MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: DUJE KRALJEV BROJ INDEKSA:	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. ~~20~~ 15

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\, ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi \vec{\mathbf{j}} + 2\sin\phi \vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: 35

1) $f'''(t) - f''(t) = \cos(t)$

$s^3F(s) - s^2\underset{0}{f(0)} - s\underset{0}{f'(0)} - \underset{0}{f''(0)} - (s^2F(s) - s\underset{0}{f(0)} - \underset{0}{f'(0)}) = \frac{s}{s^2+1}$

$s^3F(s) - s^2F(s) = \frac{s}{s^2+1}$

$F(s)(s^3 - s^2) = \frac{s}{s^2+1}$

$F(s) = \frac{\frac{s}{s^2+1}}{s^3-s^2} = \frac{s}{(s^3-s^2)(s^2+1)} = \frac{s}{s^2(s-1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds+E}{s^2+1}$

$s = As\underbrace{(s-1)(s^2+1)}_{s^3+s-s^2-1} + B\underbrace{(s-1)(s^2+1)}_{s^3+s-s^2-1} + C\underbrace{s^2(s^2+1)}_{s^4+s^2} + (Ds+E)\underbrace{s^2(s-1)}_{s^3-s^2}$

$s = 0$

$0 = B(-1)1$

$0 = -B$

$B = 0$

$s - 1 = 0$

$s = 1$

$1 = C \cdot 1 \cdot 2$

$1 = 2C$

$C = \frac{1}{2}$

$s^2 + 1 = 0$

$s^2 = -1$

$s = \sqrt{-1}$

$s = i$

$s = A(s^4 + s^2 - s^3 - s) + B(s^3 + s - s^2 - 1) + C(s^4 + s^2) + (Ds + E)(s^3 - s^2)$

$s = \cancel{As^4} + \cancel{As^2} - \cancel{As^3} - As + \cancel{Bs^3} + \cancel{Bs} - \cancel{Bs^2} - B + \cancel{Cs^4} + \cancel{Cs^2} + \cancel{Ds^4} - \cancel{Ds^3} + \cancel{Es^3} - \cancel{Es^2}$

$s = (A + C + D)s^4 + (-A + B - D + E)s^3 + (A - B + C - E)s^2 + (-A + B)s - B$

$A + C + D = 0$

$-A + B - D + E = 0$

$A - B + C - E = 0$

$-A + B = 1$

$-B = 0$

$\underline{B = 0}$

$-A + B = 1$

$-A + 0 = 1$

$\underline{A = -1}$

$A + C + D = 0$

$-1 + \frac{1}{2} + D = 0$

$D = 1 - \frac{1}{2}$

$\underline{D = \frac{1}{2}}$

$A - B + C - E = 0$

$-1 - 0 + \frac{1}{2} - E = 0$

$-E = 1 - \frac{1}{2}$

$-E = \frac{1}{2}$

$\underline{E = -\frac{1}{2}}$

$F(s) = \frac{-1}{s} + \frac{0}{s^2} + \frac{\frac{1}{2}}{s-1} + \frac{Ds + E}{s^2 + 1}$

$f(s) = -1 + \frac{1}{2} e^t + \frac{1}{2} \cos(t) - \frac{1}{2} \sin(t)$ ✓

$f'(s) = \frac{1}{2} e^t - \frac{1}{2} \sin(t) - \frac{1}{2} \cos(t)$

$f''(s) = \frac{1}{2} e^t - \frac{1}{2} \cos(t) + \frac{1}{2} \sin(t)$

$f'''(s) = \frac{1}{2} e^t + \frac{1}{2} \sin(t) + \frac{1}{2} \cos(t)$

$f'''(s) - f''(s) = \cos(t)$

$\frac{1}{2} e^t + \frac{1}{2} \sin(t) + \frac{1}{2} \cos(t) - \left(\frac{1}{2} e^t - \frac{1}{2} \cos(t) + \frac{1}{2} \sin(t)\right) = \cos(t)$

$\cancel{\frac{1}{2} e^t} + \cancel{\frac{1}{2} \sin(t)} + \frac{1}{2} \cos(t) - \cancel{\frac{1}{2} e^t} + \frac{1}{2} \cos(t) - \cancel{\frac{1}{2} \sin(t)} = \cos(t)$

$\underline{\cos(t) = \cos(t)}$

DUSO KRACOV

2) $F = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$ $F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$

$\operatorname{div} F = \frac{\partial (x^2+y^2)}{\partial x} + \frac{\partial (z)}{\partial y} + \frac{\partial (1)}{\partial z}$

$\operatorname{div} F = 2x + 0 + 0 = 2x$ ✓

$\iint\limits_{\partial K} F \cdot dS = \iiint\limits_{K} \operatorname{div} F \, dx\,dy\,dz$

$x = r\cos\varphi + 2$ ✓

$y = r\sin\varphi + 1$ ✓

$z = z$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$r \in [0, 1]$

$\varphi \in [0, 2\pi]$

$x^2 - y^2 + z^2 = R^2$

$x^2 + y^2 = r^2$

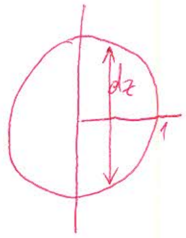
$r^2 + z^2 = 1$

$z^2 = 1 - r^2$

$z = \pm\sqrt{1-r^2}$

$z \in [-\sqrt{1-r^2}, \sqrt{1-r^2}]$

$\iiint\limits_{K} 2x \, dx\,dy\,dz = \int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2\cdot(r\cos\varphi+2)\,r\,dz\,dr\,d\varphi =$ ✓



$\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} (2r^2\cos\varphi + 4r)\,dz\,dr\,d\varphi =$

$\int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r)\left(\sqrt{1-r^2} + \sqrt{1-r^2}\right)dr\,d\varphi =$ ✓

$\int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r)\left(2\sqrt{1-r^2}\right)dr\,d\varphi =$

$\int_0^{2\pi}\int_0^1 \left(4\sqrt{1-r^2}\cdot \underbrace{r^2\cos\varphi}_{0} + 8\sqrt{1-r^2}\cdot r\right)dr\,d\varphi =$

$-2r\,dr = dt$

$\int_0^{2\pi}\int_0^1 8\sqrt{1-r^2}\cdot r\,dr\,d\varphi = \left| 1-r^2 = t \right.$ ✓

$= -\int_0^{2\pi}\int_0^1 \left(8\cdot t^{\frac{1}{2}} \cdot \frac{1}{2}\,dt\right)d\varphi = \int_0^{2\pi} -4\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\Big|_{1}^{0}\,d\varphi = \int_0^{2\pi} -4\,\frac{0}{\frac{3}{2}} - \left(-4\,\frac{1^{\frac{3}{2}}}{\frac{3}{2}}\right)d\varphi$

$\int_0^{2\pi} 4\,\frac{2}{3}\,d\varphi = \frac{8}{3}\varphi\Big|_0^{2\pi} = \frac{8}{3}0 - \frac{8}{3}2\pi = -\frac{16}{3}\pi$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**	Grupa xxoxo
IME I PREZIME: IVAN GRZUNOV BROJ INDEKSA:	POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 15

2.

$$\begin{bmatrix} x^2+y^2 \\ z \\ 1 \end{bmatrix} \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix}$$

$$\operatorname{div} w = \partial f_{1x} + \partial f_{2y} + \partial f_{3z}$$

$$\operatorname{div} w = 2x + 0 + 0$$

$$\operatorname{div} w = 2x \checkmark$$

$x = r\cos\varphi + 2$

$y = r\cos\varphi + 1$

$z = z$

$\varphi \in [0, 2\pi] \checkmark$

$z \in [-1, 1] \checkmark$

$r \in [0, \sqrt{1-z^2}] \checkmark$

$x^2 + y^2 + z^2 = r^2$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 1$

$r^2(\cos^2\varphi + \sin^2\varphi) + z^2 = 1$

$r^2 + z^2 = 1$

$r = \sqrt{1-z^2}$

$$\int_0^{2\pi}\int_0^{\sqrt{1-z^2}}\int_{-1}^{1} 2x \cdot r \, d\varphi\, dr\, dz = \int_0^{2\pi}\int_0^{\sqrt{1-z^2}} 2x \cdot r \cdot z \Big|_{-1}^{1} d\varphi\, dr = \int_0^{2\pi}\int_0^{\sqrt{1-z^2}} 2xr + 2xr \, d\varphi\, dr =$$

$$= \int_0^{2\pi}\int_0^{\sqrt{1-z^2}} 2xr\, d\varphi\, dr + \int_0^{2\pi}\int_0^{\sqrt{1-z^2}} 2xr\, d\varphi\, dr = \int_0^{2\pi} 2x\frac{r^2}{2}\Big|_0^{\sqrt{1-z^2}} d\varphi + \int_0^{2\pi} 2x\frac{r^2}{2}\Big|_0^{\sqrt{1-z^2}} d\varphi =$$

$$= \int_0^{2\pi} 2x \cdot \frac{1-z^2}{2} d\varphi + \int_0^{2\pi} 2x \frac{1-z^2}{2} d\varphi = 2x \cdot \frac{1-z^2}{2} \cdot \varphi \Big|_0^{2\pi} + 2x \cdot \frac{1-z^2}{2} \cdot \varphi \Big|_0^{2\pi} =$$

$$= 2x \cdot \frac{1-z^2}{2} 2\pi + 2x \frac{1-z^2}{2} 2\pi$$

1. $f'''(t) - f''(t) = \cos t$ $\qquad f(0) = f'(0) = f''(0) = 0$

$$s^3 F(s) - s^2 \underbrace{\cancel{f(0)}}_{0} - s \underbrace{\cancel{f'(0)}}_{0} - \underbrace{\cancel{f''(0)}}_{0} - s^2 F(s) - s \underbrace{\cancel{f(0)}}_{0} - \underbrace{\cancel{f'(0)}}_{0} = \frac{s}{s^2+1}$$

$$s^3 F(s) - s^2 F_s = \frac{s}{s^2+1}$$

$$F_s (s^3 - s^2) = \frac{s}{s^2+1} \quad \Big| \; s^3 - s^2$$

$$F_s = \frac{\frac{s}{s^2+1}}{\frac{s^3-s^2}{1}} = \frac{s}{(s^2+1)\cdot s^3 - s^2} = \frac{s}{s^3 - s^2 \cdot (s^2+1)}$$

$$= \frac{s}{s^5 + s^3 - s^4 - s^2} = \frac{s}{s^2}$$

$$\frac{s}{s^2} = \frac{A}{s^2} + \frac{B}{s} \quad \Big| \cdot s^2$$

$$s = A + Bs$$

$$\frac{s}{s^2} = \frac{1}{s^2} + \frac{0}{s}$$

$A = 1$

$B = 0$

$$\frac{s}{s^2} = \frac{1}{s^2}$$

?

GRZUNOV

4.

$x = \cos 2t$

$y = \sin 2t$ $\quad t \in [0, 3\pi]$

$z = t$

$r \in (\cos 2t, \sin 2t, t)$

$r' \in (-\sin 2t, \cos 2t, 1)$

$$\| r'_{(t)} \| = \sqrt{(-\sin 2t)^2 + (\cos 2t)^2 + 1^2}$$

$$= \sqrt{\sin^2 4t + \cos^2 4t + 1}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t) + 1}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5} \checkmark$$

$$\int_0^{3\pi} (\cos 2t + 2 \sin 2t) \cdot \sqrt{5} \, dt = \sqrt{5} \left(\int_0^{3\pi} \cos 2t \, dt + 2 \int_0^{3\pi} \sin 2t \, dt \right)$$

$$= \sqrt{5} \cdot \left(\sin 2t \Big|_0^{3\pi} + 2 \cdot (-\cos 2t \Big|_0^{3\pi} \right) \checkmark$$

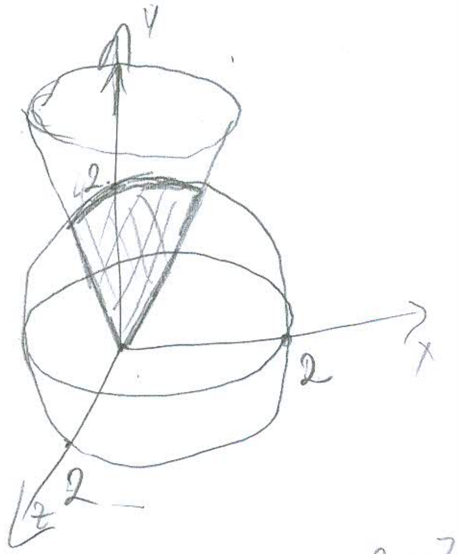
$$= \sqrt{5} \cdot (\sin 6\pi + 2 \cdot (-\cos 6\pi)) \times$$

$$= \sqrt{5} \cdot (0 - 2)$$

$$= -2\sqrt{5}$$

15

3.



$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$\in \varphi = [0, 2\pi]$

$\in z = [0, 2]$

$x^2+y^2+z^2=r^2$ $\quad \in r = [0, \sqrt{2-z^2}]$

$x^2+y^2+z^2=4$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 2$

$r^2(\cos^2\varphi+\sin^2\varphi)+z^2=2$

$r^2+z^2=2$

$r^2=2-z^2$

$r=\sqrt{2-z^2}$

$$\int_0^{2\pi}\int_0^{\sqrt{2-z^2}}\int_0^2 r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{2-z^2}} rz\Big|_0^2 dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{2-z^2}} 2r\,dr\,d\varphi =$$

$$= \int_0^{2\pi} 2\frac{r^2}{2}\Big|_0^{\sqrt{2-z^2}} d\varphi = \int_0^{2\pi} \cancel{2}\frac{2-z^2}{\cancel{2}}\,d\varphi = 2-z^2\cdot\varphi\Big|_0^{2\pi} = 2-z^2\cdot 2\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**

Grupa xxoxo
Popunjava nastavnik
Broj ↓ bodova

Ime i prezime: LOVRE KOLEGA

Broj indeksa:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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5. Izračunati $\int_{\widehat{K}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. ~~20~~

Ukupno: 0

4) $x = \cos 2t \quad y = \sin 2t \quad z = t \quad t \in [0, 3\pi]$

$x' = -\sin 2t \cdot 2 = -2\sin 2t$

$y' = \cos 2t \cdot 2 = 2\cos 2t$

$\int f(x)\cdot \|r'\|\,dt$

$\|r'\| = \sqrt{(x')^2+(y')^2+(z')^2} = \sqrt{(-2\sin 2t)^2+(2\cos 2t)^2+t^2}$ ✗

$\|r'\| = \sqrt{4\sin^2 2t + 4\cos^2 2t + t^2}$

$\|r'\| = \sqrt{4(\sin^2 2t + \cos^2 2t) + t^2}$

$\|r'\| = \sqrt{4+t^2}$

$\|r'\| = \sqrt{2^2+t^2}$

$\int_0^{3\pi} \sqrt{2^2+t^2}\,dt$ ✗

$\frac{1}{2}\left[2\sqrt{4+t^2} + t^2\ln\left(2+\sqrt{4+t^2}\right)\right]\Big|_0^{3\pi}$

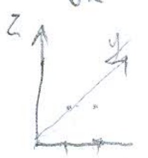
$\frac{1}{2}\left(2\sqrt{4+9\pi^2} + 9\pi^2\ln\left(2+\sqrt{4+9\pi^2}\right)\right.$

$= 9.634 + 9\pi^2\ln(11.634)$

$= 9.634 + 88.826\cdot 2.453 = 9.634 + 217.89 \simeq 227.524$

2. $\iint\limits_{\partial K} F\,ds \qquad F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix} \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix} \qquad r = 1 \qquad T(2,1,0)$

$x^2+y^2+z^2 = r^2$



$\partial x = 2x$
$\partial y = 0$
$\partial z = 0$

$\begin{bmatrix} 2x \\ 0 \\ 0 \end{bmatrix}$ ✓

$\to x = r\cos\varphi \qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi \qquad z \in [-1, 1]$

$z = z \qquad r \in [0, \sqrt{1-z^2}]$ ✗

$x^2+y^2+z^2=0$

$(r\cos\varphi)^2+(r\sin\varphi)^2+z^2=1$

$r^2\cos^2\varphi+r^2\sin^2\varphi+z^2=1$

$r^2(\underbrace{\cos^2\varphi+\sin^2\varphi}_{1})+z^2=1$

$r^2+z^2=1$

$r^2=1-z^2$

$r=\sqrt{1-z^2}$

$$\iint_{\partial K} F\,ds = \int_0^{2\pi}\int_{-1}^{1}\int_0^{\sqrt{1-z^2}} 2x\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_{-1}^{1}\int_0^{\sqrt{1-z^2}} 2r\cos\varphi\cdot r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_{-1}^{1}\int_0^{\sqrt{1-z^2}} 2r^2\cos\varphi\,r\,dr\,d\ldots$$

$$= \int_0^{2\pi}\int_{-1}^{1} 2\frac{r^3}{3}\,dz\,d\varphi\Big|^{\sqrt{1-z^2}}$$

1. $f'''(t)-f''(t)=\cos t \qquad f(0)=f'(0)=f''(0)=0$

$s^3F(s)-s^2f(0)-sf'(0)-f''(0)-(s^2F(s)-sf(0)-f'(0))=\frac{s}{s^2+1}$

$s^3F(s)-s^2F(s)=\frac{s}{s^2+1} \qquad s^2(s-1) \qquad (s^2+1) \quad (s+1)(s-1) \quad (s^2+1)$

$F(s)(s^3-s^2)=\frac{s}{(s^2+1)} \quad / : s^3-s^2$

s^2+1

$F_s=\frac{s}{s^2(s^2+1)(s-1)} \qquad (s^2+1)$

$F_s=\frac{s}{s^2(s+1)\cancel{(s-1)}\cancel{(s-1)}}=\frac{s}{s^2(s+1)}$

$s^3-s^2=\frac{A}{s^2}+\frac{Bs+C}{s+1}$

$-1=A$

$0=$

$\cancel{0}$

KOLEGA

3. $z>0$ $\quad x^2+y^2+z^2=4 \quad r=2$

$x^2+y^2=z^2$

$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$

$\varphi\in[0,2\pi]$

$z\in[-2,2]$

$r\in[0,\sqrt{4-z^2}]$

×

$(r\cos\varphi)^2+(r\sin\varphi)^2+z^2=4$

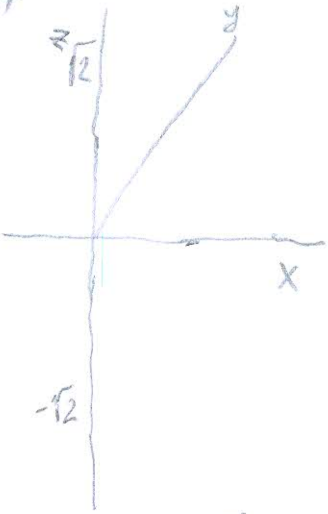
$r^2\cos^2\varphi+r^2\sin^2\varphi+z^2=4$

$r^2(\underbrace{\cos^2\varphi+\sin^2\varphi}_{1})+z^2=4$

$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$



$z^2+z^2=4$

$2z^2=4 \,/:2$

$z^2=2$

$z=\sqrt{2}$

$x^2+y^2=(\sqrt{2})^{\cancel{2}}$

$y^2=2-x^2$

$y=\sqrt{2-x^2}$

$x^2+(\sqrt{2-x^2})^{\cancel{2}}+(\sqrt{2})^2=4$

$x^2+2-x^2+2=4$

$$\int_0^2\int_{\sqrt{4-x^2}}^{\sqrt{2-x^2}}\int_{-\sqrt{2}}^{\sqrt{2}} dz\,dy\,dx$$

$x^2+y^2+(\sqrt{2})^{\cancel{2}}=4$

$x^2+y^2=4$

$y^2=4-x^2$

$y=\sqrt{4-x^2}$

$y=\sqrt{4-x^2}$

$y=\sqrt{2-x^2}$

$\sqrt{2-x^2}=\sqrt{4-x^2} \,/^2$

$2-x^2=4-x^2$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**		Grupa xxoxo POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: STIPE ŠPANJA	**BROJ INDEKSA:** 17-2-0018-2020	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f''(t) = \cos(t)\,,\quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. ~~20~~

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4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\,\mathrm{d}s$. ~~20~~

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Ukupno: 0

$f(0)=0$
$f'(0)=0$
$f''(0)=0$

1. $s^3F(s) - \cancel{s^2f(0)}^{0} - \cancel{sf'(0)}^{0} - \cancel{f''(0)}^{0} - s^2F(s) + \cancel{sf(0)}^{0} + \cancel{f'(0)}^{0} = \dfrac{s}{s^2+1}$

$s^3F(s) - s^2F(s) = \dfrac{s}{s^2+1}$

$F(s)(s^3 - s^2) = \dfrac{s}{s^2+1} \quad /:(s^3-s^2)$

$F(s) = \dfrac{s}{(s^2+1)(s^3-s^2)} = \dfrac{s}{s^2(s-1)(s^2+1)}$

$\dfrac{s}{s^2(s-1)(s^2+1)} = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s-1} + \dfrac{Ds+E}{s^2+1} \quad /\cdot s^2(s-1)(s^2+1)$

$s = A(s-1)(s^2+1) + Bs(s-1)(s^2+1) + Cs^2(s^2+1) + (Ds+E)(s^3-s^2)$

$s = A(s^3+s-s^2-1) + Bs(s^3+s-s^2-1) + Cs^4 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$

$s = As^3 + As - As^2 - A + Bs^4 + Bs^2 - Bs^3 - Bs + Cs^4 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$

$0 = B + C + D \rightarrow C + D = 1,\ D = 1 - C$

$0 = A - B - D + E \rightarrow 0 = 1 - D + E \Rightarrow -D + E = -1$

$0 = A + B + C - E \rightarrow 0 = -1 + C - E \Leftrightarrow C - E = 1$

$1 = A - B \rightarrow 1 = -B,\ \boxed{B = -1}$

$0 = -A$

$\boxed{A = 0}$

$-D + E = -1$
$C - E = 1$
$-1 + C + E = 1$
$C - E = 1$
$C + E = 2$
$C - E = 1 \quad /+$
$2C = 3$
$\boxed{C = \frac{3}{2}}$

$E = C - 1$
$\boxed{E = \frac{3}{2} - 1 = \frac{1}{2}}$

$D = 1 - C$
$D = 1 - \frac{3}{2}$
$\boxed{D = -\frac{1}{2}}$

→

$$F(s) = -\frac{1}{s} + \frac{3}{2}\frac{1}{s-1} + \frac{-\frac{1}{2}s + \frac{1}{2}}{s^2+1}$$

$$F(s) = -\frac{1}{s} + \frac{3}{2}\frac{1}{s-1} - \frac{1}{2}\frac{s}{s^2+1} + \frac{1}{2}\frac{1}{s^2+1}$$

$$F(s) = -1 + \frac{3}{2}e^t - \frac{1}{2}\cos t + \frac{1}{2}\sin t$$

$f(s) =$

$$f'(s) = \frac{3}{2}e^t + \frac{1}{2}\sin t + \frac{1}{2}\cos t$$

$$f''(s) = \frac{3}{2}e^t + \frac{1}{2}\cos t - \frac{1}{2}\sin t$$

$$f'''(s) = \frac{3}{2}e^t - \frac{1}{2}\sin t - \frac{1}{2}\cos t$$

$$f(0) = -1 + \frac{3}{2} - \frac{1}{2} = 0$$

$$f'(0) = \frac{3}{2} + \frac{1}{2} = 2$$

NE ODGOVARA POSTAVLJENIM UVJETIMA

5. $\int y\,dx + y\,dy$ $\qquad r(\varphi) = 2\cos\varphi\, j + 2\sin\varphi\, k$

$$r(\varphi) = \begin{Bmatrix} 2\cos\varphi \\ 2\sin\varphi \end{Bmatrix} \qquad r'(\varphi) = \begin{Bmatrix} -2\sin\varphi \\ 2\cos\varphi \end{Bmatrix}$$

$$W = \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} \qquad \operatorname{rot} W = \begin{bmatrix} \cancel{\partial x} \\ \partial y \\ \partial z \end{bmatrix} \times \begin{bmatrix} \cancel{y} \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0-0 \\ 0-0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{matrix} \partial x & y \\ \partial y & y \end{matrix}$$

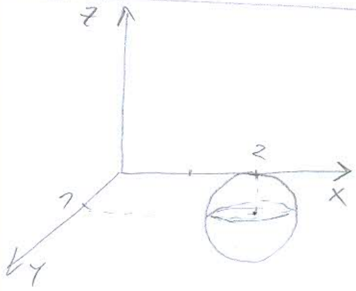
$$\int_0^{2\pi} \begin{bmatrix} -2\sin\varphi \\ 2\cos\varphi \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} d\varphi = 0 \int_0^{2\pi} d\varphi = 0$$

---

2. $F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ $\qquad T(2,1,0)$, $r = 1$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$



$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$$F' \begin{pmatrix} 2x + 2y \\ 1 \\ 0 \end{pmatrix}$$

$$\|F'\| = \sqrt{4\cos^2\varphi + 4\sin^2\varphi + 1 + 0} = \sqrt{4+1} = \sqrt{5}$$

$$\sqrt{5} \int_0^{2\pi} d\varphi \int_0^1 r\,dr = \sqrt{5} \cdot \frac{1}{2} \int_0^{2\pi} d\varphi = \frac{\sqrt{5}}{\cancel{2}} \cdot \cancel{2}\pi = \sqrt{5}\,\pi$$

3. $z > 0$ $\quad x^2+y^2+z^2=4 \quad x^2+y^2=z^2$

STIPE ŠPANJA

$r^2+z^2=4 \quad r^2=z^2$

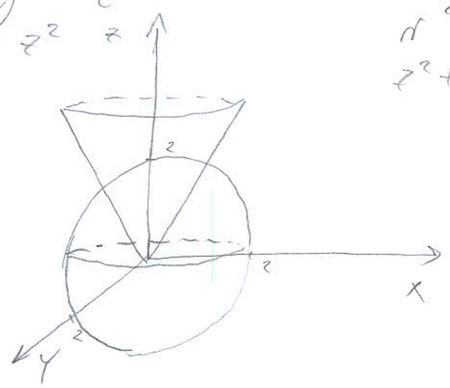
$r^2 = 4-z^2$

$r=\sqrt{4-z^2}$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$



$r^2+z^2=4$

$z^2+z^2=4$

$2z^2=4$

$z^2=2$

$z=\sqrt{2}$

$z \in [0,\sqrt{2}]$ ×

$\varphi \in [0, 2\pi]$

$r \in [0,\sqrt{4-z^2}]$ ×

$$V = \int_0^{2\pi} d\varphi \int_0^{\sqrt{2}} dz \int_0^{\sqrt{4-z^2}} r\,dr = \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{\sqrt{2}} (4-z^2)\,dz$$

$$= \frac{1}{2}\int_0^{2\pi} d\varphi \left(4z-\frac{z^3}{3}\right)\Big|_0^{\sqrt{2}} = \frac{10\sqrt{2}}{3}\int_0^{2\pi} d\varphi = \frac{10\sqrt{2}}{3}\cdot 2\pi \approx 29.61$$

---

4. $t \in [0, 3\pi]$ $\quad \int (x+2y)\,ds$ $\quad \sin^2 x + \cos^2 x = 1$

$$r(t) = \begin{Bmatrix} \cos 2t \\ \sin 2t \\ t \end{Bmatrix} \qquad r'(t) = \begin{Bmatrix} -\sin 2t \\ \cos 2t \\ 1 \end{Bmatrix}$$ × ×

$$\|r'(t)\| = \sqrt{\sin^2 2t + \cos^2 2t + 1} = \sqrt{1+1} = \sqrt{2}$$ ×

$$\sqrt{2}\int_0^{3\pi} (\cos 2t + 2\sin 2t)\,dt = \sqrt{2}\Bigg(\underbrace{\int_0^{3\pi} \cos 2t\,dt}_{I} + 2\underbrace{\int_0^{3\pi} \sin 2t\,dt}_{II}\Bigg)$$ ×

$$I \quad \int \cos 2t\,dt = \begin{Bmatrix} 2t = x \\ 2dt = dx \\ dt = \frac{1}{2}dx \end{Bmatrix} = \frac{1}{2}\int \cos x\,dx = \frac{1}{2}\sin x = \frac{1}{2}\sin 2t$$

$$II \quad \int \sin 2t\,dt = \begin{Bmatrix} 2t = x \\ 2dt = dx \\ dt = \frac{1}{2}dx \end{Bmatrix} = \frac{1}{2}\int \sin x\,dx = -\frac{1}{2}\cos x = \frac{1}{2}\cos 2t$$

$$\sqrt{2}\left(\frac{1}{2}\sin 2t\Big|_0^{3\pi} + \cancel{2}\left(-\frac{1}{\cancel{2}}\right)\cos 2t\Big|_0^{3\pi}\right) = \sqrt{2}\left(\frac{1}{\cancel{2}}\cdot 0 - (\cancel{1-1})\right)$$

$= 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**	Grupa xxoxo
Ime i prezime: VEDRAN DEGAŠ **Broj indeksa:** 52206	Popunjava nastavnik Broj ↓ bodova

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Ukupno: ~~0~~

① $f'''(t) - f''(t) = \cos(t)$

$f(0) = 0$

$f'(0) = 0$

$f''(0) = 0$

$s^3 F(s) - s^2 \cancel{f(0)} - s\cancel{f'(0)} - \cancel{f''(0)} - sF(s) - sf(0) - f'(0) = \frac{s}{s^2+1}$

$s^3 F(s) - sF(s) = \frac{s}{s^2+1}$

$F(s)(s^3 - s) = \frac{s}{s^2+1}$

$F(s) = \frac{s}{(s^3-s)(s^2+1)} = \frac{s}{s(s^2-1)(s^2+1)} = \frac{s}{s(s-1)(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{(s-1)} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$

$s = A(s^2-1)(s^2+1) + B(s^2+s)(s^2+1) + C(s^2-s)(s^2+1) + (Ds+E)(s^3-s)$

$s = A(s^4 - s^2 + s^2 - 1) + B(s^4 + s^3 + s^2 + s) + C(s^4 - s^3 + s^2 - s) + Ds^4 + Es^3 - Ds^2 - Es$

$s = As^4 - A + Bs^4 + Bs^3 + Bs^2 + Bs + Cs^4 - Cs^3 + Cs^2 - Cs + Ds^4 + Es^3 - Ds^2 - Es$

$0 = \frac{1}{4} - \frac{1}{4} - D$

$D = 0$

$0 = A + B + C + D$

$0 = B - C + E$

$0 = B + C - D$

$1 = B - C - E$

$0 = A$

$0 = B + C + D$

$\underline{0 = B + C - D}$

$0\ 0 = 2B + 2C$

$0 = \frac{2}{4} + 2C \quad | \cdot 4$

$0 = 2 + 8C$

$-8C = 2 \quad |(-1)$

$8C = -2$, $C = -\frac{1}{4}$

$0 = B - C + E$

$\underline{1 = B - C - E}$

$1 = 2B - 2C$

$\underline{0 = 2B + 2C}$

$1 = 4B$

$B = \frac{1}{4}$

$1 = \frac{1}{4} - \left(\frac{1}{4}\right) - E$

$1 = \frac{1}{2} - E$

$E = \frac{1}{2} - 1$

$E = -\frac{1}{2}$

$A=0$ $\quad F(s)=\frac{A}{s}+\frac{B}{s-1}+\frac{C}{s+1}+\frac{Ds+E}{s^2+1}$

$B=\frac{1}{4}$

$F(s)=\frac{1}{4}\frac{1}{s-1}-\frac{1}{4}\frac{1}{s+1}-\frac{1}{2}\frac{1}{s^2+1}$

$C=-\frac{1}{4}$

$D=0$

$E=-\frac{1}{2}$

$F(t)=\frac{1}{4}e^{t}-\frac{1}{4}e^{-t}-\frac{1}{2}\sin(t)$ ✗

③

KROGLA

$x^2+y^2+z^2=4$

$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$

STOŽEC

$x^2+y^2=z^2$

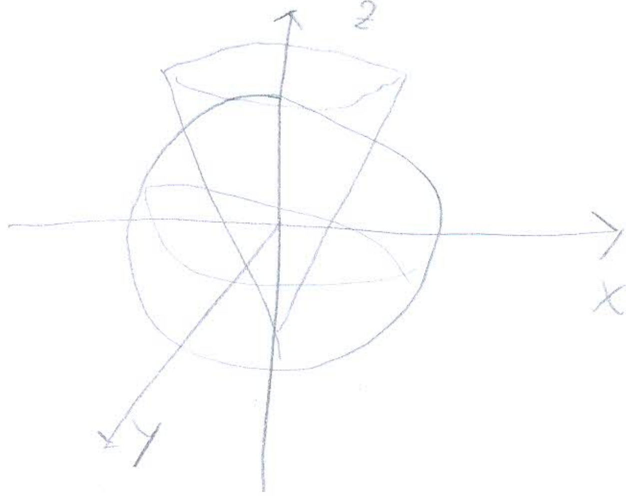
$r^2=z^2$

$r=z$

$z>0$

UNOD KUGLE

IZNOD STOŽCA



$z^2=4-z^2$

$2z^2=4$

$z^2=2$

$z=\pm\sqrt{2}$

$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_z^{\sqrt{4-z}} r\,dr\,d\varphi\,dz$

∅

$\int_0^{2\pi}d\varphi\int_0^{\sqrt{2}}dz\int_z^{\sqrt{4-z}}r\,dr=\int_0^{2\pi}d\varphi\int_0^{\sqrt{2}}dz\;\sqrt{4-z}-z\;dr$

$\int_0^{2\pi}d\varphi\int_0^{\sqrt{2}}dz\;\;4-z^2-z\;=\;∅$

$\varphi\in[0,2\pi]$

$z\in[0,\sqrt{2}]$

$r\in[z,\sqrt{4-z^2}]$

④ $\quad x=2\cos t \qquad y=2\sin t \qquad z=t$

$t\in[0,3\cdot2\pi]$

$r(t)=\begin{bmatrix}2\cos t\\2\sin t\\t\end{bmatrix} \qquad r'(t)=\begin{bmatrix}-2\sin t\\2\cos t\\1\end{bmatrix}$

$|r'(t)|=\sqrt{(2\sin t)^2+(2\cos t)^2+1^2}$

$=\sqrt{4\sin^2t+4\cos^2t+1}=\sqrt{5}$ ✓

$r'\,dt \quad \int_0^{6\pi}\sqrt{5}\,dt=\sqrt{5}\,t\Big|_0^{6\pi}=6\sqrt{5}\,\pi$ ✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**	Grupa xxoxo
Ime i prezime: LUKA Hukjev **Broj indeksa:** 58079	Popunjava nastavnik Broj ↓ bodova

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Ukupno: 0

$\cancel{x^2+y^2+z^2=4}$

3.

$x^2+y^2+z^2=4$

$z>0$

$R=\sqrt{4}$

$R=2$

$r^2+z^2=4 \ /\sqrt{\ }$

$r+z=2$

$\boxed{r=2-z}$

$r\in[0,2-z]$ ✗

$z\in[0,2]$ ✗

$\varphi\in[0,2\pi]$ ✓

$\int_0^{2\pi}\int_0^{2}\int_0^{2-z} r\,dr\,dz\,d\varphi \rightarrow$ KUGLA

$\cancel{\int_0^{2\pi}\int_0^{2}\left(\frac{r^2}{2}\Big/_0^{2-z}\right)dz\,d\varphi}$

$\cancel{\int_0^{2\pi}\int_0^{2}\left(\frac{(2-z)^2}{2}\right)dz\,d\varphi}$

$\int_0^{2-z}\int_0^{2\pi}\int_0^{2} dz\,d\varphi\,r\,dr$

$\int_0^{2-z}\int_0^{2\pi}\int_0^{2} f\,dz\,d\varphi\,r\,dr$

$\int\int\int\left(z\Big/_0^2\right)d\varphi\,r\,dr$

$\int_0^{2-z}\int_0^{2\pi} 2\,d\varphi\,r\,dr$

$\int_0^{2-z}\left(2\int_0^{2\pi} d\varphi\right)r\,dr$

$\int_0^{2-z}\left(2\left(\varphi\Big/_0^{2\pi}\right)\right)r\,dr$

$\int_0^{2-z}\left(2(2\pi)\right)r\,dr$

$\int_0^{2-z} 4\pi\,r\,dr$

$4\pi\int_0^{2-z} r\,dr = 4\pi\left(\frac{r^2}{2}\Big/_0^{2-z}\right)$

$= 4\pi\left(\frac{(2-z)^2}{2}\right)$

$= 4\pi\frac{(2-z)^2}{\cancel{2}} = \boxed{2\pi(2-z)^2}$

④ $x = \cos 2t$ $\in [0, 3\pi]$ $\int (x+2y)\,ds$

$y = \sin 2t$

$z = t$

$x = r\cos t$

$y = r\sin t$

$r(t) = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix}$ $r'(t) = \begin{bmatrix} -\sin 2t \cdot 2 \\ \cos 2t \cdot 2 \\ 1 \end{bmatrix}$

$|r'(t)| = \sqrt{(-2\sin 2t)^2 + (2\cos^2 t)^2 + 1^2}$

$= \sqrt{(4\sin 4^2 t) + (4\cos 4^2 t) + 1}$

$= \sqrt{4(\underbrace{(\sin 4^2 t) + (4\cos 4^2 t)}_{1}) + 1}$

$= \sqrt{5}\,dt$ ✓

$\int_0^{3\pi} \sqrt{5}\,dt = \sqrt{5}\int_0^{3\pi} dt = \sqrt{5}\left(t\Big|_0^{3\pi}\right) = \sqrt{5}(3\pi - 0)$ ✗

$\boxed{= 3\pi\sqrt{5}}$

② $\iint F \cdot dS$ $F = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ $r = 1$ $T(2, 1, 0)$

$x = r\cos t + 2 = \cos t + 2$ ✗

$y = r\sin t + 1 = \sin t + 1$ ✗

$z = z = 0$

$r(t) = \begin{bmatrix} \cos t + 2 \\ \sin t + 1 \\ 0 \end{bmatrix}$ $r'(t) \begin{bmatrix} -\sin t \\ \cos t \\ 0 \end{bmatrix}$ $|r'(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2}$

$= \sqrt{\sin^2 t + \cos^2 t}$

$= \sqrt{1} = \underline{1\,dt}$ ✗

$\int_0^{1}\int_0^{\pi} dt$ $\int_0^1 dt$

$\int_0^1 dt = t\Big|_0^1 = \boxed{1}$

$\begin{bmatrix} x + y^2 \\ z \\ 1 \end{bmatrix}$ $F' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$1 + 0 + 0 = 1$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: DANIJEL KAPOVIĆ **BROJ INDEKSA:** 52590-2005	POPUNJAVA NASTAVNIK Broj ↓ bodova

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$s^3F(s) - s^2F(s) = \frac{s}{s^2+1}$

$F(s)(s^3 - s^2) = \frac{s}{s^2+1}$

$F(s) = \frac{s}{s^2(s-1)(s^2+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s-1} + \frac{Ds+E}{s^2+1}$

$s = A\underbrace{(s-1)(s^2+1)}_{s^3+s-s^2-1} + Bs(s-1)(s^2+1) + Cs(s^2+1) + \underbrace{Ds^3+Es^2(s-1)}_{Ds^4-Ds^3+Es^3-Es^2}$

$s = As^3 + As - As^2 - A + Bs^4 + Bs^2 - Bs^3 - Bs + Cs^3 + Cs + Ds^4 - Ds^3 + Es^3 - Es^2$

$0 = B + D \rightarrow$

$0 = A - B + C - D + E =$

$0 = -A + B - E \rightarrow$

$1 = A - B + C \rightarrow$

$0 = -A \rightarrow A = 0,$

?

(4) $r(t) = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix}$ $\quad r'(t) = \begin{bmatrix} -\sin 2t \\ \cos 2t \\ 1 \end{bmatrix}$ $\quad t \in (0, 3\pi)$

$$\|r'(t)\| = \sqrt{(-\sin 2t)^2 + (\cos 2t)^2 + 1^2} = \sqrt{\sin^2 4t + \cos^2 4t + 1}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t) + 1} = \sqrt{4 \cdot 1 + 1} = \sqrt{5} \checkmark$$

$$\int_0^{3\pi} \sqrt{5}\, dx = \sqrt{5} \cdot 3\pi - 0 \Big|_0^{3\pi} = 3\pi\sqrt{5}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** ANTE DUŠEVIĆ **BROJ INDEKSA:** 57641	Grupa xxoxo POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\, ds$. ~~20~~

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: Ø

1.) $f'''(t) - f''(t) = \cos(t)$ $\qquad f(0) = f'(0) = f''(0) = 0$

$x'''(t) - x''(t) = \cos(t)$ $\qquad x(0) = x'(0) = x''(0) = 0$

$$s^3X(s) - s^2\cancel{x(0)}^{0} - s\cancel{x'(0)}^{0} - \cancel{x''(0)}^{0} - s^2X(s) - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{0} = \frac{s}{s^2+1}$$

$$s^3X(s) - s^2X(s) = \frac{s}{s^2+1}$$

$$X(s)(s^3 - s^2) = \frac{s}{s^2+1} \quad / : (s^3 - s^2)$$

$$X(s) = \frac{s}{(s^2+1)(s^3-s^2)} = \frac{s}{s(s^2+1)(s^2-s)} =$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+1} + \frac{Ds+E}{s^2-s} \quad / \cdot s(s^2+1)(s^2-s)$$

$$s = A(s^2+1)(s^2-s) + (Bs+C)(s(s^2-s)) + (Ds+E)(s(s^2+1)) =$$

$$= A(s^4-s) + (Bs+C)\cdot(s^3-s^2) + (Ds+E)\cdot(s^3+s) =$$

$$= As^4 - As + Bs^4 - Bs^3 + Cs^3 - Cs^2 + Ds^4 + Ds^2 + Es^3 + Es \Longrightarrow$$

NASTAVAK $\underline{\underline{1}}$. [illegible]:

$$= As^4 - As + Bs^4 - Bs^3 + Cs^3 - Cs^2 + Ds^4 + Ds^2 + Es^3 + Es$$

$$0 = A + B + D \Rightarrow$$

$$0 = -B + C + E \Rightarrow$$

$$0 = -C + D \Rightarrow C = D$$

$$1 = -A + E$$

$= \cos 2t \qquad y = \sin 2t$

$(x+2y)\,ds$

$$w = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix} \qquad \operatorname{div} w \begin{bmatrix} -\sin 2t \\ \cos 2t \\ 1 \end{bmatrix}$$

$$r\,dt = \int_0^{3\pi} \sqrt{1}\,dt$$

$$\|r'\| = \sqrt{((\cos 2t)')^2 + ((\sin 2t)')^2 + ((t)')^2}$$

$$= \sqrt{(-\sin 2t)^2 + (\cos 2t)^2 + (1)^2}$$

$$= \sqrt{(\sin^2 4t + \cos^2 4t) + 1}$$

$$= \sqrt{1}$$

$3\sqrt{1}\,\pi$

$z > 0$

$x^2 + y^2 + z^2 = 4$

$x^2 + y^2 = z^2$

2. $F = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ $r = 1$ $T(2, 1, 0)$

$$\iint_{\partial K} F \cdot dS$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: ŠIME MATANOVIĆ **BROJ INDEKSA:** 57655	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2 + y^2 + z^2 = 4$, a iznad stošca $x^2 + y^2 = z^2$. ~~20~~

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y)\, ds$. ~~20~~

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: 0

4. $x = \cos 2t$, $y = \sin 2t$, $z = t$ $\quad t \in [0, 3\pi]$

$$\|r\| = \begin{bmatrix} \cos 2t \\ \sin 2t \\ z = t \end{bmatrix}, \quad \varphi = \sqrt{x^2 + y^2 + z^2}$$

$(\cos 2t)' = 2(\cos t)' \, 2$

$$|r'| = \begin{bmatrix} -2\sin 2t \\ 2\cos 2t \\ t \end{bmatrix} \quad \varphi = \sqrt{-2\sin 2t + 2\cos 2t + t^2}$$

$$\varphi \sqrt{-2 \cdot 2 + 2 \cdot 2 + t}$$

$$\varphi \sqrt{-4 + 4 + t}$$

$$\varphi = \sqrt{t}$$

$$\int_S (x + 2y)\, ds =$$

$f(0) = 0$

$f'(0) = 0$

1. $f'''(t) - f''(t) = \cos t_0$ $\quad f''(0) = 0$

$$s^3F(s) - s^2\cancel{f(0)} - \cancel{sf'(0)} - \cancel{f''(0)} - s^2F(s) - sf(0) - \cancel{f'(0)} = \frac{s}{s^2+2^2}$$

$$s^3F(s) - s^2F(s) = \frac{s}{s^2+1^2}$$

$$F_s\left(s^3 - s^2\right) = \frac{s}{s^2+1^2}$$

$$F_s = \frac{s^2+1}{s(s^3-s^2)} = \frac{s^2+1}{s^4-s^3} = \frac{s^2+1}{s^3(s-1)} = \frac{A}{s^3} + \frac{B_s+x}{s^2-1} +$$

0

SIME MATANOVIĆ

3. $x^2+y^2+z^2=4$

$r^2+z^2=4 \quad /\sqrt{}$

$r^2=4-z^2 \quad /\sqrt{}$

$r=\sqrt{4-z}$

$x^2+y^2=z^2$

$r^2=z^2 \quad /\sqrt{}$

$r=\sqrt{z}$

$x \in [0, \sqrt{4-z})$ ×

$y \in [\sqrt{z}, 0]$ ×

$\varphi \in [0, 2\pi]$

$r=$

$x=r\cos\varphi$

$y=r\sin\varphi$

$x=\sqrt{4-z}\cos\varphi$

$y=\sqrt{z}\sin\varphi$

$\emptyset$

s^4-s^3

s

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: DINO KURIĆ **BROJ INDEKSA:** 56192-2008	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y)\, \mathrm{d}s$. 20

5. Izračunati $\int_{\widehat{K}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: ~~0~~

1) $f'''(t) - f''(t) = \cos t \qquad f(0) = f'(0) = f''(0) = 0$

$$s^3 F(s) - \cancel{s^2 f(0)} - \cancel{s f'(0)} - \cancel{f''(0)} - s^2 F(s) - \cancel{s f(0)} - \cancel{f'(0)} = \frac{s}{s^2 + 1^2}$$

$$s^3 F(s) - s^2 F(s) = \frac{s}{s^2 + 1^2}$$

$$F(s)(s^3 - s^2) = \frac{s}{s^2 + 1^2} \quad / \, (s^3 - s^2)$$

$$F(s) = \frac{\frac{s}{s^2 + 1^2}}{\frac{s}{1}}$$

?

$$F(s) = \frac{s}{s^3 + s^2}$$

$F(s) =$

3) $x = \cos 2t$

$r[\cos 2t, \sin 2t, t]$

$y = \sin 2t$

$r'(t) = [- \sin 2$

$z = t$

$t \in \langle 0, 2\pi)$

2) $F \begin{bmatrix} x^2 + y^2 \\ z \\ 1 \end{bmatrix}$ $\partial x = 2x$

$\partial y = 0$

$\partial z = 0$

$r = 1$

$T(2, 1, 0]$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Jure Pavić

BROJ INDEKSA: 51894

Grupa xxoxo

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$	~~20~~
2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom.	~~20~~
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4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\, ds$.	~~20~~
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Ukupno:	~~0~~

1. $f'''(t) - f''(t) = \cos(t) \qquad f(0) = f'(0) = f''(0) = 0$

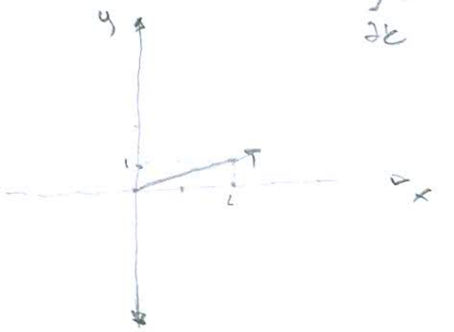
$f'''(t) - f''(t) = \cos(t) = f'''(0) = f''(0) = f(0)\cos(t)$

2. $\iint\limits_{\partial K} F \cdot ds$ K radijus 1

$F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ $\iint\limits_{\partial K} F \cdot ds \iint \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix} \cdot ds = \iint\limits_{\partial K} \begin{pmatrix} 2^2+1^2 \\ z \\ 1 \end{pmatrix} \cdot ds = \iint\limits_{\partial K}$

$T(2,1,0)$

$\int\limits_{\partial K} F \cdot \int ds \int \begin{pmatrix} 2^2+1^2 \\ z \\ 1 \end{pmatrix} \cdot \int ds$



③ $z>0,\ x^2+y^2+z^2=4 \quad , \quad x^2+y^2=z^2$

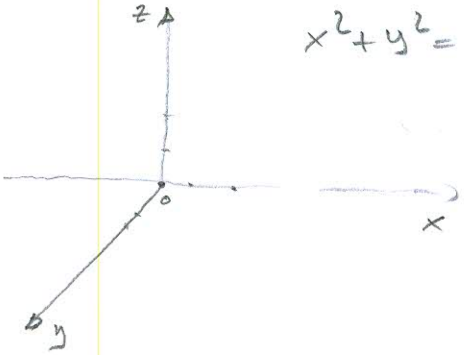
$x^2+y^2+z^2=4$

$(x^2+y^2)+(x^2+y^2)=4$

$2x^2+2y^2=4 \quad /:2$

$x^2+y^2=\underline{\underline{2}}$

$\varnothing$



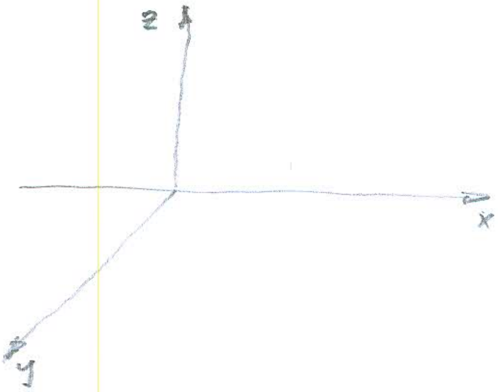
④ $x=\cos 2(t)$
$y=\sin 2(t)$
$z=t$

$f(s)\cdot\big(\cos 2(t)+2\sin 2(t)\big)\,ds$

$t\in|0,3\pi|$

$f(s)\,(x+2y)\,ds$

$\varnothing$



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: Antonio Mužanović **BROJ INDEKSA:** 17-2-0031-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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Ukupno: $\cancel{0}$

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$f(0) = f'(0) = f''(0) = 0$