MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: **BROJ INDEKSA:**	Grupa xxoox
	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20
$$x'''(t)+x'(t)=0, \quad x(0)=x'(0)=x''(0)=1.$$
4. Neka je C cilindar zadan sa $C=\{(x,y,z):(x+2)^2+(y-3)^2\le 1, -1\le z\le 1\}$. Izračunati plošni integral 20
$$\iint\limits_{\widehat{\partial C}} 2x\,dydz$$
5. Izračunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx+\ln x\cos y\,dy$ 20

Ukupno:

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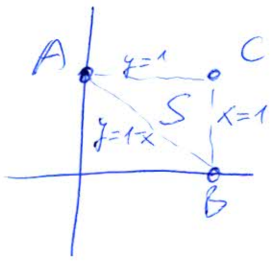
①



$$\iint\limits_S e^{x+y}dxdy=\int_0^1\int_{1-x}^1 \underbrace{e^{x+y}}_{=e^x e^y}dydx=\int_0^1 e^x\left[e^y\right]_{1-x}^1dx$$
$$=\int_0^1 e^x(e^1-e^{1-x})dx=\int_0^1 e\cdot e^x dx-\int_0^1 e\,dx=e\left[e^x\right]_0^1-e=e^2-2e$$

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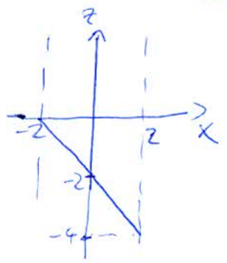
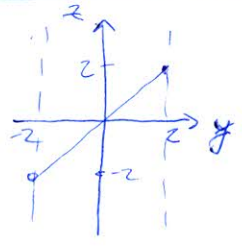
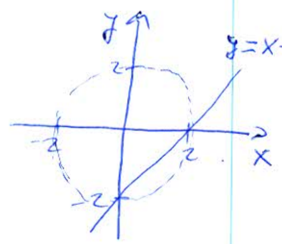
②







$y=x-2$ ← NA NJEMU SE SIJEKU RAVNINE $\begin{cases} z=y \\ z=x-2 \end{cases}$

IZNAD I LIJEVO OD PRAVCA $y=x-2$ "NAD" xy-RAVNINOM PLOHA $z=y$ JE IZNAD PLOHE $z=x-2$. VOLUMEN TOG DIJELA PROSTORA → V_1

$$V_1=\int_0^{\frac{3\pi}{2}}\int_0^2\int_{r\cos\varphi-2}^{r\sin\varphi} r\,dz\,dr\,d\varphi+\int_0^2\int_{x-2}^0\int_{x-2}^{y} dz\,dy\,dx=\ldots=5\frac{1}{3}+6\pi+\frac{4}{3}=6\frac{2}{3}+6\pi$$

ostatak:

$$V_2=\int_{\frac{3\pi}{2}}^{2\pi}\int_0^2\int_{r\sin\varphi}^{r\cos\varphi-2} r\,dz\,dr\,d\varphi-\int_0^2\int_{x-2}^0\int_y^{x-2} dz\,dy\,dx=\ldots=6\frac{2}{3}-2\pi$$

UKUPNO:

$$V=13\frac{1}{3}+4\pi$$

3)

$x'''(t) + x'(t) = 0$ $\qquad x(0)=1,\ x'(0)=1,\ x''(0)=1$

$$\Downarrow$$

$$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + sF(s) - f(0) = 0$$

$$s^3F(s) - s^2 - s - 1 + sF(s) - 1 = 0$$

$$F(s)\left[s^3+s\right] = s^2+s+2 \Rightarrow F(s) = \frac{s^2+s+2}{s(s^2+1)}$$

RASTAV NA PARCIJALNE RAZLOMKE: $F(s) = \frac{2}{s} + \frac{-s+1}{s^2+1} \Rightarrow x(t) = 2 - \cos t + \sin t$; $x(0)=1$

PROVJERA: $x'(t) = \sin t + \cos t \ \ldots\ x'(0)=1$

$x''(t) = \cos t - \sin t \ \ldots\ x''(0) = 1$

$x'''(t) = -\sin t - \cos t$

ODJ: $(-\sin t - \cos t) + (\sin t + \cos t) = 0$

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4)

$W = \begin{bmatrix} 2x \\ 0 \\ 0 \end{bmatrix}$ $\qquad \operatorname{div} W = 2$

TM. O DIVERGENCIJI:

$$\iint_{\partial C} 2x\, dy\,dz = \iiint_C 2\, dx\,dy\,dz = \left\{ \begin{array}{l} x = r\cos\varphi + 2 \\ y = r\sin\varphi - 3 \\ z = z \end{array} \middle| \begin{array}{l} r\in[0,1] \\ \varphi\in[0,2\pi] \\ z\in[-1,1] \end{array} \right\}$$

$$= \int_0^{2\pi}\int_0^1\int_{-1}^1 r\, dz\,dr\,d\varphi = 4\pi\left(\frac{r^2}{2}\right)\Big|_0^1 = 2\pi$$

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5)

PROVJERA DAJE POTENCIJALNO POLJE: $W = \begin{pmatrix} \frac{\sin y}{x} \\ \ln x \cos y \end{pmatrix} = -\operatorname{grad} f \Rightarrow$

$\partial_x f = -\frac{\sin y}{x} \Rightarrow f = -\int \frac{\sin y}{x} dy$

$\Rightarrow f(x,y) = -\sin y \ln x + C(y)$

$\partial_y f = -\ln x \cos y$

$\Rightarrow -\cos y \ln x + C'(y) = -\ln x \cos y$

$\Rightarrow C'(y) = 0 \Rightarrow f(x,y) = -\sin y \ln x + C$

$$\Rightarrow \int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y\, dy = f(1,0) - f(e,\pi)$$

$$= -\underbrace{\sin 0}_{=0} \ln 1 + \underbrace{\sin\pi}_{=0} \ln e$$

$$= 0$$