

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa
xxoxo
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: DUŠE KRALJEV

BROJ INDEKSA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

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2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom.

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3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2 + y^2 + z^2 = 4$, a iznad stošca $x^2 + y^2 = z^2$.

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4. Zadana je kružna uzvojnica (spirala) S s jednadžbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y) ds$.

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5. Izračunati $\int_{\hat{K}} y dx + y dy$ gdje je \hat{K} krivulja dana parametrizacijom $r(\varphi) = 2 \cos \varphi \vec{j} + 2 \sin \varphi \vec{k}$. Koristiti Stokesovu formulu.

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Ukupno:

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$$1) f'''(t) - f''(t) = \cos(t)$$

$$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0) - (s^2 F(s) - s f(0) - f'(0)) = \frac{s}{s^2 + 1}$$

$$s^3 F(s) - s^2 F(s) = \frac{s}{s^2 + 1}$$

$$F(s)(s^3 - s^2) = \frac{s}{s^2 + 1}$$

$$F(s) = \frac{s}{s^3 - s^2} = \frac{s}{s^2(s-1)(s+1)} = \frac{s}{s^2(s-1)(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds+E}{s^2+1}$$

$$s = A s \frac{(s-1)(s+1)}{s^3+s-s^2-1} + B \frac{(s-1)(s+1)}{s^2+s-s^2-1} + C \frac{s^2}{s^2+s} + (Ds+E) \frac{s^2}{s^3-s^2}$$

$$s=0 \quad s-1=0 \quad s+1=0$$

$$0 = B(-1) \cdot 1 \quad s=1 \quad s^2 = -1$$

$$0 = -B \quad 1 = C \cdot 1 \cdot 2 \quad s = \sqrt{-1}$$

$$B = 0 \quad 1 = 2C \quad s = i$$

$$C = \frac{1}{2}$$

$$S = A(s^4 + s^2 - s^3 - s) + B(s^3 + s - s^2 - 1) + C(s^4 + s^2) + (Ds + E)(s^2 - s)$$

$$S = \cancel{As^4} + \cancel{As^2} - \cancel{As^3} - \cancel{As} + \cancel{Bs^3} + \cancel{Bs} - \cancel{Bs^2} - B + \cancel{Cs^4} + \cancel{Cs^2} + \cancel{Ds^4} - \cancel{Ds^3} + \cancel{Es^3} - \cancel{Es^2}$$

$$S = (A+C+D)s^4 + (-A+B-D+E)s^3 + (A-B+C-E)s^2 + (-A+B)s - B$$

$$A+C+D=0$$

$$-A+B=1$$

$$A+C+D=0$$

$$A-B+C-E=0$$

$$-A+B-D+E=0$$

$$-A+0=1$$

$$-1 + \frac{1}{2} + D = 0$$

$$-1 - 0 + \frac{1}{2} - E = 0$$

$$A-B+C-E=0$$

$$\underline{A = -1}$$

$$D = 1 - \frac{1}{2}$$

$$-E = 1 - \frac{1}{2}$$

$$-A+B=1$$

$$\underline{D = \frac{1}{2}}$$

$$-E = \frac{1}{2}$$

$$-B=0$$

$$\underline{E = -\frac{1}{2}}$$

$$\underline{B=0}$$

$$F(s) = \frac{-1}{s} + \frac{0}{s^2} + \frac{\frac{1}{2}}{s-1} + \frac{Ds+E}{s^2+1}$$

$$f(s) = -1 + \frac{1}{2} e^t + \frac{1}{2} \cos(t) - \frac{1}{2} \sin(t) \quad \checkmark$$

$$f'(s) = \frac{1}{2} e^t - \frac{1}{2} \sin(t) - \frac{1}{2} \cos(t)$$

$$f''(s) = \frac{1}{2} e^t - \frac{1}{2} \cos(t) + \frac{1}{2} \sin(t)$$

$$f'''(s) = \frac{1}{2} e^t + \frac{1}{2} \sin(t) + \frac{1}{2} \cos(t)$$

$$f'''(s) - f''(s) = \cos(t)$$

$$\frac{1}{2} e^t + \frac{1}{2} \sin(t) + \frac{1}{2} \cos(t) - \left(\frac{1}{2} e^t - \frac{1}{2} \cos(t) + \frac{1}{2} \sin(t) \right) = \cos(t)$$

$$\frac{1}{2} e^t + \frac{1}{2} \sin(t) + \frac{1}{2} \cos(t) - \frac{1}{2} e^t + \frac{1}{2} \cos(t) - \frac{1}{2} \sin(t) = \cos(t)$$

$$\underline{\cos(t) = \cos(t)}$$

2) $F = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} F = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$
 $\text{div } F = \frac{\partial(x^2 + y^2)}{\partial x} + \frac{\partial(z)}{\partial y} + \frac{\partial(1)}{\partial z}$

$\text{div } F = 2x + 0 + 0 = 2x$ ✓

$x = r \cos \varphi + z$ ✓
 $y = r \sin \varphi + 1$ ✓

$z = z$

$dx dy dz = r dr d\varphi dz$

$r \in [0, 1]$

$\varphi \in [0, 2\pi]$

$x^2 - y^2 + z^2 = r^2$
 $x^2 + y^2 = r^2$

$r^2 + z^2 = 1$
 $z^2 = 1 - r^2$

$z = \pm \sqrt{1 - r^2}$
 $z \in [-\sqrt{1 - r^2}, \sqrt{1 - r^2}]$

$\iint_K F \cdot dS = \iiint_K \text{div } F dx dy dz$

$\iiint_K 2x dx dy dz = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2 \cdot (r \cos \varphi + z) r dz dr d\varphi =$ ✓

$\int_0^{2\pi} \int_0^1 \int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} (2r^2 \cos \varphi + 4r) dz dr d\varphi =$

$\int_0^{2\pi} \int_0^1 (2r^2 \cos \varphi + 4r) (2\sqrt{1-r^2}) dr d\varphi =$ ✓

$\int_0^{2\pi} \int_0^1 (2r^2 \cos \varphi + 4r) (2\sqrt{1-r^2}) dr d\varphi =$

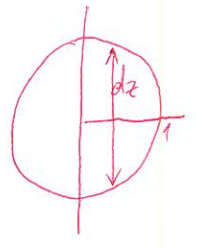
$\int_0^{2\pi} \int_0^1 (4\sqrt{1-r^2} \cdot r^2 \cos \varphi + 8\sqrt{1-r^2} \cdot r) dr d\varphi =$

$\int_0^{2\pi} \int_0^1 8\sqrt{1-r^2} \cdot r dr d\varphi = \int_0^{2\pi} \int_0^1 4(1-r^2)^{\frac{1}{2}} dr d\varphi =$ ✓

$-2r dr = dt$

$\int_0^{2\pi} \left(8 \cdot \left[\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} dt \right] \right) d\varphi = \int_0^{2\pi} -4 \cdot \frac{1}{\frac{3}{2}} \Big|_{\frac{1}{2}}^{\frac{3}{2}} d\varphi = \int_0^{2\pi} -4 \cdot \frac{2}{3} \left(\frac{1}{2} - \frac{3}{2} \right) d\varphi =$

$\int_0^{2\pi} 4 \cdot \frac{2}{3} d\varphi = \frac{8}{3} \varphi \Big|_0^{2\pi} = \frac{8}{3} 0 - \frac{8}{3} 2\pi = -\frac{16}{3} \pi$



odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: IVAN GRŽUNOV

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Ukupno:

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2.

$$\begin{bmatrix} x^2 + y^2 \\ z \\ 1 \end{bmatrix} \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix}$$

$$\text{div } w = \int f_{1x} + \int f_{2y} + \int f_{3z}$$

$$\text{div } w = 2x + 0 + 0$$

$$\text{div } w = 2x \checkmark$$

$$x = r \cos \varphi + 2$$

$$\varphi \in [0, 2\pi] \checkmark$$

$$y = r \sin \varphi + 1$$

$$z \in [-1, 1] \checkmark$$

$$z = z$$

$$r \in [0, \sqrt{1-z^2}] \checkmark$$

$$x^2 + y^2 + z^2 = r^2$$

$$r^2 \cos^2 \varphi + r^2 \sin^2 \varphi + z^2 = 1$$

$$r^2 (\cos^2 \varphi + \sin^2 \varphi) + z^2 = 1$$

$$r^2 + z^2 = 1$$

$$r = \sqrt{1-z^2}$$

$$\int_0^{2\pi} \int_{-1}^1 \int_0^{\sqrt{1-z^2}} 2x \cdot r \, dr \, d\varphi \, dz = \int_0^{2\pi} \int_{-1}^1 \int_0^{\sqrt{1-z^2}} 2x \cdot r \, dr \, d\varphi \, dz =$$

$$\int_0^{2\pi} \int_{-1}^1 \int_0^{\sqrt{1-z^2}} 2x \cdot r \, dr \, d\varphi \, dz$$

$$\int_0^{2\pi} \int_{-1}^1 2x \cdot r \cdot z \, d\varphi \, dz = \int_0^{2\pi} \int_{-1}^1 2x \cdot \frac{r^2}{2} \Big|_0^{\sqrt{1-z^2}} \, d\varphi \, dz =$$

$$= \int_0^{2\pi} \int_{-1}^1 2x \cdot \frac{1-z^2}{2} \, d\varphi \, dz + \int_0^{2\pi} \int_{-1}^1 2x \cdot \frac{1-z^2}{2} \, d\varphi \, dz = \int_0^{2\pi} -2x \cdot \frac{1-z^2}{2} \Big|_{-1}^1 \, d\varphi + \int_0^{2\pi} 2x \cdot \frac{1-z^2}{2} \Big|_{-1}^1 \, d\varphi =$$

$$= \int_0^{2\pi} 2x \cdot \frac{1-z^2}{2} \, d\varphi + \int_0^{2\pi} 2x \cdot \frac{1-z^2}{2} \, d\varphi = 2x \cdot \frac{1-z^2}{2} \cdot \varphi \Big|_0^{2\pi} + 2x \cdot \frac{1-z^2}{2} \cdot \varphi \Big|_0^{2\pi} =$$

$$= 2x \cdot \frac{1-z^2}{2} \cdot 2\pi + 2x \cdot \frac{1-z^2}{2} \cdot 2\pi$$

$$1. f'''(t) - f''(t) = \cos t$$

$$f(0) = f'(0) = f''(0) = 0$$

$$s^3 F(s) - \underbrace{s^2 f(0)}_0 - \underbrace{s f'(0)}_0 - \underbrace{f''(0)}_0 - s^2 F(s) - \underbrace{s f(0)}_0 - \underbrace{f'(0)}_0 = \frac{s}{s^2+1}$$

$$s^3 F(s) - s^2 F(s) = \frac{s}{s^2+1}$$

$$F(s)(s^3 - s^2) = \frac{s}{s^2+1} \quad | \cdot s^2$$

$$F(s) = \frac{\frac{s}{s^2+1}}{\frac{s^3-s^2}{1}} = \frac{s}{(s^2+1) \cdot s^3-s^2} = \frac{s}{s^3-s^2 \cdot (s^2+1)}$$

$$= \frac{s}{s^5+s^3-s^4-s^2} = \frac{s}{s^2}$$

$$\frac{s}{s^2} = \frac{A}{s^2} + \frac{B}{s} \quad | \cdot s^2$$

$$s = A + Bs$$

$$A = 1$$

$$B = 0$$

$$\frac{s}{s^2} = \frac{1}{s^2} + \frac{0}{s}$$

$$\frac{s}{s^2} = \frac{1}{s^2}$$

