MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: BORIS PUDELKO

BROJ INDEKSA:

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}\,dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15

2. Izracunati volumen tijela omeđenog valjkom $x^2 + y^2 = 4$ i ravninama $z = y$ i $z = x - 2$. ~~20~~

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t) + x'(t) = 0, \quad x(0) = x'(0) = x''(0) = 1.$$

4. Neka je C cilindar zadan sa $C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \le 1, -1 \le z \le 1\}$. Izračunati plošni integral ~~20~~

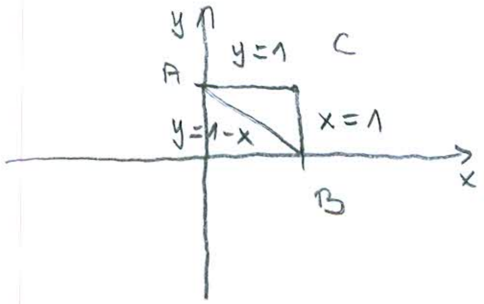
$$\iint\limits_{\partial C} 2x\,dy dz$$

5. Izračunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y\,dy$ 20

Ukupno: 35

1.) $\iint\limits_S e^{x+y}\,dx dy$

$A(0,1)$
$B(1,0)$
$C(1,1)$



$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

AB: $(y-1)(1-0) = (0-1)(x-0)$

$y-1 = -x$

$y = 1-x$

AC: $(y-1)(1-0) = (1-1)(x-0)$

$y-1 = 0$

$y = 1$

BC: $(y-0)(1-1) = (1-0)(x-1)$

$0 = x-1$

$x = 1$

$$\int_0^1 \int_{1-x}^1 e^{x+y}\,dx dy =$$

15

2.) Valjak: $x^2+y^2=4 \Rightarrow r=2$

Ravnina: $z=y \Rightarrow z=\sin\varphi$

$z=x-2 \Rightarrow z=\cos\varphi-2$

$x=\cos\varphi$

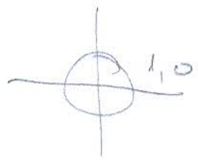
$y=\sin\varphi$

$r\in[0,2]$

$z\in[\sin\varphi,\cos\varphi-2]$

$\varphi\in[0,2\pi]$

$$V=\int_0^{2\pi}\int_{\sin\varphi}^{\cos\varphi-2}\int_0^{2} r\,dr\,dz\,d\varphi=\int_0^{2\pi}\int_{\sin\varphi}^{\cos\varphi-2}\left[\frac{r^2}{2}\right]_0^2 dz\,d\varphi$$



$$=2\int_0^{2\pi}[z]_{\sin\varphi}^{\cos\varphi-2}\,d\varphi=2\int_0^{2\pi}\cos\varphi-2-\sin\varphi\,d\varphi$$

$$=2\left(\underbrace{[\sin\varphi]_0^{2\pi}}_{=0}-\underbrace{2[\varphi]_0^{2\pi}}_{=4\pi}-\underbrace{[-\cos\varphi]_0^{2\pi}}_{=0}\right)=-8\pi$$ <0

3.) $x'''(t)+x'(t)=0$

$x(0)=1$

$x'(0)=1$

$x''(0)=1$

$$s^3X(s)-s^2\underbrace{x(0)}_{1}-s\underbrace{x'(0)}_{1}-\underbrace{x''(0)}_{1}+s^2X(s)-s\underbrace{x(0)}_{1}-\underbrace{x'(0)}_{1}=0$$

$$s^3X(s)-s^2-s-1+s^2X(s)-s-1=0$$

$$X(s)(s^3+s^2)=s^2+2s+2$$

$$X(s)=\frac{s^2+2s+2}{s^2(s+1)}$$

$$\frac{s^2+2s+2}{s^2(s+1)}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+1}$$

$$s^2+2s+2=A(s+1)+Bs(s+1)+Cs^2$$

$$s^2+2s+2=As+A+Bs^2+Bs+Cs^2$$

BORIS KUDELKO

3.)

$1 = B + C \Rightarrow \boxed{C = 1}$

$2 = A + B \Rightarrow B = 2 - A \Rightarrow \boxed{B = 0}$

$2 = A \Rightarrow \boxed{A = 2}$

$X(s) = 2 \cdot \frac{1}{s^2} + \frac{1}{s+1}$

$x(s) = 2t + e^{-t}$

PROVJERA : $x(0) = 1$
$x'(0) = 1$
$x''(0) = 1$
$x'''(t) + x'(t) \neq 0$ ↯

4) $\iint_S w \, r \, |ds = \iint_D w(r) \, \vec{n}$ $\quad w = \begin{pmatrix} 2x \\ 0 \\ 0 \end{pmatrix} \quad r = \begin{pmatrix} \cos u \\ \sin u \\ v \end{pmatrix}$

$\vec{n} = \frac{dr}{du} \times \frac{dr}{dv} = \begin{pmatrix} -\sin u \\ \cos u \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos u \\ \sin u \\ 0 \end{pmatrix}$

$-\sin u \quad 0$
$\cos u \quad 0$
$\quad 0$

OVO JE RAČUN SAMO ZA PLAST BEZ BAZA, A K TOME I POGREŠNO UVRŠTAVANJE GRANICA

$\iint_D \begin{pmatrix} 2x \\ 0 \\ 0 \end{pmatrix} \cdot \vec{n} = \iint_D 2\cos^2 u = \int_{-1}^{1} \int_{(u+2)^2+(v-3)} 2\cos^2 u \, du dv$ ✗

5) $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$

$\frac{df}{dx} = \frac{1}{x} \cdot \sin y \; / \int dx$

$f(x,y) = \ln|x| \sin y + c(y)$

$\frac{df}{dy} = \ln x \cos y$

$\ln|x| \cos y + \frac{df}{c(y)} = \ln |x| \cos y \; / \int dy$

$c(y) = 0$

$f(x,y) = \ln|x| \sin y$

$f(e,\pi) - f(1,0) = \ln|e| \sin\pi - \ln|1| \sin 0$

$= 0$ ✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa XXOOX
IME I PREZIME: LUKA MARĐETKO BROJ INDEKSA:	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15

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$$\iint\limits_{\partial C} 2x \, dydz$$

5. Izračunati $\int\limits_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}dx + \ln x \cos y \, dy$ 20

Ukupno: 35

1.

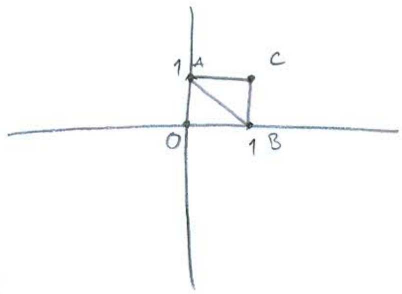
$$\iint e^{x+y} dx\,dy \qquad A(0,1) \quad B(1,0) \quad C(1,1)$$

AB (0,1) (1,0)

$$y-1=\frac{0-1}{1-0}(x-0)$$

$$y-1=-x$$

$$y=-x+1$$



BC (1,0) (1,1)

$$y-0=\frac{1-0}{1-1}(x-1)$$

$$y=0$$

AC (0,1) (1,1)

$$y-1=\frac{1-1}{1-0}(x-0)$$

$$y=1$$

$$\Rightarrow \int\limits_0^1 \int\limits_{-x+1}^1 e^{x+y}\,dy\,dx \quad ✓ \quad 15$$

DALJE . . .

$''(t) + x'(t) = 0$

$(s) - s^2 f(0) - s f'(0) - f''(0) + sF(s) - f(0) = 0$

$F(s) - s^2 - s - 1 + sF(s) - 1 = 0$

$(s^3 + s) = s^2 + s + 2$

$$\frac{s^2+s+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$s^2 + s + 2 = A \cdot (s^2+1) + Bs + C \cdot (s)$

$s^2 + s + 2 = As^2 + A + Bs^2 + Cs$

$1 = A + B \quad \Rightarrow \quad 1 = 2 + B$

$\boxed{1 = C} \qquad -B = 2 - 1$

$\boxed{2 = A} \qquad \boxed{B = -1}$

$(s) = 2 - \cos t + \sin t$

$x'''(t) = -\sin t - \cos t$

ODJ:

$x'''(t) + x'(t) = (-\sin t - \cos t) +$
$+ (\sin t + \cos t) = 0$

$x(0) = 1$
$x'(0) = 1$
$x''(0) = 1$

$'(t) = \sin t + \cos t$
$''(t) = \cos t - \sin t$

Valjak $x^2 + y^2 = 4$, ravninama $z = y$ $z = x - 2$

$V = ?$ $\qquad z = y \rightarrow r\sin\varphi$

$r^2 = 4 \qquad z = x - 2 \rightarrow r\cos\varphi - 2$

$r = 2$

$$V = \int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_{r\cos\varphi - 2}^{r\sin\varphi} dy \quad \times$$

VIDI RJEŠENJE...

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** IME I PREZIME: Lovre Nikitović BROJ INDEKSA: 17-2-0035-2010	Grupa xxoox POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~
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$$\iint\limits_{\partial C} 2x\,dydz$$

5. Izračunati $\int\limits_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}dx+\ln x\cos y\,dy$ ~~20~~

Ukupno: 20

3.

$s^3X(s)-s^2x(0)-sx'(0)-x''(0)+(sX(s)-x(0))=0$

$x(0)=x'(0)=x''(0)=1$

$s^3X(s)-\underbrace{s^2x(0)}_{1}-\underbrace{sx'(0)}_{1}-\underbrace{x''(0)}_{1}+sX(s)-\underbrace{x(0)}_{1}=0$

$s^3X(s)-s^2-s-1+sX(s)-1=0$

$s^3X(s)+sX(s)=s^2+s+2$

$X(s)(s^3+s)=s^2+s+2$

$X(s)=\frac{s^2+s+2}{s^3+s}=\frac{s^2+s+2}{s(s^2+1)}=\frac{A}{s}+\frac{Bs+C}{s^2+1}\Big/ s,s^2+1)$

$s^2+s+2=A(s^2+1)+(Bs+C)s$

$s^2+s+2=As^2+A+Bs^2+Cs$

$=(A+B)s^2+Cs+A$

$\boxed{A=2}$ $\boxed{C=1}$

$A+B=1$

$2+B=1$

$\boxed{B=-1}$

$X(s)=\frac{2}{s}+\frac{-s+1}{s^2+1}$

$X(s)=\frac{2}{s}-\frac{s}{s^2+1}+\frac{1}{s^2+1}$

$\boxed{X(s)=2-\cos(t)+\sin(t)}$ ✓

5.
$$\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y\,dy$$

$$\begin{bmatrix} \frac{\sin y}{x} \\ \ln x \cos y \end{bmatrix} = -\operatorname{grad} f \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -\frac{\sin y}{x} \Rightarrow f = \int -\frac{\sin y}{x}\,dx + c(y) \Rightarrow f = \cos y + c(y)$$?

$$\frac{\partial f}{\partial y} = -\ln x \cos y = \frac{\partial c(y)}{c(y)} =$$

NA DRUGOM MJESTU DANA JE DRUGA FORMULA.

$$V = \int_0^{2\pi}\int_0^2\int_{r\cos\varphi-2}^{r\sin\varphi} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^2 (r\sin\varphi - r\cos\varphi - 2)\,r\,dr\,d\varphi$$

$$V = \int_0^{2\pi}\int_0^2 (r^2\sin\varphi - r^2\cos\varphi - 2r)\,dr\,d\varphi$$

$$V = \int_0^{2\pi} \left(\frac{r^3}{3}\sin\varphi - \frac{r^3}{3}\cos\varphi - 2\cdot\frac{r^2}{2}\right)\Big|_0^2 d\varphi$$

$$V = \int_0^{2\pi} \left(\frac{2^3}{3}\sin\varphi - \frac{2^3}{3}\cos\varphi - 2\cdot\frac{2^2}{2}\right) d\varphi$$

$$V = \int_0^{2\pi} \left(\frac{8}{3}\sin\varphi - \frac{8}{3}\cos\varphi - 4\right) d\varphi$$

$$V = -\frac{8}{3}\cos\varphi - \frac{8}{3}\sin\varphi - 4\varphi \Big|_0^{2\pi}$$

$$V = -\frac{8}{3}\cos 2\pi - \frac{8}{3}\sin 2\pi - 4\cdot 2\pi - \left(-\frac{8}{3}\cos 0 - \frac{8}{3}\sin 0 - 4\cdot 0\right)$$

$$V = -\frac{8}{3} - 0 - 8\pi - \left(-\frac{8}{3} - 0 - 0\right)$$

$$V = -\frac{8}{3} - 8\pi + \frac{8}{3} = -8\pi$$

(1.) $x^2+y^2=4$ $\quad z=y$ $\quad z=x-2$ LOVRE NIKITOVIĆ

$S(0,0,0)$

prijelazom na
cilindrične koordinate

$x=r\cos\varphi$
$y=r\sin\varphi$

$\varphi\in[0,2\pi]$ $\quad r\in[0,2]$
$dx\,dy\,dz=r\,dr\,d\varphi\,dz$

$$V=\int_0^{2\pi}\int_0^2\int_{r\sin\varphi}^{r\cos\varphi-2} r\,dz\,dr\,d\varphi$$

$x^2+y^2=4$
$x^2+y^2=r^2$

$$V=\int_0^{2\pi}\int_0^2 (r\cos\varphi-2-r\sin\varphi)\,r\,dr\,d\varphi$$

$r^2=4$
$r=\sqrt{4}$
$r=2$

$$V=\int_0^{2\pi}\int_0^2 (r^2\cos\varphi-2r-r^2\sin\varphi)\,dr\,d\varphi$$

$z\in[r\sin\varphi,\ r\cos\varphi-2]$

$$V=\int_0^{2\pi}\left(\frac{r^3}{3}\cos\varphi-2\cdot\frac{r^2}{2}-\frac{r^3}{3}\sin\varphi\right)\Big|_0^2\,d\varphi$$

$$V=\int_0^{2\pi}\left(\frac{2^3}{3}\cos\varphi-2\cdot2-\frac{2^3}{3}\sin\varphi\right)d\varphi$$

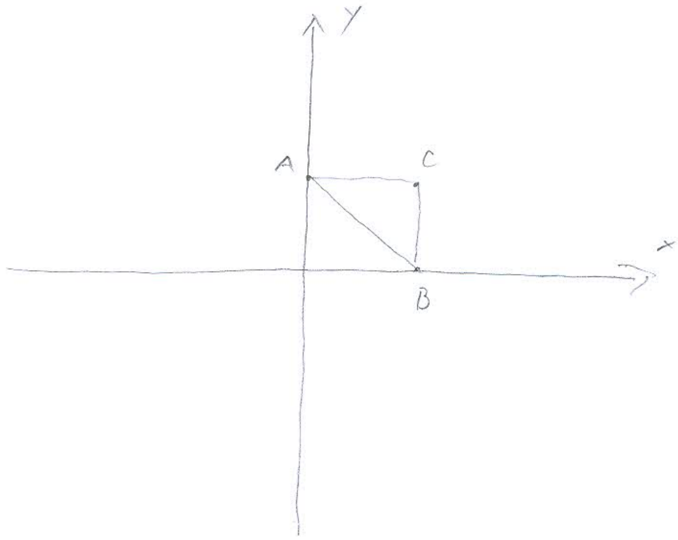
$$V=\int_0^{2\pi}\left(\frac{8}{3}\cos\varphi-4-\frac{8}{3}\sin\varphi\right)d\varphi=\left(\frac{8}{3}\sin\varphi-4\varphi+\frac{8}{3}\cos\varphi\right)\Big|_0^{2\pi}$$

$$V=\frac{8}{3}\sin2\pi-4\cdot2\pi+\frac{8}{3}\cos2\pi-\left(\frac{8}{3}\sin0-4\cdot0+\frac{8}{3}\cos0\right)$$

$$V=\frac{8}{3}\cdot0-8\pi+\frac{8}{3}\cdot1-\left(\frac{8}{3}\cdot0-0+\frac{8}{3}\right)$$

$$V=0-8\pi+\cancel{\frac{8}{3}}-\cancel{\frac{8}{3}}=-8\pi$$

1. $\overset{x_1\ y_1}{A(0,1)}$ $\overset{x_1\ y_1}{B(1,0)}$ $\overset{x_2\ y_2}{C(1,1)}$



$(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$

$\overline{AB}$... $(y-1)(1-0)=(0-1)(x-0)$

$(y-1)\cdot 1=-1(x-0)$

$y-1=-x$

$y=-x+1$

$$\iint_S e^{x+y}\,dx\,dy$$

$\overline{AC}$... $(y-1)(1-0)=(1-1)(x-0)$

$y-1(1)=0(x-0)$

$y-1=0$

$y=1$

$\overset{x_1\ y_1}{B(1,0)}$ $\overset{x_2\ y_2}{C(1,1)}$

$\overline{BC}$... $(y-0)\underbrace{(1-1)}_{=0}=(1-0)(x-1)$

$y=1(x-1)$ ×

$y=x-1$

$e^{x\cdot 2x}$ e^{2x^2}

$$\int_0^1\int_{-x+1}^{1} e^{x+y}\,dy\,dx=\int_0^1 e^x\cdot e^y \Big|_{-x+1}^{1} dx$$

$1+(-x-1)$

$1+x+1$

$2x$

$$\int_0^1 e^x(1+x+1)\cdot e^y(1+x+1)\,dx$$

$$\int_0^1 e^x(2x)\cdot e^y(2x)\,dx$$

$$2xe^x\cdot 2xe^y\Big|_0^1$$

$2\cdot 1e^1\cdot 2\cdot 1e^y-2\cdot 0e^0\cdot 2\cdot 0e^y$

$2e\cdot 2e^y$ $4e^2y$

4. $z[-1,1]$ $x[-1,1]$ LOVRE NIKITOVIĆ

$$W = \begin{bmatrix} 2x \\ 0 \\ 0 \end{bmatrix} \checkmark \qquad \operatorname{div} W = \frac{\partial(2x)}{\partial x} + \frac{\partial(0)}{\partial y} + \frac{\partial(0)}{\partial z}$$

$$\operatorname{div} W = 2 + 0 + 0 = 2 \checkmark$$

$$\iint_{\partial C} 2x \, dy\, dz = \iiint_K \operatorname{div} W \, dx\, dy\, dz = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 2 \, dz\, dy\, dx \times$$

$$= \int_{-1}^{1}\int_{-1}^{1} 2\,(1-1)\, dy\, dz = \int_{-1}^{1}\int_{-1}^{1} 0 \, dy\, dz$$

$$\boxed{= 0}$$

$$(x+2)^2 = x^2 + 4x + 4$$

$$= -1^2 + 4(-1) + 4$$

$$= -1 - 4 + 4$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: BERNARDO KOTLAR **BROJ INDEKSA:** 11-2-0015-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15

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Ukupno: 15

1.

$\iint_S e^{x+y} dx dy$

$A(0,1)$ ~~$B(1,0)$~~ $C(1,1)$

$\int_0^1 \int_{-x+1}^{x+1} e^{x+y} dx dy$

$y \quad x$

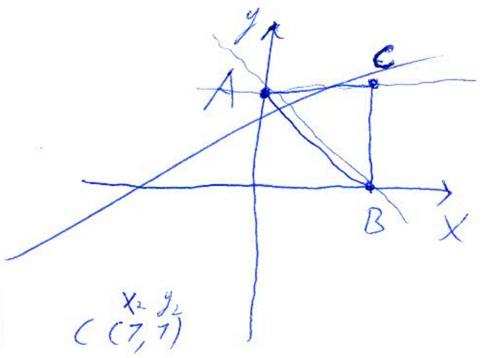
$\int_0^1 \int_{-x+1}^{x+1} e^x \cdot e^y \, dx \, dy$

$\int_0^1 e^x \Big|_{y-1}^{y+1} \cdot e^y \, dy$

$\int_0^1 (e^{x+1} - e^{-x+1}) \cdot e^y \, dy$

$(e^{x+1} - e^{-x+1}) \cdot e^y \Big|_0^1$

$(e^{x+1} - e^{-x+1}) \cdot (e^1 - e^0)$



$\overline{AC}$ $\quad \overset{x_1 \, y_1}{(0,1)}$ $\quad C \overset{x_2 \, y_2}{(1,1)}$

$y - y_1 = \frac{x_2 - x_1}{y_2 - y_1} (x - y_1)$

$y - 1 = \frac{1-0}{1-1} \cdot (x - 0)$

$y = x + 1$

$\overline{AB}$ $\quad A \overset{x_1 \, y_1}{(0,1)}$ $\quad B \overset{x_2 \, y_2}{(1,0)}$

$y - 1 = \frac{1-0}{0-1} (x - 0)$

$y = -x + 1$

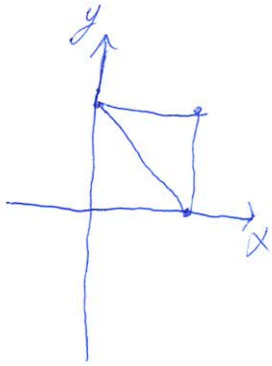
1.

$y-1=-x$

$y=-x+1$

$BC \quad x=1$

$AC \quad y=1$



$$\iint_S e^{x+y}\,dy\,dx = \int_0^1\int_{-x+1}^1 e^{x+y}\,dx\,dy$$

1,5

$$\int_0^1 e^{x+y}\Big|_{-x+1}^{1}\,dx$$

$$=\int_0^1 e^{x+1}-e^{x-x+1}\,dx$$

$$=\left(\cancel{\int_0^1 e^{x+1}-e^1\,dx}\right)$$

$$\int_0^1 e^{x+1}\,dx-\int_0^1 e^1\,dx$$

$$e^{x+1}\Big|_0^1-e^x\Big|_0^1$$

$$e^{1+1}-e^1-(e^1-e^1)$$

$$e^2-e^0-e$$

$\int e^{x+y}\,dy \quad x+y=t \;/'$

$dy=dt$

$\int e^t\,dt=e^t$

$\Rightarrow e^{x+y}$

$\int e^{x+y}\,dx \quad x+1=t \;/'$

$e^t\,dt=et \quad dx=dt$

$\int e^{x+1}\,dx=e^{x+1}$

~ARDO KOTLAR

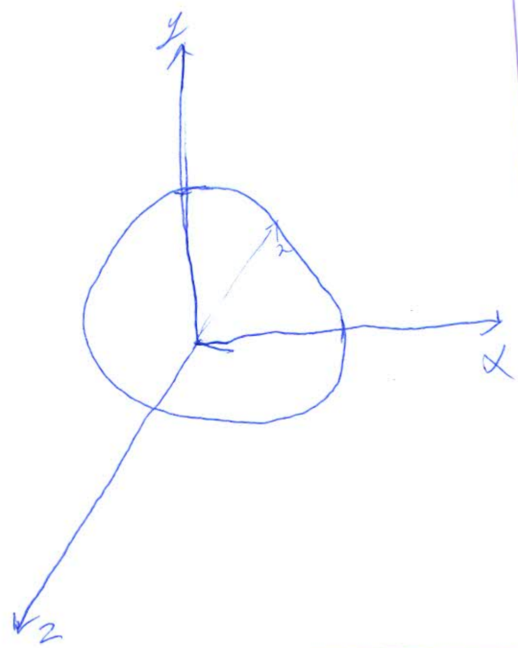
$x^2+y^2=4$ $\quad z=y \quad z=x-2$

$x+y=2$

$x=2-y$

$y=2-x$

$\iint f($



$r=\sqrt{4}=2$

$$\int_0^{2\pi}\int_0^2\int_{r\cos\varphi}^{r\sin\varphi} r\,dz\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^2 r_z \Big|_{r\cos\varphi-2}^{r\sin\varphi} dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^2 r\cdot(r\sin\varphi - r\cos\varphi-2)\,dr\,d\varphi$$

$$\int_0^{2\pi} \cancel{-r\cos y - r\sin z \Big|_0^2}\, dy$$

$$\cancel{\int_0^{2\pi}\int_0^2 r\cdot(\sin\varphi - \cos\varphi-2)\,dr\,dy}$$

$$\int_0^{2\pi}\int_0^2 r^2\sin\varphi - r^2\cos\varphi-2\,dr\,d\varphi$$

$$\int_0^{2\pi} \frac{r^{2+1}}{2+1}\cos\varphi - \frac{r^{2+1}}{2+1}\sin\varphi-2 \Big|_0^2 d\varphi$$

$z_2 = y = r\sin\varphi$

$z_1 = x-2 = r\cos\varphi-2$

$$\int_0^{2\pi}\left(-\frac{2^3}{3}\cos\varphi-\frac{2^3}{3}\sin\varphi-2\right)-\underbrace{\left(-\frac{0^3}{3}\cdot\cos\varphi-\frac{0^3}{3}\sin\varphi-2\right)}_{0}$$

$$\int_0^{2\pi}\frac{8}{3}\cos\varphi-\frac{8}{3}\sin\varphi-2\,d\varphi$$

$$\frac{8}{3}\int_0^{2\pi}\cos\varphi-\sin\varphi-2\,d\varphi$$

$$\frac{8}{3}\sin\varphi+\cos\varphi-2\Big|_0^{2\pi}$$

$$\frac{8}{3}\cdot\left[(\sin 2\pi+\cos 2\pi-2)-(\sin 0+\cos 0-2)\right]$$

$$=-\frac{8}{3}+8\pi+\frac{8}{3}=8\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**

Grupa xxoox

Ime i prezime: ANTE BOTICA **Broj indeksa:** 17-1-0019-2010

Popunjava nastavnik
Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. ~~20~~
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$$x'''(t)+x'(t)=0, \quad x(0)=x'(0)=x''(0)=1.$$
4. Neka je C cilindar zadan sa $C=\{(x,y,z):(x+2)^2+(y-3)^2\le 1, -1\le z\le 1\}$. Izračunati plošni integral 20
$$\iint\limits_{\partial C} 2x\,dydz$$
5. Izračunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx+\ln x\cos y\,dy$ 20

Ukupno: 15

(3) $L(x'''(t)) + L(x'(t)) = 0$

za $x(0)=1$
$x'(0)=1$
$x''(0)=1$

$s^3X(t) - \overset{s^2}{s^2}\,\overset{1}{x(0)} - \overset{s}{s}\,\overset{1}{x'(0)} - \overset{1}{x''(0)} + \left[sX(t) - \overset{1}{x(0)}\right] = 0$

$s^3X(t) - s^2 - s - 1 + sX(t) - 1 = 0$

$s^3X(t) + sX(t) - s^2 - s - 2 = 0$

$s^3X(t) + sX(t) = s^2+s+2$

$X(t)(s^3+s) = s^2+s+2$

$s(s^2+1)=0$

$X(t) = \dfrac{s^2+s+2}{s^3+s} = \dfrac{s^2+s+2}{s(s^2+1)}$

$s_1 = 0$
A

$s^2+1=0$
$s^2=-1$
$Bs+C$

$\dfrac{s^2+s+2}{s(s^2+1)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+1}$

ANTO BOTICA

$$A = 0 \quad B = 2 \quad C = 0$$

Za $S = 0$

$-0 = C$

Za $S = -1$

$$\frac{1 - 1 + 2}{-1(1+1)} = \frac{A}{-1} + \frac{-B + C}{1+1} \Rightarrow \frac{2}{-2} = -A - \frac{B}{2} + \frac{C}{2} \quad C = 0$$

$$-1 = -A - \frac{B}{2} \Big/ \cdot 2 \Rightarrow -2 = -2A - B$$

$$\Rightarrow B = -2A + 2 \quad (1)$$

$S = 1$

$$\frac{1 + 1 + 2}{1(1+1)} = A + \frac{B + C}{2} \Rightarrow \frac{4}{2} = A + \frac{B}{2} + \frac{C}{2} \Rightarrow 2 = A + \frac{B}{2} \Big/ \cdot 2$$

$$4 = 2A + B$$

$$B = 4 - 2A \quad (2)$$

$= 2$

$$\frac{4 + 2 + 2}{2(4+1)} = \frac{A}{2} + \frac{2B + C}{4+1}$$

$$\frac{8}{10} = \frac{A}{2} + \frac{2B}{5} + \frac{C}{5}$$

$$= \frac{A}{2} + \frac{2B}{5} \Big/ \cdot 5$$

$$= \frac{5}{2}A + 2B \Big/ : 2$$

$$= \frac{5}{4}A + B$$

$$= 2 - \frac{5}{4}A \quad (3)$$

$(3) \to (1)$

$$B = -2A + 2$$

$$B = 2 - \frac{5}{4}A$$

$$2 - \frac{5}{4}A = -2A + 2 \Big/ \cdot 4$$

$$8 - 5A = -8A + 8$$

$$-5A + 8A = 8 - 8$$

$$3A = 0$$

$$A = 0$$

$$B = 0 + 2$$

$$B = 2$$

$$(6)\ \frac{p}{p^2 + a^2} = \cos at$$

$a = 1$

$$X(t) = \frac{0}{s} + \frac{2s + 0}{s^2 + 1} \Rightarrow X(t) = \frac{2s}{s^2+1} = 2\left(\frac{s}{s^2+1}\right)$$

$$\boxed{x'(t) = 2\cos t}$$

$x(t) = 2\cos t$

$x'(t) = -2\sin t$

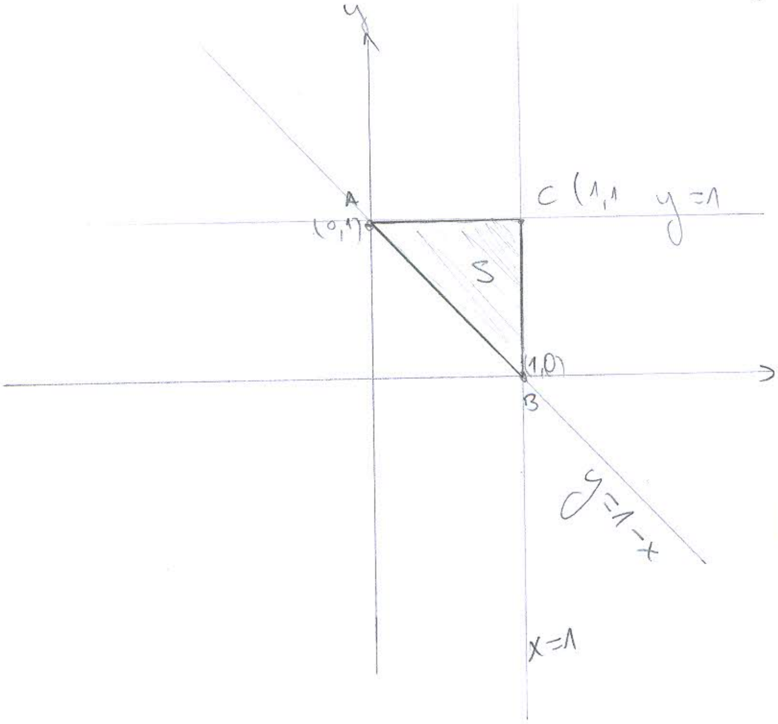
$x(0) = 2$

$x'(0) = 0$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

① $\iint_S e^{x+y}\,dx\,dy \quad ; \quad S : \Delta$

	x	y
A =	(0,	1)
B	(1,	0)
C	(1,	1)



$p_{AB} \cdots y - 1 = \frac{0-1}{1-0}(x - 0)$

$y - 1 = \frac{-1}{1}(x)$

$y - 1 = -1x$

$\boxed{y = 1 - x} \quad ; \quad x = 1 - y$

$p_{AC} \cdots y - 1 = \frac{1-1}{1-0}(x - 0)$

$y - 1 = 0$

$\boxed{y = 1}$

$p_{BC} \cdots \boxed{x = 1}$

$$I = \int_0^1 \int_{1-y}^1 e^{x+y}\,dx\,dy = \int_0^1 e^{x+y}\Big|_{1-y}^{1} dy$$

15 ✓

$$= \int_0^1 (e^{1-y} - e^{1-y-y})\,dy = \int_0^1 e^{1-y} - e^1\,dy = \int_0^1 e^{1-y}\,dy - \int_0^1 e^1\,dy$$

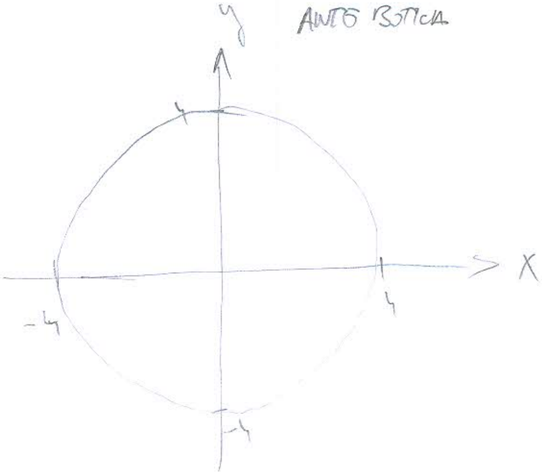
×

$$= e^{1-y}\Big|_0^1 - e = \left[e^{1-1} - e^{1-0}\right] - e = e^0 - e^1 - e^1 = 1 - 2e^1 = -4{,}436563\ldots$$

×

ANTE BOTICA

(2) $x^2+y^2=4$ valjak $R=r$; $\underline{r=4}$

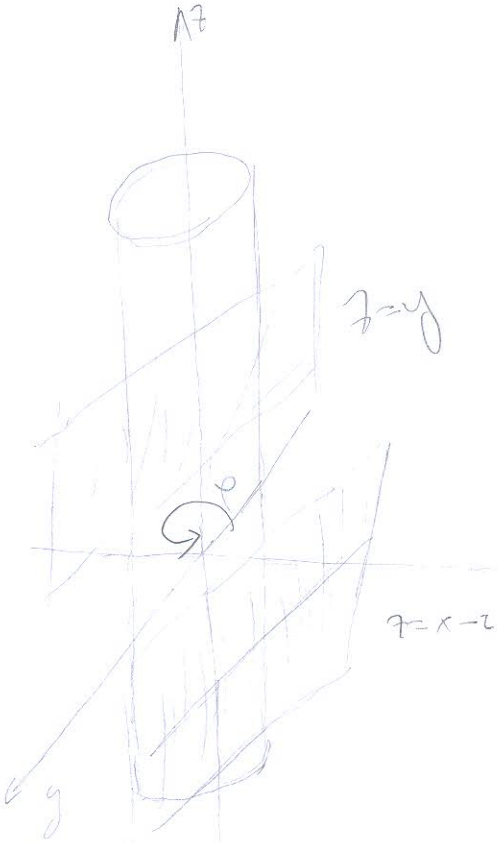


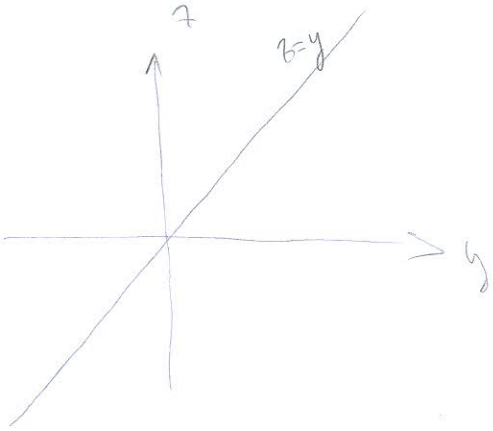
$z=y$

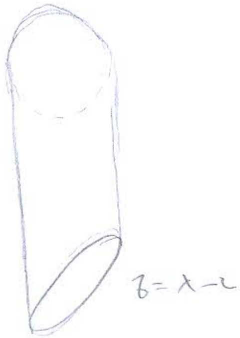
$z=x-2$

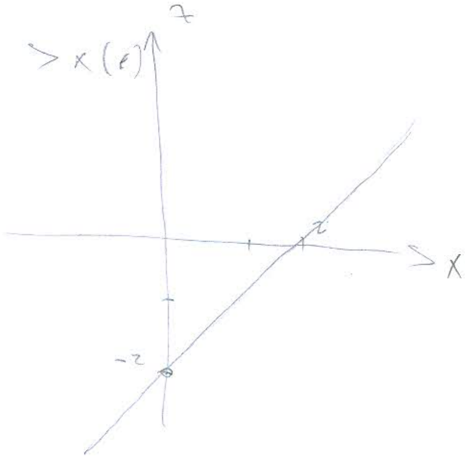
$x=r\cos\varphi$

$y=r\sin\varphi$









$z\in[x-2\,;\,y] \Rightarrow [r\cos\varphi-2\,;\,r\sin\varphi]$

$\varphi\in[0,2\pi]$

$r\in[0,4]$

$z=x-2$

$z=0 \quad x=2$

$x=0 \quad z=-2$

$$\int_0^{2\pi} d\varphi\int_0^{4} r\,dr\int_{r\cos\varphi-2}^{r\sin\varphi} dz\; 1\, r\,dr\,d\varphi\,dz \quad \times$$

$$=\int_0^{2\pi} d\varphi\int_0^4 r\,dr\; z\Big|_{r\cos\varphi-2}^{r\sin\varphi} = \int_0^{2\pi} d\varphi\int_0^4 (r\sin\varphi - r\cos\varphi+2)\,r\,dr$$

$$=\int_0^{2\pi} d\varphi\int_0^4 r^2\sin\varphi - r^2\cos\varphi + 2r\;dr = \int_0^{2\pi} d\varphi\left(\frac{r^3}{3}\sin\varphi\Big|_0^4 - \frac{r^3}{3}\cos\varphi\Big|_0^4 + \frac{\cancel{2}r^2}{\cancel{2}}\Big|_0^4\right)$$

$$=\int_0^{2\pi}\left(\frac{4^3-0^3}{3}\sin\varphi - \frac{4^3-0^3}{3}\cos\varphi + 4^2-0^2\right)d\varphi = \int_0^{2\pi}\left(\frac{64}{3}\sin\varphi - \frac{64}{3}\cos\varphi + 16\right)d\varphi$$

(2) $$\int_0^{2\pi} \frac{64}{3} \sin\varphi \, d\varphi - \int_0^{2\pi} \frac{64}{3} \cos\varphi \, d\varphi + \int_0^{2\pi} 16 \, d\varphi = -\frac{64}{3} \cos\varphi \Big|_0^{2\pi} - \frac{64}{3} \sin\varphi \Big|_0^{2\pi} + 16\varphi \Big|_0^{2\pi}$$

$$= -\frac{64}{3}\cancel{\left[\overset{1}{\cos 2\pi} - \overset{1}{\cos 0}\right]} - \frac{64}{3}\cancel{\left[\overset{0}{\sin 2\pi} - \overset{0}{\sin 0}\right]} + 16 \cdot 2\pi$$

$$= 16 \cdot 2\pi = \underline{\underline{32\pi}}$$

$$\boxed{V = 32\pi}$$

volumen tijela je 32π

(6.) $$\int_{1,0}^{e,\pi} \frac{\sin y}{x} \, dx + \ln x \cos y \, dy$$

$\varnothing$

ANTE BOTICA

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: Toma Medić BROJ INDEKSA: 17-2-0052	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t)+x'(t)=0, \quad x(0)=x'(0)=x''(0)=1.$$

4. Neka je C cilindar zadan sa $C=\{(x,y,z):(x+2)^2+(y-3)^2\leq 1, -1\leq z\leq 1\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} 2x \, dydz$$

5. Izračunati $\int\limits_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ 20

Ukupno: 15

3.) $s^3X(s)-s^2x(0)-sx'(0)-x''(0)+sX(s)-x(0)=0$

$s^3X(s)-s^2-s-1+sX(s)-1=0$

$s^3X(s)-sX(s)=s^2+s+2$

$X(s)(s^3-s)=s^2+s+2 \quad /:(s^3-s)$

$X(s)=\frac{s^2+s+2}{s^3-s}=\frac{s^2+s+2}{s(s^2-1)}$

$\frac{s^2+s+2}{s(s^2-1)}=\frac{A}{s}+\frac{Bs+C}{s^2-1} \quad /\cdot s(s^2-1)$

$s^2+s+2=A(s^2-1)+(Bs+C)s$

$s^2+s+2=As^2-A+Bs^2+Cs$

$s^2+s+2=(A+B)s^2+Cs-A$

$A+B=1 \qquad -2+B=1$

$-A=2 \qquad B=3$

$A=-2 \qquad C=1$

$$X(s) = \frac{-2}{s} + \frac{3s+1}{s^2-1}$$

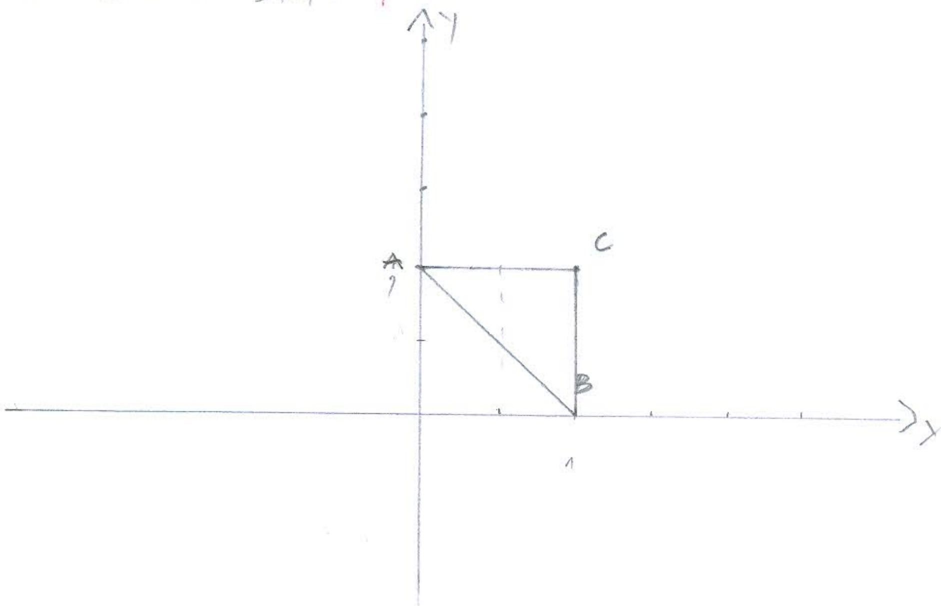
$$X(s) = -2\,\frac{1}{s} + 3\,\frac{s}{s^2-1} + \frac{1}{s^2-1}$$

$$X(s) = -2 + 3\cos t + \sin t$$

$x'(t) = -3\sin t + \cos t$ ... $x'(0) = 1$

$x''(t) = -3\cos t - \sin t$ ... $x''(0) = 3$

1.) $A(0,1)$
$B(1,0)$
$C(1,1)$



$1°$

$\overline{AB} \ldots (x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$\overline{AB} \ldots (1-0)(y-1) = (0-1)(x-0)$

$\overline{AB} \ldots y-1 = -x$

$\overline{AB} \ldots y = -x+1$

$2°$

$\overline{BC} \ldots (x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$\overline{BC} \ldots x = 1$

DALJE ...

$3°$

$\overline{AC} \ldots y = 1$

Toma Medić

$\iint e^{x+y}\,dx\,dy$

15

$$\int_0^1 \int_{-x+1}^{y=1} e^{x+y}\,dy\,dx = \int_0^1 \int_{-x+1}^{y=1} e^{x}\,dx + \int_0^1 \int_{-x+1}^{y=1} e^{y}\,dy =$$

$$= \int_0^1 e^{-x+1}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15

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$$x'''(t)+x'(t)=0, \quad x(0)=x'(0)=x''(0)=1.$$

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$$\iint_{\partial C} 2x\, dy dz$$

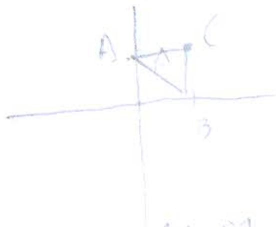
5. Izračunati $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y\, dy$ 20

Ukupno: 15

$x(0)=x'$

3. $x'''(t)+x'(t)=0$

$s^3X(s)-s^2x(0)-sx'-x''(0)+sX(s)-x(0)=0 \rightarrow$



1. $(x_2-x_1)\cdot(y-y_1)=(y_2-y_1)\cdot(x-x_1)$

$(1-0)\cdot(y-1)=(0-1)\cdot(x-0)$

$y-1=-x$

$y=-x+1$

$\overline{CA}=(0-1)\cdot(y-1)=(1-1)\cdot$

$(y-1)$

$y=1$

BC

$(1-1)\cdot(y-0)=(1-0)\cdot(x-1)$

$0=x-1$

$x=1$

$$p = \iint e^{x+y}\,dx\,dy$$

$e^x \cdot e^y$

$$p = \int_0^1 \int_{-x+1}^{1} e^{x+y}\,dy\,dx$$

$$p = \int_0^1 \int_{-x+1}^{1} e^x \cdot e^y\,dy\,dx$$

$\underline{15}$

$$p = \int_0^1 e^x \cdot e^y \Big/_{-x+1}^{1}\,dx$$

$1-(-x+1)$

$$p = \int_0^1 e^x \cdot e^x\,dx$$

$$p = e^x \cdot e^x \Big/_0^1$$

$$p = e^1 \cdot e^1$$

$$p = e^2$$

$$p = 7.38$$

$1 \rightarrow \int^3 x(1) - \int^2 x(0)$

$$p = \int_0^1 e^{x^2}\,dx$$

$$p = \int_0^1 e^{\frac{x^3}{3}}$$

$$p = \int e^x \cdot e^x$$

$$p = e^x \cdot e^x \Big/_0^1$$

$$p = e^1 \cdot e^1$$

$p = 7.$

3. $s^3X(s) - s^2x(0) - sx'(0) - x''(0) + sX(s) - x(0) = 0$ žemie

$x(0)=1,\ x'(0)=1,\ x''(0)=1,\ x(0)=1$

$s^3X(s) - s^2 - s - 1 + sX(s) - 1 = 0$

$s^3X(s) - s^2 - s + sX(s) - 2 = 0$

$X(s)\cdot(s^3+s) - s^2 - s - 2 = 0$

$X(s)\cdot(s^3+s) = s^2+s+2$

$X(s) = \frac{s^2+s+2}{s^3+s}$

$X(s) = \frac{s^2+s+2}{s\cdot(s^2+1)}$

$X(s) = \frac{A}{s} + \frac{Bs+C}{s^2+1}$ / $\cdot$ množ

$x(s) = \frac{0}{s} \cdot \frac{s+1}{s^2+1}$

$X(s) = 0$

$s^3+s = As^2 + A + Bs^2 + Cs$

$s\cdot(s^2+1) = As^2+A+Bs^2+Cs$

$C=1$ $A=0$

$B=1$

?

s^3+s^2

$s(s^2+1)=0$

$A+B=1$ $Cs=0$

$A=0$ $B=1$

Firne Zeni

4

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa XXOOX
IME I PREZIME: BRUNO LIPOTILA BROJ INDEKSA: 54960	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. — ~~20~~

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3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20

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$$\iint\limits_{\partial C} 2x \, dy dz$$

5. Izračunati $\int\limits_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ — 20

Ukupno: 0

(2)

$x^2 + y^2 = 4 \qquad z = y \quad z = x - 2$

$\varphi\ (0, 2\pi)$

$r\ (0, 2)$

$z\ (r\sin\varphi, r\cos\varphi + 2)$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$\int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_{r\sin\varphi}^{r\cos\varphi+2} dz$$

$\times$

$$\int_0^{2\pi} d\varphi \int_0^2 r\,(r\cos\varphi + 2 - r\sin\varphi)$$

$$\int_0^{2\pi} d\varphi \int_0^2 (r^2\cos\varphi + 2r) - r^2\sin\varphi$$

$$\int_0^{2\pi} d\varphi \left(\frac{r^3}{3}\cos\varphi + \cancel{2}\frac{r^2}{\cancel{2}}\right) - \left(\frac{r^3}{3}\sin\varphi\right)\Bigg|_0^2$$

$$\int_0^{2\pi} \left(\frac{2^3}{3}\cos\varphi + 2^2\right) - \left(\frac{2^3}{3}\sin\varphi\right)$$

$$\int_0^{2\pi} \left(\frac{8}{3}\cos\varphi + 4\right) - \left(\frac{8}{3}\sin\varphi\right)$$

$$\frac{8}{3}\sin\varphi + 4\varphi + \frac{8}{3}\cos\varphi \Big|_0^{2\pi}$$

$$\frac{8}{3}\sin 2\pi + 4\cdot 2\pi + \frac{8}{3}\cos 2\pi - \left(\frac{8}{3}\sin 0 - 4\cdot 0 + \frac{8}{3}\cos 0\right)$$

$$\frac{8}{3}\cdot 0 + 8\pi + \frac{8}{3}\cdot 1 - \left(0 - 0 + \frac{8}{3}\right)$$

$$= 8\pi + \cancel{\frac{8}{3}} - \cancel{\frac{8}{3}}$$

$$= 8\pi$$

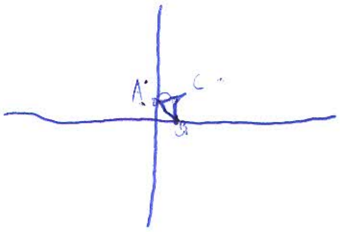
---

1. $\iint e^{x+y}$

$A(0,1)$ (x_1, y_1)

$B(1,0)$ (x_1, y_1)

$C(1,1)$ (x_2, y_2)



$$\int_0^1 \int_{-x+1}^{x+1} e^x + e^y \, dx\, dy$$ ×

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\vec{AB}$

$$y - 1 = \frac{0-1}{1-0}(x-0)$$

$$y - 1 = \frac{-1}{1}(x-0)$$

$$y - 1 = -1(x-0)$$

$$y - 1 = -x$$

$$\boxed{y = -x+1}$$

$\vec{AC}$

$$y - 1 = \frac{1-1}{1-0}(x-0)$$

$$y - 1 = 1(x-0)$$

$$y - 1 = x \quad \boxed{y = x+1}$$

(4)

LIPOTICA

$\{(x,y,z): (x+2)^2 + (y-3)^2 \leq 1, -1 \leq z \leq 1\}$

$\iint 2x \, dy \, dz$

$\frac{\partial \varphi}{\partial x} - \frac{\partial \eta}{\partial y} =$

$2(x+2)$

$\int_{-1}^{1}\int_{0}^{1} 2x \cdot 2(x+2)$ ×

$\int_{-1}^{1}\int_{0}^{1} 2x \cdot 2x + 4$

$\int_{-1}^{1}\int_{0}^{1} 4x^2 + 4$

$\int_{-1}^{1} 4\frac{x^3}{3} + 4x \Big|_{0}^{1}$

$\int_{-1}^{1} 4\frac{1^3}{3} + 4$

$\int_{-1}^{1} \frac{4}{3} + 4 \, dz$

$\int_{-1}^{1} \frac{16}{3} \, dz$

$\frac{16}{3} z \Big|_{-1}^{1}$

$\frac{16}{3} \cdot 1 - (\frac{16}{3} \cdot (-1))$

$\frac{16}{3} + \frac{16}{3}$

$= \frac{32}{3}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: JOSIP KALEBIĆ BROJ INDEKSA: 56776-2008	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}\mathrm{d}x\mathrm{d}y$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. ~~20~~
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t)+x'(t)=0, \quad x(0)=x'(0)=x''(0)=1.$$

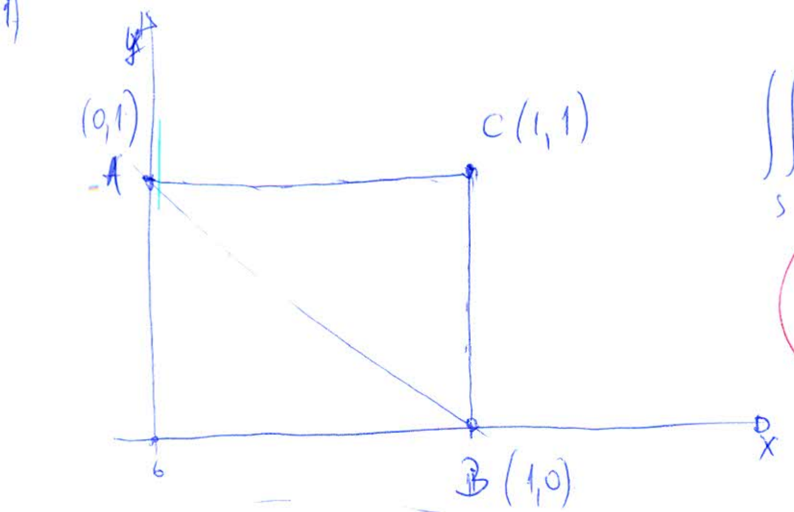
4. Neka je C cilindar zadan sa $C=\left\{(x,y,z) : (x+2)^2+(y-3)^2 \le 1, -1\le z\le 1\right\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} 2x\,\mathrm{d}y\mathrm{d}z$$

5. Izračunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,\mathrm{d}x+\ln x\cos y\,\mathrm{d}y$ 20

Ukupno: 0

1)



$$\iint_S e^{x+y}\,dx\,dy$$

$$I=\int_0^1\left(\int_0^1 \left(e^{x+y}\right)dx\right)dy$$

$$I=\int_0^1\left(\int_0^1 \left(e^{x+y}\right)dx\right)dy \quad \begin{cases} x+y=t \\ dx+y=dt \\ dx=dt-y \end{cases}$$

$$=\int_0^1\left(\int_0^1 \left(e^{t}\right)\cdot dt-y\right)dy$$

$$=\int_0^1\left(\left[e^t\Big|_0^1\right]-y\right)dy$$

$$=\int_0^1\left(\left(e^1-e^0\right)-y\right)dy$$

$$=\int_0^1 (e-y)\,dy$$

$$=\left[(e-y)\Big|_0^1\right]=(e-1)-(e-0)$$

$$=e-1-e$$

$$=-1$$

?

(3)

$$X'''(t) + X'(t) = 0 \qquad X'(0) = X(0) = X''(0) = 1$$

$$s^3 \mathcal{X}(t) - s^2 \cdot X(0) - s \cdot X'(0) - X''(0) + s \cdot \mathcal{X}(t) - X(0) = 0$$

$$s^3 \mathcal{X}(t) - s^2 - s - 1 + s \cdot \mathcal{X}(t) - 1 = 0$$

$$\mathcal{X}(t)\left(s^3 + s\right) - s^2 - s - 2 = 0$$

$$\mathcal{X}(t)\left(s^3 + s\right) = s^2 + s + 2$$

$$\mathcal{X}(t) = \frac{s^2 + s + 2}{s^3 + s} = \frac{s^2 + s + 2}{s(s^2 + 1)}$$

ovo je ok!

molim vas pogledajte iako je prekrižemo

(prekrižemo je greškom)

$$\frac{s^2 + s + 2}{s(s^2+1)} = \frac{A}{s} + \frac{B}{(s^2+1)} + \frac{C}{s(s^2+1)}$$

$$\frac{s^2 + s + 2}{s(s^2+1)} = \frac{A(s^2+1) + B \cdot s + C}{s(s^2+1)} = \frac{As^2 + A + Bs + C}{s(s^2+1)}$$

$$\frac{s^2 + s + 2}{s(s^2+1)} = \frac{As^2 + Bs + A + C}{s(s^2+1)}$$

$$\Rightarrow A = 1$$

$$B = 1$$

$$A + C = 2$$

$$C = 2 - A = 1$$

$$\mathcal{X}(s) \quad \frac{s^2 + s + 2}{s(s^2+1)} = \frac{1}{s} + \frac{1}{s^2+1} + \frac{1}{s(s^2+1)}$$

$$x(s) = L^{-1}\left(\mathcal{X}(s)\right) = \frac{1}{s} + \frac{1}{s^2+1^2} + \frac{1}{s} \cdot \frac{1}{(s^2+1^2)}$$

$$= 1 + \sin(s) + 1 \cdot \sin(s)$$

$$= 1 + \sin(s) + \sin(s) = 1 + 2 \cdot \sin(s)$$

!

JOSIP KALEBIĆ

3. NASTAVAK

$$X(t) = \frac{s^2+s+2}{s(s^2+1)} = \frac{s^2+s+2}{s(s-1)(s-1)}$$

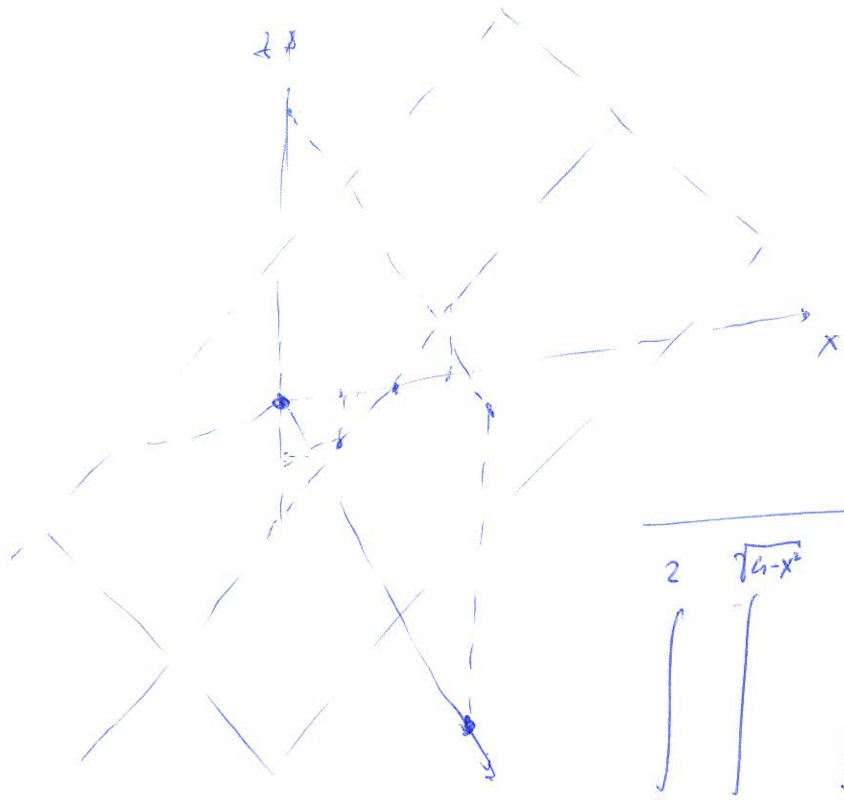
$$= \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-1}$$

$$\frac{s^2+s+2}{s(s-1)(s-1)} = \frac{A(s-1)(s-1) + B\,s(s-1) + C(s-1)\cdot s}{s(s-1)(s-1)}$$

$$= \underline{\qquad\qquad}$$

---

2. volumen valjka. $x^2+y^2=4$; $z=y$ $z=x-2$



$$D = \left\{ (x,y,z) : \begin{array}{l} x\ (0,2) \\ y\ \left(-\sqrt{4-x^2},\ \sqrt{4-x^2}\right) \\ z = (-2, 0) \end{array} \right\}$$

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-2}^{0} \left(\sqrt{4-x^2}\right) dx\, dy\, dz$$

$x = r\cdot \sin\varphi$
$y = r\cdot\cos\varphi$

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-2}^{0} \left(\sqrt{4-r^2\sin^2\varphi}\right)\cdot r\cdot dr\cdot d\varphi$$

$$\int_0^2 \int_0^{\pi} \left(\sqrt{4-r^2 \sin^2\varphi}\right) r \cdot dr \cdot d\varphi$$

$$4-r^2 \cdot \sin^2\varphi = t \; /'$$

$$-2 \cdot r \cdot \cos^2\varphi \, dr = dt$$

$$dr = \frac{dt}{-2 \cdot r \cdot \cos^2\varphi}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: IVAN ŠIKIĆ

BROJ INDEKSA: 17-1-0014-2010

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20
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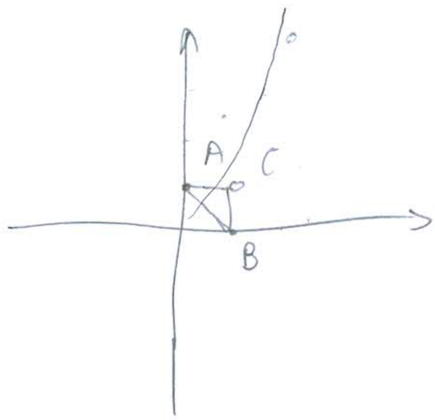
Ukupno:

1°



$\overline{AC}$

$\overline{CB}$

AC:

$$x - x_1 = \frac{y_2 - y_1}{x_2 - x_1}(y - y_1)$$

$$x - 0 = \frac{1-1}{1-0}($$

$$\underline{x = 0}$$

BC:

$$x - 1 = \frac{1-0}{1-1}$$

$$x = 0$$

$$\int_0^1 \int_0^0 e^{x+y} dx dy$$

$$\int_0^1 \int_0^0 e^x \cdot e^y dx dy = 0$$

PREMA TEOREMU O DIVERGENCIJI SLIJEDI

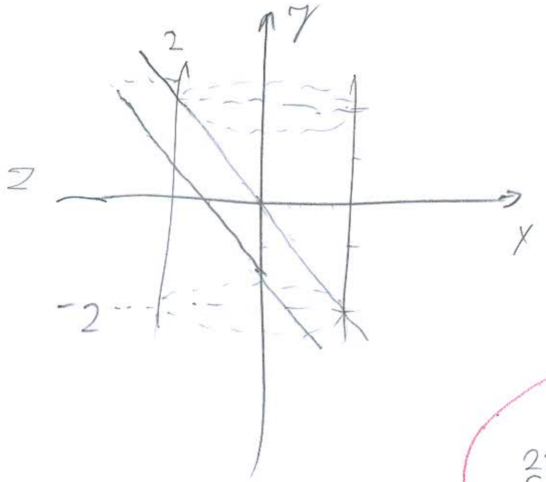
$$\iint\limits_{\partial K} (w / ds) = \iiint (\operatorname{div} V) = 0$$

2. $x^2+y^2=4$

$z=y$

$z=x-2$

$z+2=x$



$x^2+y^2=r^2$

$|r|=2$

$x=r\cos\varphi$

$y=r\sin\varphi$

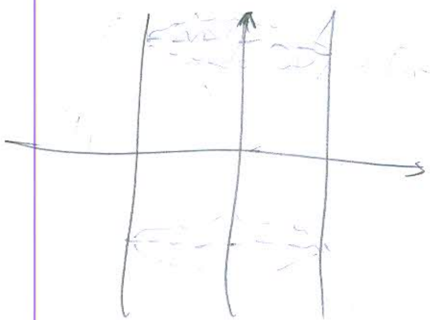
$y=2\sin\varphi$

$x=2\cos\varphi-2$

$$\int_0^{2\pi} d\varphi \int_{-2}^{2} dr \int_0^{\sqrt{z+2}} 2\sin\varphi\cdot(2\cos\varphi-2)\, r\,dr\,d\varphi=$$

4. $C(x,y,z)$ $\quad (x+2)^2+(y-3)^2\le 1 \quad , -1\le z\le 1$

$z\in[-1,1]$



$$\iint_{\partial C} 2x\,dy\,dz$$

$$\int_0^{2\pi} d\varphi \int_{-1}^{1} 2(\cos\varphi+2)\, r\,dr\,d\varphi=$$

$x=1\cos\varphi+2$

$y=1\sin\varphi\cancel{+}3$

$$\int_0^{2\pi} 6 = 12\pi$$

$\cos\varphi+2$

$\sin\varphi-3$

IVAN ŠIKIĆ 17-1-0014-2010

5. $$\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y\,dy$$

$$\begin{bmatrix} \frac{\sin y}{x} \\ \ln x \cos y \end{bmatrix}$$

?

$$r' = \begin{bmatrix} 0 \\ \\ \end{bmatrix}$$

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IME I PREZIME: NINO MIKULANDRA **BROJ INDEKSA:** 57645	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y}dxdy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~

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Ukupno: ~~0~~

2) $x^2+y^2=4 \qquad z=y$ i $z=x-2$

$r^2\cos^2\varphi+r^2\sin^2\varphi=4 \qquad z=y$

$r^2(\underbrace{\cos^2\varphi+\sin^2\varphi}_{=1})=4 \qquad z=x-2$

$r^2=4/\sqrt{\ }$

$r=\sqrt{4}$

$r\in[0,\sqrt{4}]$

$z\in[y,x-2]$

$\varphi\in[0,2\pi]$

$$\int_0^{2\pi}\int_y^{x-2}\int_0^{\sqrt{4}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}d\varphi\int_y^{x-2}\frac{r^2}{2}dr\int_0^{\sqrt{4}}dz =$$

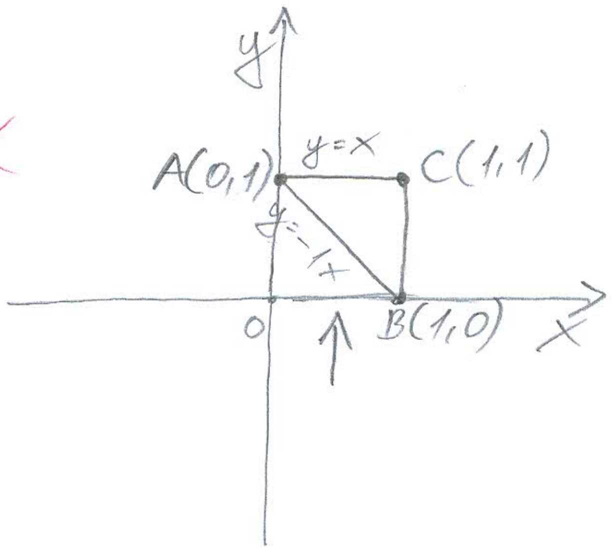
NINO MIKULANDRA 57645

1.) $\iint\limits_S e^{x+y}\,dx\,dy$

$A(0,1),\ B(1,0),\ C(1,1)$

$$\int_1^1\int_{-1x}^{x} e^{x+y}\,dx\,dy =$$

$$\int_1^1 e^{x+y}\,dx + \int_{-1x}^{x} e^{x+y}\,dy =$$



$A(\overset{x_1}{0},\overset{y_1}{1})$

$C(\overset{x_2}{1},\overset{y_2}{1})$

$$y-y_1\ \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-1\ \frac{1-1}{1-0}(x-0)$$

$$\underline{\underline{y=x}}$$

$A(\overset{x_1}{0},\overset{y_1}{1})$

$B(\overset{x_2}{1},\overset{y_2}{0})$

$$y-y_1\ \frac{y_2-y_1}{x_2-y_1}(x-x_1)$$

$$y-1\ \frac{0-1}{1-0}(x-0)$$

$$y-1\ \frac{-1}{1}(x-0)$$

$$y-1\cdot(-1)(x-0)$$

$$y=1\cdot(-1)(x-0)$$

$$\underline{\underline{y=-1x}}$$

5.) $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y\, dy$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = -\operatorname{grad} \varphi = \begin{bmatrix} -\frac{\partial \varphi}{\partial x} \\ -\frac{\partial \varphi}{\partial y} \end{bmatrix}$$

$$d_x \varphi = -\ln x \cos y \;/ \int dx$$

$$\varphi = -\ln x \cos y \int dx$$

$$\varphi = -\ln x^2 = -\ln x^2 + C(y)$$

$$d_y \varphi = -\frac{\sin y}{x} \;/ \int dx$$

$$d_y(-\ln x^2 + C(y)) - \frac{\sin y}{x}$$