MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** DOMAGOJ KNEŽEVIĆ **BROJ INDEKSA:**	Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \le 1$. ~~20~~

3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. ~~20~~ 15

4. Izračunati integral funkcije $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$ 20

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom.

Ukupno: 35

(1.) $f'''(t) + f''(t) = \sin(t)$ $\quad f'(0) = 0$ $\quad f(0) = 1$ $\quad f''(0) = 1$

$s^3 F(s) - s^2 f(0) - s\cancel{f'(0)}^{0} - f''(0) + s^2 F(s) - s f(0) - \cancel{f'(0)}^{0} = \frac{1}{s^2+1}$

$F(s)[s^3+s^2] - s^2 - 1 - s = \frac{1}{s^2+1}$ $\qquad \frac{1}{s^2+1} + s^2 + s + 1 = \frac{1+s^4+s^2+s^3+s+s^2+1}{s^2+1}$

$F(s)[s^3+s^2] = \frac{s^4+s^2+s^2+1+s^3+s+1}{s^2+1} = \frac{s^4+s^3+2s^2+s+2}{s^2+1}$

$F(s) = \frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \Big/ \cdot s^2(s+1)(s^2+1)$

$s^4+s^3+2s^2+s+2 = As(s^3+s+s^2+1) + B(s^3+s^2+s+1) + Cs^2(s^2+1) + (Ds+E)(s^3+s^2)$

$s^4+s^3+2s^2+s+2 = As^4+As^2+As^3+As+Bs^3+Bs^2+Bs+B+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$

(0) $\boxed{B=2}$

(1) $A+B=1$
$\boxed{A=-1}$

(2) $A+B+C+E=2$
$C+E=1$

(3) $A+D+E=1$
$D+E=2$
$D=2-E$
$\boxed{D=2}$

(4)
$A+C+D=1$
$C+D=2$

$C + \cancel{2} - E = \cancel{2}$

$C+E=0$
$C+E=1$ +

$\boxed{C=1}$

$C+E=1$
$\cancel{1}+E=\cancel{1}$
$\boxed{E=0}$

$F(s) = -\frac{1}{s} + 2\frac{1}{s^2} + \frac{1}{s+1} + 2\frac{s}{s^2+1}$

$\boxed{F(s) = -1 + 2t + e^{-t} + 2\cos t}$

$f'(t) = 2 - e^{-t} - 2\sin t$

PROVJERA:
$f(0) = 2$
$f'(0) = 1$

(2.) $x^2+y^2=5z \qquad z \le 1$

$x = r\cos\varphi \qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi \qquad r \in [0, \sqrt{5z}]$

$z = z \qquad z \in [0, 1]$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 5z$

$r^2(\cos^2\varphi + \sin^2\varphi) = 5z$

$r^2 = 5z \;/\sqrt{\ } \quad \Rightarrow \quad r=0 \qquad 5z=0$

$z=0$

$r = \sqrt{5z}$

$$P = \int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{5z}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^{1} \left(\frac{r^2}{2}\Big|_0^{\sqrt{5z}}\right) dz\,d\varphi = \int_0^{2\pi}\int_0^{1} \frac{5}{2} z\,dz\,d\varphi = \int_0^{2\pi}\left(\frac{5}{2}\cdot\frac{1}{2}z^2\Big|_0^1\right)d\varphi$$

$$= \int_0^{2\pi} \frac{5}{4}\,d\varphi = \frac{5}{4}\varphi\Big|_0^{2\pi} = \frac{5}{4}\cdot 2\pi = \frac{5}{2}\pi$$

DOMAGOJ KNEŽEVIĆ

3.

$A(-2,4)$

$B(10,5)$

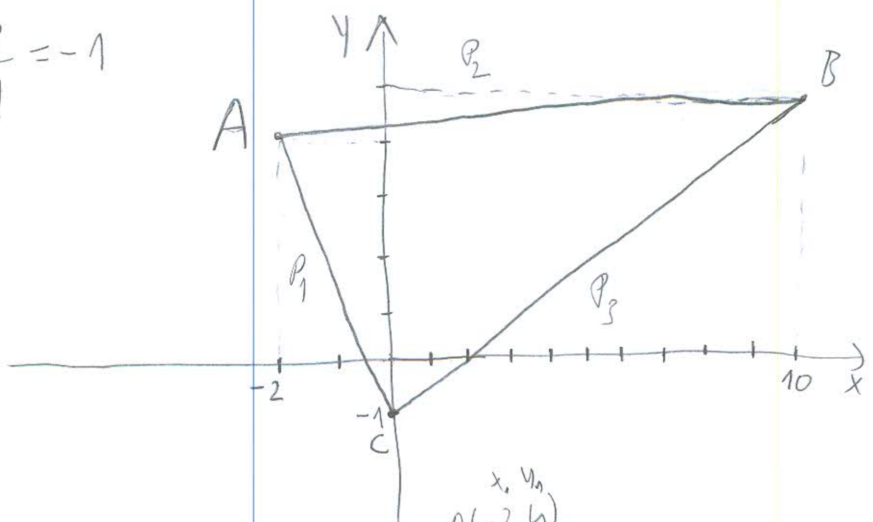
$C(0,-1)$

$$I = \oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$$

$P = x^2 - y$

$Q = \sin(y^3)$

$\frac{\partial Q}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = -1$



$$I = \iint (0+1)\,dy\,dx = \iint dy\,dx$$

$I = I_1 + I_2$

$$I_1 = \int_{-2}^{0}\int_{p_1}^{p_2} dy\,dx$$ ✓

$$I_2 = \int_{0}^{10}\int_{p_3}^{p_2} dy\,dx$$ ✓

15

$A(-2,4)$

$B(10,5)$

$y - 4 = \frac{5-4}{10+2}(x+2)$

$y = \frac{1}{12}x + \frac{1}{6} + 4$

$p_2 \ldots\ y = \frac{1}{12}x + \frac{25}{6}$

$B(10,5)$

$C(0,-1)$

$y - 5 = \frac{-1-5}{0-10}(x-10)$

$y = \frac{6}{10}x - 6 + 5$

$p_3 \ldots\ y = \frac{6}{10}x - 1$

$A(-2,4)$

$C(0,-1)$

$y - 4 = \frac{-1-4}{0+2}(x+2)$

$y = -\frac{5}{2}x - 5 + 4$

$p_1 \ldots\ y = -\frac{5}{2}x - 1$

4.) $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$ $\qquad \varphi \in [0, \frac{3\pi}{2}]$ $\quad \underline{r = 2}$ $\quad x = r\cos\varphi$

$r \in [0, 2]$ $\qquad y = r\sin\varphi$

$$\sqrt{r^2\cos^2\varphi + r^2\sin^2\varphi} = \sqrt{x^2+y^2}$$

$$I = \int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{\sqrt{r^2(\cos^2\varphi + \sin^2\varphi)}}\, r\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{\sqrt{r^{\cancel{2}}}}\, r\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}}\int \frac{2}{\cancel{r}}\cdot \cancel{r}\,dr\,d\varphi$$

$$= \int_0^{\frac{3\pi}{2}}\int_0^2 2\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}}\left(2r\Big|_0^2\right)d\varphi = \int_0^{\frac{3\pi}{2}} 4\,d\varphi = 4\varphi\Big|_0^{\frac{3\pi}{2}} = \overset{2}{\cancel{4}}\cdot\frac{3\pi}{\cancel{2}}$$

$$\boxed{I = 6\pi}$$

✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**	Grupa xxoxx
Ime i prezime: Lovro Sorić **Broj indeksa:** 57638-2009	Popunjava nastavnik Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. ~~20~~
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   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 20

2) $x^2+y^2=5z, \quad z \leq 1$

$$\begin{bmatrix} r\cos x \\ r\sin y \\ z \end{bmatrix} \qquad r^2 = 5z \qquad z \in [0,1]$$

$$r = \sqrt{5z} \qquad \varphi \in [0, 2\pi]$$

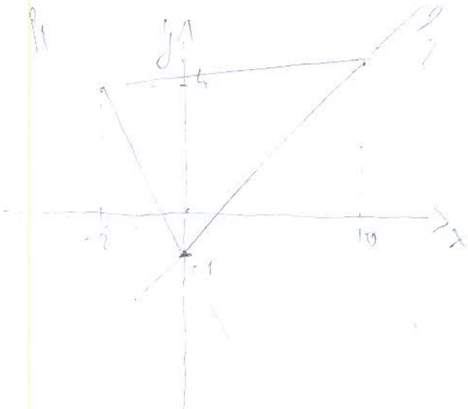
$$r = [0, \sqrt{5z}]$$

$$I = \int_0^{2\pi}\int_0^1\int_0^{\sqrt{5z}} r\,dr\,d\varphi\,dz = 2\pi \int_0^1 \frac{r^2}{2} \Big/_0^{\sqrt{5z}} dz = 2\pi \int_0^1 \frac{1}{2} 5z\,dz$$

$$= 2\pi \frac{1}{2} \int_0^1 5z\,dz = \pi \cdot 5 \frac{z^2}{2} \Big/_0^1 = \frac{5}{2}\pi \cdot \frac{1}{2} = \frac{5\pi}{4}$$

③ $A(-2,4)\ B(10,5)\ C(0,-1)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$$



$p_1 \cdots\quad y-4=\frac{5-4}{10+2}\cdot(x+2)$

$y-4=\frac{1}{12}\cdot(x+2)$

$y-4=\frac{1}{12}x+\frac{\cancel{2}^{1}}{\cancel{12}_{6}}$

$y-4=\frac{1}{12}x+\frac{1}{6}$

$y=\frac{1}{12}x+\frac{5}{2}$

$p_2 \cdots\quad y-5=\frac{-1-5}{0-10}\cdot(x-10)$

$y-5=\frac{-\cancel{6}^{3}}{-\cancel{10}_{5}}\cdot(x-10)$

$y-5=-\frac{3}{5}x-6$

$y=-\frac{3}{5}x-1$

?

④

$$\sigma(x,y)=\frac{2}{\sqrt{x^2+y^2}}$$

$x=\cos\varphi \qquad \varphi\in[0,\frac{3\pi}{2}]$

$y=\sin\varphi \qquad r\in[0,2]$

$$I=\int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{\underbrace{\sqrt{(\cos\varphi)^2+(\sin\varphi)^2}}_{1}}\,r\,dr\,d\varphi=\int_0^{\frac{3\pi}{2}}\int_0^2 2r\,dr\,d\varphi$$

$$I=\int_0^{\frac{3\pi}{2}}2\,\frac{r^2}{2}\Big/_0^2\,d\varphi=\int_0^{\frac{3\pi}{2}}4\,d\varphi=4\varphi\Big/_0^{\frac{3\pi}{2}}=4\cdot\frac{3\pi}{2}=\underline{6\pi}$$

✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Mateja Mitrović

BROJ INDEKSA:

Grupa xxoxx
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. ~~20~~
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C} (x^2 - y)\, dx + \sin(y^3)\, dy$. ~~20~~
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   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: Ø

$f'''(t) = s^3F(s) - s^2 \underbrace{f(0)}_{1} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{1} = s^3F(s) - 2$ f'(0

$f''(t) = s^2F(s) - s\underbrace{f(0)}_{1} - \underbrace{f'(0)}_{0} = s^2F(s) - 1$

$s^3F(s) - 2 + s^2F(s) - 1 = \frac{a}{s^2+a^2}$

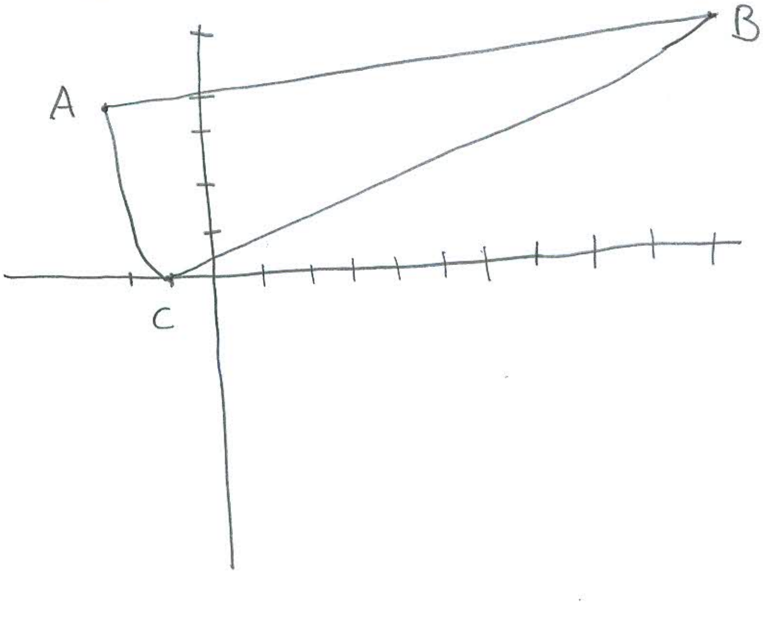
$s^3F(s) + s^2F(s) = \frac{-3}{s^2+9}$

$(s^3+s^2)F(s) = \frac{-3}{s^2+9}$

$F(s) = \frac{s^3+s^2}{\frac{-3}{s^2+9}}$ $F(s) = \frac{-3}{s^3+9}$ $F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3}$

?

2.



$\oint\limits_{ABC} \left(\frac{x^2}{x}dx - \frac{y}{x}dx\right) + \left(\sin^{dy} \frac{y^3}{y} dy\right)$

$\oint\limits_{\overleftarrow{ABC}}$

Ø

$x^2+y^2=5z \quad z\leq 1$

$x^2+y^2=5\cdot 1$

$x^2+y^2=5$

$(x+y)^2=5$

$x_1=0 \; y_1=0$

$x^2+y^2=5$

$x^2=5y^2 /:5$

$x^2=\frac{y^2}{5} /\sqrt{}$

$x=\sqrt{\frac{y}{5}}$

$\int_0^5 (x+y+z)\,dx=\int_0^{\frac{y}{5}} x\,dx+\int_0^1 y\,dx+\int_0^1 d\,dz=$

$\varnothing$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: MATEJ ČURK **BROJ INDEKSA:** 57331	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20

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Ukupno: 0

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: Igor Brajica BROJ INDEKSA: 52803-2005	POPUNJAVA NASTAVNIK Broj ↓ bodova

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   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: $\cancel{0}$

(1) $f'''(t) + f''(t) = \sin(t)$

$f'(0) = 0$

$\cancel{f(0) = 0}$

$f''(0) = 1$

$$s^3 F(s) - \cancel{s^2 f(0)} - \cancel{s f'(0)} - f''(0) + s^2 F(s) - \cancel{s f(0)} - \cancel{f'(0)} = \frac{1}{s^2+1}$$

$$s^3 F(s) - 1 + s^2 F(s) = \frac{1}{s^2+1}$$

$$s^3 F(s) + s^2 F(s) = \frac{1}{s^2+1} + 1$$

$$F(s)(s^3 + s) = \frac{1 + s^2 + 1}{s^2 + 1}$$

$$F(s) = \frac{s^2 + 2}{(s^2+1)(s^3+s^2)}$$

$$F(s) = \frac{s^2 + 2}{s^2(s+1)(s^2+1)}$$

Igor Brajrca

$$F(s) = \frac{s^2+2}{s^2(s+1)(s^2+1)}$$

$$s^2+2 = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$$

s^3+s+s^2+1

$$s^2+2 = As^2(s+1)(s^2+1) + Bs(s+1)(s^2+1) + Cs^4 + Cs^2 +$$
$$Ds^4 + Ds^2 + Es^3 + Es^2$$

$$s^2+2 = As^3 + As^3 + As$$

?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Domagoj Nekić **BROJ INDEKSA:** 17-2-0028-2010

Grupa xxoxx

POPUNJAVA NASTAVNIK

Broj ↓ bodova

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   Pomoć: $\int y \sin y\,dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: $\emptyset$

3. $A(-2,4)$ $B(10,5)$ $C(0,-1)$

$$y - y_1 = \frac{x_2 - x_1}{y_2 - y_1}(x - x_1)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**

Ime i prezime: ŠKIBOLA MATIJA

Broj indeksa: 54961 2007

Grupa xxoxx

Popunjava nastavnik

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~
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   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom.

Ukupno: Ø

$x'''(t) + x''(t) = \sin t \quad x'(0) = 0,\ x(0) = x''(0) = 1$

$s^3 f(s) - s^2 f(0) - s f'(0) - f''(0) + s^2 f(s) - s f(0) - f'(0) = \frac{1}{s^2+1}$

$s^3 f(s) - s^2 \cdot 1 - s \cdot 0 - 1 + s^2 f(s) - s \cdot 1 - 0 = \frac{1}{s^2+1}$

$s^3 f(s) - s^2 - 1 + s^2 f(s) - s = \frac{1}{s^2+1}$

$f(s)(s^3+s^2) = \frac{1}{s^2+1} + s^2 + s + 1 \Rightarrow \frac{1 + (s^2+1)(s^2+s+1)}{s^2+1} \Rightarrow \frac{1 + s^4 + s^3 + s^2 + s^2 + s + 1}{s^2+1}$

$f(s)(s^3+s^2) = \frac{s^4+s^3+2s^2+s+2}{s^2+1}$

$f(s) = \dfrac{\frac{s^4+s^3+2s^2+s+2}{s^2+1}}{\frac{s^3+s^2}{1}} = \frac{s^4+s^3+2s^2+s+2}{(s^2+1)(s^3+s^2)} = \frac{s^4+s^3+2s^2+s+2}{(s^2+1)s^2(s+1)}$

$f(s) = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2} + \frac{E}{s+1}$

$f(s) = (As+B)(s^2)(s+1) + (Cs+D)(s^2+1)(s+1) + E(s^2+1)(s^2)$

$= (As^3+Bs^2)(s+1) + (Cs^3+Cs+Ds^2+D)(s+1) + (Es^2+E)(s^2)$

$= As^4 + As^3 + Bs^3 + Bs^2 + Cs^4 + Cs^2 + Ds^3 + Ds + Cs^3 + Cs + Ds^2 + D + Es^4 + Es^2$

$s^4+s^3+2s^2+s+2 = s^4(A+C+E) + s^3(A+B+D+C) + s^2(B+C+D+E) + s(D+C) + D$

$A + C + E = 1$

$A + B + C + D = 1$

$B + C + D + E = 2$

$D + C = 1 \Rightarrow C + 2 = 1$

$D = 2$ $C = -1$

$B + C + D + E = 2$

$B - 1 + 2 + E = 2$

$B + 1 + E = 2$

$\boxed{B = 1 - E}$

$A + B + C + D = 1$

$A + B - 1 + 2 = 1$

$A + B + 1 = 1$

$\boxed{A = -B}$

$A + C + E = 1$

$-B - 1 + E = 1$

$-1 + E - 1 + E = 1$

$-2 + 2E = 1$

$2E = 3$

$E = \frac{3}{2}$

$B + C + D + E = 2$

$B - 1 + 2 + \frac{3}{2} = 2$

$B = 2 + 1 - 2 - \frac{3}{2}$

$B = 1 - \frac{3}{2}$

$B = -\frac{1}{2}$

$A + B + C + D = 1$

$A - \frac{1}{2} - 1 + 2 = 1$

$A = 1 + \frac{1}{2} + 1 - 2$

$A = \frac{1}{2}$

$$\frac{As + B}{s^2 + 1} + \frac{Cs + D}{s^2} + \frac{E}{s + 1} = \frac{\frac{1}{2}s - \frac{1}{2}}{s^2 + 1} + \frac{-s + 2}{s^2} + \frac{\frac{3}{2}}{s + 1}$$

$$\frac{1}{2} \cdot \frac{s}{s^2 + 1} - \frac{1}{2} \cdot \frac{1}{s^2 + 1} - \frac{s}{1} \cdot \frac{2}{s^2} + \frac{3}{2} \cdot \frac{1}{s + 1}$$

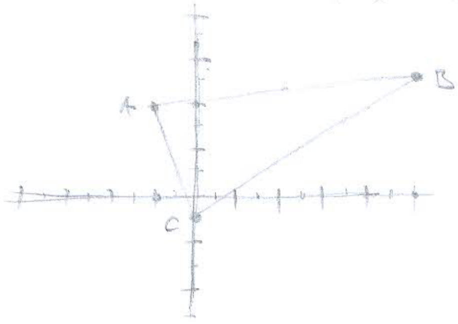
$$\frac{1}{2}\cos(t) - \frac{1}{2}\sin(t) - \frac{2}{2}\sin(t)$$

PROVJERA

$f(0) = \frac{1}{2}$ ϟ

MATIJA SKIBOLA

3. $A(-2,4)$ $B(10,5)$ $C(0,-1)$



AB

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-4=\frac{5-4}{10+2}(x+2)$$

$$y-4=\frac{1}{12}(x+2)$$

$$y-4=\frac{1}{12}x+\frac{1}{6}$$

$$y=\frac{1}{12}x+\frac{1}{6}+4$$

$$\boxed{y=\frac{1}{12}x+\frac{25}{6}}$$

AC

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-4=\frac{-1-4}{0+2}(x+2)$$

$$y-4=\frac{-5}{2}(x+2)$$

$$y-4=-\frac{5}{2}x-10$$

$$\boxed{y=-\frac{5}{2}x-6}$$

BC

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-5=\frac{-1-5}{0-10}(x-10)$$

$$y-5=\frac{-6}{-10}(x-10)$$

$$y-5=\frac{3}{5}(x-10)$$

$$y-5=\frac{3}{5}x-2 \quad \boxed{y=\frac{3}{5}x+3}$$

$$\int_{-2}^{0}\int_{-\frac{5}{2}x-6}^{\frac{1}{12}x+\frac{25}{6}}(x^2-y)\,dx+\sin(y^3)\,dy \quad \int_{0}^{10}\int_{\frac{3}{5}x+3}^{\frac{1}{12}x+\frac{25}{6}}(x^2-y)\,dx+\sin(y^3)\,dy$$

×

2. $x^2+y^2=5z \qquad z\leq 1$

$r^2=z \qquad \varphi\in[0,2\pi]$

$r^2=5z \qquad r\in[0,5\sqrt{z}]$

$r=5\sqrt{z}$

$z\in[0,1]$

$$\int_0^{2\pi}\int_0^{1}\int_0^{5\sqrt{z}} r\,dr\,d\varphi\,dz=\int_0^{2\pi}d\varphi\int_0^{1}dz\int_0^{5\sqrt{z}}r\,dr$$

×

$x=r\cos\varphi$

$y=r\sin\varphi$

$(r\sin\varphi)^2+(r\cos\varphi)^2=5z$

$r(\sin^2\varphi+\cos^2\varphi)=5z$

$r=5z$

$$\int_0^{2\pi}\int_0^{1}\int_0^{5z} r\,dr\,d\varphi\,dz=\int_0^{2\pi}d\varphi\int_0^{1}dz\int_0^{5z}r\,dr$$

$$=\int_0^{2\pi}d\varphi\int_0^{1}dz\left.\frac{r^2}{2}\right|_0^{5z}=\int_0^{2\pi}d\varphi\int_0^{1}dz\,\frac{1}{2}\,25z^2=\int_0^{2\pi}d\varphi\int_0^{1}dz\,\frac{25}{2}z^2$$

$$\int_0^{2\pi}d\varphi\,\frac{25}{2}\int_0^{1}z^2dz=\int d\varphi\,\frac{25}{2}\left.\frac{z^3}{3}\right|_0^1=\int_0^{2\pi}d\varphi\,\frac{25}{2}\cdot\frac{1}{3}=\int_0^{2\pi}d\varphi\,\frac{25}{6}$$

$$=\frac{25}{6}(2\pi-0)=\frac{25}{3}\pi$$

4. $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$ $\quad r=2 \quad \theta \in \left[0, \frac{3\pi}{2}\right]$

$r \in [0,2]$

$x = r\cos\theta$

$y = r\sin\theta \quad f(x,y) = \int_0^{\frac{3\pi}{2}}\int_0^{2} \frac{2}{\sqrt{x^2+y^2}}$

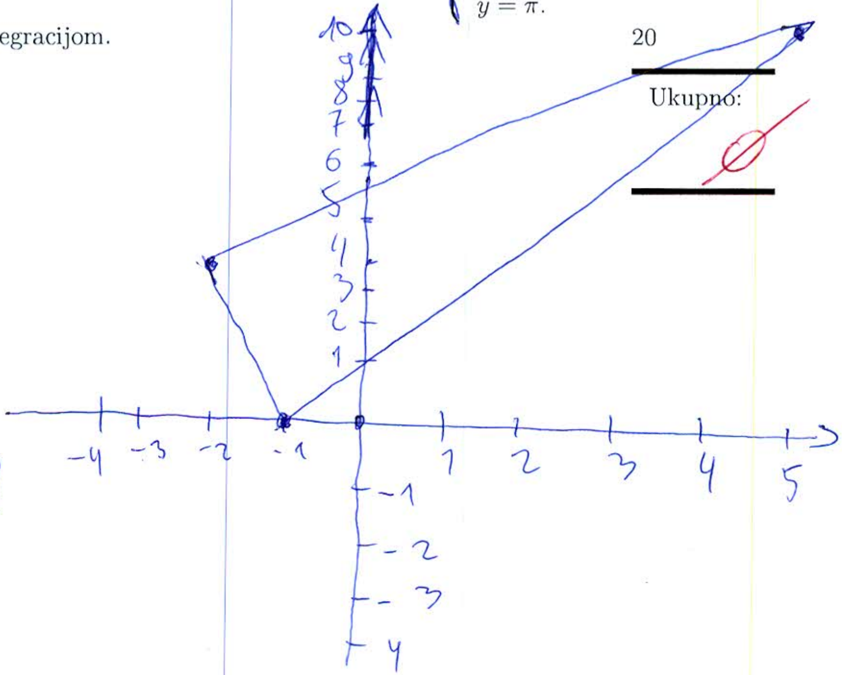
X

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** IME I PREZIME: STIPE VULIĆ BROJ INDEKSA: 57663-2009	Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20
3. Zadan je trokut s vrhovima $A(-2, 4)$ $B(10, 5)$ i $C(0, -1)$. Izracunati $\oint\limits_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. 20
4. Izračunati integral funkcije $f(x, y) = \dfrac{2}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x, y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: Ø



3.

$A(-2,4)$ $B(10,5)$ $C(0,-1)$

$\oint\limits_{ABC} (x^2-y)dx + \sin(y^3)dy = ?$

$P(x,y) = (x^2-y)dx$

$Q(x,y) = \sin(y^3)dy$

EGZAKTNA DIFERENCIJALNA JEDNADŽBA

$\dfrac{\partial P}{\partial y} = -1$ $\qquad \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}$

$\dfrac{\partial Q}{\partial x} = 0$ $\qquad = 0 - 1 = -1$

$$A\begin{pmatrix} x_1 & y_1 \\ -2, & 4\end{pmatrix} \quad B\begin{pmatrix} x_2 & y_2 \\ 10, & 5\end{pmatrix}$$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-4=\frac{5-4}{10+2}(x+1)$$

$$y-4=\frac{1}{12}(x+1)$$

$$y-4=\frac{1}{12}x+\frac{1}{12}$$

$$y=\frac{1}{12}x+\frac{1}{12}+4$$

$$y=\frac{1}{12}x+\frac{49}{12}$$

$$B\begin{pmatrix} x_1 & y_1 \\ 10, & 5\end{pmatrix} \quad C\begin{pmatrix} x_2 & y_2 \\ 0, & -1\end{pmatrix}$$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-5=\frac{-1-5}{0-10}(x-10)$$

$$y-5=\frac{-6}{-10}(x-10)$$

$$y-5=\frac{6}{10}x-1$$

$$y=\frac{6}{10}x-1+5$$

$$y=\frac{6}{10}x-4$$

$$=\int_{-2}^{5}\int_{\frac{1}{12}x+\frac{49}{12}}^{\frac{6}{10}x-4}(-1)\,dx\,dy=$$

×

① LAPLACEOVA TRANSFORMACIJA

$$f'''(t)+f''(t)=\sin t,\ f'(0)=0,\ f(0)=f''(0)=1$$

$$s^3F(s)-s^2\cancel{f(0)}^{1}-s\cancel{f'(0)}^{0}-\cancel{f''(0)}^{1}+s^2F(s)-s\cancel{f(0)}^{1}-\cancel{f'(0)}^{0}=\frac{a}{s^2+a^2}$$

$$s^3f(s)+s^2F(s)-s^2-1-1=\frac{a}{s^2+a^2}$$

$$s^3f(s)+s^2F(s)=s^2+1+1$$

Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: Toni Mika **BROJ INDEKSA:** 53277	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

3. Zadan je trokut s vrhovima $A(-2, 4)$ $B(10, 5)$ i $C(0, -1)$. Izracunati $\oint\limits_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. ✓ ~~20~~

4. Izračunati integral funkcije $f(x, y) = \dfrac{2}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. ~~20~~

5. Odrediti integral funkcije $f(x, y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$.

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 0

1) $s^3F(s) - s^2 \underbrace{f(0)}_{1} - s \underbrace{f'(0)}_{0} - f''(0) + s^2F(s) - s f(0) - f'(0)$

$s^3F(s) - s^2 + 1 + s^2F(s) - s = -\cos t$

$s^3F(s) + s^2F(s) - s^2 + 1 - s = -\cos t$

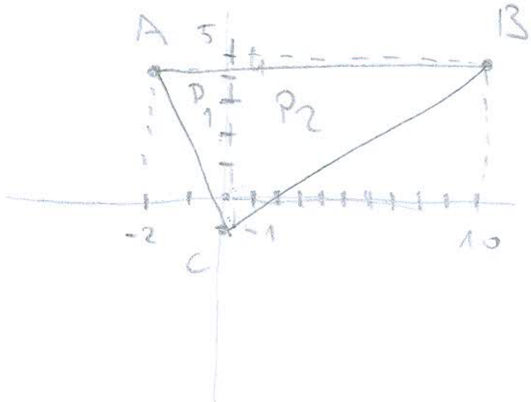
$s^3F(s) + s^2F(s) = -\cos t + s^2 + s - 1 =$

?

3) $A(-2,4)$ $\quad \int (x^2-y^2)dx + \sin(y^3)\,dy$

$B(10,5)$

$C(0,-1)$



$$\overline{AB} = y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$= y-4 = \frac{5-4}{10+2}(x+2)$$

$$y-4 = \frac{1}{12}(x+2)$$

$$y-4 = \frac{1}{12}x+\frac{1}{6}$$

$$y = \frac{1}{12}x+\frac{1}{6}+4$$

$$y = \frac{1x+2+48}{12}$$

$$y = \frac{1}{12}x+\frac{25}{6}$$

$$\overline{AC} = y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$= y-4 = \frac{-1-4}{0+2}(x+2)$$

$$y-4 = \frac{-5}{2}(x+2)$$

$$y-4 = -\frac{5}{2}x-5$$

$$y = -\frac{5}{2}x-5+4$$

$$y = -\frac{5}{2}x-1$$

$$BC = y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$= y-5 = \frac{-1-5}{-1-10}(x-5)$$

$$y-5 = \frac{-6}{-11}(x-5)$$

$$y-5 = \frac{6}{11}x-\frac{30}{11}$$

$$y = \frac{6}{11}x-\frac{30}{11}+5$$

$$y = \frac{6}{11}x-\frac{30+55}{11}$$

$$y = \frac{6}{11}x-\frac{85}{1}$$

$$P_1 = \int_{-2}^{-1}\int_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}}(x^2-y^2)dx$$

$$P_2 = \int_{-1}^{10}\int_{\frac{6}{11}x-\frac{25}{11}}^{\frac{1}{12}x+\frac{25}{6}}\sin(y^3)\,dy$$

$$P = \int_{-2}^{-1}\int_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}}(x^2-y^2)dx + \int_{-1}^{10}\int_{\frac{6}{11}x-\frac{85}{11}}^{\frac{1}{12}x+\frac{25}{6}}\sin(y^3)\,dy =$$

4. $x = r\cos + x$
$y = r\cos + y$

$x = 2\cos + x$
$y = 2\cos + y$

$$\int_0^2\int_0^{\frac{3\pi}{2}} \frac{r\ 2}{\sqrt{x^2+y^2}} =$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**	Grupa xxoxx
Ime i prezime: MARIN MAKAŠ **Broj indeksa:** 57651	Popunjava nastavnik Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. ~~20~~
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint_{A\widehat{B}C} (x^2 - y)\,dx + \sin(y^3)\,dy$. 20
4. Izračunati integral funkcije $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$.

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: $\emptyset$

1. $f'''(t) + f''(t) = \sin(t)$

$f'(0) = 0$

$f(0) = 1$

$f''(0) = 1$

$$s^3 F(s) - s^2 f(0) - s\cancel{f'(0)} - f''(0) + s^2 F(s) - s f(0) - \cancel{f'(0)} = \frac{1}{s^2+1}$$

$$s^3 F(s) - s^2 - 1 + s^2 F(s) - s = \frac{1}{s^2+1}$$

$$s^3 F(s) + s^2 F(s) = \frac{1}{s^2+1} + s^2 + 1 + s$$

$$F(s)(s^3+s^2) = \frac{1 + s^2(s^2+1) + 1(s^2+1) + s(s^2+1)}{s^2+1}$$

$$F(s)(s^3+s^2) = \frac{1 + s^4 + s^2 + s^2 + 1 + s^3 + s}{s^2+1}$$

$$F(s)(s^3+s^2) = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2(s+1)(s^2+1)}$$

$$= \frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$$

$$s^4+s^3+2s^2+s+2 = A(s+1)(s^2+1) + Bs(s+1)(s^2+1) + Cs^2(s^2+1) + (Ds+E)(s+1)s^2$$

$$s^4+s^3+2s^2+s+2 = A(s^3+s+s^2+1) + Bs(s^3+s+s^2+1) + Cs^4+Cs^2+(Ds+E)(s^3+s^2)$$

$$s^4+s^3+2s^2+s+2 = As^3+As+As^2+A+Bs^4+Bs^2+Bs^3+Bs+Cs^4+Cs^2+Ds^4+Ds^3+Es^3 + Es^2$$

$1 = B+C+D$

$1 = A+B+D+E$

$2 = A+B+C+E \rightarrow 2-2+1 = C+E$ $\quad \boxed{-C=E}$

$1 = A+B \rightarrow 1-2=B$

$\boxed{2=A}$ $\quad \boxed{B=-1}$

$1+1 = -\cancel{E}+D \longrightarrow 2 = -E+1$ $\qquad 1 = -1+C+1$

$1-2+1 = \cancel{E}+D$ $\qquad 2-1 = -E$ $\qquad 1+1-1 = C$

$2 = 2D \mid \cdot \frac{1}{2}$ $\qquad -E = 1 \mid \cdot(-1)$ $\qquad \boxed{C=1}$

$2D = 2 \mid \cdot \frac{1}{2}$ $\qquad \boxed{E=-1}$

$\boxed{D=1}$

$$= \frac{2}{s^2} - \frac{1}{s} + \frac{1}{s+1} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$= 2\cdot t - 1 + e^{-t} + \cos(t) - \sin(t)$$

PROVJERA:

$f(0)=1$

$f'(t) = 2 - e^{-t} - \sin t - \cos t$ $\qquad f'(0)=0$

$f''(t) = e^{-t} - \cos t + \sin t$ $\qquad f''(0)=0$

MARAŠ

2. Površina oplošja paraboloida

$x^2 + y^2 = 5z$, $z \leq 1$

$r^2 = 5$

$r = \sqrt{5}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{5}]$

$z \in [0, 1]$

$$\int_0^{2\pi}\int_0^{\sqrt{5z}}\int_0^{1} r \, dr \, d\varphi \, dz$$

$$= \int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} dz \left(\frac{r^2}{2}\right)\Big|_0^1 = \frac{1}{2}\int_0^{2\pi} d\varphi \; z \Big|_0^{\sqrt{5}} = \frac{1}{2} \cdot \sqrt{5} \; \varphi \Big|_0^{2\pi}$$

$$= \frac{1}{2} \cdot \sqrt{5} \cdot 2\pi \approx 7$$