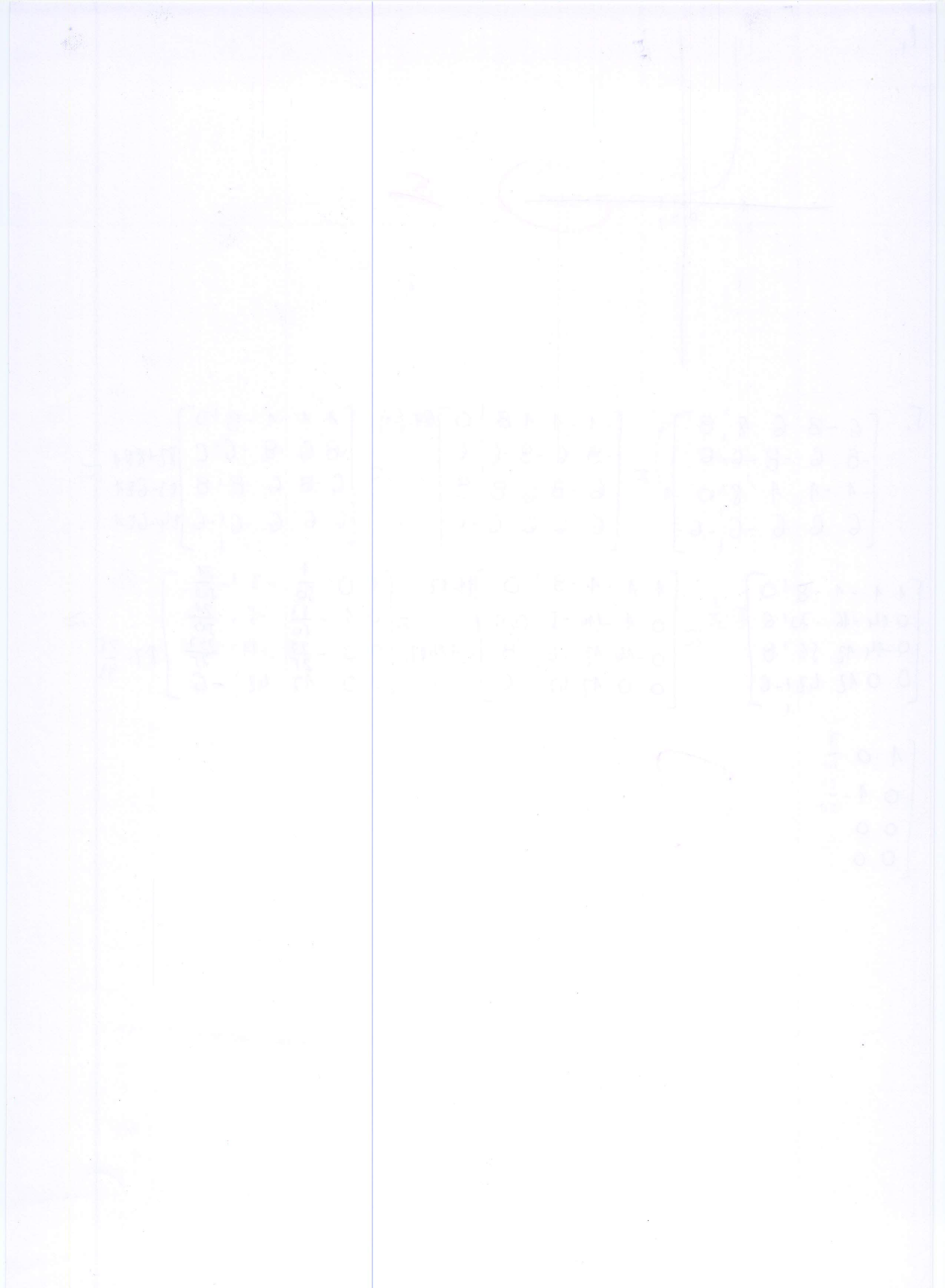


5

5. $\begin{bmatrix} 6 & -8 & 6 & 8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{bmatrix} \approx \begin{bmatrix} -1 & -1 & 1 & 8 & 0 \\ -8 & 6 & -8 & -6 & 6 \\ 6 & -8 & 6 & 8 & 8 \\ 6 & 6 & 6 & -6 & -6 \end{bmatrix} R1 \cdot (-1) \approx \begin{bmatrix} 1 & 1 & -1 & -8 & 0 \\ -8 & 6 & -8 & -6 & 6 \\ 6 & -8 & 6 & 8 & 8 \\ 6 & 6 & 6 & -6 & -6 \end{bmatrix} \begin{matrix} R2+8R1 \\ R3-6R1 \\ R4-6R1 \end{matrix} \approx$

$\begin{bmatrix} 1 & 1 & -1 & -8 & 0 \\ 0 & 14 & -16 & -70 & 6 \\ 0 & -14 & 12 & 56 & 8 \\ 0 & 0 & 12 & 42 & -6 \end{bmatrix} R2 \cdot \frac{1}{14} \approx \begin{bmatrix} 1 & 1 & -1 & -8 & 0 \\ 0 & 1 & -1,14 & -5 & 0,42 \\ 0 & -14 & 12 & 56 & 8 \\ 0 & 0 & 12 & 42 & -6 \end{bmatrix} \begin{matrix} R1-R2 \\ \\ R3+14R2 \\ \\ \end{matrix} \approx \begin{bmatrix} 1 & 0 & \frac{7}{50} & -3 & -\frac{21}{50} \\ 0 & 1 & -\frac{57}{50} & -5 & \frac{21}{50} \\ 0 & 0 & -\frac{99}{25} & -14 & \frac{147}{25} \\ 0 & 0 & 12 & 42 & -6 \end{bmatrix} R3 \cdot \frac{25}{99} \approx$

$\begin{bmatrix} 1 & 0 & \frac{7}{50} \\ 0 & 1 & -\frac{57}{50} \\ 0 & 0 & \\ 0 & 0 & \end{bmatrix}$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ G3

IME I PREZIME: TOMISLAV BOLONJA **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{z+2i} = 3i$. Na kraju provjeriti rješenja. 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 6A - 8B + 6C + 8D &= 8 \\ -8A + 6B - 8C - 6D &= 6 \\ -A - B + C + 8D &= 0 \\ 6A + 6B + 6C - 6D &= -6 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 + 4x + 2} - x$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{x+2}{x^2-4}$. 15(graf)
5. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \arctan(x^2)$. 2+4+6+8
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Ukupno: 0

1) $\frac{|z|}{2+2i} = 3i$ ✗

2)

$$\left[\begin{array}{cccc|c} 6 & -8 & 6 & 8 & 8 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{\text{I}-\text{III}}{\sim} \left[\begin{array}{cccc|c} 5 & -7 & 5 & 0 & 8 \\ -9 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{\text{IV}-\text{I}}{\sim} \left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ -8 & 6 & -8 & -6 & 6 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{\text{II}+\text{IV}}{\sim} \left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ -2 & 12 & -2 & -12 & 0 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{\text{II}-\text{III}}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ -3 & -13 & -3 & -6 & 14 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{\text{II}+\text{I}}{\sim} \left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ 2 & 0 & 2 & -12 & 0 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{\text{II}-\text{III}}{\sim} \left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ 3 & 1 & 1 & -20 & 0 \\ -1 & -1 & 1 & 8 & 0 \\ 6 & 6 & 6 & -6 & -6 \end{array}\right] \overset{1:(-6)}{\sim} \left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ 3 & 1 & 1 & -20 & 0 \\ -1 & -1 & 1 & 8 & 0 \\ -1 & -1 & -1 & 1 & 1 \end{array}\right] \overset{\text{III}+\text{II}}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ 3 & 1 & 1 & -20 & 0 \\ -2 & 0 & 2 & -12 & 0 \\ -1 & -1 & -1 & 1 & 1 \end{array}\right] \overset{\text{III}+\text{IV}}{\sim} \left[\begin{array}{cccc|c} 1 & 13 & 1 & -6 & -14 \\ 3 & 1 & 1 & -20 & 0 \\ -3 & -1 & 1 & -11 & 1 \\ -1 & -1 & -1 & 1 & 1 \end{array}\right]$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ G3

IME I PREZIME: Alexandru Colesca **BROJ INDEKSA:** 14-1-0088-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0