MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: BORIS PUDELKO **BROJ INDEKSA:**

Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20 15

2. Izracunati volumen tijela omeđenog valjkom $x^2 + y^2 = 4$ i ravninama $z = y$ i $z = x - 2$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x'(0) = x''(0) = 1.$$

4. Neka je C cilindar zadan sa $C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \leq 1, -1 \leq z \leq 1\}$. Izračunati plošni integral 20

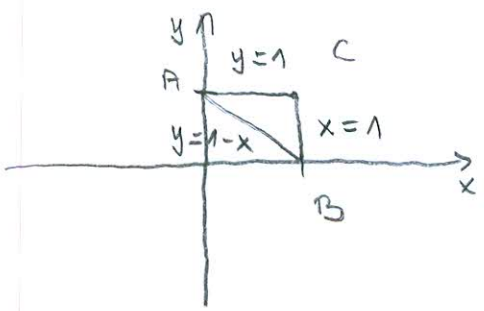
$$\iint_{\partial C} 2x \, dy dz$$

5. Izračunati $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ 20

Ukupno: 35

1.) $\iint_S e^{x+y} dx dy$

$A(0,1)$
$B(1,0)$
$C(1,1)$



$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

AB: $(y-1)(1-0) = (0-1)(x-0)$

$y-1 = -x$

$y = 1-x$

AC: $(y-1)(1-0) = (1-1)(x-0)$

$y-1 = 0$

$y = 1$

BC: $(y-0)(1-1) = (1-0)(x-1)$

$0 = x-1$

$x = 1$

$\int_0^1 \int_{1-x}^1 e^{x+y} dx dy =$

15

2.) Valjak: $x^2+y^2=4 \Rightarrow r=2$

Rovnina: $z=y \Rightarrow z=\sin\rho$

$z=x-2 \Rightarrow z=\cos\rho-2$

$x=\cos\rho$

$y=\sin\rho$

$r\in[0,2]$

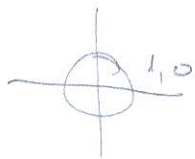
$z\in[\sin\rho,\cos\rho-2]$

$\rho\in[0,2\pi]$

$$V=\int_0^{2\pi}\int_{\sin\rho}^{\cos\rho-2}\int_0^2 r\,dr\,dz\,d\rho = \int_0^{2\pi}\int_{\sin\rho}^{\cos\rho-2}\left[\frac{r^2}{2}\right]_0^2 dz\,d\rho$$

$$= 2\int_0^{2\pi}\left[z\right]_{\sin\rho}^{\cos\rho-2} d\rho = 2\int_0^{2\pi}\cos\rho-2-\sin\rho\,d\rho$$

$$= 2\Big(\underbrace{\left[\sin\rho\right]_0^{2\pi}}_{=0} - \underbrace{2\left[\rho\right]_0^{2\pi}}_{=4\pi} - \underbrace{\left[-\cos\rho\right]_0^{2\pi}}_{=0}\Big) = -8\pi < 0$$



3.) $x'''(t)+x'(t)=0$

$x(0)=1$

$x'(0)=1$

$x''(0)=1$

$$s^3X(s)-s^2\underbrace{x(0)}_{1}-s\underbrace{x'(0)}_{1}-\underbrace{x''(0)}_{1}+s^2X(s)-s\underbrace{x(0)}_{1}-\underbrace{x'(0)}_{1}=0$$

$$s^3X(s)-s^2-s-1+s^2X(s)-s-1=0$$

$$X(s)\left(s^3+s^2\right)=s^2+2s+2$$

$$X(s)=\frac{s^2+2s+2}{s^2(s+1)}$$

$$\frac{s^2+2s+2}{s^2(s+1)}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+1}$$

$$s^2+2s+2=A(s+1)+Bs(s+1)+Cs^2$$

$$s^2+2s+2=As+A+Bs^2+Bs+Cs^2$$

3.)

$1 = B + C \Rightarrow \boxed{C = 1}$

$2 = A + B \Rightarrow B = 2 - A \Rightarrow \boxed{B = 0}$

$2 = A \Rightarrow \boxed{A = 2}$

$$X(s) = 2 \cdot \frac{1}{s^2} + \frac{1}{s+1}$$

$$x(s) = 2t + e^{-t}$$

PROVJERA: $x(0) = 1$

$x'(0) = 1$

$x''(0) = 1$

$x'''(t) + x'(t) \neq 0$ ↯

4) $\iint_S w_r \, ds = \iint_D w(r) \vec{n}$

$$w = \begin{pmatrix} 2x \\ 0 \\ 0 \end{pmatrix} \qquad r = \begin{pmatrix} \cos u \\ \sin u \\ v \end{pmatrix}$$

$$\vec{n} = \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = \begin{pmatrix} -\sin u \\ \cos u \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos u \\ \sin u \\ 0 \end{pmatrix}$$

$-\sin u \cdot 0$

$\cos u \cdot 0$

$$\iint_D \begin{pmatrix} 2x \\ 0 \\ 0 \end{pmatrix} \cdot \vec{n} = \iint_D 2\cos^2 u = \int_{-1}^{1} \int_{(u+2)^2+(v-3)} 2\cos^2 u \, du\,dv$$

OVO JE RAČUN SAMO ZA PLAŠT BEZ BAZA, A K TOME I POGREŠNO UVRŠTAVANJE GRANICA

5) $$\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$$

$$\frac{df}{dx} = \frac{1}{x} \cdot \sin y \,/\int dx$$

$$f(x,y) = \ln|x| \sin y + c(y)$$

$$\frac{df}{dy} = \ln x \cos y$$

$$\ln|x| \cos y + \frac{df}{c(y)} = \ln|x| \cos y \,/\int dy$$

$c(y) = 0$

$f(x,y) = \ln|x| \sin y$

$f(e,\pi) - f(1,0) = \ln|e| \sin\pi - \ln|1| \sin 0$

$= 0$ ✓

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IME I PREZIME: LUKA MARĐETKO BROJ INDEKSA:

Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

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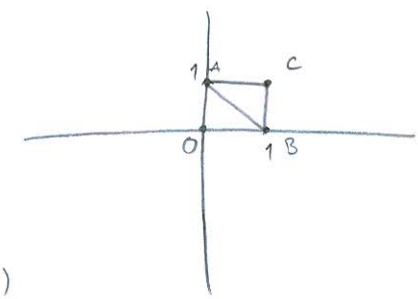
1. $\iint e^{x+y} dx dy$ $A(0,1)$ $B(1,0)$ $C(1,1)$



AB (0,1) (1,0)

$y - 1 = \frac{0-1}{1-0}(x-0)$

$y - 1 = -x$

$y = -x + 1$

BC (1,0) (1,1)

$y - 0 = \frac{1-0}{1-1}(x-1)$

$y = 0$

AC (0,1) (1,1)

$y - 1 = \frac{1-1}{1-0}(x-0)$

$y = 1$

$\Rightarrow \int_0^1 \int_{-x+1}^1 e^{x+y} dy dx$ ✓ 15

RAČJE...

$'(t) + x'(t) = 0$

$(s) - s^2 f(0) - s f'(0) - f''(0) + sF(s) - f(0) = 0$

$F(s) - s^2 - s - 1 + sF(s) - 1 = 0$

$^3+s) = s^2+s+2$

$$\frac{s^2+s+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$s^2+s+2 = A\cdot(s^2+1) + Bs+C\cdot(s)$$

$$s^2+s+2 = As^2 + A + Bs^2 + Cs$$

$1 = A+B \Rightarrow 1 = 2+B$

$\boxed{1 = C}$ $\quad -B = 2-1$

$\boxed{2 = A}$ $\quad \boxed{B = -1}$

$(s) = \dfrac{2}{s} - \dfrac{\cos t + \sin t}{s^2+1}$ ✓

$'(t) = \sin t + \cos t$

$''(t) = \cos t - \sin t$

$x'''(t) = -\sin t - \cos t$

ODJ:

$x'''(t) + x'(t) = (-\sin t - \cos t) + (\sin t + \cos t) = 0$

$x(0) = 1$

$x'(0) = 1$

$x''(0) = 1$

Valjak $x^2+y^2=4$, ravninama $z=y$ $z=x-2$

$V = ?$

$r^2 = 4$

$r = 2$

$z = y \Rightarrow r\sin\varphi$

$z = x-2 \Rightarrow r\cos\varphi - 2$

$$V = \int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_{r\cos\varphi-2}^{r\sin\varphi} dy$$

VIDI RJEŠENJE...