

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa
XXXXX
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: DOMAGOJ KNEŽEVIĆ

BROJ INDEKSA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

3. Zadan je trokut s vrhovima $A(-2, 4)$, $B(10, 5)$ i $C(0, -1)$. Izračunati $\oint_{\widehat{ABC}} (x^2 - y) dx + \sin(y^3) dy$. 20

4. Izračunati integral funkcije $f(x, y) = \frac{2}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x, y) = -y$ na području X koje je ograničeno krivuljama $X \dots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$
Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno:

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1. $y'''(t) + y''(t) = \sin(t)$ $y'(0) = 0$ $y(0) = 1$ $y''(0) = 1$

$$s^3 F(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 F(s) - s y(0) - y'(0) = \frac{1}{s^2 + 1}$$

$$F(s)[s^3 + s^2] - s^2 - 1 - s = \frac{1}{s^2 + 1}$$

$$\frac{1}{s^2 + 1} + s^2 + s + 1 = \frac{1 + s^4 + s^2 + s^3 + s + s^2 + 1}{s^2 + 1}$$

$$F(s)[s^3 + s^2] = \frac{s^4 + s^2 + s^2 + 1 + s^3 + s + 1}{s^2 + 1} = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2 + 1}$$

$$F(s) = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$$

$$s^4 + s^3 + 2s^2 + s + 2 = As(s^3 + s^2 + 1) + B(s^3 + s^2 + s + 1) + Cs^2(s^2 + 1) + (Ds + E)(s^3 + s^2)$$

$$s^4 + s^3 + 2s^2 + s + 2 = \underline{A}s^4 + \underline{A}s^3 + \underline{A}s^2 + \underline{A}s + \underline{B}s^3 + \underline{B}s^2 + \underline{B}s + \underline{B} + \underline{C}s^4 + \underline{C}s^2 + \underline{D}s^4 + \underline{D}s^3 + \underline{E}s^3 + \underline{E}s^2$$

(1) $B = 2$

(2) $A + B = 1$

$A = -1$

(3) $A + B + C + E = 2$

$C + E = 1$

(4) $A + D + E = 1$

$D + E = 2$

$D = 2 - E$

$D = 2$

(5)

$A + C + D = 1$

$C + D = 2$

$C + 2 - E = 2$

$C + E = 0$
 $C + E = 1$

$C = 1$

$C + E = 1$

$E = 0$

$E = 0$

$$F(s) = -\frac{1}{s} + 2\frac{1}{s^2} + \frac{1}{s+1} + 2\frac{s}{s^2+1}$$

$$F(s) = -1 + 2t + e^{-t} + 2\cos t$$

$$f'(t) = 2 - e^{-t} - 2\sin t$$

PROVJERA:

$f(0) = 2$

$f'(0) = 1$

$$(2) \quad x^2 + y^2 = 5z \quad z \leq 1$$

$$\begin{aligned} x &= r \cos \varphi \\ y &= r \sin \varphi \\ z &= z \end{aligned}$$

$$\begin{aligned} \varphi &\in [0, 2\pi] \\ r &\in [0, \sqrt{5z}] \\ z &\in [0, 1] \end{aligned}$$

$$r^2 \cos^2 \varphi + r^2 \sin^2 \varphi = 5z$$

$$r^2 (\cos^2 \varphi + \sin^2 \varphi) = 5z$$

$$r^2 = 5z / \sqrt{\quad} \Rightarrow r = 0 \quad \begin{matrix} 5z = 0 \\ z = 0 \end{matrix}$$

$$r = \sqrt{5z}$$

$$\begin{aligned} P &= \int_0^{2\pi} \int_0^1 \int_0^{\sqrt{5z}} r \, dr \, dz \, d\varphi = \int_0^{2\pi} \int_0^1 \left(\frac{r^2}{2} \right) \Big|_0^{\sqrt{5z}} dz \, d\varphi = \int_0^{2\pi} \int_0^1 \frac{5}{2} z \, dz \, d\varphi = \int_0^{2\pi} \left(\frac{5}{2} \cdot \frac{1}{2} z^2 \Big|_0^1 \right) d\varphi \\ &= \int_0^{2\pi} \frac{5}{4} d\varphi = \frac{5}{4} \varphi \Big|_0^{2\pi} = \frac{5}{4} \cdot 2\pi = \frac{5}{2} \pi \end{aligned}$$

3. $A(-2, 4)$
 $B(10, 5)$
 $C(0, -1)$

$$\bar{I} = \oint_{ABC} (x^2 - y) dx + \sin(y^3) dy$$

$$P = x^2 - y$$

$$\frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial P}{\partial y} = -1$$

$$Q = \sin(y^3)$$

$$\bar{I} = \iint (0 - 1) dy dx = - \iint dy dx$$

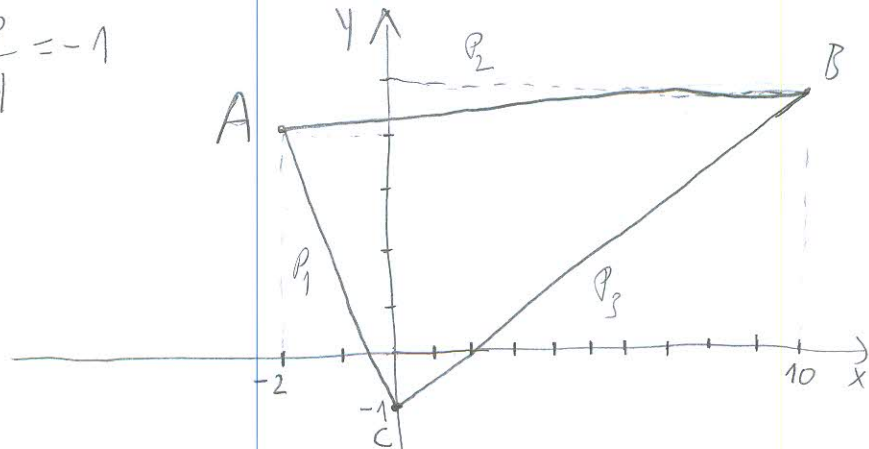
$$\bar{I} = I_1 + I_2$$

$$I_1 = \int_{-2}^0 \int_{P_1}^{P_2} dy dx \quad \checkmark$$

$$I_2 = \int_0^{10} \int_{P_3}^{P_2} dy dx \quad \checkmark$$

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DOMAGOJ KNEŽEVIĆ



$$\begin{matrix} x_1 & y_1 \\ A & (-2, 4) \\ x_2 & y_2 \\ B & (10, 5) \end{matrix}$$

$$y - 4 = \frac{5 - 4}{10 - (-2)} (x + 2)$$

$$y = \frac{1}{12}x + \frac{1}{6} + 4$$

$$P_2 \dots y = \frac{1}{12}x + \frac{25}{6}$$

$$\begin{matrix} x_1 & y_1 \\ B & (10, 5) \\ x_2 & y_2 \\ C & (0, -1) \end{matrix}$$

$$y - 5 = \frac{-1 - 5}{0 - 10} (x - 10)$$

$$y = \frac{6}{10}x - 6 + 5$$

$$P_3 \dots y = \frac{6}{10}x - 1$$

$$\begin{matrix} x_1 & y_1 \\ A & (-2, 4) \\ x_2 & y_2 \\ C & (0, -1) \end{matrix}$$

$$y - 4 = \frac{-1 - 4}{0 - (-2)} (x + 2)$$

$$y = -\frac{5}{2}x - 5 + 4$$

$$P_1 \dots y = -\frac{5}{2}x + 1$$

$$(4.) f(x, y) = \frac{2}{\sqrt{x^2 + y^2}}$$

$$\varphi \in [0, \frac{3\pi}{2}]$$

$$n = 2$$

$$x = r \cos \varphi$$

$$r \in [0, 2]$$

$$y = r \sin \varphi$$

$$\sqrt{r^2 \cos^2 \varphi + r^2 \sin^2 \varphi} = \sqrt{x^2 + y^2}$$

$$I = \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{\sqrt{r^2(\cos^2 \varphi + \sin^2 \varphi)}} r dr d\varphi = \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{\sqrt{r^2}} r dr d\varphi = \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{r} \cdot r dr d\varphi$$

$$= \int_0^{\frac{3\pi}{2}} \int_0^2 2 dr d\varphi = \int_0^{\frac{3\pi}{2}} (2r \Big|_0^2) d\varphi = \int_0^{\frac{3\pi}{2}} 4 d\varphi = 4\varphi \Big|_0^{\frac{3\pi}{2}} = 4 \cdot \frac{3\pi}{2}$$

$$I = 6\pi$$



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IME I PREZIME: *Louvo Sonec*

BROJ INDEKSA: *57638-2009*

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Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom.

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Ukupno:

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2. $x^2 + y^2 = 5z, z \leq 1$

$$\begin{bmatrix} r \cos \varphi \\ r \sin \varphi \\ z \end{bmatrix}$$

$$r^2 = 5z$$

$$r = \sqrt{5z}$$

$$z \in [0, 1]$$

$$\varphi \in [0, 2\pi]$$

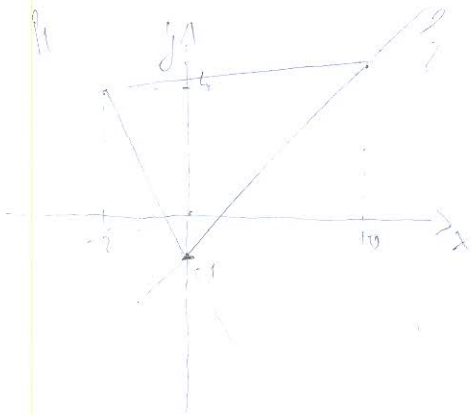
$$r \in [0, \sqrt{5z}]$$

$$I = \int_0^{2\pi} \int_0^1 \int_0^{\sqrt{5z}} r dr d\varphi dz = 2\pi \int_0^1 \frac{r^2}{2} \Big|_0^{\sqrt{5z}} dz = 2\pi \int_0^1 \frac{1}{2} 5z dz$$

$$= 2\pi \frac{1}{2} \int_0^1 5z dz = \pi \cdot 5 \frac{z^2}{2} \Big|_0^1 = \frac{5}{2} \pi \cdot \frac{1}{2} = \frac{5\pi}{4}$$

③ $A(-2, 4)$ $B(10, 5)$ $C(0, -1)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$



$$p_1 \dots y - 4 = \frac{5 - 4}{10 - 2} \cdot (x + 2)$$

$$y - 4 = \frac{1}{12} \cdot (x + 2)$$

$$y - 4 = \frac{1}{12}x + \frac{2}{12}$$

$$y - 4 = \frac{1}{12}x + \frac{1}{6}$$

$$y = \frac{1}{12}x + \frac{5}{6}$$

$$p_2 \dots y - 5 = \frac{-1 - 5}{0 - 10} \cdot (x - 10)$$

$$y - 5 = \frac{-6}{-10} \cdot (x - 10)$$

$$y - 5 = -\frac{3}{5}x + 6$$

$$y = -\frac{3}{5}x + 11$$

④

$$f(x, y) = \frac{2}{\sqrt{x^2 + y^2}}$$

$$x = \cos \varphi \quad \varphi \in [0, \frac{3\pi}{2}]$$

$$y = \sin \varphi \quad r \in [0, 2]$$

$$I = \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{\sqrt{(\cos \varphi)^2 + (\sin \varphi)^2}} r dr d\varphi = \int_0^{\frac{3\pi}{2}} \int_0^2 2r dr d\varphi$$

$$I = \int_0^{\frac{3\pi}{2}} 2 \cdot \frac{r^2}{2} \Big|_0^2 d\varphi = \int_0^{\frac{3\pi}{2}} 4 d\varphi = 4\varphi \Big|_0^{\frac{3\pi}{2}} = 4 \cdot \frac{3\pi}{2} = \underline{\underline{6\pi}}$$

