

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

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BROJ INDEKSA:

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

ustmeni (graf) Kocor

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

7/2

- Riješiti jednačbu:  $z^3 - (1-i)^5 = 0$ . Prikaži rješenja u kompleksnoj ravni!
- Odrediti domenu i sve asimptote funkcije  $f(x) = x - \sqrt{x^2 - 4}$ .
- Ispitati domenu, (ne)parnost i zaktivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ .
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira.
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje:

$$\begin{aligned} x + 2y - z + u &= 8 \\ 2x + 5y - z + 2u &= 8 \\ 3x - y - 2z + u &= 8 \\ x - y + 3z - 5u &= 8 \end{aligned}$$

6. Izračunati:  $\lim_{x \rightarrow 1} e^{\frac{1}{x^2-1}}$

12+3

5+15

5+5+10

20 (graf)

15

10

Ukupno:

48

①  $z^3 - (1-i)^5 = 0$

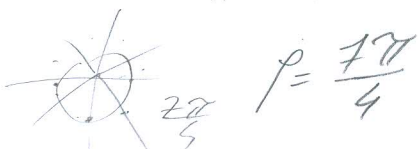
$z^3 = (1-i)^5$

$z^3 = -4 + i4$

$\sqrt[3]{w} = \sqrt[3]{r} \left( \cos \frac{\varphi + k2\pi}{n} + i \sin \frac{\varphi + k2\pi}{n} \right)$

$1-i$   
 $\arctan \varphi = \frac{y}{x} = \frac{-1}{1} = -1$

$r = \sqrt{16+16} = 4\sqrt{2}$   
 $\varphi = \arctan \frac{4}{-4} = \frac{4}{-4} + \pi = -1$   
 $\varphi = \frac{7\pi}{4}$   
 $\sqrt[3]{w} = \sqrt[3]{4\sqrt{2}} \left( \cos \frac{2.356 + k2\pi}{3} + i \sin \frac{2.356 + k2\pi}{3} \right)$



$\arctan \varphi = \frac{4}{-4} + \pi = -1$

$z_1 = \sqrt[3]{4\sqrt{2}} \left( \cos \frac{2.356}{3} + i \sin \frac{2.356}{3} \right)$

$w = \sqrt{x^2 + y^2}$   
 $w = \sqrt{2}$

$(x < 0) \rightarrow \varphi = \frac{7\pi}{4}$   
 $z_1 = 1.25 + i 1.25$

$k=1 \dots z_2 = \sqrt[3]{4\sqrt{2}} \left( \cos \frac{2.356 + 2\pi}{3} + i \sin \frac{2.356 + 2\pi}{3} \right)$

$w = (\sqrt{2}) \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$

$w = (\sqrt{2})^5 \left( \cos 5 \frac{7\pi}{4} + i \sin 5 \frac{7\pi}{4} \right)$

$w = (\sqrt{2})^5 \left( \cos \frac{35\pi}{4} + i \sin \frac{35\pi}{4} \right)$

$w = (\sqrt{2})^5 (-0.7071 + i 0.7071)$

$w = -3.9999 + i 3.9999$

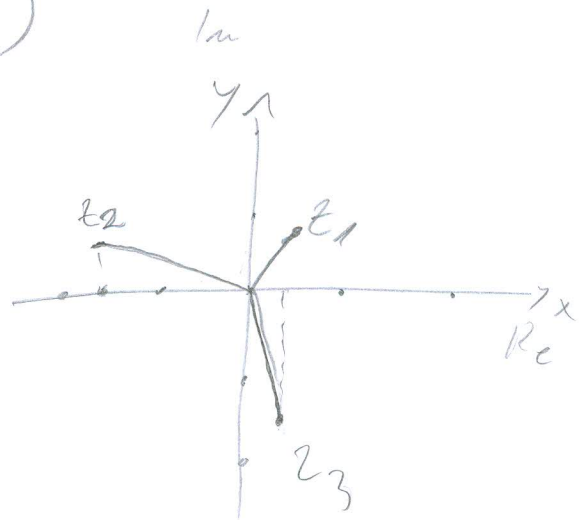
$z_2 = \sqrt[3]{4\sqrt{2}} (-0.9664 + i 0.258)$

$z_2 = -1.7217 + i 0.459$

$$k=2 \quad z_3 = \sqrt[3]{4\sqrt{2}} \left( \cos \frac{2.356 + 4\pi}{3} + i \sin \frac{2.356 + 4\pi}{3} \right)$$

$$z_3 = \sqrt[3]{4\sqrt{2}} (0.2587 + i - 0.966)$$

$$z_3 = 0.461 + i - 1.7212$$



(2)  $f(x) = x - \sqrt{x^2 - 4}$

$$x^2 - 4 \geq 0$$

$$x^2 \geq 4$$

$$x \geq 2$$

$$x \geq -2$$

$$D_f = \langle -\infty, -2 \rangle \cup \langle 2, +\infty \rangle \checkmark$$



V.A

$$\lim_{x \rightarrow -2^-} x - \sqrt{x^2 - 4} = -2 - \sqrt{0^+} = -2 - 0 = -2$$

vermoet V.A  $\checkmark$

H.A

$$\lim_{x \rightarrow +\infty} x - \sqrt{x^2 - 4} = \frac{x - \sqrt{x^2 - 4}}{1} \cdot \frac{x + \sqrt{x^2 - 4}}{x + \sqrt{x^2 - 4}}$$

$$= \frac{x^2 - x^2 + 4}{x + \sqrt{x^2 - 4}} = \frac{4}{x + \sqrt{x^2 - 4}} \cdot \frac{1}{x}$$

$$= \frac{4}{x} = \frac{0}{\infty} = 0 \checkmark$$

$$\lim_{x \rightarrow -\infty} (-x) - \sqrt{(-x)^2 - 4} = \frac{(-x) - \sqrt{(-x)^2 - 4}}{(-x) + \sqrt{(-x)^2 - 4}} \quad \times$$

$$\lim_{x \rightarrow -\infty} = \frac{x^2 - x^2 + 4}{-x + \sqrt{x^2 - 4}} \cdot \frac{1}{x} = \frac{4}{-x + \sqrt{x^2 - 4}} \cdot \frac{1}{x} = \frac{4}{-1 + \sqrt{1 - \frac{4}{x^2}}} \cdot \frac{1}{x} = \frac{0}{0}$$

(3)  $g(x) = \ln(4-x^2)$

100% PASTUOVU 2. list

$4-x^2$   
 $4-x$

$4-x^2 > 0$

$4 > x^2$

$x^2 < 4$

$x < 2$

$x < -2$

$D_f = \langle -2, 2 \rangle$  ✓ 5

$g(-x) = g(x)$

$g(-x) = \ln(4-(-x)^2)$

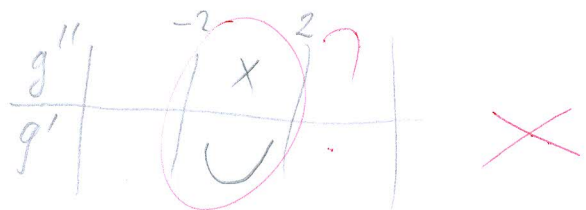
$g(x) = \ln(4-x^2) = f(x)$

funkcija  
parna ✓ 5

$g'' = \frac{2x}{4-x^2}$

$= \frac{(2x) \cdot (4-x^2) - (2x) \cdot (-2x)}{(4-x^2)^2}$

$= \frac{2 \cdot (4-x^2) - (2x) \cdot (-2x)}{(4-x^2)^2} = \frac{8-2x^2-4x^2}{(4-x^2)^2} = \frac{8-2x^2-4x^2}{(4-x^2)^2}$



funkcija je konveksna  
za  $x \in \langle -2, 2 \rangle$

$g' = \ln(4-x^2)'$

$= \frac{1}{4-x^2} \cdot (4-x^2)'$

$= \frac{1}{4-x^2} \cdot (-2x)$

$= \frac{2x}{4-x^2}$  ✗

$g(x)$	-	+	-	+
$g'(x)$	-	↑	-	

funkcija raste za  $x \in \langle -2, 2 \rangle$

$8-2-4 = +$   
 $8-2x^2-4x^2$   
 $4^2-2 \cdot 4 \cdot x^2+x^4$  ✗  
 $4-8$

