MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **IME I PREZIME:** RJEŠENJE **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r=1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$? 15

2. Izračunati dvostruki integral: $\iint\limits_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\le 1 \text{ i } x\ge y\}$. 20

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x+\mathrm{d}y)$. 15

4. Izračunati volumen paraboloida omeđenog plohama: $z=\frac{x^2}{3}+\frac{y^2}{3}$, $z=1$. 15

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z=\frac{x^2}{3}+\frac{y^2}{3}$ što leži iznad područja $D\ldots x^2+y^2\le 1$. Nije potrebno računati površinu baze. 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t)+2y''(t)+y'(t)=0,\quad y(0)=2,\ y'(0)=0,\ y''(0)=0.$$

Ukupno:

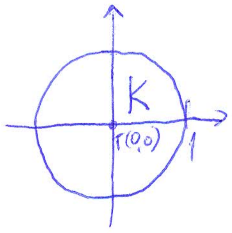
Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int \frac{\mathrm{d}x}{x}=\ln\|x\|+C$	$\int \sinh x\,\mathrm{d}x=\cosh x+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x=\sinh x+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int \sin x\,\mathrm{d}x=-\cos x+C$	$\int \tanh x\,\mathrm{d}x=\ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \coth x\,\mathrm{d}x=\ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x=-\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x}=\tanh x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x=\ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x}=-\coth x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

1.



$$\int_{\partial K} xy\, ds = ?$$

PARAMETRIZACIJA KRUŽNICE: $\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\cos t\\ \sin t\end{pmatrix}, \quad t\in[0,2\pi].$

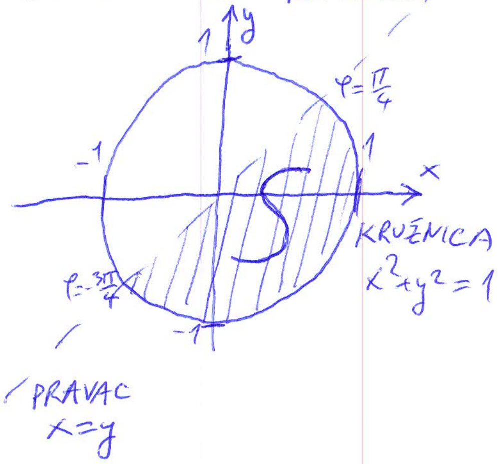
$$\Rightarrow \begin{pmatrix}\dot x\\ \dot y\end{pmatrix} = \begin{pmatrix}-\sin t\\ \cos t\end{pmatrix}$$

$$\int_{\partial K} xy\, ds = \int_0^{2\pi} \cos t\cdot \sin t\cdot \underbrace{\sqrt{(-\sin t)^2+(\cos t)^2}}_{=1}\, dt = \int_0^{2\pi} \cos t \sin t\, dt = 0$$

---

2. $S = \{(x,y)\in\mathbb{R}^2 : x^2+y^2\le 1,\ x\ge y\}$

SKICA U RAVNINI



$$\iint_S xy\, dx\,dy = \left\{\begin{matrix}\text{POLARNE KOORDINATE:}\\ x = r\cos\varphi\\ y = r\sin\varphi\\ r\in[0,1]\\ \varphi\in\left[-\frac{3\pi}{4},\frac{\pi}{4}\right]\end{matrix}\right\} = \int_0^1\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} r\cos\varphi\, r\sin\varphi\, r\, d\varphi\, dr$$

$$= \underbrace{\int_0^1 r^3\, dr}_{=\left[\frac{r^4}{4}\right]_0^1} \underbrace{\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \cos\varphi \sin\varphi\, d\varphi}_{=\left[\frac{\sin^2\varphi}{2}\right]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}} = \frac{1}{4}\underbrace{\left(\frac{\overbrace{\sin^2\frac{\pi}{4}}^{\left(\frac{\sqrt{2}}{2}\right)^2} - \overbrace{\sin^2\left(-\frac{3\pi}{4}\right)}^{\left(-\frac{\sqrt{2}}{2}\right)^2}}{2}\right)}_{=0} = 0$$

---

3. $$\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy) = ?$$

PO ZAPISU PREPOZNAJEMO DA JE RIJEČ O KRIVULJNOM INTEGRALU (U POTENCIJALNOM POLJU) KOJI NE OVISI O PUTU INTEGRACIJE.

TRAŽIMO POTENCIJAL $f:\mathbb{R}^2\to\mathbb{R}$ TAKO DA

$$\begin{pmatrix}x+y\\ x+y\end{pmatrix} = -\operatorname{grad} f = -\begin{pmatrix}\partial_x f\\ \partial_y f\end{pmatrix}$$

IZ $\partial_x f = -x-y \Rightarrow f = \int -x-y\, dx$

$\Rightarrow f(x,y) = -\frac{x^2}{2} - xy + C(y)$

IZ $\partial_y f = -x-y \Rightarrow \cancel{-x} + C'(y) = \cancel{-x} - y$

$\Rightarrow C'(y) = -y \Rightarrow C(y) = \int -y\, dy = -\frac{y^2}{2} + c$

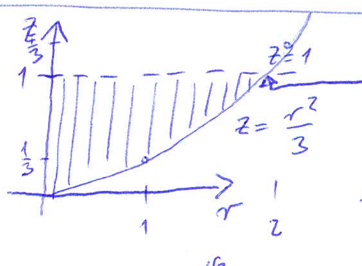
$\Rightarrow f(x,y) = -\frac{x^2}{2} - xy - \frac{y^2}{2} + C$

$$= \int_{(-1,2)}^{(2,3)} (x+y)\,dx + (x+y)\,dy = f(-1,2) - f(2,3)$$

$$= -\frac{1}{2} + 2 - \cancel{\frac{4}{2}} - \left(-\cancel{\frac{4}{2}} - 6 - \frac{9}{2}\right) = 12$$

---

4. PARABOLOID $\left\{\begin{matrix} z = \frac{x^2}{3} + \frac{y^2}{3}\\ z = 1\end{matrix}\right.$ $\xrightarrow[\text{KOORDINATE}]{\text{CILINDRIČNE}}$ $\begin{matrix} z = \frac{r^2}{3}\\ z = 1\end{matrix}$



SJECIŠTE

$1 = \frac{r^2}{3} \Rightarrow r = \sqrt{3}$

$\Rightarrow \left\{\begin{matrix} r\in[0,\sqrt{3}]\\ z\in\left[\frac{r^2}{3}, 1\right]\end{matrix}\right.$

VOLUMEN:

$$V = \int_0^{2\pi}\int_0^{\sqrt{3}}\int_{\frac{r^2}{3}}^{1} r\, dz\, dr\, d\varphi = 2\pi\int_0^{\sqrt{3}} r\left(1 - \frac{r^2}{3}\right) dr = 2\pi\left[\frac{r^2}{2} - \frac{r^4}{12}\right]_0^{\sqrt{3}}$$

$$= 2\pi\left(\frac{3}{2} - \frac{\cancel{9}^{\,3}}{\cancel{12}_{\,4}}\right) = \frac{3\pi}{2}$$

5. OPLOŠJE PARABOLOIDA

$$z = \frac{x^2}{3} + \frac{y^2}{3}$$

IZNAD $D \dots \underbrace{x^2+y^2 \le 1}$

↳ OVO JE KRUG

$\Rightarrow$ NORMALA:

$$\left.\begin{aligned} \partial_x z &= \frac{2x}{3} \\ \partial_y z &= \frac{2y}{3} \end{aligned}\right\} \Rightarrow \vec{n}(x,y) = \begin{pmatrix} \frac{2x}{3} \\ \frac{2y}{3} \end{pmatrix},$$

DULJINA NORMALE:

$$\|\vec{n}\| = \sqrt{\frac{4x^2}{9} + \frac{4y^2}{9}} = \frac{2}{3}\sqrt{x^2+y^2}$$

POVRŠINA:

$$P = \iint_D \|\vec{n}(x,y)\|\, dx\,dy = \iint_{x^2+y^2 \le 1} \frac{2}{3}\sqrt{x^2+y^2}\, dx\,dy = \left\{\begin{aligned} &\text{POLARNE KOORDINATE:} \\ &x = r\cos\varphi \\ &y = r\sin\varphi \\ &r \in [0,1] \\ &\varphi \in [0, 2\pi] \end{aligned}\right\} = \int_0^{2\pi}\int_0^1 \frac{2}{3}\sqrt{(r\cos\varphi)^2+(r\sin\varphi)^2}\cdot r\, dr\, d\varphi$$

$$= \frac{2}{3}\int_0^{2\pi}\int_0^1 \sqrt{r^2(\cos^2\varphi+\sin^2\varphi)}\cdot r\, dr\, d\varphi = \frac{2}{3}\int_0^{2\pi}\int_0^1 r^2\, dr\, d\varphi = \frac{2}{3}\cdot 2\pi \cdot \left[\frac{r^3}{3}\right]_0^1 = \frac{4\pi}{9}$$

---

6. $y'''(t) + 2y''(t) + y'(t) = 0, \quad y(0) = 2, \; y'(0) = 0, \; y''(0) = 0$

$$s^3 Y(s) - s^2 \underbrace{y(0)}_{=2} - s\underbrace{y'(0)}_{=0} - \underbrace{y''(0)}_{=0} + 2\left[s^2 Y(s) - s\underbrace{y(0)}_{=2} - \underbrace{y'(0)}_{=0}\right] + sY(s) - \underbrace{y(0)}_{=2} = 0$$

$$Y(s)\left[s^3 + 2s^2 + s\right] = 2 + 4s + 2s^2$$

$$Y(s) = \frac{2(s+1)^2}{s(s+1)^2} = \frac{2}{s}$$

$$y(t) = 2$$

ISPALO JE PRELAGANO!

PROVJERA: $y'(t) = 0, \; y''(t) = 0, \; y'''(t) = 0$

ODJ: $0 + 2\cdot 0 + 0 = 0$ ✓

UVJETI:
$$\left|\begin{aligned} &y(0) = 2 \; ✓ \\ &y'(0) = 0 \; ✓ \\ &y''(0) = 0 \; ✓ \end{aligned}\right.$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: MARIN MARAŠ **BROJ INDEKSA:** 57651

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$? ~~15~~ 12
2. Izračunati dvostruki integral: $\iint\limits_S xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 \,:\, x^2 + y^2 \le 1 \text{ i } x \ge y\}$. 20
3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x + \mathrm{d}y)$. ~~15~~ 14
4. Izračunati volumen paraboloida omeđenog plohama: $z = \frac{x^2}{3} + \frac{y^2}{3}$, $z = 1$. 15
5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \le 1$. Nije potrebno računati površinu baze. ~~15~~
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 2y''(t) + y'(t) = 0, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 0.$$

Ukupno: 61

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \ne -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

MARIN MARAŠ

1. $r = 1$ $\quad T(0,0)$ $\quad \int_{\partial K} xy \, ds$

$x = r\cos t \to \cos t$ ✓

$y = r\sin t \to \sin t$ ✓

$t \in [0, 2\pi]$ ✓

$r(t) = (\cos t, \sin t)$ ✓

$r'(t) = (-\sin t, \cos t)$ ✓

$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2}$

$= \sqrt{1}$

$= 1$ ✓

$$= \int_0^{2\pi} \cos t \sin t \, dt$$ ✓

$$= (-\sin t)\cdot \cos t \Big|_0^{2\pi} = (-\sin 2\pi)\cdot \cos 2\pi - (-\sin(0))\cos 0$$ ✗

$= 0$

12

3. $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy)$

$w \begin{bmatrix} x+y \\ x+y \end{bmatrix} = -\text{grad} f$ ✓

$\partial_x f = -(x+y) \; / \int dx$

$f = -\frac{x^2}{2} - yx + C(y)$ ✓

$\partial_y f = -(x+y)$

$\partial_y \left(-\frac{x^2}{2} - yx + C(y)\right) = -x - y$

$-\cancel{x} + \frac{\partial C(y)}{\partial y} = -\cancel{x} - y \; / \int dy$

$C(y) = -\frac{y^2}{2}$ ✓

$f = -\frac{x^2}{2} - yx - \frac{y^2}{2}$ ✓

$f(\underset{x}{-1}, \underset{y}{2}) - f(\underset{x}{2}, \underset{y}{3})$ ✓ $= -\frac{1}{2} + 2 - \cancel{\frac{4}{2}} + \cancel{\frac{4}{2}} + 6 + \frac{9}{2} = 12$

$= 13$ ✗

14

4. $\frac{x^2}{3} + \frac{y^2}{3} = z \quad \Big| \cdot 3$

$x^2 + y^2 = 3z \qquad z = 1$

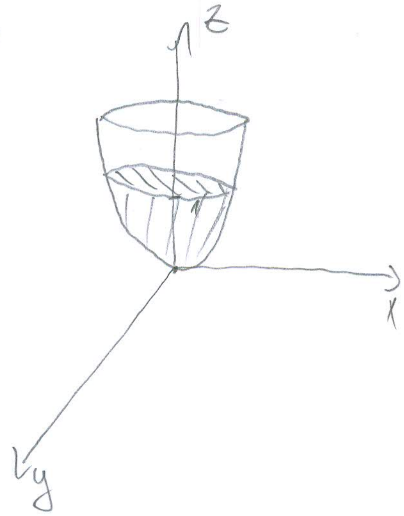
$r^2 = 3z \qquad 3z = r^2 \quad \Big| \cdot \frac{1}{3}$

$r = \sqrt{3} \qquad z = \frac{r^2}{3}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{3}]$

$z \in \left[\frac{r^2}{3}, 1\right]$



$$V = \int_0^{2\pi} d\varphi \int_0^{\sqrt{3}} r\,dr \int_{\frac{r^2}{3}}^{1} dz$$

$$= \int_0^{2\pi} d\varphi \int_0^{\sqrt{3}} r\left(1 - \frac{r^2}{3}\right) dr$$

$$= \int_0^{2\pi} d\varphi \int_0^{\sqrt{3}} \left(r - \frac{r^3}{3}\right) dr$$

$$= \int_0^{2\pi} d\varphi \left(\frac{r^2}{2} - \frac{r^4}{12}\right)\Bigg|_0^{\sqrt{3}}$$

$$= \frac{3}{4}\int_0^{2\pi} d\varphi = \frac{3}{4}(\varphi)\Big|_0^{2\pi} = \frac{3}{4}\cdot(2\pi - 0) = \frac{3}{4}\cdot 2\pi$$

$$= \frac{3}{2}\pi$$

15

MAKAŠ MARIN

6. $y'''(t) + 2y''(t) + y'(t) = 0$ $\qquad$ $y(0) = 2$

$y'(0) = 0$

$y''(0) = 0$

$$s^3F(s) - s^2\overset{2}{\overset{\shortparallel}{y(0)}} - s\cancel{y'(0)} - \cancel{y''(0)} + 2\left(s^2F(s) - s\overset{2}{\overset{\shortparallel}{y(0)}} - \cancel{y'(0)}\right) + sF(s) - \overset{2}{\overset{\shortparallel}{y(0)}} = 0$$

$$s^3F(s) - 2s^2 + 2s^2F(s) - 2s + sF(s) - 2 = 0$$

$$s^3F(s) + 2s^2F(s) + sF(s) = 2s^2 + 2s + 2$$

$$F(s)\left(s^3 + 2s^2 + s\right) = 2s^2 + 2s + 2$$

$$F(s) = \frac{2s^2 + 2s + 2}{(s^3 + 2s^2 + s)}$$

$$F(s) = \frac{2s^2 + 2s + 2}{(s^2+s)(s+1)}$$

$$F(s) = \frac{2s^2 + 2s + 2}{s(s+1)(s+1)}$$

$$\frac{2s^2 + 2s + 2}{s(s+1)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+1}$$

$$= A(s^2 + 2s + 1) + Bs(s+1) + Cs(s+1)$$

$$= As^2 + 2As + A + Bs^2 + Bs + Cs^2 + Cs$$

$2 = A + B + C$

$2 = 2A + B + C$

$\boxed{2 = A}$

$B + C = 2$

$2B = C \,/\cdot \frac{1}{2}$

$B = \frac{1}{2}C$

$\boxed{B = 0}$

$\rightarrow 2 + \frac{1}{2}C + C = 2$

$\frac{1}{2}C + C = 2 - 2$

$\frac{3}{2}C = 0 \,/\cdot \frac{3}{2}$

$\boxed{C = 0}$

$$= 2 \cdot \frac{1}{s} + \frac{0}{s+1} + \frac{0}{s+1}$$

$$= 2 \;//$$ ✓

20

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **IME I PREZIME:** BRUNO CIPOTICA **BROJ INDEKSA:** 54960	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$? 15

2. Izračunati dvostruki integral: $\iint\limits_{S} xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 \,:\, x^2 + y^2 \leq 1 \text{ i } x \geq y\}$. ~~20~~

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x + \mathrm{d}y)$. 15

4. Izračunati volumen paraboloida omeđenog plohama: $z = \frac{x^2}{3} + \frac{y^2}{3}$, $z = 1$. ~~15~~

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \leq 1$. Nije potrebno računati površinu baze. 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) + 2y''(t) + y'(t) = 0, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 0.$$

Ukupno: 30

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

MATEMATIKA 3 BRUNO LIPOTICA

3 $\int_{(-1,2)}^{(2,3)} (x+y)(dx+dy)$

$\frac{x^2}{2} + xy + \frac{y^2}{2}$

$\int x+y \, dx = \frac{x^2}{2} + xy$

$\int x+y \, dy = y\cancel{x}^{=0} + \frac{y^2}{2}$

$\frac{2^2}{2} + 2\cdot 3 + \frac{3^2}{2} - \left(\frac{1}{2}^2 + (-1\cdot 2) + \frac{2^2}{2}\right.$

$2 + 6 + \frac{9}{2} - \left(\frac{1}{2} - \cancel{2} + \cancel{2}\right)$

$= 8 + \frac{9}{2} - \left(\frac{1}{2}\right)$

$= 8 + 4 = \underline{\underline{12}}$ ✓

15

④ $z = \frac{x^2}{3} + \frac{y^2}{3}, \quad z = 1$

$\int_0^{2\pi} d\varphi \int_0^1 dz \int_0^{\sqrt{\frac{z}{3}}} r\,dr$ $\emptyset$

$\varphi (0, 2\pi)$

$r \left(0, \sqrt{\frac{z}{3}}\right)$

$z \in (0, 1)$

$\int_0^{2\pi} d\varphi \int_0^1 dz \; \frac{r^2}{2} \Big|_0^{\sqrt{\frac{z}{3}}}$

$\int_0^{2\pi} d\varphi \int_0^1 dz \; \frac{1}{3} \frac{\sqrt{z}^{\,2}}{2}$

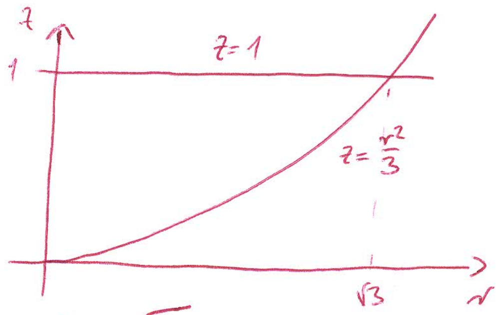
$\frac{1}{3} \int_0^{2\pi} \int_0^1 dz \frac{1}{2} z$

$\frac{1}{6} \int_0^{2\pi} \frac{z^2}{2} \Big|_0^1$

$\frac{1}{6} \int_0^{2\pi} \frac{1^2}{2}$

$\frac{1}{6} \cdot \frac{1}{2} \varphi \Big|_0^{2\pi}$

$\frac{1}{12} \cdot 2\pi = \frac{1}{6}\pi$



$V = \int_0^{2\pi} \int_0^1 \int_0^{\sqrt{3z}} r \; dr\, dz\, d\varphi$

(2)

$$\iint_S xy \, dx \, dy$$

$$\int_0^{4\pi} \int_0^{7} (r\cos\varphi \cdot r\sin\varphi)$$

$x = r\cos\varphi$

$y = r\sin\varphi$

(1) $r = 1 \quad T(0,0)$

BRUNO LIPOTICA

$$\int xy\, ds$$

$x = \cos\varphi$
$y = \sin\varphi$

$$\begin{vmatrix} -\sin\varphi \\ \cos\varphi \end{vmatrix}$$

$$\sqrt{\sin^2\varphi + \cos^2\varphi}$$

$$\sqrt{1}$$

$$= 1$$

$$\int_0^{2\pi} (\cos t \cdot \sin t) \cdot 1\, dt$$

$$\int_0^{2\pi} \sin \frac{2t}{2}\, dt$$

$$\frac{1}{2} \int_0^{2\pi} \sin 2t$$

$$-\frac{1}{2} \cos 2t \Big|_0^{2\pi}$$

$$-\frac{1}{2} \cos 2 \cdot 2\pi - \left(-\frac{1}{2} \cos 0\right)$$

$$-\frac{1}{2} \cos 4\pi - \left(-\frac{1}{2} \cos 0\right)$$

$$-\frac{1}{2} \cdot 1 - \left(-\frac{1}{2} \cdot 1\right)$$

$$-\frac{1}{2} - \left(-\frac{1}{2}\right)$$

$$-\frac{1}{2} + \frac{1}{2} = 0$$

15

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: MATIJA ŠKIBOLA **BROJ INDEKSA:** 54961/2007

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy \, ds$? ~~15~~ 12
2. Izračunati dvostruki integral: $\iint\limits_S xy \, dx dy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1 \text{ i } x \ge y\}$. 20
3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(dx + dy)$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = \frac{x^2}{3} + \frac{y^2}{3}$, $z = 1$. 15
5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = \frac{x^2}{3} + \frac{y^2}{3}$ što leži iznad područja $D \ldots x^2 + y^2 \le 1$. Nije potrebno računati površinu baze. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) + 2y''(t) + y'(t) = 0, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 0.$$

Ukupno: 12

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

6. $y'''(t) + 2y''(t) + y'(t) = 0$ $\quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 0$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 2\left[s^2F(s) - sf(0) - f'(0)\right] + sF(s) - f(0) = 0$

$s^3F(s) - s^2\cdot 2 - s\cdot 0 - 0 + 2\left[s^2F(s) - s\cdot 2 - s\cdot 0\right] + sF(s) - 2 = 0$

$s^3F(s) - 2s^2 + 2\left[s^2F(s) - 2s\right] + sF(s) - 2 = 0$

$s^3F(s) - 2s^2 + 2s^2F(s) - 4s + sF(s) - 2 = 0$

$F(s)(s^3 + 2s^2 + s) = 2 + 4s + 2s^2$?

$F(s) = \frac{2s^2+4s+2}{s^3+2s^2+s} \Rightarrow \frac{2s^3+4s+2}{s(s^2+2s+1)}$

?

$2s^3+4s+2 = \frac{A}{s} + \frac{Bs+C}{s^2+2s+1} \Rightarrow As^2 + 2As + A + Bs^2 + Cs$

?

$2s^3+4s+2 = s^2(A+B) + s(2A+C) + A$

$A + B = 0$ × $\Rightarrow 2 + B = 0$ $\quad 2A + C = 4$

$2A + C = 4$ ✓ $\quad B = -2$ $\quad 4 + C = 4$

$A = 2$ ✓ $\quad$ × $\quad C = 0$

$\frac{A}{s} + \frac{Bs+C}{s^2+2s+1} = \frac{2}{s} - \frac{2s}{s^2+2s+1} = 2 - \frac{2}{s^2} + \frac{1}{2}\cdot\frac{s}{s+1}$ ×

$= 2 - 2t \cdot \frac{1}{2} + \frac{s}{s+1}$

$= 2 - 2t + \frac{1}{2}\cdot\cos(t)$ × $\quad$ 0

TREBALI STE PROVJERITI UVRŠTAVANJEM U ODJ I POČETNE UVJETE.

Škibola Mat DS1

1. $x = r\cos t$ $\begin{bmatrix} r\cos t \\ r\sin t \end{bmatrix}$ $r' \begin{bmatrix} 1\cos t \\ 1\sin t \end{bmatrix}$ $\begin{bmatrix} -1\sin t \\ 1\cos t \end{bmatrix}$

$y = r\sin t$

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2} = \sqrt{(1\sin^2 t) + (1\cos^2 t)} = \sqrt{1(\sin^2 t + \cos^2 t)} = \sqrt{1} = 1$$ ✓

$$\int_0^{2\pi} xy\, ds = \int_0^{2\pi} (r\cos t)(r\sin t)\, 1\, dt = \int_0^{2\pi} (\cos t)(\sin t)\, dt$$ ✓ 12

$$= \int_0^{2\pi} \cos t\, dt \int_0^{2\pi} \sin t\, dt = \int_0^{2\pi} \cos t\, dt + \cos t\Big/_0^{2\pi} = \int_0^{2\pi} \cos t\, dt \cdot 1(\cos 2\pi - \cos 0)$$

$$= \sin t\Big/_0^{2\pi} = 1(\sin 2\pi - \sin 0) = \boxed{0}$$

INTEGRAL UMNOŽKE ≠ UMNOŽAK INTEGRALA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. IME I PREZIME: STIPE VULIĆ BROJ INDEKSA: 57663-2009	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r=1$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} xy\,\mathrm{d}s$? ~~15~~ 12

2. Izračunati dvostruki integral: $\iint\limits_{S} xy\,\mathrm{d}x\mathrm{d}y$, gdje je $S=\{(x,y)\in\mathbb{R}^2 \,:\, x^2+y^2\le 1 \text{ i } x\ge y\}$. ~~20~~

3. Izračunati $\int\limits_{(-1,2)}^{(2,3)} (x+y)\,(\mathrm{d}x+\mathrm{d}y)$. 15

4. Izračunati volumen paraboloida omeđenog plohama: $z=\frac{x^2}{3}+\frac{y^2}{3}$, $z=1$. ~~15~~

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z=\frac{x^2}{3}+\frac{y^2}{3}$ što leži iznad područja $D\ldots x^2+y^2\le 1$. Nije potrebno računati površinu baze. 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~ ~~0~~

$$y'''(t)+2y''(t)+y'(t)=0,\quad y(0)=2,\; y'(0)=0,\; y''(0)=0.$$

Ukupno: 12

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int \frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int \sinh x\,\mathrm{d}x=\cosh x+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x=\sinh x+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int \sin x\,\mathrm{d}x=-\cos x+C$	$\int \tanh x\,\mathrm{d}x=\ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \coth x\,\mathrm{d}x=\ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x}=\tanh x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x}=-\coth x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

STIPE VULIĆ 57663-2009

$\boxed{ds = dt}$

① KRUG $r=1$ $T(0,0)$

$\int_{\partial K} xy\,ds = ?$

$r(t) = (\overset{x}{\cos t}, \overset{y}{\sin t})$

$r'(t) = (-\sin t, \cos t)$

$= \int_0^{2\pi} \cos t \cdot \sin t \cdot 1\,dt$ ✓

$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2}$

$\|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t}$

$\|r'(t)\| = \sqrt{1} = 1$ ✓

$= \int_0^{2\pi} \cos t\,dt \cdot \int_0^{2\pi} \sin t\,dt$ ✗

$= \sin t \Big|_0^{2\pi} \cdot -(\cos t)\Big|_0^{2\pi}$

$\boxed{\text{INTEGRAL UMNOŠKA NIJE UMNOŽAK INTEGRALA}}$

$= (\sin 2\pi - \sin 0) \cdot (-\cos 2\pi - \cos 0)$

12

$= 0$

⑥ LAPLACEOVA TRANSFORMACIJA

$y'''(t) + 2y''(t) + y'(t) = 0$

~~$y''(t) + 2y'(t) = 0$~~ !!!

$y(0) = 2$, $y'(0) = 0$, $y''(0) = 0$

JEDNAČBA PRELAZI NA OVAJ OBLIK

$s^3 Y(s) - s^2 \cancel{y(0)}^{2} - s\cancel{y'(0)}^{0} - \cancel{y''(0)}^{0} + 2 \cdot s^2 Y(s) - s\cancel{y(0)}^{2}$

$- \cancel{y'(0)}^{0} + sY(s) - \cancel{y(0)}^{2} = 0$ ✗

$s^3 Y(s) - 2s^2 + 2s^2 Y(s) - 2s + sY(s) - 2 = 0$

$s^3 Y(s) + 2s^2 Y(s) + sY(s) = 2s^2 + 2s + 2$

$S^3 Y(s) + 2S^2 Y(s) + SY(s) = 2s^2 + 2s + 2$

$Y(s)(s^3 + 2S^2 + S) = 2s^2 + 2s + 2$

$$Y(s) = \frac{2s^2 + 2s + 2}{2(S^3 + S^2 + S)}$$

$$Y(s) = \frac{2s^2 + 2s + 2}{\cancel{2}\,2S(S^2 + 1 + 1)} = \frac{A}{2S} + \frac{Bs + C}{S^2 + 1 + 1} \Big/ \cdot \text{NAZNVIK}$$

$$Y(S) = \frac{2s^2 + 2s + 2}{2S(S^2 + 2)} = \frac{A}{2S} + \frac{Bs + C}{S^2 + 2} \Big/ \cdot \text{NAZNVIK}$$

$2S^2 + 2s + 2 = A(s^2 + 1 + 1) + Bs + C(s^2 + 1 + 1)$

$2S^2 + 2s + 2 = As^2 + 2A + \cancel{2}Bs^3 + 2\cancel{s}BS + Cs^2 + 2C$

uz s^3 = $B = 0$

uz s^2 = $2A + 2C = 2$

uz s^1 – $2B = 2$

uz s^0 =

$\cancel{\ }\ \cancel{\ }2C = 2$

$2C = 2\cancel{4}$

$\cancel{4}C = \cancel{8}\cancel{2}$

$C = \cancel{2}$

$C = 1$

$2A + \cancel{\ } = 2$

$2A + \cancel{\ } = 2$

$2A + \cancel{0} = 2$

$2A = 2$

$A = 1$

$$f(s) = \frac{A}{2S} + \frac{Bs + C}{S^2 + S + 1}$$

$$f(s) = \frac{1}{2S} + \frac{0s + \cancel{1}}{S^2 + S + 1} = \frac{1}{2S} + \cancel{0}\,\frac{1}{S^2 + S\cancel{s} + 1} = \frac{1}{2S} + \frac{1}{S^2 \cancel{\ }} + \frac{1}{S+1}$$

$f(s) = \cancel{\ }$ $= 2 + \cos(t)_u + t + e^{-t}$ //

~~0~~

X

BILO BI DOBRO DA STE POKUŠALI PROVJERITI

5.

$$z = \frac{x^2}{3} + \frac{y^2}{3}, \quad x^2 + y^2 \leq 1$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

GDJE JE $S = \{(x,y) \in R^2 : x^2+y^2 \leq 1 \text{ i } x \geq y\}$

$= \int_0^{2\pi} \int_0^1 xy \, dx \, dy = \int_0^{2\pi} \int_0^1 x \, dx + \int_0^{2\pi} \int_0^1 y \, dy$

$= 2\pi \int_0^1 x \, dx + 2\pi \int_0^1 y \, dy$

$= 2\pi \cdot \frac{1}{2} x^2 \Big|_0^1 + 2\pi \cdot \frac{1}{2} y^2 \Big|_0^1$

$= 2\pi \cdot \frac{1}{2}(1^2 - 0^2) + 2\pi \cdot \frac{1}{2}(1^2 - 0^2)$

$= \frac{2\pi}{2} \cdot 1 - 0 + \frac{2\pi}{2} \cdot (1-0)$

$= \frac{2\pi}{2} + \frac{2\pi}{2} = \frac{4\pi}{2} = 2\pi$

IZRAČUNATI

$\int_{(1,2)}^{(2,3)} (x+y)(dx+dy)$

?

VOLUMEN PARABOLOIDA

$\frac{x^2}{3} + \frac{y^2}{3}, \; z = 1$ $= \frac{r^2 \cos^2 \varphi}{3} + \frac{r^2 \sin^2 \varphi}{3}$

$\cos\varphi$ $\varphi \in (0, 2\pi)$ $V = \int_0^{2\pi} \int_0^1 \frac{r^2 \cos^2 \varphi}{3} + \frac{r^2 \sin^2 \varphi}{3} \cdot r \, dz \, dr \, d\varphi$

$\sin\varphi$

$r = \pm z$

$z = \pm r$