MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ S5

IME I PREZIME: MARINO ŽUBČIĆ **BROJ INDEKSA:** 17-2-0216-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

40

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 5i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 = -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = 1 \\ 3x_1 & & & - & x_3 & + & x_4 = -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = -1 \end{array}$$

3. Odrediti kada je $\frac{x+1}{\sqrt{x^2-x}} + 1 > 0$ i obavezno provjeriti rješenje. 13+2

4. Za funkciju: $f(x) = \sqrt{x^2+7x+7}$ treba:
   (a) pronaći drugu derivaciju 10
   (b) na temelju ispitivanja toka funkcije skicirati graf 20(graf)

5. Odrediti i provjeriti rješenje $\lim_{x\to+\infty} \frac{x^2}{e^{2x}} =$ 8+2

6. Riješiti jednadžbu $e^x = \arccos x$ grafičkom metodom. *Provjeriti uvrštavanjem!* 10+5

Ukupno:

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(2)

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} -1 & -1 & 2 & -2 & 0 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right]$$

1R - 3R; 1R · (-1); 1R · (-2) + 2R; 1R · (-3) + 3R; 1R · (-2) + 4R

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & -3 & 4 & -7 & 1 \\ 0 & -3 & 5 & -5 & -1 \\ 0 & 0 & 2 & 1 & -1 \end{array}\right] 1:(-3) \sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 1 & -4/3 & 7/3 & -1/3 \\ 0 & -3 & 5 & -5 & -1 \\ 0 & 0 & 2 & 1 & -1 \end{array}\right] 2R \cdot 3 + 3R \sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 1 & -4/3 & 7/3 & -1/3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 2 & 1 & -1 \end{array}\right] 3R \cdot (-2) + 4R$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 1 & -4/3 & 7/3 & -1/3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & -3 & 3 \end{array}\right] 1:(-3) \sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 2 & 0 \\ 0 & 1 & -4/3 & 7/3 & -1/3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 2 \\ 0 & 1 & -4/3 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

4R · (-2) + 3R; 4R · (-7/3) + 2R; 4R · (-2) + 1R; 3R · 4/3 + 2R; 3R · (-2) + 1R

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

2R · (-1) + 1R

✓

PROVJERA:

① $2x_1 - x_2 + x_3 - x_4 = -1$

$2\cdot 0 - 2 + 0 + 1 = -1$

$0 - 2 + 0 + 1 = -1$

$-1 = -1$ ✓

② $2x_1 - x_2 - 3x_4 = 1$

$2\cdot 0 - 2 - 3\cdot(-1) = 1$

$0 - 2 + 3 = 1$

$1 = 1$ ✓

③ $3x_1 - x_3 + x_4 = -1$

$3\cdot 0 - 0 + (-1) = -1$

$-1 = -1$ ✓

④ $2x_1 + 2x_2 - 2x_3 + 5x_4 = -1$

$2\cdot 0 + 2\cdot 2 - 2\cdot 0 + 5\cdot(-1) = -1$

$4 - 5 = -1$

$-1 = -1$ ✓

⑥ $e^x = \arccos x$

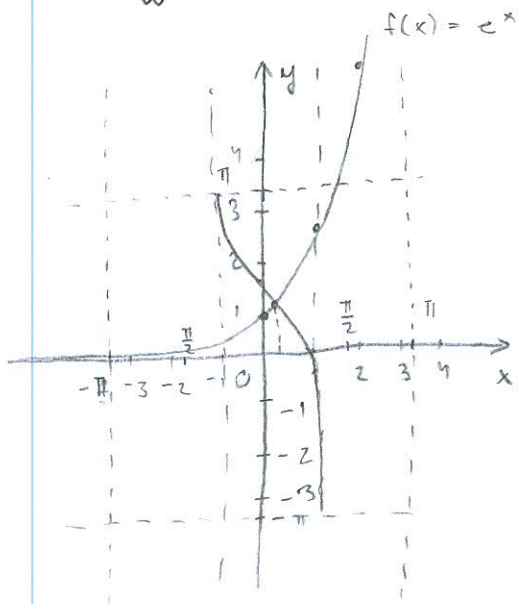
$f(x) = e^x$ $\quad g(x) = \arccos x$

$f(0) = 1$ $\quad g(-1) = \pi$

$f(1) = 2.71$ $\quad g(0) = \frac{\pi}{2}$

$f(2) = 7.39$ $\quad g(1) = 0$

$f(3) = 20.09$



PROVJERA

$x \approx 0.27$

$e^{0.27} = \arccos 0.27$

$1.30 = 1.30$ ✓

⑤ $\lim_{x\to\infty} \frac{x^2}{e^{2x}} = \left[\frac{\infty}{\infty}\right]^{L'H} = \frac{2x}{e^{2x}\cdot(2x)'} = \frac{2x}{e^{2x}\cdot 2} = \frac{2x}{2e^{2x}} = \frac{x}{e^{2x}}$ 2

③ $\frac{x+1}{\sqrt{x^2-x}} + 1 > 0$

$D_f = +R \setminus [0, 1]$

$x^2 - x \geq 0$

$x(x-1) \geq 0$

$x_1 = 0$

$x_2 - 1 > 0$

$x_2 > 1$

PROVJERA: $x = 2$

$\frac{2+1}{\sqrt{2^2-2}} + 1 > 0$

$\frac{3}{\sqrt{2}} + 1 > 0$

$3.12 > 0$

$x = 3$

$\frac{3+1}{\sqrt{3^2-3}} + 1 > 0$

$\frac{4}{\sqrt{6}} + 1 > 0$

$1.63 > 0$

④ $f(x) = \sqrt{x^2+7x+7}$

a)

$f'(x) = \frac{1}{2\sqrt{x^2+7x+7}} \cdot (x^2+7x+7)'$

$f'(x) = \frac{1}{2\sqrt{x^2+7x+7}} \cdot (2x+7)$

$f'(x) = \frac{2x+7}{2\sqrt{x^2+7x+7}}$

$f''(x) = \frac{(2x+7)'\cdot(2\sqrt{x^2+7x+7}) - (2x+7)(2\sqrt{x^2+7x+7})'}{(2\sqrt{x^2+7x+7})^2}$

$f''(x) = \frac{2(2\sqrt{x^2+7x+7}) - (2x+7)\left(2\cdot\frac{1}{2\sqrt{x^2+7x+7}}\cdot(x^2+7x+7)'\right)}{(2\sqrt{x^2+7x+7})^2}$

$f''(x) = \frac{2(2\sqrt{x^2+7x+7}) - (2x+7)\cdot\frac{2x+7}{2\sqrt{x^2+7x+7}}}{(2\sqrt{x^2+7x+7})^2}$

$f''(x) = \frac{2(2\sqrt{x^2+7x+7}) - \frac{(2x+7)^2}{2\sqrt{x^2+7x+7}}}{(2\sqrt{x^2+7x+7})^2}$ ✓

① $\left|\frac{z}{2i}\right|^2 = z + 5i$

$\frac{z^2}{4} = z + 5i \quad / \cdot 4$

$z^2 = 4z + 5i$

$z^2 - 4z = 5i$

$z(z-4) = 5i$

$z \cdot 5i = 5i$

$z = 1$

④ b) $f(x) = \sqrt{x^2+7x+7}$ Df:

$x^2 + 7x + 7 \geq 0$

$x^2 + 7x \geq -7$

$x(x+7) \geq -7$

$x_1 = -7$

$x + 7 > -7$

$x \geq -7 - 7$

$x \geq -14$

♣10

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POPUNJAVA NASTAVNIK Broj ↓ bodova

0

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Ukupno:

(2)

$$\begin{aligned} 2x_1 - x_2 + x_3 - x_4 &= -1 \\ 2x_1 - x_2 \quad\quad - 3x_4 &= 1 \\ 3x_1 \quad\quad - x_3 + x_4 &= -1 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 &= -1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] /:2 \sim \left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] /\cdot(-2) /\cdot(-3) /\cdot(-2)$$

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