MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. 20

4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$.

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno:

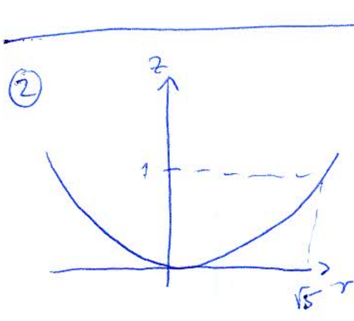
(1) $s^3F(s) - s^2\underbrace{f(0)}_{=1} - s\underbrace{f'(0)}_{=0} - \underbrace{f''(0)}_{=1} + s^2F(s) - s\underbrace{f(0)}_{=1} - \underbrace{f'(0)}_{=0} = \dfrac{1}{s^2+1}$

$(s^3+s^2)F(s) = s^2+s+1+\dfrac{1}{s^2+1}$, $F(s) = \dfrac{s^2+s+1}{s^2(s+1)} + \dfrac{1}{s^2(s+1)(s^2+1)}$

RASTAV NA PARCIJALNE RAZLOMKE $\Rightarrow F(s) = \dfrac{2}{s^2} - \dfrac{1}{s} + \dfrac{3}{2}\cdot\dfrac{1}{s+1} + \dfrac{-\frac{1}{2}+\frac{1}{2}s}{1+s^2}$

INVERZNI LAPLACE $\Rightarrow f(t) = 2t - 1 + \frac{3}{2}e^{-t} + \frac{1}{2}(\cos t - \sin t)$

(2)



$r^2 = 5z$

$z = \dfrac{r^2}{5}$

$r = \sqrt{5z}$

GORNJA PLOHA JE KRUG RADIJUSA $\sqrt{5}$, $P_1 = (\sqrt{5})^2 \cdot \pi = 5\pi$

DONJA PLOHA JE PARABOLOID SA PARAMETRIZACIJOM $r(u,v) = \cancel{\begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ \frac{r^2}{5} \end{pmatrix}} = \begin{pmatrix} u \\ v \\ \frac{u^2+v^2}{5} \end{pmatrix}$

$\vec{n} = \dfrac{\partial r}{\partial u} \times \dfrac{\partial r}{\partial v} = \begin{pmatrix} 1 \\ 0 \\ \frac{2}{5}u \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{2}{5}v \end{pmatrix} = \begin{pmatrix} -\frac{2}{5}u \\ -\frac{2}{5}v \\ 1 \end{pmatrix}$, $\|\vec{n}\| = \sqrt{\frac{4}{25}(u^2+r^2)+1}$

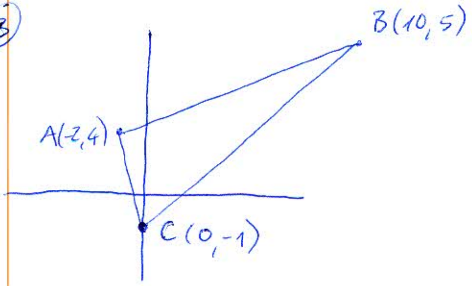
POVRŠINA PLOHE PARABOLOIDA

$P_2 = \iint\limits_{\substack{\text{KRUG} \\ \text{RADIJUSA} \\ R=\sqrt{5}}} \|\vec{n}\| = \int_0^{\sqrt{5}}\int_0^{2\pi} \sqrt{\frac{4}{25}r^2+1} \cdot r\,d\varphi\,dr = 2\pi\left[\frac{25}{2\cdot 4}\,\frac{\left(\frac{4}{25}r^2+1\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\sqrt{5}} = \ldots = \frac{25\pi}{6}\left(\left(\frac{9}{5}\right)^{\frac{3}{2}} - 1\right)$

4

$$\int_0^{\frac{3\pi}{2}}\int_0^{2} \frac{2}{\cancel{r}}\cdot \cancel{r}\,dr\,d\varphi = 2\cdot 2\cdot \frac{3\pi}{2} = 6\pi$$

3

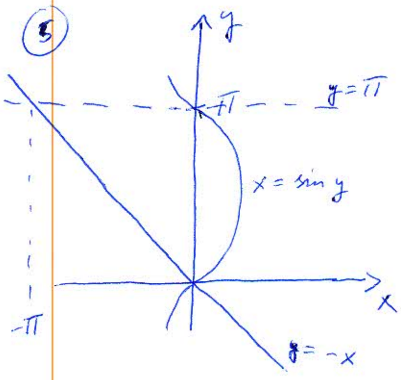


GREENOVA FORMULA:

$$\oint_{\overleftarrow{ABC}} \underbrace{(x^2-y)}_{P}\,dx + \underbrace{\sin(y^3)}_{Q}\,dy = \iint_{ABC} \underbrace{\frac{\partial}{\partial x}\sin(y^3)}_{=0} - \underbrace{\frac{\partial}{\partial y}(x^2-y)}_{=-1}\,dx\,dy$$

$$= \iint_{ABC} dx\,dy = \ldots = 31$$

5



$$I = \int_0^{\pi}\int_{-y}^{\sin y} -y \quad dx\,dy = -\int_0^{\pi} y\sin y + y^2\,dy = \left[\frac{y^3}{3} - t\cos t + \sin t\right]_0^{\pi}$$

$$= \frac{\pi^3}{3} + \pi$$