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| MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** | Grupa xxoxo |
| **Ime i prezime:** **Broj indeksa:** | Popunjava nastavnik<br>Broj ↓ bodova |

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i $\partial K$ rub kugle $K$ radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2=4$, a iznad stošca $x^2+y^2=z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) $S$ s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\, ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi \vec{\mathbf{j}} + 2\sin \varphi \vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno:

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① $\underbrace{f'''(t) - f''(t) = \cos t}$ $\quad f(0)=0,\ f'(0)=0,\ f''(0)=0$

$\Downarrow$

$$s^3F(s) - s^2\underbrace{f(0)}_{=0} - s\underbrace{f'(0)}_{=0} - \underbrace{f''(0)}_{=0} - \left[s^2F(s) - s\underbrace{f(0)}_{=0} - \underbrace{f'(0)}_{=0}\right] = \frac{s}{s^2+1}$$

$$(s^3 - s^2)F(s) = \frac{s}{s^2+1} \Rightarrow F(s) = \frac{\cancel{s}}{s^{\cancel{2}}(s-1)(s^2+1)} = \frac{1}{s(s-1)(s^2+1)}$$

RASTAV NA PARC. RAZLOMKE ... $\quad F(s) = -\frac{1}{s} + \frac{1}{2}\frac{1}{s-1} + \frac{\frac{1}{2}s - \frac{1}{2}}{s^2+1}$

$$f(t) = \mathcal{L}^{-1}[F(s)] = -1 + \frac{1}{2}e^t + \frac{1}{2}\cos t - \frac{1}{2}\sin t$$

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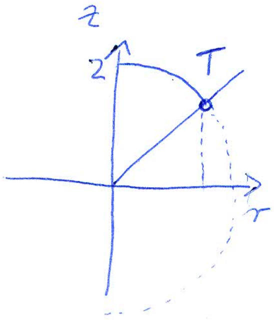
② PLOŠNI INTEGRAL VEKTORSKE FUNKCIJE PO ZATVORENOJ PLOHI PO TEOREMU O DIVERGENCIJI: $\Rightarrow \iiint_K \underbrace{\operatorname{div} F}_{=2x}\, dxdydz = \iiint_K 2x\, dxdydz$

PRELAZAK NA CILINDRIČNE KOORDINATE $\begin{cases} x = r\cos\varphi + 2 \\ y = r\sin\varphi + 1 \\ z = z \end{cases}$ PARAMETRIZACIJA KUGLE $\begin{cases} \varphi \in [0, 2\pi] \\ r \in [0,1] \\ z \in [-\sqrt{1-r^2}, \sqrt{1-r^2}] \end{cases}$

$$\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2(r\cos\varphi + 2)\, r\, dz\, dr\, d\varphi = \underbrace{\int_0^{2\pi}\cos\varphi \int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2r^2\, dz\, dr\, d\varphi}_{=0} + 4\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r\, dz\, dr\, d\varphi =$$

$$= 8\pi \int_0^1 2r\sqrt{1-r^2}\, dr = 8\pi \left[-\frac{(1-r^2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{8\pi}{3}\cdot 2 = \frac{16\pi}{3}$$

(3)




U točki T sijeku se $z^2+r^2=4$ sa $r=z$

$\Rightarrow$ $2r^2=4$

$r^2=2$

$r=\sqrt{2}$

$\varphi\in[0,2\pi]$

$r\in[0,\sqrt{2}]$

$z\in[r,\sqrt{4-r^2}]$

$$V=\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} r\,dz\,dr\,d\varphi=\ldots=2\pi\left(\frac{13}{3}-\frac{4\sqrt{2}}{3}\right)$$

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(4)

$$\int_S (x+2y)\,ds=\left\{ r(t)=\begin{pmatrix}\cos 2t\\ \sin 2t\\ t\end{pmatrix},\ \dot r(t)=\begin{pmatrix}-2\sin 2t\\ 2\cos 2t\\ 1\end{pmatrix},\ \|\dot r(t)\|=\sqrt{4\sin^2(2t)+4\cos^2(2t)+1}=\sqrt{5}\right\}$$

$$=\int_0^{3\pi}\left(\cos(2t)+2\sin(2t)\right)\cdot\sqrt{5}\,dt=\ldots=0$$

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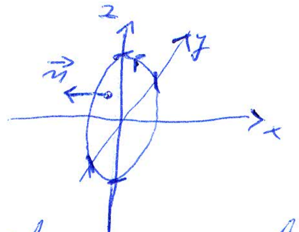
(5)

$$\int_{\vec K} y\,dx+y\,dy=\left\{\begin{array}{l}\vec K \ldots\ r(\varphi)=2\cos\varphi\,\vec j+2\sin\varphi\,\vec k\\ r(\varphi)=\begin{pmatrix}0\\ 2\cos\varphi\\ 2\sin\varphi\end{pmatrix},\ \dot r(\varphi)=\begin{pmatrix}0\\ -2\sin\varphi\\ 2\cos\varphi\end{pmatrix}\end{array}\right\}$$

STOKESOVA FORMULA VRIJEDI ZA ZATVORENE KRIVULJE PA SE STOGA $\varphi\in[0,2\pi]$




$$w=\begin{pmatrix}y\\ y\\ 0\end{pmatrix}\Rightarrow \operatorname{rot} w=\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix}\times\begin{pmatrix}y\\ y\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 0\\ -1\end{pmatrix}$$

$\vec n$ jedinična normala na skicirani krug $=\begin{pmatrix}-1\\ 0\\ 0\end{pmatrix}$

$$\Rightarrow \iint_{\substack{\text{skicirani}\\ \text{krug}}} \operatorname{rot} w\,dS=\iint_{\text{domena}} \underbrace{\begin{pmatrix}0\\ 0\\ -1\end{pmatrix}\cdot\begin{pmatrix}-1\\ 0\\ 0\end{pmatrix}}_{=0}=0$$

domena $=\{(u,v):\ u^2+v^2\le 4\}$

skicirani krug $=\{(x,y,z):\ x=0,\ y^2+z^2\le 4\}$

parametrizacija $r(u,v)=\begin{pmatrix}0\\ v\\ u\end{pmatrix}$ usklađena s normalom $\vec n=\begin{pmatrix}-1\\ 0\\ 0\end{pmatrix}$