MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: TOMI' PASTUOVIC **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

usmeni Grof Kocor

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^3 - (1-i)^5 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-4}$. 5+15 8
3. Ispitati domenu, (ne)parnost i zaktivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-3}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf) 10
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 8 \\ 2x + 5y - z + 2u &= 8 \\ 3x - y - 2z + u &= 8 \\ x - y + 3z - 5u &= 8 \end{aligned}$$

6. Izračunati: $\lim\limits_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno: 48

① $z^3 - (1-i)^5 = 0$

$z^3 = (1-i)^5$

$z^3 = -4 + i4$

$\sqrt[3]{w} = \sqrt[3]{r}\left(\cos\frac{\varphi + k2\pi}{n} + i\sin\frac{\varphi + k2\pi}{n}\right)$

$1 - i$

$r = \sqrt{16+16}$

$r = \sqrt{32} = \sqrt{16\cdot 2} = 4\sqrt{2}$

$\operatorname{arctg}\varphi = \frac{y}{x} = \frac{-1}{1} = -1$

$\varphi = \frac{7\pi}{4}$

$\operatorname{arctg}\varphi = \frac{\varphi}{-4} + \pi = -1$

$= 2.356$

$(x<0)$

$w = \sqrt{x^2 + y^2}$

$w = \sqrt{2}$

$k = 0 \ldots z_1 = \sqrt[3]{4\sqrt{2}}\left(\cos\frac{2.356}{3} + i\sin\frac{2.356}{3}\right)$

$z_1 = \sqrt[3]{4\sqrt{2}}(0.7071 + i\,0.7071)$

$z_1 = 1.25 + i\,1.25$

$k = 1 \ldots z_2 = \sqrt[3]{4\sqrt{2}}\left(\cos\frac{2.356 + 2\pi}{3} + i\sin\frac{2.356 + 2\pi}{3}\right)$

$z_2 = \sqrt[3]{4\sqrt{2}}(-0.966 + i\,0.258)$

$z_2 = -1.2122 + 0.459$

$w = (\sqrt{2})\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$w = (\sqrt{2})^5\left(\cos 5\frac{7\pi}{4} + i\sin 5\frac{7\pi}{4}\right)$

$w = (\sqrt{2})^5\left(\cos\frac{35\pi}{4} + i\sin\frac{35\pi}{4}\right)$

$w = (\sqrt{2})^5(-0.7071 + i\,0.7071)$

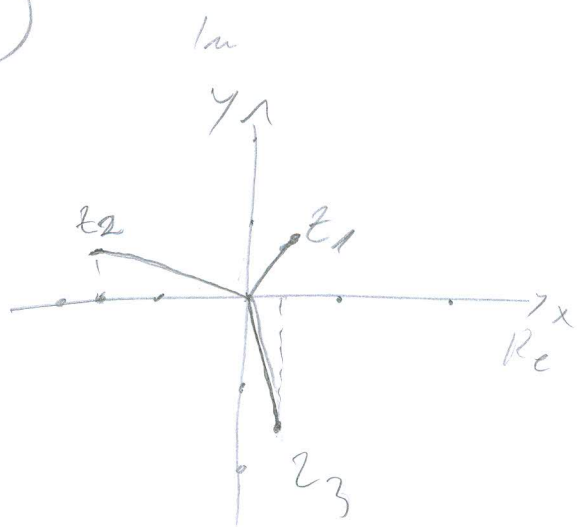
$w = -3.9999 + i\,3.999$

$k=2$

$$z_3 = \sqrt[3]{4\sqrt{2}}\left(\cos\frac{2.356+4\pi}{3} + i\sin\frac{2.356+4\pi}{3}\right)$$

$$z_3 = \sqrt[3]{4\sqrt{2}}\,(0.2587 + i\,(-0.966))$$

$$z_3 = 0.461 + i\,(-1.722)$$



② $f(x) = x - \sqrt{x^2-4}$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x \geq 2$

$x \geq -2$

$D_f = \langle -\infty, -2\rangle \cup \langle 2, +\infty\rangle$ ✓

V. A

$$\lim_{x\to -2^-} x - \sqrt{x^2-4} = -2 - \sqrt{0^+}$$

$$= -2 - 0 = -2$$

nemoi V. A ✓

H. A

$$\lim_{x\to+\infty} = x - \sqrt{x^2-4} = \frac{x-\sqrt{x^2-4}}{1}\cdot\frac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}}$$

$$= \frac{x^2 - x^2 - 4}{x+\sqrt{x^2-4}} = \frac{-4\ /:x}{x+\sqrt{x^2-4}\ /:x}$$

$$= \frac{-\frac{4}{x}}{1+\sqrt{1-\frac{4}{x^2}}} = \frac{0}{2} = 0$$ ✓

$$\lim_{x\to-\infty} = \frac{-x - \sqrt{(x)^2-4}\cdot(-x)+\sqrt{x^2-4}}{(-x)+\sqrt{x^2-4}}$$ ×

$$\lim_{x\to-\infty} = \frac{x^2 - x^2 - 4\ /:x}{-x+\sqrt{x^2-4}\ /:x} = \frac{-\frac{4}{x}}{-1+\sqrt{1-\frac{4}{x^2}}}$$

$$= \frac{0}{0} =$$

(3) $g(x) = \ln(4 - x^2)$

$4 - x^2 > 0$

$4 > x^2$

$x^2 < 4$

$x < 2$

$x < -2$

$Df: \langle -2, 2 \rangle$ ✓ 5

$g'(-x) = g'(x)$

$g(-x) = \ln(4 - (-x)^2)$

$g(-x) = \ln(4 - x^2) = g(x)$

funkcija parna ✓ 5

$g' = \ln(4 - x^2)'$

$= \frac{1}{4 - x^2} \cdot (4 - x^2)'$

$= \frac{1}{4 - x^2} \cdot (-2x)$

$= \frac{2x}{4 - x^2}$ ✗

	$-\infty$	-2		2	$+\infty$
$g'(x)$		−	+		−
$g(x)$		−	↗		−

funkcija rastuća za $x \in \langle -2, 2 \rangle$

$g'' = \left(\frac{2x}{4 - x^2}\right)'$

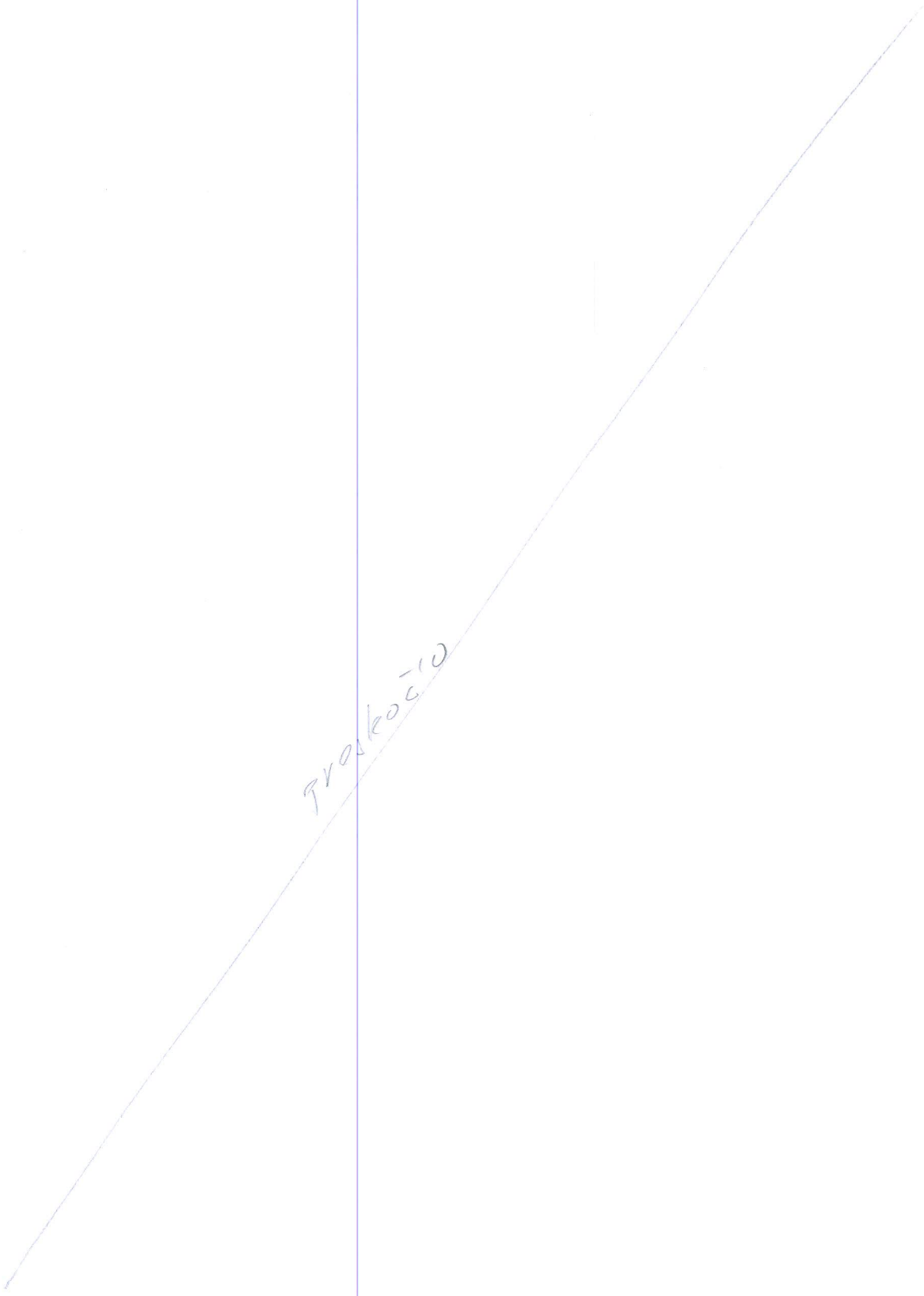
$= \frac{(2x)' \cdot (4 - x^2) - (2x) \cdot (4 - x^2)'}{(4 - x^2)^2}$

$= \frac{2 \cdot (4 - x^2) - (2x) \cdot (2x)}{(4 - x^2)^2} = \frac{8 - 2x^2 - 4x^2}{(4 - x^2)^2} = \frac{8 - 2x^2 - 4x^2}{4^2 - 2 \cdot 4 \cdot x^2 + x^4}$ ✗

	$-\infty$	-2		2	$+\infty$
g''			x		
g'		⌣		⌢	

✗

funkcija je konveksna za $x \in \langle -2, 2 \rangle$


Proiekcio

Tomislav Pasičević
3. list

5)
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \begin{array}{l} \\ -2 \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & -1 & 2 & -8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \begin{array}{l} \\ \\ -3 \cdot I \\ -I \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & -1 & 2 & -8 \\ 0 & -7 & 1 & -2 & -16 \\ 0 & -3 & 4 & 2 & 0 \end{array}\right] \begin{array}{l} \\ \\ +II \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 7 & -7 & 12 & -8 \\ 0 & 0 & -6 & 10 & -24 \\ 0 & -3 & 4 & 2 & 0 \end{array}\right] \begin{array}{l} \\ :7 \\ \\ \cdot III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & -1 & \frac{12}{7} & -\frac{8}{7} \\ 0 & 0 & -6 & 10 & -24 \\ 0 & 0 & -24 & 20 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ -24 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & -1 & \frac{12}{7} & -\frac{8}{7} \\ 0 & 0 & -6 & 10 & -24 \\ 0 & 0 & -24 & 20 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ :2 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & -1 & \frac{12}{7} & -\frac{8}{7} \\ 0 & 0 & -6 & 10 & -24 \\ 0 & 0 & -12 & 10 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ -IV \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & -1 & \frac{12}{7} & -\frac{8}{7} \\ 0 & 0 & 6 & 0 & -24 \\ 0 & 0 & -12 & 10 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & -1 & \frac{12}{7} & -\frac{8}{7} \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & -12 & 10 & 0 \end{array}\right] \begin{array}{l} \\ \\ +III \\ +10 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & 0 & \frac{12}{7} & -\frac{36}{7} \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & -\frac{6}{5} & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot 11 \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & 0 & \frac{12}{7} & -\frac{36}{7} \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & \frac{12}{7} & 0 \end{array}\right]$$

$$\frac{-8}{2} - \frac{4}{11} \cdot \frac{7}{2} = \frac{-8}{7} - \frac{88}{11} \quad \cdots = \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{17}{7} & \frac{136}{56} \\ 0 & 1 & 0 & \frac{12}{7} & -\frac{36}{7} \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot III \end{array} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & \frac{68}{7} \\ 0 & 0 & 0 & 0 & \frac{144}{7} \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\frac{12}{10} = \frac{6}{5}$$

Matrica nema rješenja

Jednadžba nema realna rješenja. ✗

⑥ $\lim_{x\to 1} e^{\frac{1}{x^2-1}} \overset{\frac{1}{0}=\infty}{=} \lim_{x\to 1} e^{\frac{1}{0}} = \lim_{x\to 1} e^{\infty} = 1$

$x \neq 1$

nema rješenja ✗

$x \neq \emptyset$

⑤

$h(x) = \frac{x^2-2x-3}{x^2+1}$

$h'(x) = \left(\frac{x^2-2x-3}{x^2+1}\right)' = \frac{(x^2-2x-3)'\cdot(x^2+1) - (x^2-2x-3)(x^2+1)'}{(x^2+1)^2}$

$D: \mathbb{R} \setminus \{-1, 1\}$ ✗

$= \frac{(2x-2)\cdot(x^2+1) - (x^2-2x-3)\cdot(2x)}{x^4+2x^2+1}$

NT: $x^2-2x-3=0$

$= \frac{(2x^3-2x-2x^2-2) - (2x^3-4x^2-6x)}{x^4+2x^2+1}$

$y_{1,2} = \frac{2 \pm \sqrt{4-4\cdot(-3)}}{2}$

$= \frac{2x^3-2x^2-2-4x^2-6x}{x^4+2x^2+1}$

$y_{1,2} = \frac{2 \pm \sqrt{16}}{2} =$

$= \frac{-6x^2-8x-2}{x^4+2x^2+1}$

$y_1 = \frac{2+4}{2} = \frac{6}{2} = 3$ ✓

TOVO NIJE UOPĆE KRITIČNA TOČKA

$y_2 = \frac{2-4}{2} = \frac{-2}{2} = -1$ ✓

	$-\infty$		-1		1		$+\infty$
$h'(x)$		−	0	+	0	−	
$h(x)$	↘		Min	↗	MAX	↘	

$h(x)$ pada, $x \in \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle$

$h(x)$ raste $x \in \langle -1, 1 \rangle$

$NT_1(3, 0)$

$NT_2(-1, 0)$

GDJE SE TO OGLEDA NA GRAFU?

$-6x^2-8x-2=0$

$-24 \quad -16 \quad -2$

$-24 \pm 16 \quad -2$

$y_{1,2} = \frac{8 \pm \sqrt{64-4\cdot 6\cdot(-2)}}{-36}$

V.A

$\lim_{x\to -1} \frac{x^2-2x-3}{x^2+1} = \frac{1+2-3}{2} = \frac{0}{2} = 0$ nema V.A

$y_{1,2} = \frac{8 \pm \sqrt{}}{-36}$

H.A

$y_1 = \frac{8+4}{-36} = \frac{12}{36} = \frac{2}{6} = \frac{1}{3}$

$\lim_{x\to+\infty} \frac{x^2-2x-3}{x^2+1} = \frac{1-\frac{2}{x}-\frac{3}{x^2}}{1+\frac{1}{x^2}} = \frac{1}{1} = 1$ ✓

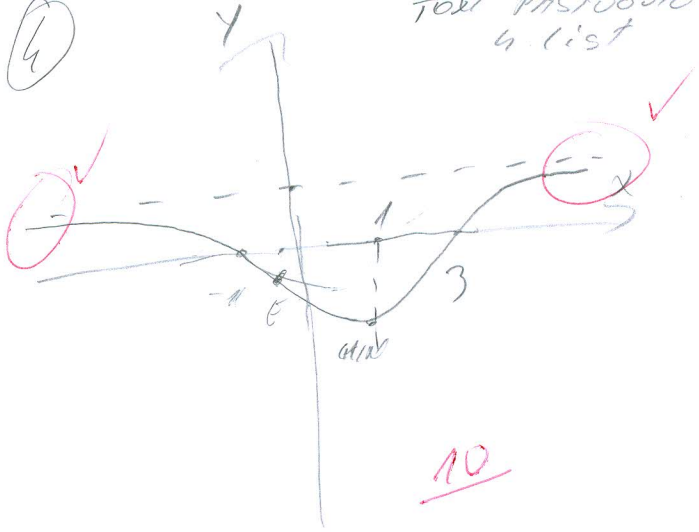
$x_2 = \frac{8-4}{-36} = \frac{4}{-36} = -\frac{1}{9}$

$E\left(-\frac{1}{3}, -2\right)$

$\lim_{x\to-\infty} \frac{x^2+2x-3}{x^2+1} = \frac{1+\frac{2}{x}-\frac{3}{x^2}}{1+\frac{1}{x^2}} = \frac{1}{1} = 1$

$E\left(-\frac{1}{9}, \frac{-112}{81}\right)$

4



10

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IME I PREZIME: TENA KRUPOTIĆ **BROJ INDEKSA:** 5970

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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Ukupno: 37

(1) $f(x) = x - \sqrt{x^2-4}$

DOMENA

a) $x^2-4 \geq 0$

$x^2 \geq 4 \quad /\sqrt{\ }$

$x \geq 2$

a) DOMENA ✗

b) ASIMPTOTE

V.A.

b) $\lim\limits_{x\to 2^-} x - \sqrt{x^2-4} = 2 - \sqrt{2^2-4} = 2-0 = 2$

$\lim\limits_{x\to 2^+} x - \sqrt{x^2-4} = 2 - \sqrt{2^2-4} = 2-0 = 2$

NEMA VERTIKALNIH ASIMPTOTA

POGLEDAJTE: ~~GDJE VAS~~ KAKO RIJEŠAVATI NEJEDNADŽBE?

H.A.

$\lim\limits_{x\to\infty} x - \sqrt{x^2-4} = x - \sqrt{x^2-4} \cdot \dfrac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}} = \dfrac{x^2-(x^2-4)}{x+\sqrt{x^2-4}}$

$= \dfrac{x^2-x^2+4}{x+\sqrt{x^2-4}} = \dfrac{4\,/:x^2}{x+\sqrt{x^2-4}\,/:x^2} = \dfrac{0}{1} = 0$

$\lim\limits_{x\to-\infty} x - \sqrt{x^2-4} = 0$

$y = 0$

ZAŠTO ISPITUJETE KOSE ASIMPTOTE, AKO STE NAŠLI HORIZONTALNE?

MOŽE LI FUNKCIJA IMATI HORIZONTALNE I KOSE ASIMPTOTE ZAJEDNO?

KOSA

$k = \lim\limits_{x\to\pm\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x-\sqrt{x^2-4}\,/:x}{x\quad /:x} = \dfrac{1-1}{1} = \dfrac{0}{1} = 0$

$l = \lim\limits_{x\to\pm\infty} f(x) - kx = x - \sqrt{x^2-4} - 0 = x - \sqrt{x^2-4} \cdot \dfrac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}}$

$= \dfrac{x^2-(x^2-4)}{x+\sqrt{x^2-4}} = \dfrac{4}{x+\sqrt{x^2-4}}$

$y = kx + l$

$y = 0 \cdot x + 0$

$y = 0$

NEMA KOSIH ASIMPTOTA $= \dfrac{4\,/:x^2}{x+\sqrt{x^2-4}\,/:x} = \dfrac{0}{1} = 0$

② $g(x) = \ln(4-x^2)$

a) $4-x^2 > 0$

$-x^2 > 4 \mid \cdot(-)$

$x^2 > -4 \mid\mid \sqrt{}$

$x > \sqrt{-4} \Rightarrow$ kod korijena nema minusa

$D(f): \mathbb{R} \Rightarrow$ domena funkcije su realni brojevi

✗

KAKO SE RJEŠAVAJU NEJEDNADŽBE ?

b) $f(x) = f(-x)$

$\ln(4-(-x^2))$

$\ln(4-x^2) \Rightarrow$ funkcija parna ✓

c) $f'(x) = \ln(4-x^2) = \frac{1}{4-x^2} \cdot (4-x^2)'$

$= \frac{1}{4-x^2} \cdot (-2x) = \frac{2x}{4-x^2} \mid\mid$ ✗

$f''(x) = \frac{2}{4-x^2} = \frac{(2)'\cdot(4-x^2) - (2)\cdot(4-x^2)'}{(4-x^2)^2} =$

$= \frac{-2\cdot(-2x)}{(4-x^2)^2} = \frac{4x}{(4-x^2)^2} \mid\mid$ ✗

$f(x) = 0 \quad 4x = 0 \quad x = 4$

$f(0) = \frac{4\cdot 0}{(4-0^2)^2} = \frac{0}{16} = 0$

$T(0,0)$

e) $\lim_{x\to 1} e^{\frac{1}{x^2-1}} = \lim_{x\to 1} e^{\frac{1}{1^2-1}} = \lim_{x\to 1} e^{\frac{1}{1-1}} (4,0)$

$= \lim_{x\to 1} = e^{\frac{1}{0}} = e^{0} = 1 \mid\mid$ ✗

$\frac{1}{0} \neq 0$

⑤ $x+2y-z+4=8$

$2x+5y-z+24=8$

$3x-y-2z+4=8$

$x-y+3z-5=8$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \; | \cdot(-2)+\text{II}R_1, \cdot(-3)+\text{III}R_1, \cdot(-1)+\text{IV}R_1 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & -7 & 1 & -2 & -16 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \; | \cdot 7+\text{III}R_2, \cdot 3+\text{IV}R_2 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 8 & -2 & -72 \\ 0 & 0 & 7 & -6 & -24 \end{array}\right] \; | \cdot 8 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & -\frac{1}{4} & -9 \\ 0 & 0 & 7 & -6 & -24 \end{array}\right] \; | \cdot(-7)+\text{IV}R \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & -\frac{1}{4} & -9 \\ 0 & 0 & 0 & -\frac{17}{4} & 39 \end{array}\right] \; | \cdot \left(-\frac{4}{17}\right) \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & -\frac{1}{4} & -9 \\ 0 & 0 & 0 & 1 & -\frac{156}{17} \end{array}\right] \; | \cdot \frac{1}{4}+\text{III}R_3 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & 0 & -\frac{192}{17} \\ 0 & 0 & 0 & 1 & -\frac{156}{17} \end{array}\right] \; | \cdot(-1) \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & \frac{292}{17} \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & 0 & -\frac{192}{17} \\ 0 & 0 & 0 & 1 & -\frac{156}{17} \end{array}\right] \; | \cdot(-1)+\text{II}R_3, \cdot 1+\text{I}R_3 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & \frac{100}{17} \\ 0 & 1 & 0 & 0 & \frac{136}{17} \\ 0 & 0 & 1 & 0 & -\frac{192}{17} \\ 0 & 0 & 0 & 1 & -\frac{156}{17} \end{array}\right] \; | \cdot(-2)+\text{I}R \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{12}{17} \\ 0 & 1 & 0 & 0 & \frac{156}{17} \\ 0 & 0 & 1 & 0 & -\frac{192}{17} \\ 0 & 0 & 0 & 1 & -\frac{156}{17} \end{array}\right] \begin{matrix} x \\ y \\ z \\ t \end{matrix}$$

⇒ PROVERA

$x + 2y - z + 4 = 8$

$$\frac{-12}{17} + 2\cdot\left(\frac{56}{17}\right) - \left(-\frac{152}{17}\right) + \left(-\frac{104}{17}\right) = 8$$

$8 = 8$ ✓

$2x + 5y - z + 2y = 8$

$$2\cdot\left(\frac{-12}{17}\right) + 5\cdot\left(\frac{56}{17}\right) - \left(-\frac{152}{17}\right) + 2\cdot\left(-\frac{104}{17}\right) = 8$$

$8 = 8$ ✓

$3x - y - 2z + 4 = 8$

$$3\cdot\left(\frac{-12}{17}\right) - \frac{56}{17} - 2\cdot\left(\frac{-152}{17}\right) + \left(-\frac{104}{17}\right)$$

$8 = 8$ ✓

$x - y + 3z - 5y = 8$

$$\frac{-12}{17} - \frac{56}{17} + 3\cdot\left(\frac{-152}{17}\right) - 5\cdot\left(\frac{-56}{17}\right)$$

$8 = 8$ ✓

TENA KRUPTO

④ TOK:

$$\frac{x^2-2x+3}{x^2+1}$$

① DOMENA

$x^2+1 \neq 0$

$x^2 \neq -1 \; / \sqrt{}$

$x \neq \pm 1$

② ASIMPTOTE V.A

$\lim_{x\to 1^-} \frac{x^2-2x+3}{x^2+1} = \frac{1^2-2\cdot 1+3}{1^2+1} = 1$

$\lim_{x\to 1^+} \frac{x^2-2x+3}{x^2+1} = 1$

$\lim_{x\to -1^-} \frac{x^2-2x+3}{x^2+1} = \frac{(-1)^2-2(-1)+3}{(-1)^2+1} = \frac{1+2+3}{1+1} = \frac{6}{2} = 3$ funkcija ni ni parna ni liha, neparna

$\lim_{x\to -1^+} \frac{x^2-2x+3}{x^2+1} = 3$

H.A

$\lim_{x\to\infty} \frac{x^2-2x+3}{x^2+1} \; / :x^2 = \frac{\frac{x^2}{x^2}-\frac{2x}{x^2}+\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \frac{1}{1+0} = \frac{1}{1} = 1$

$\lim_{x\to\infty} \frac{x^2-2x+3}{x^2+1} = 1$ $\quad \boxed{y=1}$ = HORIZONTALNA ASIMPTOTA

K.A

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \left[\frac{\frac{x^2-2x+3}{x^2+1}}{\frac{x}{1}}\right] = \frac{x^2-2x+3}{x(x^2+1)} = \frac{x^2-2x+3 \; / :x^3}{x^3+x \; / :x} = \frac{0}{1} = 0$

$l = \lim_{x\to\infty} f(x) - kx = \lim_{x\to\infty} \frac{x^2-2x+3}{x^2+1} \; / :x^2 = \frac{1}{1} = 1$

$y = kx + l$

$y = 0 + 1 \Rightarrow$ nema kose

⑤ MULTOCKE

$x^2-2x+3=0$

$x_{1,2} = \frac{2 \pm \sqrt{4-12}}{2} = \frac{2 \pm \sqrt{-8}}{2}$

⇓

nema multocke zbog minusa ispod korijena

⑤ DERIVACIJA

$f'(x) = \frac{x^2-2x+3}{x^2+1} = \frac{(x^2-2x+3)'\cdot(x^2+1)-(x^2-2x+3)(x^2+1)'}{(x^2+1)^2}$

$= \frac{(2x-2)(x^2+1)-(x^2-2x+3)\cdot 2x}{(x^2+1)^2}$

$= \frac{2x^3+2x-2x^2-2-(2x^3-4x^2+6x)}{(x^2+1)^2}$

$= \frac{2x^3+2x-2x^2-2-2x^3+4x^2-6x}{(x^2+1)^2}$

$= \frac{2x^2-4x-2}{(x^2+1)^2}$ ✗ =?

$f'(x)=0$

$2x^2-4x=0$

$x(2x-4)$

$x_1=0$ $\quad 2x-4=0$, $2x=4 \; /:2$, $x=2$

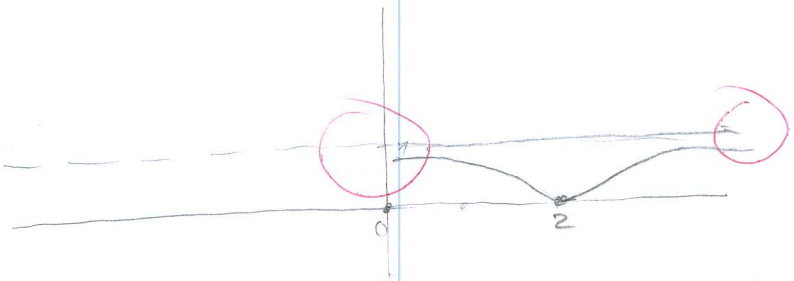
$f(0) = \frac{x^2-2x+3}{x^2+1} = 3$

	$-\infty$	-1	0	1	2	$2{,}5$	3	4	$+\infty$
$f(x)$	$-\infty$	∄	0	$-$	0	$+$	∄	$+$	$+\infty$
	↘			↘		↗		↗	

$$\frac{2x^2-4x}{(x^2+1)^2}$$

LOK. MIN

$f'(2)=0$



10

$$k=\lim_{x\to-\infty}\frac{f(x)}{x}=\frac{\frac{x^2-2x-3}{x^2+1}}{x}=\left|x\to-x\right|=\frac{\frac{x^2+2x-3}{-x^2-1}}{\frac{-x}{1}}=\frac{x^2+2x-3}{-x(x^2+1)}=\frac{x^2+2x-3\ /:x^3}{-x^3-x\ /:x^3}=\frac{0}{-1}=0$$

$$q=\lim_{x\to-\infty}f(x)-kx=\frac{x^2-2x-3}{x^2+1}-0=\left|x\to-x\right|=\frac{x^2+2x-3\ /:x^2}{x^2+1\ /:x^2}=\frac{1}{1}=1$$

$y=kx+q$

$y=0x+1$

$y=0+1 \Rightarrow$ nem kör

① $z^3-(1-i)^5=0$ $\quad k=0,1,2$

$z^3-i=0$

$|w|=\sqrt{x^2+y^2}$

$=\sqrt{0^2+1^2}$

$=\sqrt{1}$ ✗

$|w=1|$

$\operatorname{tg}\varphi=\frac{y}{x}$

$\operatorname{tg}\varphi=\frac{1}{0}$?

$\varphi=0$

$k=0$ $\quad w=\sqrt[3]{1}\left(\cos\frac{0+0\cdot2\pi}{3}+\sin i\,\frac{0+0\cdot2\pi}{3}\right)$

$w=1\left(\cos\frac{0+0}{3}+\sin i\,\frac{0+0}{3}\right)$

$w=1\left(\cos\frac{0}{3}+\sin i\,\frac{0}{3}\right)$

$w=(1+0i)$

$k=1$ $\quad w=1\left(\cos\frac{0+1\cdot2\pi}{3}+\sin i\,\frac{0+1\cdot2\pi}{3}\right)$

$w=1\left(\cos\frac{0+2\pi}{3}+\sin i\,\frac{0+2\pi}{3}\right)$

$w=0{,}999+0{,}036i$

$k=2$ $\quad w=1\left(\cos\frac{0+4\pi}{3}+\sin i\,\frac{0+4\pi}{3}\right)$

$w=0{,}997+0{,}073i$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{H}2$

IME I PREZIME: FILIP GOSPIĆ **BROJ INDEKSA:** 58010-2009

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^3 - (1-i)^5 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-4}$. 5+15
3. Ispitati domenu, (ne)parnost i zaktivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-3}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 8 \\ 2x + 5y - z + 2u &= 8 \\ 3x - y - 2z + u &= 8 \\ x - y + 3z - 5u &= 8 \end{aligned}$$

6. Izračunati: $\lim\limits_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno: 20

1. $z^3-(1-i)^5=0$

$z^3=(1-i)^5$

$z^3=(1-i)^2\cdot(1-i)^3$

$z^3=(1-2i+i^2)(1+3i+3i^2+i^3)$

$z^3=(1-2i-1)(1+3i-3-i)$

$z^3=-2i(-2-2i)$

$z^3=4i+4i^2$

$z^3=-4+4i$

$z=\sqrt[3]{-4+4i}$

$|z|=\sqrt{x^2+y^2}$

$=\sqrt{(-4)^2+4^2}$

$=\sqrt{16+16}$

$=\sqrt{32}$

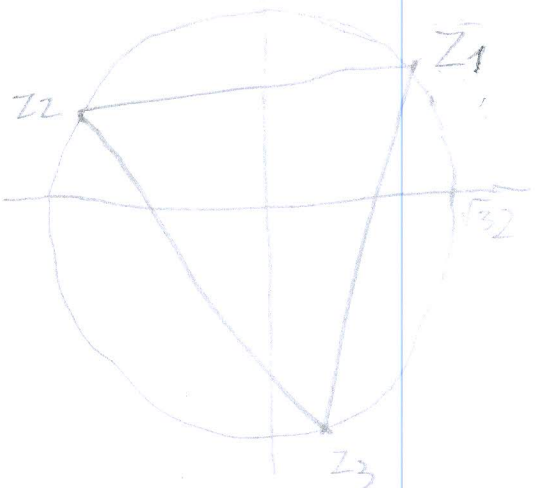
$\varphi=\frac{\pi}{4}=\frac{3\pi}{4}$

$\sqrt[3]{z}=\sqrt{32}\left[\cos\frac{\frac{3\pi}{4}+2k\pi}{3}+i\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right]$

$k=0,\ \sqrt[3]{z_1}=\sqrt{32}\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]$

$k=1,\ \sqrt[3]{z_2}=\sqrt{32}\left[\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right]$

$k=2,\ \sqrt[3]{z_3}=\sqrt{32}\left[\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right]$



2. $f(x) = x - \sqrt{x^2-4}$

1° DOMENA

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x_1 \geq -2$

$x_2 \geq 2$

$x \in [2, +\infty\rangle$ ✗

2° $y = 0$

V. A.

$\lim_{x \to 0} x - \sqrt{x^2-4} = \sqrt{-4}$ ✗ nema

$\lim_{x \to 0^+} x - \sqrt{x^2-4} = \sqrt{-4}$ ✗ nema

H. A

$\lim_{x \to -\infty} x - \sqrt{x^2-4} \left\{ \begin{matrix} x = -x \\ x \to \infty \end{matrix} \right\} = \lim_{x \to \infty} -x - \sqrt{(-x)^2-4} \cdot \frac{-x + \sqrt{(-x)^2-4}}{-x + \sqrt{(-x)^2-4}} = \frac{-x^2 - (-x)^2 + 4}{-x + \sqrt{(-x)^2-4}} = \frac{0}{\infty}$

$\lim_{x \to -\infty} \frac{\frac{4}{x}}{\frac{-x}{x} + \sqrt{\frac{x^2}{x^2} - \frac{4}{x^2}}} = \frac{0}{2} = 0$ ✗ nema

K. A.

$k = \frac{f(x)}{x} = \frac{x - \sqrt{x^2-4}}{x} \Big/ x = \frac{1 - \sqrt{\frac{x^2}{x^2} - \frac{4}{x^2}}}{1} = 1 - 1 = 0$

3. $g(x) = \ln(4 - x^2)$

$4 - x^2 > 0$

$-x^2 > -4$

$x^2 < 4$

$x_1 < 2$

$x_2 < -2$

$x \in \langle -\infty, -2 \rangle \cup \langle 2, 4 \rangle$ ✗

$g(-x) = \ln(4 + x^2)$ parna ✓

$g'(x) = \frac{1}{4 - x^2} \cdot (-2x)$

$g'(x) = \frac{-2x}{4 - x^2}$

$g''(x) = \frac{(-2x)' \cdot (4 - x^2) - (-2x) \cdot (4 - x^2)'}{(4 - x^2)^2}$

$g''(x) = \frac{-2(4 + x^2) + 2x \cdot (-2x)}{(4 - x^2)^2}$

$g''(x) = \frac{-8 - 2x^2 - 4x^2}{(4 - x^2)^2}$ ✗

$4 + x^2 - 2x^2 - x = 0$

$2x^2 - x^2 + 4 = 0$

$x^2 = t$

$t_{1,2} = \frac{-1 \pm \sqrt{1 - 32}}{4}$

$\frac{1 + \sqrt{33}}{4}$ $\frac{1 - \sqrt{33}}{4}$

$\cup \cup \left| \cap \right.$

$h(x)=\frac{x^2-2x-3}{x^2+1}$

1°

$x^2+1\neq 0$ nema rešenje $x\in\{R\}$

$x^2=-1$

2° n.v.

$x^2-2x-3=0$

$x_{1,2}=\frac{2\pm\sqrt{4+12}}{2}$

$x_1=\frac{2+4}{2}=3 \quad x_2=\frac{2-4}{2}=-1$

$x_1(3,0) \quad x_2(-1,0) \quad y(0,-3)$

3° H.A.

$\lim_{x\to\infty}\frac{x^2-2x-3}{x^2+1}\left\{\begin{matrix}x=-\infty\\-\infty=\infty\end{matrix}\right\}=\lim_{x\to-\infty}\frac{-x^2+2x-3\,/:x^2}{-x^2+1\,/:x^2}=\frac{-\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}{-\frac{x^2}{x^2}+\frac{1}{x^2}}=\frac{-1}{-1}=1$

$\lim_{x\to\infty}\frac{x^2-2x-3\,/:x^2}{x^2+1\,/:x^2}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}=\frac{1}{1}=1$

K.A.

$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{\frac{x^2-2x-3}{x^2+1}}{x}=\lim_{x\to\infty}\frac{x^2-2x-3}{x(x^2+1)}\overset{\infty}{=}\lim_{x\to\infty}\frac{x^2-2x-3\,/:x^3}{x^3+x\,/:x^3}=\lim_{x\to\infty}\frac{\frac{x^2}{x^3}-\frac{2x}{x^3}-\frac{3}{x^3}}{\frac{x^3}{x^3}+\frac{x}{x^3}}=0$ nema K.A.

4°

$f'(x)=\frac{(x^2-2x-3)'(x^2+1)-(x^2-2x-3)(x^2+1)'}{(x^2+1)^2}=\frac{(2x-2)(x^2+1)-2x(x^2-2x-3)}{(x^2+1)^2}=\frac{2x^3+2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2}$

$f'(x)=\frac{2x^2+8x-2}{(x^2+1)^2}$

$2x^2+8x-2=0\,/:2$

$x^2+4x-1=0$

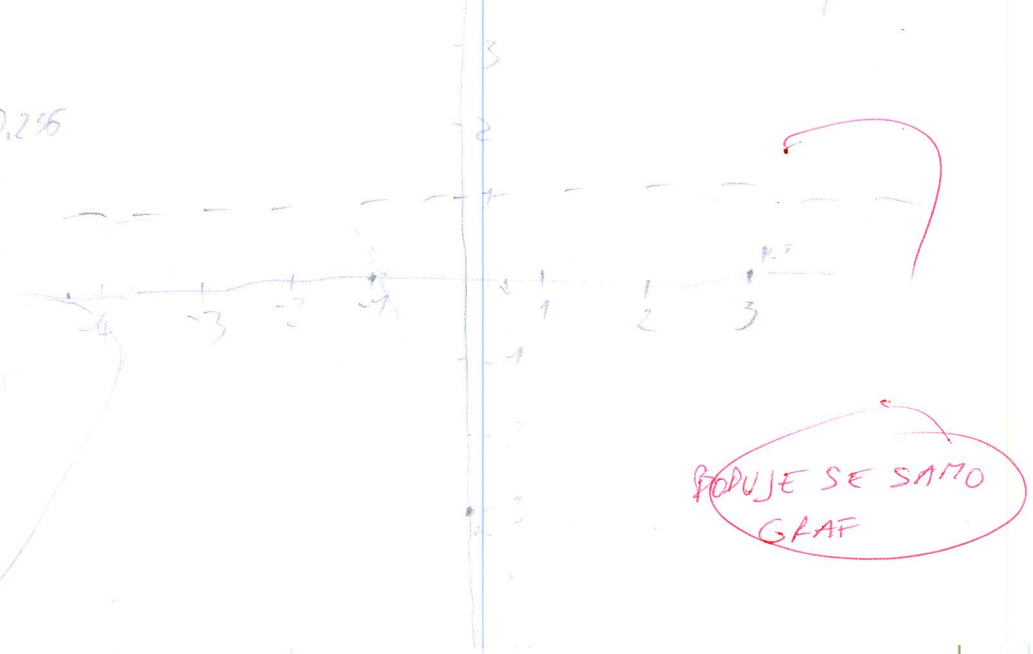
$x_{1,2}=\frac{-4\pm\sqrt{16+4}}{2}$ $\quad -4.236 \quad 0.236$

$x_1=\frac{-4-2\sqrt{5}}{2} \quad x_2=\frac{-4+2\sqrt{5}}{2}$

	$-\infty$		$\frac{-4-2\sqrt{5}}{2}$		$\frac{-4+2\sqrt{5}}{2}$		$+\infty$
	-5		0		1		
$f'(x)$	$+$		$-$		$+$		
$f(x)$	↗		↘		↗		
			M		m		

$M\left(\frac{-4-2\sqrt{5}}{2}, -1+\sqrt{5}\right)$ -1.236

$m\left(\frac{-4+2\sqrt{5}}{2}, -1-\sqrt{5}\right)$ -3.236



RADUJE SE SAMO GRAF

5.

$$\left[\begin{array}{cccc:c} 1 & 2 & -1 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \\ R_4-R_1 \end{array} \sim \left[\begin{array}{cccc:c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & -7 & 1 & -2 & -16 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ \\ R_3+7R_2 \\ R_4+3R_2 \end{array}$$

$$\left[\begin{array}{cccc:c} 1 & 0 & -3 & 1 & 24 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 8 & -2 & -72 \\ 0 & 0 & 7 & -6 & -24 \end{array}\right] \cdot \frac{1}{8} \sim \left[\begin{array}{cccc:c} 1 & 0 & -3 & 1 & 24 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & -1/4 & -9 \\ 0 & 0 & 7 & -6 & -24 \end{array}\right] \begin{array}{l} R_1+3R_3 \\ R_2-R_3 \\ \\ R_4-7R_4 \end{array} \sim \left[\begin{array}{cccc:c} 1 & 0 & 0 & 1/4 & -3 \\ 0 & 1 & 0 & 1/4 & 1 \\ 0 & 0 & 1 & -1/4 & -9 \\ 0 & 0 & 0 & -17/4 & 39 \end{array}\right]$$

?

6.

$$\lim_{x\to 1} e^{\frac{1}{x^2-1}} = \lim_{x\to 1} e^{\frac{1}{1-1}} = \lim_{x\to 1} e^{\frac{1}{0}} = \lim_{x\to 1} e^{\infty} = \infty$$ ✗

$\frac{1}{0} \neq \infty$

$\frac{1}{0_+} = +\infty$

$\frac{1}{0_-} = -\infty$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: Krešimir Lalaina **BROJ INDEKSA:** 571 181/2009

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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6. Izračunati: $\lim\limits_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno: 15

(2) $f(x) = x - \sqrt{x^2-4} \qquad x^2-4 \geq 0$

$D_f = \langle -\infty, -2] \cup [2, +\infty \rangle$

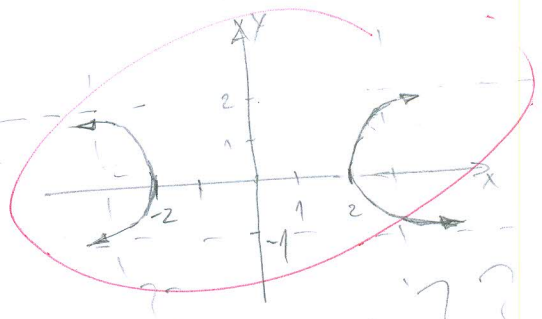
$\lim\limits_{x\to-\infty} x - \sqrt{x^2-4} = \left[\begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix}\right] = \lim\limits_{x\to+\infty} -x - \sqrt{x^2-4} = -\infty - \infty = -x - \sqrt{x^2-4} \,/ :x^2 = 0 - \sqrt{1} = -1$

$\lim\limits_{x\to-2} x - \sqrt{x^2-4} = -2$

$\lim\limits_{x\to 2^+} x - \sqrt{x^2-4} = 2$

$\lim\limits_{x\to+\infty} x - \sqrt{x^2-4} = \infty - \infty = x - \sqrt{x^2-x} \,/ :x^2 = \frac{x}{x^2} - \sqrt{1} = -1$

LKA?



FUNKCIJA?

$$\begin{aligned} x+2y-z+u &= 8\\ 2x+5y-z+2u &= 8\\ 3x-y-2z+u &= 8\\ x-y+3z-5u &= 8 \end{aligned}$$

$$\left[\begin{array}{cccc;c} 1 & 2 & -1 & 1 & 8\\ 2 & 5 & -1 & 2 & 8\\ 3 & -1 & -2 & 1 & 8\\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \begin{matrix} \\ R_2-2R_1\\ R_3-3R_1\\ R_4-R_1 \end{matrix} \sim \left[\begin{array}{cccc;c} 1 & 2 & -1 & 1 & 8\\ 0 & 1 & 1 & 0 & -8\\ 0 & -7 & 1 & -2 & -16\\ 0 & -1 & 4 & -6 & 0 \end{array}\right] \begin{matrix} R_1-2R_2\\ \\ R_3+7R_2\\ R_4+R_2 \end{matrix}$$

$$\left[\begin{array}{cccc;c} 1 & 0 & -3 & 1 & 24\\ 0 & 1 & 1 & 0 & -8\\ 0 & 0 & -6 & -2 & -72\\ 0 & 0 & 5 & -6 & -8 \end{array}\right] R_3\cdot\left(-\frac{1}{6}\right) \sim \left[\begin{array}{cccc;c} 1 & 0 & -3 & 1 & 24\\ 0 & 1 & 1 & 0 & -8\\ 0 & 0 & 1 & \frac{2}{6} & 12\\ 0 & 0 & 5 & -6 & -8 \end{array}\right] \begin{matrix} R_1+3R_3\\ R_2-R_3\\ \\ R_4-5R_3 \end{matrix} \sim \left[\begin{array}{cccc;c} 1 & 0 & 0 & 2 & 60\\ 0 & 1 & 0 & -\frac{1}{3} & -20\\ 0 & 0 & 1 & \frac{1}{3} & 12\\ 0 & 0 & 0 & -\frac{23}{3} & -68 \end{array}\right] R_4\cdot\left(-\frac{3}{23}\right)$$

$$\left[\begin{array}{cccc;c} 1 & 0 & 0 & 2 & 60\\ 0 & 1 & 0 & -\frac{1}{3} & -20\\ 0 & 0 & 1 & \frac{1}{3} & 12\\ 0 & 0 & 0 & 1 & \frac{204}{23} \end{array}\right] \begin{matrix} R_1-2R_4\\ R_2+\frac{1}{3}R_4\\ R_3-\frac{1}{3}R_4\\ \\ \end{matrix} \sim \left[\begin{array}{cccc;c} x & y & z & u & \\ 1 & 0 & 0 & 0 & \frac{372}{23}\\ 0 & 1 & 0 & 0 & -\frac{1176}{69}\\ 0 & 0 & 1 & 0 & \frac{624}{69}\\ 0 & 0 & 0 & 1 & \frac{204}{23} \end{array}\right]$$

$$\frac{68}{1}\cdot\left(-\frac{3}{23}\right)$$

$$\frac{204}{23}$$

$$\left[\frac{60}{1}-\frac{408}{23}\right]$$

$$-\frac{20}{1}+\left[\frac{1}{3}\cdot\frac{204}{23}\right]$$

$$-\frac{20}{1}+\frac{204}{69}=\frac{-1380+204}{69}=-\frac{1176}{69}$$

$$\frac{372}{23}+2\cdot\left(\frac{-1176}{69}\right)-\left(\frac{624}{69}\right)+\left(\frac{204}{23}\right)=8\ \checkmark$$

$$2\left(\frac{372}{23}\right)+5\left(\frac{-1176}{69}\right)-\left(\frac{624}{69}\right)+2\cdot\left(\frac{204}{23}\right)=8\ \checkmark$$

OSTALE PROVJERE ?

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(3) $g(x)=\ln(4-x^2)$ $\quad 4-x^2>0$

$D_f(g)=\langle -2,2\rangle$ ✓

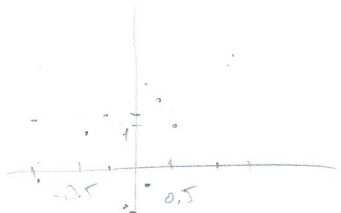
NEPARNOST

$g(x)=g(-x)$?

$g(x)\ \ln(4-x^2)=\ln(4-(-x)^2)$

Funkcija $g(x)$ je PARNA ✓

preslikava se preko y osi



$$\frac{12}{1}-\frac{204}{69}=\frac{828}{69}-\frac{624}{69}$$

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IME I PREZIME: Ante Podišić **BROJ INDEKSA:** 17-1-0114-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
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4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-3}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 8 \\ 2x + 5y - z + 2u &= 8 \\ 3x - y - 2z + u &= 8 \\ x - y + 3z - 5u &= 8 \end{aligned}$$

6. Izračunati: $\lim\limits_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno: 5

(1.) $z^3 - (1-i)^5 = 0$

$z^3 = (1-i)^5$

$z^3 = \sqrt[5]{1-i}$

$z^3 = \sqrt[5]{1} - \sqrt[5]{i}$ ✗

$z^3 = 1 - (i)^{-5}$

$z^3 = 1 - (-(i)^5)$

$z^3 = 1 + i$

$z = \sqrt[3]{1+i}$

$|w| = \sqrt{1^2+1^2}$

$= \sqrt{1}$

$= 1$

$\operatorname{tg}\gamma = \frac{1}{1} \Rightarrow \operatorname{tg}\gamma = 1/\operatorname{arc\,tg}$

$\gamma = \frac{1}{4}\pi$

$k = 0, 1, 2$

za $k = 0$

$z_1 = \sqrt{1} \cdot \left(\cos\frac{\frac{1}{4}\pi + 2k\pi}{3} + i\sin\frac{\frac{1}{4}\pi + 2k\pi}{3}\right)$

$z_1 = 1 \cdot \left(\cos\frac{\frac{1}{4}\pi}{3} + i\sin\frac{\frac{1}{4}\pi}{3}\right)$

$z_1 = \frac{\sqrt{6}+\sqrt{2}}{4} + i\,\frac{\sqrt{6}-\sqrt{2}}{4}$

za $k = 1$

$z_2 = 1 \cdot \left(\cos\frac{\frac{1}{4}\pi + 2\pi}{3} + i\sin\frac{\frac{1}{4}\pi + 2\pi}{3}\right)$

$z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$z_3 = 1 \cdot \left(\cos\frac{\frac{1}{4}\pi + 4\pi}{3} + i\sin\frac{\frac{1}{4}\pi + 4\pi}{3}\right)$

$z_3 = \frac{-\sqrt{6}+\sqrt{2}}{4} - \frac{\sqrt{6}+\sqrt{2}}{4}i$

② $f(x) = x - \sqrt{x^2-4}$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x = \pm\sqrt{4}$

$x_1 = 2, \; x_2 = -2$

$Df(x) = \langle -\infty, -2] \cup [2, +\infty\rangle$ ✓5

Asimptote

V.A

$\lim\limits_{x \to 2^-} x - \sqrt{x^2-4} = \lim\limits_{x \to 2^-} 2 - \sqrt{2^2-4} = 2^+$

$\lim\limits_{x \to 2^+} x - \sqrt{x^2-4} = \lim\limits_{x \to 2^+} 2 - \sqrt{2^2-4} = 2^-$

H.A

$\lim\limits_{x \to \infty^-} x - \sqrt{x^2-4} = \frac{x}{x^2} - \sqrt{\frac{x^2}{x^2} - \frac{4}{x^2}} = -\sqrt{1} = -1$ D.H.A 1

KOSA

$y = kx + l$

$k = \lim\limits_{x \to \infty} \frac{f(x)}{x} = \frac{x - \sqrt{x^2-4}\,/:x^2}{x \quad /:x^2} = \frac{\frac{x}{x^2} - \sqrt{\frac{x^2}{x^2} - \frac{4}{x^2}}}{\frac{x}{x^2}}$ ($\frac{x}{x^2} \to 0$; $\frac{x^2}{x^2} \to 1$; $\frac{4}{x^2} \to 0$; $\sqrt{1-0} = 1$; $\frac{x}{x^2} \to 0$)

$k = \frac{-1}{0}$ Kosa nema!

(3.) $g(x) = \ln(4 - x^2)$

$4 - x^2 > 0 \quad / \cdot (-1)$ $\quad Dg(x) = \langle -\infty, 2 \rangle \cup \langle 2, +\infty \rangle$ ✗

$-4 + x^2 > 0$

$x^2 > 4$

$x = \pm 2$

Parnost neparnost

$g(-x) = \ln(4 - (-x)^2) = -\ln(4 - x^2)$ Niti parna, niti neparna ✗

$g'(x) = \ln(4 - x^2)$

$g'(x) = \frac{1}{4 - x^2} \cdot (4 - x^2)'$ $\quad g'(x) = \frac{-2x}{4 - x^2} = 0$

$= \frac{1}{4 - x^2} \cdot (-2x)$ $\quad -2x = 0$ $\quad 4 - x^2 = 0$

$= \frac{-2x}{4 - x^2}$ $\quad x = 2$ $\quad x^2 = 4$

$x_1 = -2, \; x_2 = 2$

	$-\infty$	-2	2	$+\infty$
$g'(x)$	∩	∪	∩	

✗

(4.)

Domena

$h(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$ $\quad D(x) = \langle -\infty, -1] \cup [1, +\infty \rangle$

$x^2 + 1 \geq 0$

$x^2 \geq 1$

$x_1 = -1, \; x_2 = 1$

Asimptote

V. A.

$\lim_{x \to 1^-} \frac{x^2 - 2x - 3}{x^2 + 1} = \lim_{x \to 1^-} \frac{1 - 2 - 3}{1 + 1} = \frac{-4}{1} = -4^+$

$\lim_{x \to 1^+} \frac{x^2 - 2x - 3}{x^2 + 1} = \lim_{x \to 1^+} \frac{1 - 2 - 3}{1 + 1} = \frac{-4}{1} = -4^-$

H.A

$$\lim_{x\to\infty}\frac{x^2-2x-3}{x^2+1}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}=\frac{1}{1}=1$$

$\frac{x^2}{x^2}=1$, $\frac{2x}{x^2}=0$, $\frac{3}{x^2}=0$, $\frac{1}{x^2}=0$

D. H. A - 1 //

KOSA

$y=kx+l$

$$k=\lim_{x\to\infty}\frac{\frac{x^2-2x-3}{x^2+1}}{\frac{x}{1}}=\lim_{x\to\infty}\frac{x^2-2x-3}{x\cdot(x^2+1)}=\lim_{x\to\infty}\frac{\frac{x^2}{x^3}-\frac{2x}{x^3}-\frac{3}{x^3}}{\frac{x}{x^3}\cdot\left(\frac{x^2}{x^3}+\frac{1}{x^3}\right)}=\frac{1}{0}=0$$

$\frac{x^2}{x^3}=\frac{1}{x}=0$, $\frac{2x}{x^3}=\frac{2}{x^2}=0$, $\frac{3}{x^3}=0$, $\frac{x^2}{x^3}=\frac{1}{x}=0$, $\frac{1}{x^3}=0$

Kosa nema!

Glob. svojstva

$$f(-x)=\frac{-x^2-2(-x)-3}{-x^2+1}=-\frac{x^2-2x-3}{x^2+1}=f(x)$$

funkcija je parna!

Periodična?

Nul točke itd.

$$\frac{x^2-2x-3}{x^2+1}=0$$

$S(0,-3)$

$$f(0)=\frac{0^2-2\cdot0-3}{0^2+1}=-3$$

Derivacija

$$f'(x)=\frac{(x^2-2x-3)'\cdot(x^2+1)-(x^2-2x-3)\cdot(x^2+1)'}{(x^2+1)^2}$$

$$f'(x)=\frac{(2x-2)\cdot(x^2+1)-(x^2-2x-3)\cdot(2x)}{(x^2+1)^2}$$

$$f'(x)=\frac{2x^3+2x-2x^2-2-(2x^3-4x^2-6x)}{(x^2+1)^2}$$

$$f'(x)=\frac{2x^3+2x-2x^2-2-2x^3+4x^2+6x}{(x^2+1)^2}$$

$2x^2$

$$=\frac{2x^2+8x-2}{(x^2+1)^2}$$

//

4) nastavak na sljedećoj stranici

Ekstremi

$$f'(x) = \frac{2x^2+8x-2}{(x^2+1)^2}$$

$$2x^2+8x-2=0$$

$$x_{1,2} = \frac{-8 \pm \sqrt{64+16}}{4}$$

$$x_{1,2} = \frac{-8 \pm 4\sqrt{5}}{4}$$

$$x_1 \approx 0{,}236$$

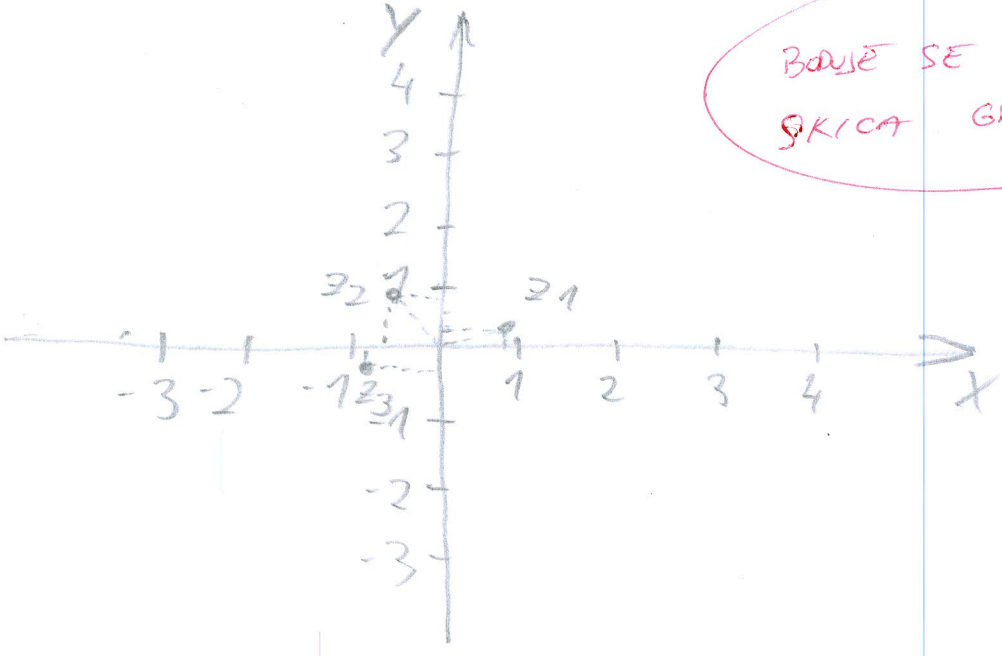
$$x_2 \approx -4{,}23$$

(6.) $\lim_{x \to 1} e^{\frac{1}{x^2-1}} = \lim_{x \to 1} e^{\frac{1}{1-1}} = \lim_{x \to 1} e^{\frac{1}{0}} \Rightarrow e^{\infty}$

(11.) $z_1 = \frac{\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i \Rightarrow x \approx 0{,}966, y \approx 0{,}26$

$z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \Rightarrow x \approx -0{,}707, y \approx 0{,}707$

$z_3 = -\frac{\sqrt{6}+\sqrt{2}}{4} - \frac{\sqrt{6}+\sqrt{2}}{4}i \Rightarrow x \approx -0{,}966, y \approx -0{,}26$



BOLJE SE SAMO SKICA GRAFA

(57)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \overset{IV \leftrightarrow II}{\sim} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \end{array}\right] \begin{array}{l} /\cdot(-3) \\ /\cdot(-2) \\ \\ \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \\ 0 & -4 & -5 & 16 & -16 \\ 0 & 7 & -7 & 12 & -8 \end{array}\right] \begin{array}{l} /\cdot(-1) \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & -3 & 4 & -6 & 8 \\ 0 & 4 & -5 & 16 & -16 \\ 0 & 7 & -7 & 12 & -8 \end{array}\right] /:(-3) \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & -\frac{4}{3} & 2 & -\frac{8}{3} \\ 0 & 4 & -5 & 16 & -16 \\ 0 & 7 & -7 & 12 & -8 \end{array}\right] \begin{array}{l} \\ /\cdot(-2)\ /\cdot(4) \\ /\cdot(-7) \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{5}{3} & -3 & \frac{40}{3} \\ 0 & 1 & -\frac{4}{3} & 2 & -\frac{8}{3} \\ 0 & 0 & \frac{1}{3} & 8 & -\frac{16}{3} \\ 0 & 0 & \frac{7}{3} & -2 & \frac{32}{3} \end{array}\right] \begin{array}{l} \\ \\ /\cdot(4)\ /\cdot(-5)\ /\cdot(-7) \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -43 & 40 \\ 0 & 1 & 0 & 34 & -24 \\ 0 & 0 & \frac{1}{3} & 8 & -\frac{16}{3} \\ 0 & 0 & 0 & -58 & 48 \end{array}\right] \begin{array}{l} \\ \\ /\cdot 3 \\ /:2 \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -43 & 40 \\ 0 & 1 & 0 & 34 & -24 \\ 0 & 0 & 1 & 24 & -16 \\ 0 & 0 & 0 & -29 & 24 \end{array}\right] /:(-29) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -43 & 40 \\ 0 & 1 & 0 & 34 & -24 \\ 0 & 0 & 1 & 24 & -16 \\ 0 & 0 & 0 & 1 & -\frac{24}{29} \end{array}\right] \begin{array}{l} \\ \\ \\ /\cdot(-24)\ /\cdot(-34)\ /\cdot 43 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{128}{29} \\ 0 & 1 & 0 & 0 & \frac{120}{29} \\ 0 & 0 & 1 & 0 & \frac{112}{29} \\ 0 & 0 & 0 & 1 & -\frac{24}{29} \end{array}\right] \begin{array}{l} x = \frac{128}{29} \\ y = \frac{120}{29} \\ z = \frac{112}{29} \\ u = -\frac{24}{29} \end{array}$$

OSTALE PROVJERE ?

Provjera

$$\frac{128}{29} + 2\cdot\frac{120}{29} - \frac{112}{29} + \left(-\frac{24}{29}\right) = 8$$

$$x + 2y - z + u = 8$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: Marko Bilušić **BROJ INDEKSA:** 17-1-0061-2011

ZAOKRUŽITI AKO ŽELITE: (ustmeni kod prof. Uglešića)

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^3 - (1-i)^5 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-4}$. 5+15
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5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 8 \\ 2x + 5y - z + 2u &= 8 \\ 3x - y - 2z + u &= 8 \\ x - y + 3z - 5u &= 8 \end{aligned}$$

6. Izračunati: $\lim_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno: 5

(1.) $z^3 - (1-i)^5 = 0$

$z^3 = (1-i)^5$

$z^3 = 1 \cdot i^5$ ✗ / $\sqrt[3]{}$

$z = \sqrt[3]{1 + i^5}$

$r = \sqrt[2]{\sqrt[3]{(1)^2 + (-1)^2}}$

$r = \sqrt[6]{1+1}$

$r = \sqrt[6]{2}$

$\cos\varphi = \frac{\pi}{4}$

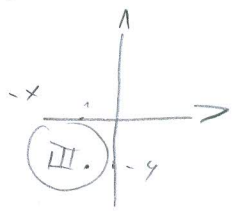
$\varphi_0 = \frac{2\pi}{2} - \frac{\pi}{4} = \frac{3\pi}{4}$

$\varphi = \frac{3\pi}{4}$

$z =$

$x = -1^5 = -1$ / : 4

$y = i^5 = -i^1 = 1$



$\frac{2\pi}{2} - \varphi$

$\frac{y}{x} = \frac{1}{-1} = -1$

2. $f(x) = x - \sqrt{x^2 - 4}$

$x^2 - 4 \geq 0$

$x^2 \geq 4 \quad / \sqrt{}$

$x \geq \sqrt{4}$?

$x \geq \pm 2$ =

$Df = \mathbb{R} \setminus \langle -\infty, -2 \rangle \cup \langle 2, +\infty \rangle$ ✓

asimptote:

Vertikalne nema //

horizontalna

$f(x) = \infty - \sqrt{\infty^2 - 4} = \infty - \infty = 0$ // ✗

$\ln x \to \infty$

kosa

3. $g(x) = \ln(4 - x^2)$

$\ln x > 0$

$4 - x^2 > 0$? POSTUPAK NIJE DOBAR.

$x^2 > -4 \quad / \sqrt{}$

$x > \pm 2$ // ✗

$Df = \mathbb{R} \setminus \langle -\infty, -2] \cup [2, +\infty \rangle$

(Marko Bilić - 17-1-0061-2011)

(4.) $h(x) = \dfrac{x^2 - 2x - 3}{x^2 + 1}$

1.) (2) – nemoguće jer je kvadrat

$x^2 + 1 \neq 0$

$Df = \mathbb{R}$ svi realni brojevi ✓

2.) a) $f(x) = 0$

$\dfrac{x^2 - 2x - 3}{x^2 + 1} = 0 \quad / \cdot (x^2 + 1)$

$1 \quad -2 \quad -3$

$x^2 - 2x - 3 = 0$

$x_{1/2} = \dfrac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \dfrac{2 \pm \sqrt{4 + 12}}{2} = \dfrac{2 \pm \sqrt{16}}{2} = \dfrac{2 \pm 4}{2}$

$x_1 = \dfrac{6}{2} = 3$

$x_2 = 1$

b) $f(0)$

$\dfrac{0 - 0 - 3}{0 + 1} = \dfrac{-3}{1} = -3$

3.) $f'(x) = \dfrac{(x^2 - 2x - 3)' \cdot (x^2 + 1) - (x^2 + 1)' \cdot (x^2 - 2x - 3)}{(x^2 + 1)^2}$

$f'(x) = \dfrac{(2x - 2) \cdot (x^2 + 1) - 2x \cdot (x^2 - 2x - 3)}{x^4 + 1}$

$f'(x) = \dfrac{2x^3 + 2x - 2x^2 - 2 - 2x^3 + 4x^2 + 6x}{x^4 + 1}$

$f'(x) = \dfrac{-6x^2 + 8x - 2}{x^4 + 1} = 0 \quad / \cdot (x^4 + 1)$

$x_{1/2} = -6x^2 + 8x - 2 = \dfrac{-8 \pm \sqrt{(-8)^2 - 4 \cdot (-6) \cdot (-2)}}{2 \cdot (-6)} = \dfrac{-8 \pm \sqrt{64 - 48}}{-12} = \dfrac{-8 \pm \sqrt{112}}{-12}$

$= \dfrac{-8 \pm 10{,}5}{-12} \quad x_1 = \dfrac{-8 + 10{,}5}{-12} = \dfrac{2{,}5}{-12} = -0{,}2$

$x_2 = \dfrac{-18{,}5}{-12} = 1{,}5$

4)

$$f''(x) = \frac{6x^2+8x-2}{x^4+1}$$

$$= \frac{(6x^2+8x-2)' \cdot (x^4+1) - (x^4+1)' \cdot (6x+8x-2)}{(x^4+1)^2} = \frac{(12x+8)\cdot(x^4+1) - 4x\cdot(6x+8x-2)}{x^8+1}$$

$$= \frac{12x^5+12x+8x^4+8-24x^2-32x^2+8x}{x^8+1} = \frac{12x^5+8x^4-56x^2+20x+8}{x^8+1}$$

$f(0{,}2)$ $\frac{+}{+}$ min $\frac{(0{,}2)^2-2(0{,}2)-3}{(0{,}2)^2+1} = \frac{0{,}4-0{,}4-3}{0{,}4+1} = \frac{-3}{1{,}4} = -2{,}1$

$M_{min}(0{,}2, -2{,}1)$

$f(-1{,}5)$ = Max $\frac{(-1{,}5)^2-2(-1{,}5)-3}{(-1{,}5)^2+1} = \frac{2{,}25+3-3}{2{,}25+1} = \frac{2{,}25}{3{,}25} = 0{,}6$

$M_{max}(-1{,}5, 0{,}6)$

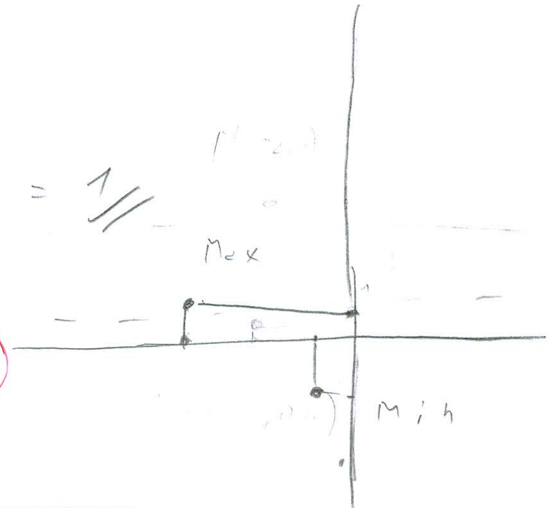
5.) Vertikalne nema

horizontalna

$\lim_{x\to\infty} f(x)$ $\frac{\infty-\infty-3}{\infty} = 0 \Rightarrow \lim_{x\to\infty} \frac{6x^2-8x-2}{x^4+1} = \frac{\infty}{\infty} = 1$

0

SKICA GRAFA SE BODUJE



5.

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \cdot -2, -3, -1 = \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & -3 & 0 & -8 \\ 0 & -7 & -5 & -2 & -16 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \cdot -2, 7, 3 \Rightarrow$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & 1 & 24 \\ 0 & 1 & -3 & 0 & -8 \\ 0 & 0 & -26 & -2 & 72 \\ 0 & 0 & -8 & -6 & -24 \end{array}\right] \begin{array}{c} \\ \\ /:2 \\ /:2 \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 5 & 1 & 24 \\ 0 & 1 & -3 & 0 & -8 \\ 0 & 0 & -13 & -1 & 36 \\ 0 & 0 & -4 & -3 & -12 \end{array}\right] \cdot -3 \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 12 \\ 0 & 1 & -3 & 0 & -8 \\ 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & -4 & -3 & -12 \end{array}\right] \cdot -3, \cdot 4$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 12 \\ 0 & 1 & 0 & -24 & -8 \\ 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & 0 & -35 & -12 \end{array}\right] /:7 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 12 \\ 0 & 1 & 0 & -24 & -8 \\ 0 & 0 & -1 & 8 & 0 \\ 0 & 0 & 0 & 5 & -1{,}7 \end{array}\right]$$

? - 0

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IME I PREZIME: IVAN TAMIĆ **BROJ INDEKSA:** 56437

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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6. Izračunati: $\lim\limits_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno:

5.

$$\begin{bmatrix} 1 & 2 & -1 & +1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{bmatrix} \begin{matrix} \\ R2-2R1 \\ R3-3R1 \\ R4-R1 \end{matrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 8 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & -7 & 1 & -2 & -16 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \begin{matrix} R1-2R2 \\ \\ R3+7R2 \\ R4+3R2 \end{matrix} \sim \begin{bmatrix} 1 & 0 & -3 & 1 & 24 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 8 & -2 & -72 \\ 0 & 0 & 7 & -6 & -24 \end{bmatrix} R3-R4 \sim$$

$$\begin{bmatrix} 1 & 0 & -3 & 1 & 24 \\ 0 & 1 & 1 & 0 & -8 \\ 0 & 0 & 1 & -8 & -72 \\ 0 & 0 & 7 & -6 & -24 \end{bmatrix} \begin{matrix} R1+3R3 \\ R2-R3 \\ \\ R4-7R3 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & -23 & 192 \\ 0 & 1 & 0 & -8 & -80 \\ 0 & 0 & 1 & -8 & -72 \\ 0 & 0 & 0 & 50 & 528 \end{bmatrix} :50 \sim \begin{bmatrix} 1 & 0 & 0 & -23 & 192 \\ 0 & 1 & 0 & -8 & -80 \\ 0 & 0 & 1 & -8 & -72 \\ 0 & 0 & 0 & 1 & 10{,}56 \end{bmatrix} \begin{matrix} R1+23R4 \\ R2+8R4 \\ R3+8R4 \\ \\ \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 434{,}88 \\ 0 & 1 & 0 & 0 & 4{,}28 \\ 0 & 0 & 1 & 0 & 12{,}28 \\ 0 & 0 & 0 & 1 & 10{,}56 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & -23 & 192 \\ 0 & 1 & 0 & -8 & -80 \\ 0 & 0 & 1 & -8 & -72 \\ 0 & 0 & 0 & 25 & 264 \end{bmatrix} R4+3R3 \sim \begin{bmatrix} 1 & 0 & 0 & -23 & 192 \\ 0 & 1 & 0 & -8 & -80 \\ 0 & 0 & 1 & -8 & -72 \\ 0 & 0 & 3 & 1 & 48 \end{bmatrix} \begin{matrix} R1+23R4 \\ R2+8R4 \\ R3+8R4 \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1296 \\ 0 & 1 & 0 & 0 & 304 \\ 0 & 0 & 1 & 0 & 312 \\ 0 & 0 & 3 & 1 & 48 \end{bmatrix} R4-3R3 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1296 \\ 0 & 1 & 0 & 0 & 304 \\ 0 & 0 & 1 & 0 & 312 \\ 0 & 0 & 0 & 1 & 888 \end{bmatrix}$$

$1296 - 304 + 3\cdot(312) - 5\cdot 888$
$936 - 4440$

SUSTAV NEMA RJEŠENJA

4. $h(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$

1. $x^2 - 2x - 3 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{a^2}$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{1}$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{1}$$

$x_1 = 2 + \sqrt{16} = 6$

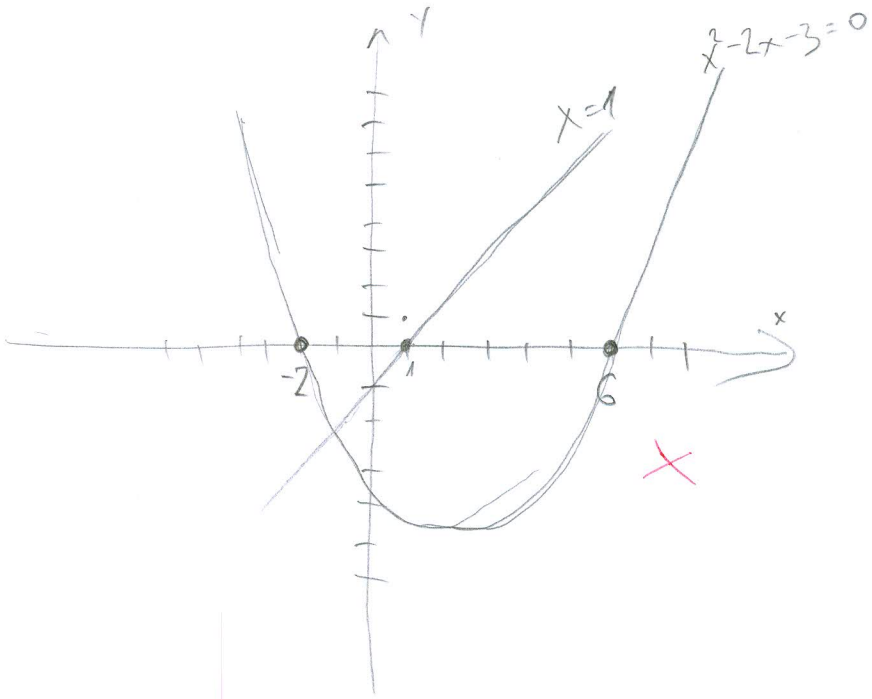
$x_2 = 2 - \sqrt{16} = -2$

2. $x^2 + 1 = 0$

$x^2 = 1$

$x = \sqrt{1}$

$x = 1$



$D(h) = \langle -\infty, -2 \rangle \cup \langle 6, +\infty \rangle$

IVAN MATIĆ

(2.) $f(x) = x - \sqrt{x^2-4}$

$D(f) = \langle -\infty, 0 \rangle \cup \langle 2, +\infty \rangle$ ×

$x = 0 \quad = 0$

$\sqrt{x^2-4} = 0$

$\sqrt{x^2} = \sqrt{4}$

$x = 2$

(6.) $\lim_{x \to 1} e^{\frac{1}{x^2-1}} = \lim_{x \to 1} e^{\frac{1}{1-1}} = \lim_{x \to 1} e^{1} = \lim_{x \to 1} e$ ×

(3.) $g(x) = \ln(4-x^2)$

$D(f) = \langle -\infty, 0 \rangle \cup \langle 2, +\infty \rangle$ ×

$\ln = 0$

$4 - x^2 = 0$

$4 = x^2$

$x = \sqrt{4}$

$x = 2$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: TONI LULIĆ **BROJ INDEKSA:** 5796

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^3 - (1-i)^5 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-4}$. 5+15
3. Ispitati domenu, (ne)parnost i zaktivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-2x-3}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 8 \\ 2x + 5y - z + 2u &= 8 \\ 3x - y - 2z + u &= 8 \\ x - y + 3z - 5u &= 8 \end{aligned}$$

6. Izračunati: $\lim\limits_{x\to 1} e^{\frac{1}{x^2-1}}$ 10

Ukupno: 0

5.
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 2 & 5 & -1 & 2 & 8 \\ 3 & -1 & -2 & 1 & 8 \\ 1 & -1 & 3 & -5 & 8 \end{array}\right] \begin{matrix} \\ \text{II}-\text{III} \\ \text{II}-\text{I} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 8 \\ 1 & 3 & 0 & 1 & 0 \\ 3 & -1 & -2 & 1 & 8 \\ -2 & 0 & 5 & -6 & 0 \end{array}\right]$$