MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ P4

IME I PREZIME: ANĐELO ŽMIRE **BROJ INDEKSA:** 0269065578

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

21.2.2013.

POPUNJAVA NASTAVNIK Broj ↓ bodova

65

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} x + 2y + 3z &= 3 \\ -2x \quad + z &= -2 \\ x + 2y - z &= 3 \\ -x + 2y + 12z &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x + 1} - \log_{10}(x^2 - x)$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 - 7}{x^2 + 5}$ 20(graf)
5. Odrediti i provjeriti:
   (a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3}$. 4+1
   (b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}}$. 8+2
6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

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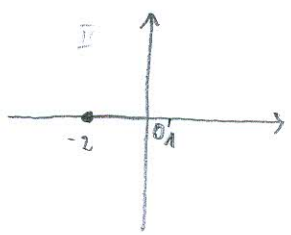
21 : 4 = 5
1



(1) $z^3 - \frac{(1-i)^6}{i^{21}} = 0$

$z^3 - \frac{[(1-i)^2]^3}{i^1} = 0$

$z^3 = \frac{[(1^2 - 2\cdot 1(-i) + i^2)]^3}{i}$

$z^3 = \frac{[(1+2i-1]^3}{i}$

$z^3 = \frac{(2i)^3}{i}$

$z^3 = \frac{-8i}{i}$

$z^3 = -8 \mid \sqrt[3]{\ }$

$z = \sqrt[3]{-8}$

$\boxed{z = -2}$

$\varphi_1 = tg^{-1}\frac{y}{x}$

$\varphi_1 = tg^{-1}\frac{0}{-2}$

$\varphi_1 = 0$

$\varphi = \pi - \varphi_1$

$\varphi = \pi$

$z_n = \sqrt[n]{r}\left(\cos\frac{\varphi + k2\pi}{n} + i\sin\frac{\varphi + k2\pi}{n}\right) \quad (k = 0,1,2)$

$z_1 = -2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= -2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -1 - \sqrt{3}i$

BRAVO

$z_2 = -2\left(\cos\frac{\pi + 2\pi}{3} + i\sin\frac{\pi + 2\pi}{3}\right)$

$= -2\left(\cos\frac{2\pi}{3} + i\sin\frac{3\pi}{3}\right)$

$= -2(\cos\pi + i\sin\pi)$

$= -2(-1 + 0) = 2$

$z_3 = -2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$= -2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1 + \sqrt{3}i$

✓

KOMPL. RAVNINA

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 3 & \\ -2 & 0 & 1 & -2 & \\ 1 & 2 & -1 & 3 & \\ -1 & 2 & 12 & 1 & \end{array}\right] \begin{array}{l} \text{I}-\text{III} \\ \text{II}+\text{III} \\ \text{III}+\text{IV} \\ \\ \end{array} \left[\begin{array}{cccc|c} 0 & 0 & 4 & 0 & \\ -1 & 2 & 0 & 1 & \\ 0 & 4 & 11 & 4 & \\ -1 & 2 & 12 & 1 & \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ -1 & 2 & 0 & 1 & \\ 0 & 4 & 11 & 4 & \\ 0 & 0 & 4 & 0 & \end{array}\right] \begin{array}{l} \text{II}+\text{I} \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & -12 & 0 & \\ 0 & 4 & 11 & 4 & \\ 0 & 0 & 4 & 0 & \end{array}\right] \begin{array}{l} \text{III}+\text{IV} \\ \\ \\ \cdot 3 \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & -12 & 0 & \\ 0 & 4 & -1 & 4 & \\ 0 & 0 & 12 & 0 & \end{array}\right] \begin{array}{l} \text{II}+\text{IV} \\ \\ \\ :12 \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & 0 & 0 & \\ 0 & 4 & -1 & 4 & \\ 0 & 0 & 1 & 0 & \end{array}\right] \begin{array}{l} \text{III}+\text{IV} \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & 0 & 0 & \\ 0 & 4 & 0 & 4 & \\ 0 & 0 & 1 & 0 & \end{array}\right] \begin{array}{l} \text{I}+12\text{III} \\ \\ :4 \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & -1 & \\ 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 1 & \\ 0 & 0 & 1 & 0 & \end{array}\right] \begin{array}{l} \text{I}+2\text{III} \\ \\ \\ \\ \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} x=1 \\ y=1 \\ z=0 \end{array}$$

$x+2y+3z=3$

$1+2\cdot 1+3\cdot 0=3$

$1+2+0=3$

$3=3$ ✓

$-2x+z=-2$

$-2\cdot 1+0=-2$

$-2=-2$

$x+2y-z=3$

$1+2-0=3$

$3=3$

③ $g(x)=\sqrt{x^2+x+1}-\log_{10}(x^2-x)$

① $\sqrt{x^2+x+1} \geq 0$

$x^2+x+1 \geq 0$

$x \in \langle -\infty, +\infty \rangle$

$D(f)=\langle -\infty, 0\rangle \cup \langle 1, +\infty\rangle$

② $x^2-x>0$

$x(x-1)>0$

$x=0$ $\quad x-1=0 \quad x=1$

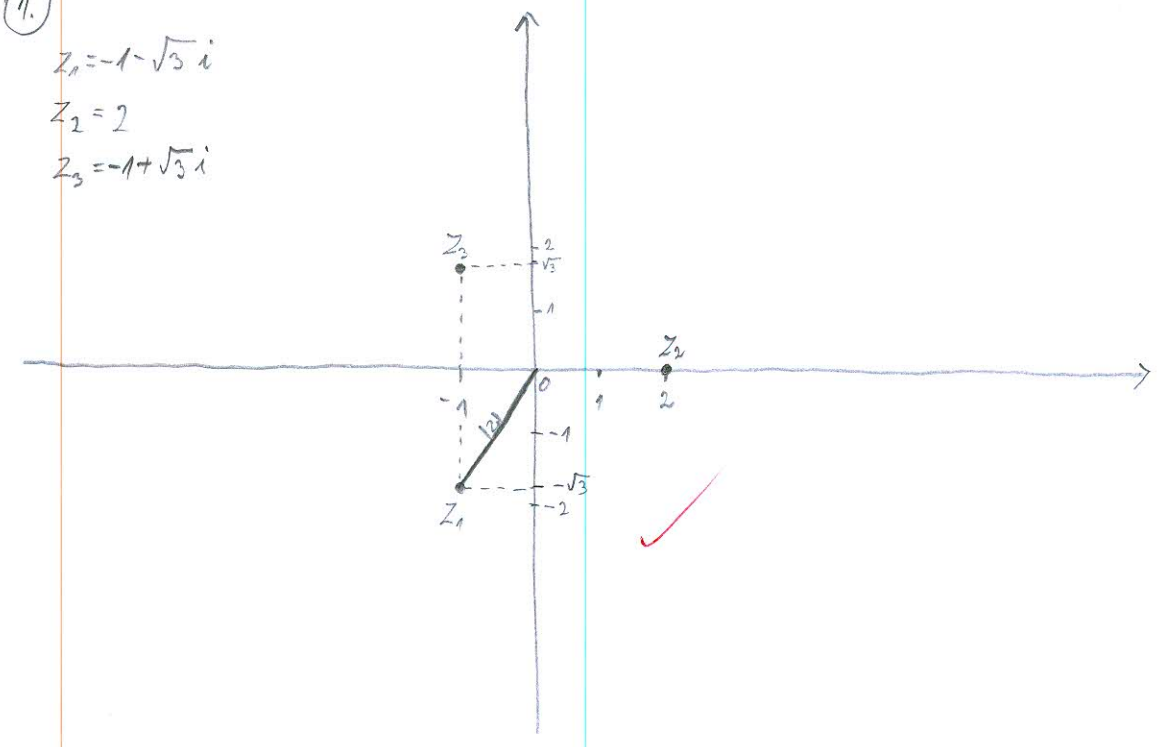
	$-\infty$		-1		0		$\frac{1}{2}$		1		2		$+\infty$
x		$-$		$-$	0	$+$		$+$		$+$		$+$	
$(x-1)$		$-$		$-$		$-$		$-$	0	$+$		$+$	
		$+$		$-$		$+$		$-$		$+$		$+$	

$x \in \langle -\infty, 0\rangle \cup \langle 1, +\infty\rangle$ ✓

① $z_1=-1-\sqrt{3}\,i$

$z_2=2$

$z_3=-1+\sqrt{3}\,i$



✓

ANDEZO OMIRE

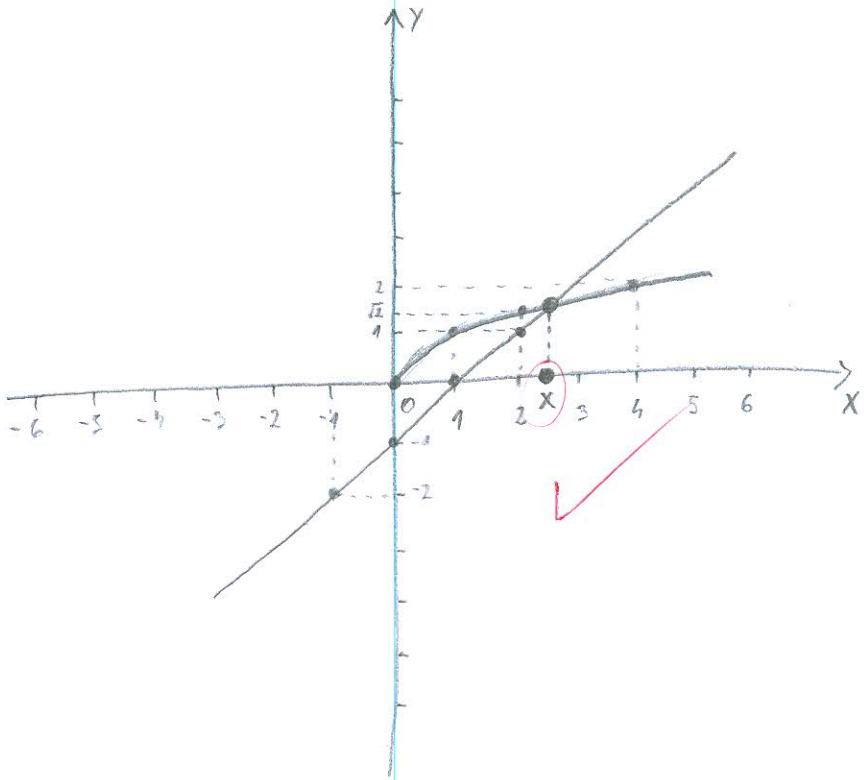
6. $x-1=\sqrt{x}$

$f(x)=x-1$

$g(x)=\sqrt{x}$

x	f(x)
1	0
-1	-2
0	-1
2	1

x	g(x)
1	1
2	≈1.41
4	2
0	0



$x=2.6$

$2.6-1\approx\sqrt{2.6}$

$1.6\approx1.6$

5. a) $\lim_{n\to\infty}\frac{\sin n / n^2}{1-n^3 / n^3}=\lim_{n\to\infty}\frac{0}{-1}=0$

b) $\lim_{n\to\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\lim_{n\to\infty}\frac{x^{\frac{1}{2}}+x^{\frac{1}{3}}+x^{\frac{1}{4}}}{\sqrt{2x+1}}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ P4

IME I PREZIME: IVAN KELAVA **BROJ INDEKSA:** 17-1-0084-11

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova
50

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: ✓ 10+5

$$\begin{aligned} x + 2y + 3z &= 3 \\ -2x \quad + z &= -2 \\ x + 2y - z &= 3 \\ -x + 2y + 12z &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x + 1} - \log_{10}(x^2 - x)$. ✓ 15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 7}{x^2 + 5}$ ✓ 20(graf)

5. Odrediti i provjeriti:

   (a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3}$. 4+1

   (b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}}$. 8+2

6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

3. 1. $x^2 + x + 1 \geq 0$

$x^2 + x + 1 = 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2}$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2}$

$x_{1,2}$ = NEMA RJ.

⇊

$D(f) = \mathbb{R}$

2. $x^2 - x > 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ $\quad x - 1 = 0$

$x = 1$

$D(f) = \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$

✓

$D(f) = \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$

(5) a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3} : n^3 = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{\frac{\sin n}{n^3}}{\frac{1}{n^3} - \frac{n^3}{n^3}} = \frac{0}{1} = 0 \Rightarrow \frac{\sin \infty}{1-\infty}$

b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}} : x^{\frac{1}{2}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\frac{\sqrt{x}}{x^{\frac{1}{2}}} + \frac{\sqrt[3]{x}}{x^{\frac{1}{2}}} + \frac{\sqrt[4]{x}}{x^{\frac{1}{2}}}}{\frac{\sqrt{2x+1}}{x^{\frac{1}{2}}}} = \lim_{x\to\infty} \frac{\sqrt{\frac{x}{x}} + \sqrt[3]{\frac{x}{x^3}} + \sqrt[4]{\frac{x}{x^2}}}{\sqrt{\frac{2x+1}{x}}} =$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{x}{x}}^{1} + \sqrt[3]{\frac{x}{x^3}}^{0} + \sqrt[4]{\frac{x}{x^2}}^{0}}{\sqrt{\frac{2x}{x} + \frac{1}{x^0}}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

(2)
$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 3 \\ -2 & 0 & 1 & 1 & -2 \\ 1 & 2 & -1 & 1 & 3 \\ -1 & 2 & 12 & 1 & 1 \end{array}\right] \begin{array}{l} \\ +2R1 \\ -R1 \\ +R1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 3 \\ 0 & 4 & 7 & 1 & 4 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 4 & 15 & 1 & 4 \end{array}\right] :4 \simeq \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 3 \\ 0 & 1 & \frac{7}{4} & 1 & 1 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 4 & 15 & 1 & 4 \end{array}\right] \begin{array}{l} -2R2 \\ \\ \\ -4R2 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 1 & \frac{7}{4} & 1 & 1 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 8 & 1 & 0 \end{array}\right] :-4 \sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 1 & \frac{7}{4} & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 8 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R4-8R3 \\ R2-\frac{7}{4}R3 \\ R1+\frac{1}{2}R3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} x \\ y \\ z \end{array} \quad = \text{RANG MATRICE } \underline{\underline{3}}$$

✓

$x + 2y + 3z = 3 \Rightarrow 1 + 2\cdot 1 + 3\cdot 0 = 3$ ✓

$-2x + z = -2 \Rightarrow -2\cdot 1 + 0 = -2$ ✓

$x + 2y + z = 3 \Rightarrow 1 + 2\cdot 1 - 0 = 3$ ✓

$-x + 2y + 12z = 1 \Rightarrow -1 + 2\cdot 1 + 12\cdot 0 = 1$ ✓

IVAN KELAVA

(4.) $f(x)=\frac{x^2-7}{x^2+5}$

1) DOMENA

$x^2+5\neq 0$

$x^2+5=0$

$x^2=-5$

NIJE REALAN BROJ KVADRIRAN NE DAJE NEGATIVAN BR.

$D(f)=\mathbb{R}$ ✓

2) NULTOČKE

$x^2-7=0$

$x^2=7$

$x=\pm\sqrt{7}$ ✓

3.) EKSTREMI, RAST-PAD

$f'(x)=\frac{2x(x^2+5)-2x(x^2-7)}{(x^2+5)^2}=\frac{2x^3+10x-2x^3+14x}{(x^2+5)^2}$

$f'(x)=\frac{24x}{(x^2+5)^2}$, $f'(x)=0$

$24x=0 \mid :24$

$x=0$

$y=-\frac{7}{5}$ ✓

	$-\infty$		0		$+\infty$
$f'(x)$		−	0	+	
$f(x)$		↘		↗	

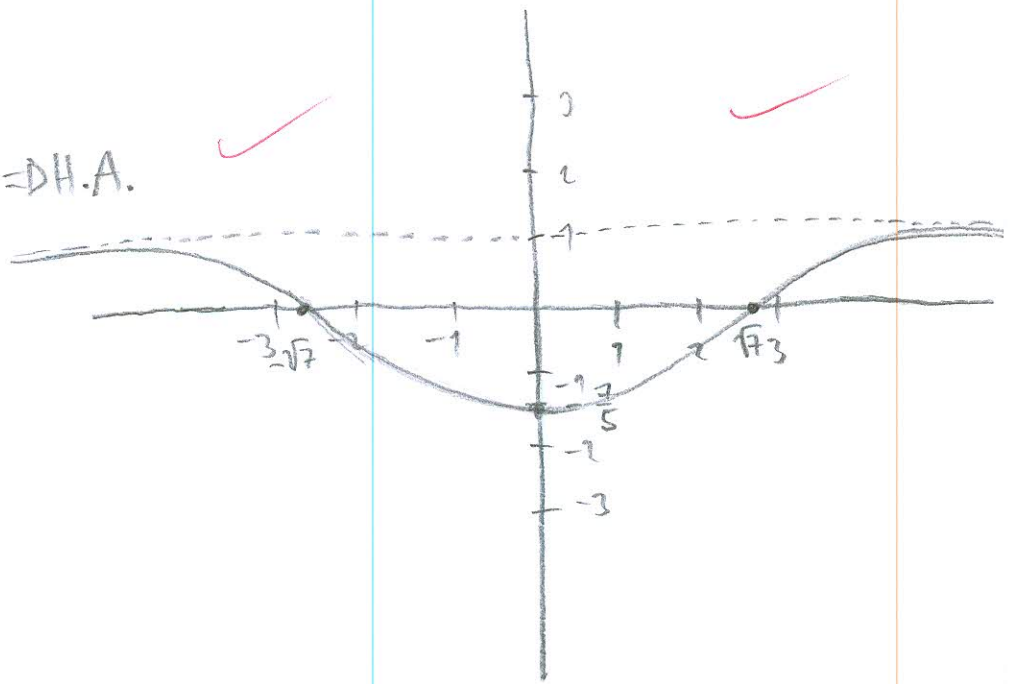
MIN.

ASIMPTOTE

$D(f)=\mathbb{R} \Rightarrow$ NEMA V.A.

ISPITUJEM H.A.

$\lim_{x\to\pm\infty}\frac{x^2-7}{x^2+5}=\frac{\frac{x^2}{x^2}-\frac{7}{x^2}}{\frac{x^2}{x^2}+\frac{5}{x^2}}=\frac{1}{1}=1=$ D.H.A.



IZ GRAFA VIDIMO DA JE PARNA FUNKCIJA

Bože, ima li te? Zašto si me ostavio?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ P4

IME I PREZIME: ANTON RAŠIĆ **BROJ INDEKSA:** 17-2-0084-2011

ZAOKRUŽITI AKO ŽELITE: [redacted] MATE KOSOR

POPUNJAVA NASTAVNIK
Broj ↓ bodova
32

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3
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6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

(2)
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ -2 & 0 & 1 & -2 \\ 1 & 2 & -1 & 3 \\ -1 & 2 & 12 & 1 \end{array}\right] \begin{array}{l} \\ R_2+2R_1 \\ R_3-R_1 \\ R_4+R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \sim \begin{array}{l} \\ \\ \\ R_4-R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ :(-4) \\ R_4+2R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] 1:(4) \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 7/4 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

BESKONAČNO MNOGO RJEŠENJA

Rješenje je parametarsko, sustav je neodređen!

(3) DOMENA $g(x) = \sqrt{x^2+x+1} - \log_{10}(x^2-x)$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 1}}{2} = \frac{-1 \pm \sqrt{3}}{2}$

$x^2 + x + 1 \geq 0$

$\left(x - \frac{-1+i\sqrt{3}}{2}\right)\left(x - \frac{-1-i\sqrt{3}}{2}\right) \geq 0$

	$-\infty$		$-\frac{1+i\sqrt{3}}{2}$		$-\frac{1-i\sqrt{3}}{2}$		∞
$x - \frac{-1-i\sqrt{3}}{2}$		−	0	+		+	
$x - \frac{-1+i\sqrt{3}}{2}$		−		−	0	+	
		+				+	

$\left\langle -\infty, -\frac{1+i\sqrt{3}}{2} \right] \cup \left[-\frac{1+i\sqrt{3}}{2}, \infty \right\rangle$

$x \in \left\langle -\infty, -\frac{1-i\sqrt{3}}{2} \right] \cup \langle 1, \infty \rangle$

$x^2 - x > 0$

$x(x-1) > 0$

x	$-\infty$		0		1	∞
$x-1$		−		−	0	+
		+				+

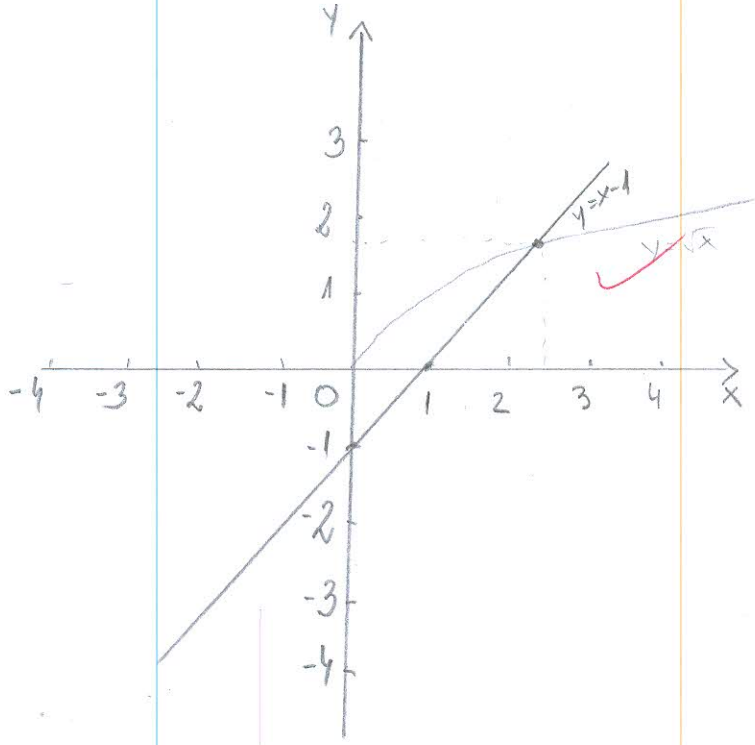
$x \in \langle -\infty, 0 \rangle \cup \langle 1, \infty \rangle$

(5) a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3} = \lim \frac{\frac{\sin n}{n^3}}{\frac{1}{n^3} - \frac{n^3}{n^3}} = \frac{0}{-1} = -\infty$

b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}} = \frac{\frac{\sqrt{x}}{x^2} + \frac{\sqrt[3]{x}}{x^2} + \frac{\sqrt[4]{x}}{x^2}}{\frac{2x}{x^2} + \frac{1}{x^2}} = \frac{0}{0} = 0$

ANTON RAŠIĆ 17-2-0084-2011 (R-Nautika)

⑥ $x-1=\sqrt{x}$



⑤ a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3} = \lim_{n\to\infty} \frac{-1}{1-n^3} = \lim_{n\to\infty} \frac{-\frac{1}{n^3}}{\frac{1}{n^3}-1} = \frac{0}{0-1} = 0$

$\lim \frac{-1}{1-n^3} = \frac{-\frac{1}{n^3}}{\frac{1}{n^3}-1} = \frac{0}{0-1} = 0$

$-1 \le \sin n \le 1$

$-\frac{1}{1-n^3} \le \frac{\sin n}{1-n^3} \le \frac{1}{1-n^3}$

b) $\lim_{x\to\infty} \frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}} = \lim_{x\to\infty} \frac{\frac{\sqrt{x}}{\sqrt{x}}+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}}{\sqrt{\frac{2x}{x}+\frac{1}{x}}} = \frac{1}{\sqrt{2}}$

③

---

① $z^3 = -\frac{(1-i)^6}{i^{21}} = 0$

$i^{21} = i^4 = 1$

$i^{21} = w$

$w = 1 - i$

$|w| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$tg\,\varphi_2 = \frac{-1}{1} = -1$

$\varphi_2 = \frac{7\pi}{4}$

$w = \sqrt{2} \cdot (\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$

$w^6 = (\sqrt{2})^6 \cdot (\cos 6 \cdot \frac{7\pi}{4} + i\sin 6 \cdot \frac{7\pi}{4})$

$w^6 = 2^3 \cdot (\cos\frac{21\pi}{2} + i\sin\frac{21\pi}{2})$

$w^6 = 8 \cdot (\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

$w^6 = 8 \cdot (0 + i \cdot 1)$

$w^6 = 8 \cdot i$

$z^3 = -\frac{8i}{i} = 0$

$z^3 - 8 = 0$

$z^3 = 8$

$z = \sqrt[3]{8}$

$z_2 = 8$

$|z_2| = \sqrt{8}^2 = 8$

$tg\,\varphi_3 = \frac{0}{8} = 0$

$\varphi_3 = 2\pi$ ili $\varphi_3 = 0$

$z^2 = 8 \cdot (\cos 0 + i\sin 0)$

$\sqrt[3]{z^2} = \sqrt[3]{8} \cdot (\cos\frac{0+2\pi}{3} + i\sin\frac{0+2\pi}{3})$

$= \sqrt[3]{8} \cdot (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

1° $k = 0$ $\sqrt[3]{z^2} = 2 \cdot (\cos 0 + i\sin 0)$

2° $k = 1$ $\sqrt[3]{z^2} = 2 \cdot (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

3° $k = 2$ $\sqrt[3]{z^2} = 2 \cdot (\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ P4

IME I PREZIME: Judela Ulloda **BROJ INDEKSA:** 17-2-0106-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

23

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} x + 2y + 3z &= 3 \\ -2x \quad + z &= -2 \\ x + 2y - z &= 3 \\ -x + 2y + 12z &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x + 1} - \log_{10}(x^2 - x)$. 15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 7}{x^2 + 5}$ 20(graf)

5. Odrediti i provjeriti:

   (a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3}$. 4+1

   (b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}}$. 8+2

6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

2. $x+2y+3z=3$
$-2x+z=-2$
$x+2y-z=3$
$-x+2y+12z=1$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & & 3 \\ -2 & 0 & 1 & & -2 \\ 1 & 2 & -1 & & 3 \\ -1 & 2 & 12 & & 1 \end{array}\right] \begin{array}{l} \text{II}+2\text{I} \\ \text{II}-\text{I} \\ \text{I}+2 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & & 3 \\ 0 & 4 & 7 & & 4 \\ 0 & 0 & -4 & & 0 \\ 0 & 4 & 15 & & 4 \end{array}\right] \text{II}-\text{I} \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & & 3 \\ 0 & 4 & 7 & & 4 \\ 0 & 0 & -4 & & 0 \\ 0 & 0 & 8 & & 0 \end{array}\right] \text{II}+2\text{I}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & & 3 \\ 0 & 4 & 7 & & 4 \\ 0 & 0 & -4 & & 0 \\ 0 & 0 & 0 & & 0 \end{array}\right]$$

$x+2y+3z=3 \Rightarrow x+2+0-3 \to x-1$
$4y+7z=4 \Rightarrow 4y+7\cdot 0-4 \to 4y-4 \mid :4 \to y=1$
$-4z=0 \Rightarrow z=0$

$x=1$
$y=1$
$z=0$

$x+2y+3z=3 \Rightarrow 1+2\cdot 1+3\cdot 0=3 \Rightarrow 3=3$
$-2x+\quad +z=-2 \Rightarrow -2\cdot 1+0=2 \Rightarrow -2=-2$
$x+2y-z=3 \Rightarrow 1+2\cdot 1-0=3 \Rightarrow 3=3$
$-x+2y+12z=1 \Rightarrow -1+2\cdot 1+12\cdot 0=1 \Rightarrow 1=1$

5. $\lim_{n\to\infty} \frac{\sin n}{1-n^3} \cdot \frac{1-n^3}{1-n^3} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{\sin n}{1-n^3} = \frac{-\cos n}{-3n^2} = \frac{\cos n \cdot 2n}{3n^2}$

$\frac{-\sin n}{6n} = -\frac{\sin}{6}$

b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{2x+1} = \frac{x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{4}} / : x^{\frac{1}{2}}}{2x^{\frac{1}{2}} + 1 \ / : x^{\frac{1}{2}}} = \frac{x + \frac{1}{x} + \frac{1}{x}}{2 \cdot 1 + \frac{1}{x^2}} = \frac{1}{2}$

3. $g(x) = \sqrt{x^2+x+1} - \log_{10}(x^2-x)$

$x^2+x+1 \geq 0$

$x^2+x+1 \geq 0$

$x = \emptyset$

$x^2 - x > 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$

$x_1 = \frac{1+\sqrt{5}}{2}$

$x_2 = \frac{1-\sqrt{5}}{2}$

$D_g = \left\langle -\infty, \frac{1-\sqrt{5}}{2} \right] \cup \left[\frac{1+\sqrt{5}}{2}, +\infty \right\rangle$

6. $x - 1 = \sqrt{x}$

$x - 1 = \sqrt{x}$

$f(x) = x - 1$

$f(0) = -1$

$f(1) = 0$

$f(2) = 1$

$f(4) = 3$

$x - 1 = \sqrt{x}$

$2 - 1$

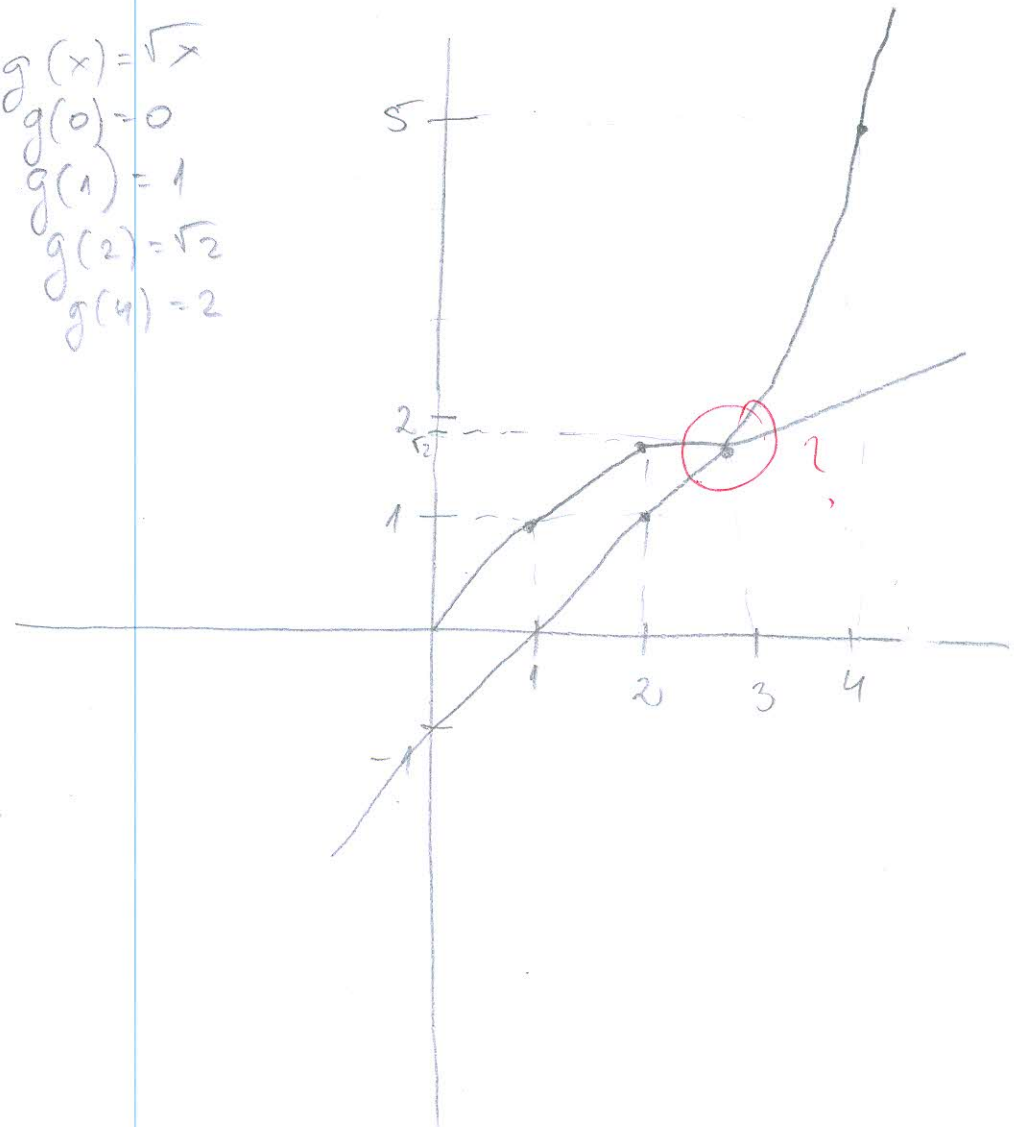
$g(x) = \sqrt{x}$

$g(0) = 0$

$g(1) = 1$

$g(2) = \sqrt{2}$

$g(4) = 2$



1. $z^3 - \left(\frac{1-i}{i^{21}}\right)^6$ $\quad i^{21} = i$

$z^3 = \frac{2+16i}{i} = 0$

$((1^2 - 3i + 3i^2 - i^3))^2 = (-2-2i)^2 =$

$= 4 + 16i + 2i^2 = 2 + 16i$

$z^3 = 16 - 2i$

$z = \sqrt[3]{16-2i}$

$w = 16 - 2i$

$\frac{2+16i}{i} \cdot \frac{-i}{-i} = \frac{-2i - 16i^2}{-i^2} = \frac{-2i+16}{1}$

$|w| = \sqrt{(16)^2 + (-2)^2} = \sqrt{256+4} = \sqrt{260} = 2\sqrt{65} \quad = 16 - 2i$

$\operatorname{tg} \varphi = \frac{y}{x} = -\frac{2}{16} = -\frac{1}{8}$ $\quad \varphi = -2\pi - \operatorname{arctg}\left|\frac{y}{x}\right|$

$\varphi = 6.1588$

$\sqrt[3]{16-2i} = \sqrt[3]{2\sqrt{65}} \cdot \left(\cos \frac{6.1588 + k \cdot 2\pi}{3} + i \sin \frac{6.1588 + k \cdot 2\pi}{3}\right)_{k=0,1,2}$

a) $k = 0$

$z_1 = 2.52636 \cdot (\cos 2.0529 + i \sin 2.0529) = 2.52636 \cdot (-0.4636 + 0.886i) = -1.1712 + 2.1878i$

b) $k = 1$

$z_2 = 2.52636 \cdot \left(\cos \frac{6.1588 + 2\pi}{3} + i \sin \frac{6.1588 + 2\pi}{3}\right) =$

$= 2.52636 (\cos 4.1473 + i \sin 4.1473) =$

$z_2 = 2.52636 \cdot (-0.5355 - 0.8445i) = -1.3529 - 2.1335i$

c) $k = 2$

$z_3 = 2.52636 \left(\cos \frac{6.1588 + 4\pi}{3} + i \sin \frac{6.1588 + 4\pi}{3}\right) =$

$z_3 = 2.52636 \cdot (\cos 6.2417 + i \sin 6.2417) =$

$z_3 = 2.52636 \cdot (0.9991 + 0.0415i) = 2.524 - 0.1048i$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{P}4$

IME I PREZIME: TEO MANDŽUKA **BROJ INDEKSA:** 6041983 11 026906 1995 3

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

20

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3

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6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

(3) $g(x) = \sqrt{x^2+x+1} - \log_{10}(x^2-x)$

I $x^2+x+1 \geq 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2}$

NEMOGUĆE RIJEŠITI

II $\log_{10}(x^2-x)$

$\log_{10} x^2 - x > 0 \;/ \log_{10}$

$x^2 - x > 1$

$x^2 - x - 1 > 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2}$

$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$x_1 = \frac{1}{2} + \frac{\sqrt{5}}{2}$ $\quad x_2 = \frac{1}{2} - \frac{\sqrt{5}}{2}$

$x_1 = 1{,}61$ $\quad x_2 = -0{,}61$

	$-\infty$		$-0{,}61$		$1{,}61$		$+\infty$
x^2-x-1		$+$	0	$-$	0	$+$	

$D_f \in \langle -\infty, -0{,}61 \rangle \cup \langle 1{,}61, +\infty \rangle$

$$f(x) = \frac{x^2-7}{x^2+5}$$

① DOMENA

$x^2+5 \neq 0$ $\quad x_{1,2} = \frac{0 \pm \sqrt{0-20}}{2}$ → NE POSTOJI RJEŠENJE

FUNKCIJA NEMA PREKIDA

$D: x \in \mathbb{R}$ ✓

② SVOJSTVA, PARNOST, NEPARNOST I PERIODIČNOST

$f(x)=0$ $\quad x_1 = 2{,}64$ $\quad N_1(-2{,}64, 0)$ $\quad f(0) = -\frac{7}{5}$ $\quad (0, -\frac{7}{5})$

$x^2 = 7$ $\quad x_2 = -2{,}64$ $\quad N_2(2{,}64, 0)$ ✓

$x_{1,2} = \pm\sqrt{7}$

$\lim_{x\to\infty} \frac{x^2-7}{x^2+7} = +$

NEMA VERTIKALNIH ASIMPTOTA

KOSE ASIMPTOTE

$k = \lim_{x\to\infty} \frac{\frac{x^2-7}{x^2+5}}{x} \to \lim_{x\to\infty} \frac{x^2-7}{x^3+5x} \quad | : x^3$

$= \frac{\frac{1}{x} - \frac{7}{x^3}}{1 + \frac{5}{x^2}} = 0$ NEMA KOSE

H.A

$\lim_{x\to\infty} \frac{x^2-7}{x^2+5} \quad | : x^2 = \frac{1-\frac{7}{x^2}}{1+\frac{5}{x^2}} = 1$ H.A = 1 ✓

$x \to \infty, \frac{x^2-7}{x^2+5} = \frac{1-\frac{7}{x^2}}{1+\frac{5}{x^2}} = 1+$

③ MONOTONOST

$f'(x) = \frac{2x(x^2+5) - (x^2-7)2x}{(x^2+5)^2}$

$f'(x) = \frac{x^2+5-x^2+7}{(x^2+5)^2}$

$f'(x) = \frac{12}{(x^2+5)^2}$

$f'(x) = 0$

$12 = 0$

NEMA STACIONARNIH TOČAKA

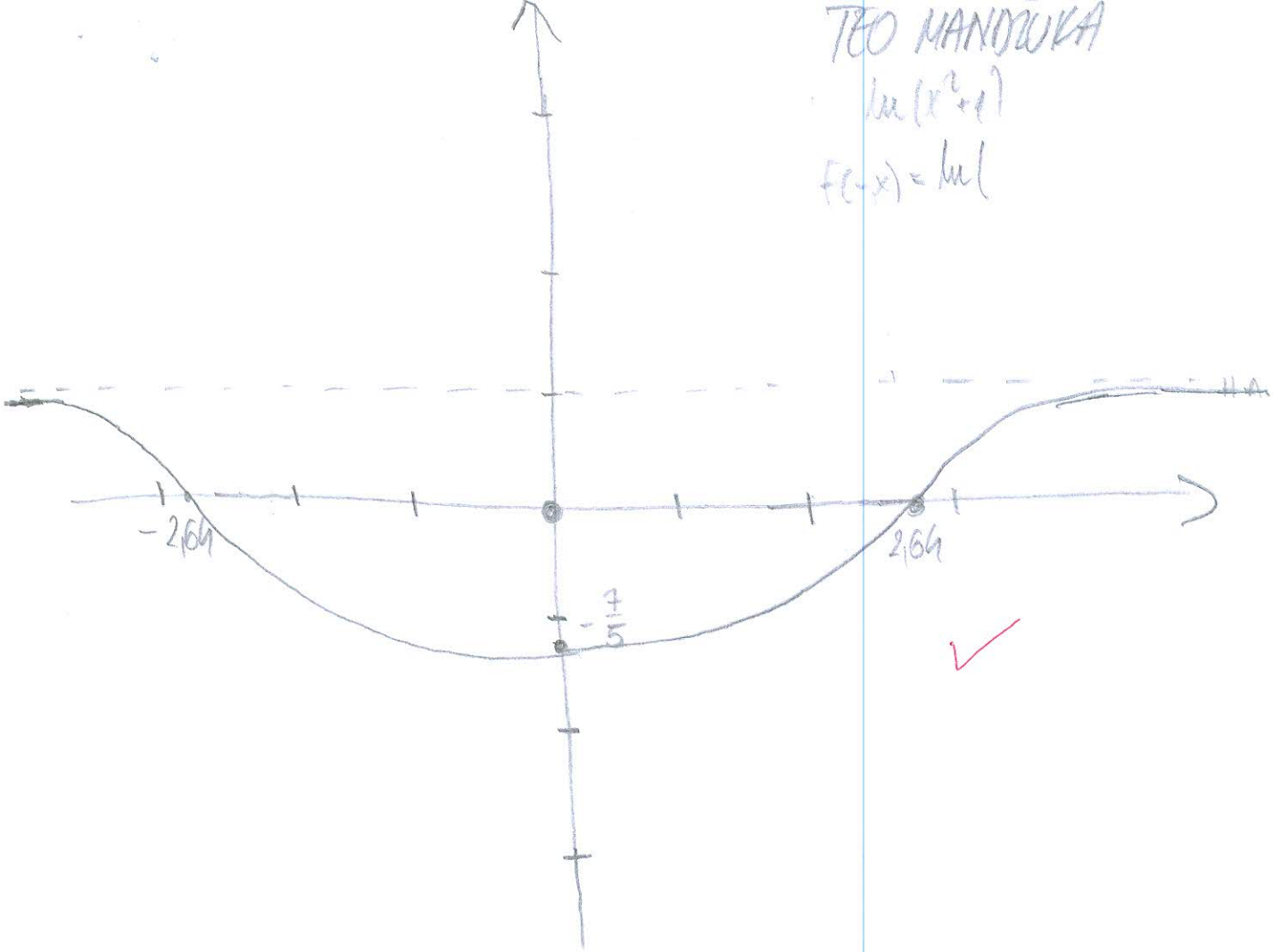
$f''(x) = \frac{0\cdot[(x^2+5)^2]^2 - [2(x^2+5)\cdot 2x]\cdot 12}{(x^2+5)^4}$

$f''(x) = \frac{-(4x^3+20x)\cdot 12}{(x^2+5)^4} = \frac{-48x^3-240x}{(x^2+5)^4}$

TEO MANDZUKA

$\ln(x^2+1)$

$f(-x) = \ln($



✓

6. a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3} = \frac{\cos n}{-3n^2} = \frac{\cos\infty}{-3\cdot\infty} = \frac{\infty}{\infty}$

b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}} = \lim_{x\to\infty} \frac{x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{4}}}{(2x+1)^{\frac{1}{2}}}$

$\lim_{x\to\infty} \frac{\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x}{\frac{1}{2}(2x+1)\cdot 2} = \lim_{x\to\infty} \frac{\frac{6x+4x+3x}{12}}{\frac{2x+1}{1}} = \frac{\frac{13x}{12}}{\frac{2x+1}{1}} = \frac{13x}{24x+12}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓ P4

IME I PREZIME: Mitrović Matin **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

20

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3

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6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

4. $\frac{x^2-7}{x^2+5}$

I $x^2+5 \neq 0$

$x^2 \neq -5 \mid \sqrt{}$

$x \neq \sqrt{-5}$

$Df(x) \in \mathbb{R}$

II $x^2 - 7 = 0$

$x_{1,2} = \frac{0 \pm \sqrt{-4 \cdot 1 \cdot -7}}{2 \cdot 1}$

$x_{1,2} = \frac{0 \pm \sqrt{28}}{2} = \frac{\pm\sqrt{28}}{2}$

$x_1 = \frac{+\sqrt{28}}{2} = 2.6458$ $\quad x_2 = \frac{-\sqrt{28}}{2} = -2.6458$

-> Nije periodična jer nije trigonometrijska

-> $f(0) = -1.4$

$\frac{0^2-7}{0^2+5} = -1.4$

-> V. A nema jer je $Df(x) \in \mathbb{R}$

-> H. A $\lim_{x\to\infty} \frac{x^2-7 \mid :x^2}{x^2+5 \mid :x^2} = \frac{1 - \frac{7}{x^2}}{1 \frac{5}{x^2}} \to 0 = \frac{1}{1} = 1$ H. A $y = 1$

-> N. K. A ima H. A

$$f'(x) = \frac{x^2-7}{x^2+5}$$

$$f'(x) = \frac{(x^2-7)' \cdot (x^2+5) - (x^2-7)' \cdot (x^2+5)}{(x^2+5)^2}$$

$$= \frac{2x(x^2+5) - (2x \cdot (x^2+5)}{(x^2+5)^2}$$

$$= \frac{2x^3+10x - (2x^3+10x)}{(x^2+5)^2} = \frac{2x^3+10x-2x^3-10x}{(x^2+5)^2} = \frac{1}{(x^2+5)^2}$$

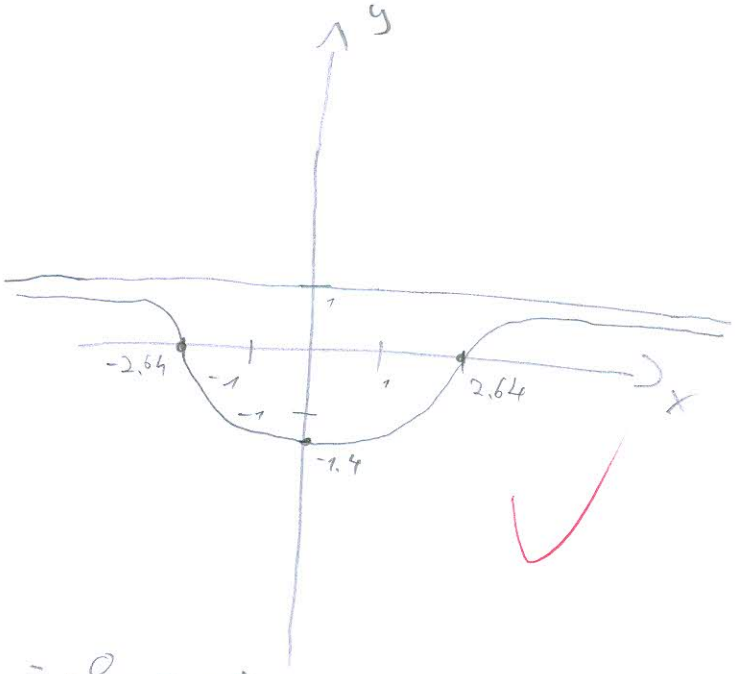
$1 \neq 0$

$$f''(x) = \frac{1}{(x^2+5)^2}$$

$$= \frac{1' \cdot (x^2+5)^2 - ((x^2+5)^2)' \cdot 1}{((x^2+5)^2)^2}$$

$$= \frac{0 \cdot (x^2+5)^2 - (2 \cdot (2x)}{(x^2+5)^4}$$

$$= \frac{-4x}{(x^2+5)^4}$$



5/

a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3} = \lim_{n\to\infty} \frac{\sin n \,/\, n^3}{1-n^3 \,/\, n^3} = \frac{0}{-1^-} = 0^+$

$\lim_{n\to-\infty} \frac{\sin n}{1-n^3} = \lim_{n\to-\infty} \frac{\sin n \,/\, n^3}{1-n^3 \,/\, n^3} = \frac{0}{-1^-} = 0^-$

b) $\lim_{x\to\infty} \frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}} = \frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}} \cdot \frac{-\sqrt{2x+1}}{-\sqrt{2x+1}}$

$i^{21} = i^{20+1} = i^{5+1}$

$= i^1$

$\boxed{i^1 = i}$

♣7 1/ $z^3 - \frac{(1-i)^6}{i^{21}} = 0$

$z^3 = \frac{(1-i)^6}{i^1}$

$z^3 = \frac{\left((1-i)^2\right)^3}{i}$

$z^3 = \frac{(1^2 + 2i + i^2)}{i}$

$z^3 = \frac{1-2i-1}{i}$

$z^3 = \frac{-2-2i}{i} \cdot \frac{-i}{-i} = \frac{+2i+2i}{-i^2}$

$z^3 = \frac{2i-2}{1}$ $\quad z^3 = 2i-2$

2) $x + 2y + 3z = 3$

$-2x + 0y + 1z = -2$

$x + 2y - z = 3$

$-x + 2y + 12z = 1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ -2 & 0 & 1 & -2 \\ 1 & 2 & -1 & 3 \\ -1 & 2 & 12 & 1 \end{array}\right] \quad (2)\ (-1)$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \quad I \leftrightarrow II$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \quad (-2)\ (-4)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] \quad \cdot(-\frac{1}{4})$$

3) $g(x) = \sqrt{x^2+x+1} - \log(x^2-x)$

I $x^2+x+1 \geq 0$

$x^2+x+1=0$

$x_{1/2} = \frac{-1 \pm \sqrt{(-1)^2-4\cdot 1\cdot 1}}{2\cdot 1}$

$= \frac{-1 \pm \sqrt{4}}{2}$

$= \frac{-1 \pm \sqrt{4}}{2}$

$x_1 = \frac{-1+\sqrt{4}}{2}$

$x_2 = \frac{-1-\sqrt{4}}{2}$

II $x^2-x > 0$

$x(x-1) > 0$

$x > 1$

$\langle 1, +\infty \rangle$

$\langle 0, +\infty \rangle$

2/

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ -2 & 0 & 1 & -2 \\ 1 & 2 & -1 & 3 \\ -1 & 2 & 12 & 1 \end{array}\right] \begin{matrix} \\ (+1) \\ \\ (+1) \end{matrix} \begin{matrix} \\ \\ \\ (-1) \end{matrix} \begin{matrix} \\ \\ \\ (+1) \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & +4 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} \\ 1:4 \\ \\ \\ \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \begin{matrix} \\ \\ \\ (-4) \end{matrix} \begin{matrix} \\ \\ \\ (+1) \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] \begin{matrix} \\ \\ \\ 1:(-4) \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] \begin{matrix} \\ \\ \\ \\ \end{matrix} \begin{matrix} \\ \\ \\ \frac{1}{2} \end{matrix} \begin{matrix} \\ \\ \\ \left(-\frac{7}{4}\right) \end{matrix} \begin{matrix} \\ \\ \\ (-8) \end{matrix} \sim$$

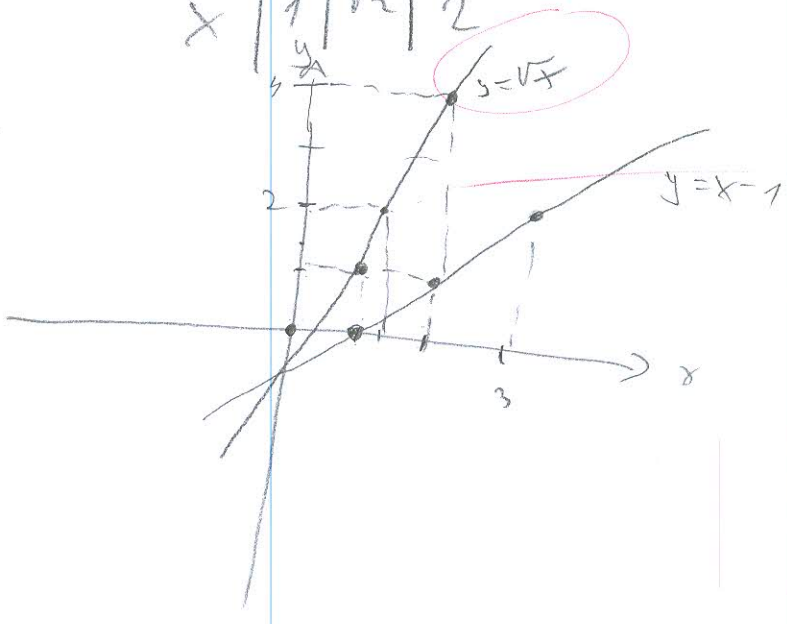
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} x \\ y \\ z \\ \\ \end{matrix}$$

6/ $x - 1 = \sqrt{x}$

$y = x - 1$ $\qquad$ $y = \sqrt{x}$

x	1	2	3
$y = x - 1$	0	1	2

y	1	2	4
x	1	$\sqrt{2}$	2



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ P4

IME I PREZIME: TOMI PAŠTUOVIĆ **BROJ INDEKSA:** 0269068933

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

ustmeni kod prof. Kuzet

POPUNJAVA NASTAVNIK
Broj ↓ bodova
15

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4. Odrediti tok funkcije $f(x) = \frac{x^2 - 7}{x^2 + 5}$ 20(graf)

5. Odrediti i provjeriti:

   (a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3}$. 4+1

   (b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}}$. 8+2

6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

(2)

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 3 \\ -2 & 0 & 1 & -2 \\ 1 & 2 & -1 & 3 \\ -1 & 2 & 12 & 1 \end{array}\right] \cdot 2+ \sim \left[\begin{array}{cccc} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 1 & 2 & -1 & 3 \\ -1 & 2 & 12 & 1 \end{array}\right] \cdot(-1)+ \sim \left[\begin{array}{cccc} 1 & 2 & 3 & 3 \\ 0 & 4 & 7 & 4 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \cdot 1:4$$

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 3 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & -4 & 0 \\ 0 & 4 & 15 & 4 \end{array}\right] \cdot(-2)+ \sim \left[\begin{array}{cccc} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{15}{4} & 1 \end{array}\right] \cdot -\frac{15}{4}+ \sim \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{15}{4} & 1 \end{array}\right] \cdot -\frac{7}{4}+ \sim \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{7}{4} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$x = 1$
$y = 1$
$z = 0$

$1 + 2 \cdot 1 + 3 \cdot 0 = 3$
$1 + 2 + 0 = 3$
$3 = 3$ ✓

$-2 \cdot 1 + 0 = -2$
$-2 = -2$ ✓

$1 + 2 - 0 = 3$
$3 = 3$ ✓

$-1 + 2 \cdot 1 + 12 \cdot 0 = 1$
$-1 + 2 + 0 = 1$
$1 = 1$ ✓

redak nadaje rješenje jer:
$0 \cdot x + 0 \cdot y + 0 \cdot z \neq 1$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ✓

(3) Domena $g(x) = \sqrt{x^2+x+1} - \log_{10}(x^2-x)$

$\log_2 8 = 3$

$\log_{10} 0 = 1$

1° $x^2+x+1 \geq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 1\cdot 1}}{2}$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$

nema realnih nultočka

$D = \mathbb{R}$

$x^2 - x \geq 0$

$x_{1,2} = \frac{1 \pm \sqrt{1-4\cdot 1\cdot 0}}{2}$

$x_{1,2} = \frac{1 \pm 1}{2}$

$x_1 = 1$

$x_2 = \frac{0}{2} = 0$

$D_x: \langle -\infty, 0] \cup [1, +\infty \rangle$

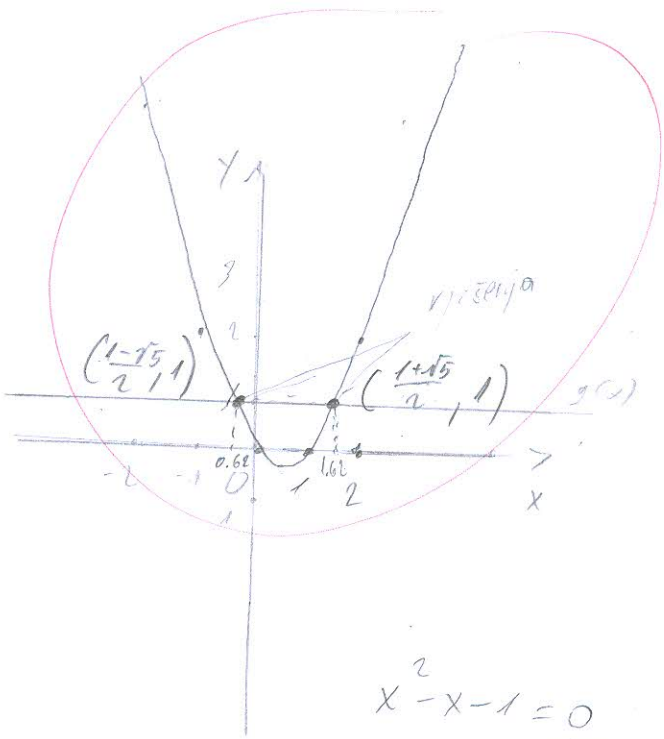
$D_g: \langle -\infty, 0] \cup [1, +\infty \rangle$

(6) $x - 1 = \sqrt{x}$

$x - \sqrt{x} = 1 \quad /^2$

$x^2 - x = 1$

$f(x) \quad g(x)$



$x^2 - x - 0 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1-4\cdot 1\cdot 0}}{2}$

$x_{1,2} = \frac{1 \pm 1}{2}$

$x_1 = \frac{2}{2} = 1$

$x_2 = \frac{0}{2} = 0$

x	-2	-1	0	1	2	$\frac{1}{2}$
x^2-x	6	2	0	0	2	

$x^2 - x - 1 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1-4\cdot 1\cdot(-1)}}{2}$

$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$x_1 = \frac{1+\sqrt{5}}{2}$ – rješenja

$x_2 = \frac{1-\sqrt{5}}{2}$ –

TONI PAŠTUOVIĆ

6) Tok f

$$f(x) = \frac{x^2-7}{x^2+5}$$

D 1° $x^2+5 \neq 0$

$x^2 \neq -5$

$D_f = \mathbb{R}$ ✓

nultočke

$$\frac{x^2-7}{x^2+5} = 0$$

$x^2 - 7 = 0$ $\quad x^2 = 7$ $\quad x = \pm\sqrt{7}$

$x_{1,2} = \frac{\sqrt{-4 \cdot 1 \cdot -7}}{2}$

$x_{1,2} = \frac{\sqrt{28}}{2} = \frac{\sqrt{7 \cdot 4}}{2} = \frac{2\sqrt{7}}{2} = \sqrt{7}$

ASIMPTOTE

H.A

$$\lim_{x \to +\infty} \frac{x^2-7}{x^2+5} = \lim_{x \to +\infty} \frac{\frac{x^2}{x^2} - \frac{7}{x^2}}{\frac{x^2}{x^2} - \frac{5}{x^2}} = \frac{-1}{1} = -1$$

$f(x)$

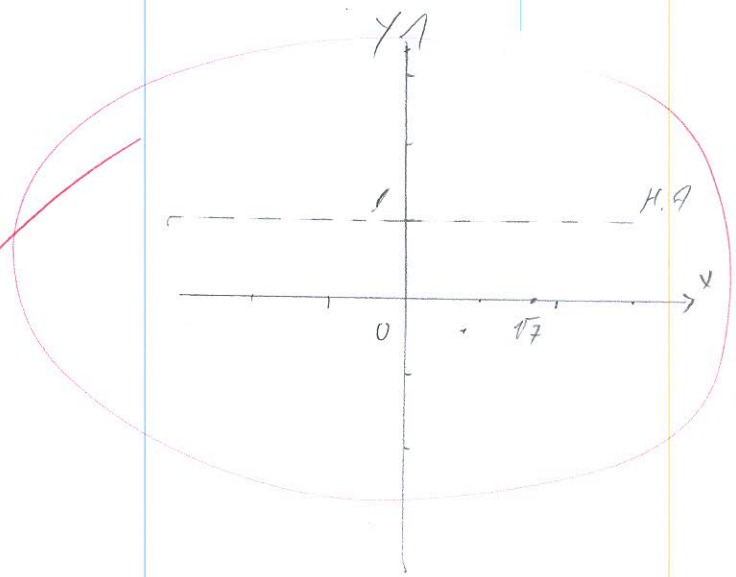
$$f'(x) = \left(\frac{x^2-7}{x^2-5}\right)'$$

$$f'(x) = \frac{(x^2-7)' \cdot (x^2-5) - (x^2-7) \cdot (x^2-5)'}{(x^2-5)^2}$$

$$f'(x) = \frac{2x \cdot (x^2-5) - (x^2-7) \cdot 2x}{x^4 - 2x^2 \cdot (-5) - 25}$$

$$f'(x) = \frac{2x^3 - 10x - 2x^3 + 14x}{x^4 + 10x^2 - 25}$$

$$= \frac{-24x}{x^4 + 10x^2 - 25}$$ ✓



⑤ $\lim_{n\to\infty} \frac{\sin n}{1-n^3} = \frac{\sin n / n^3}{\frac{1}{n^3} - 1} = \frac{0}{-1} = 0$

$\lim_{n\to\infty} \frac{\sin n}{1-n^3} \overset{L'H}{=} \frac{\cos n}{(1-n^3)'} = \frac{\cos n / n^2}{-3n^2/n^2} = \frac{\frac{\cos n}{n^2}}{-3} = \frac{0}{-3} = 0$

$n = 2$

$\frac{\sin 2}{1-8} = \frac{0.035}{-7} = -4.98 \cdot 10^{-3}$

$n = 4$

$\frac{\sin 4}{1-4^3} = \frac{0.0647}{1-64} = \frac{0.0647}{-63} = -1.107 \cdot 10^{-3}$

$n = -2$

$\frac{\sin(-2)}{1-(-2)^3} = \frac{-0.0349}{1+8} = -3.57 \cdot 10^{-3}$

⑥ $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}} \overset{L'H}{=} \frac{(x^{\frac{1}{2}} + (x^{\frac{1}{3}} + (x^{\frac{1}{4}})}{(2x+1)^{\frac{1}{2}})'} = \frac{\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{4}}}{\frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot (2x+1)'}$

$\frac{1}{4} - \frac{3}{4}$

$\frac{1}{3} - \frac{2}{3}$

$= \frac{(\frac{x}{2})^{-\frac{1}{2}} + (\frac{x}{3})^{-\frac{2}{3}} + (\frac{x}{4})^{-\frac{3}{4}}}{\frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot 2} = \frac{\frac{1}{\sqrt{\frac{x}{2}}} + \frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt[4]{\frac{x^3}{x^3}}}}{\frac{1}{\sqrt{2x+1}}} = 0$

$\sqrt[4]{\frac{x}{4}}^{-3}$

$\sqrt[4]{\frac{1}{\frac{x^3}{4^3}}} = \sqrt[4]{\frac{4^3}{x^3}}$

TOXII PASTUOVIC

1) $z^3 - \frac{(1-i)^6}{i^{21}} = 0$ $\quad i^{21} = i$ $\quad 21 : 4 = 5$ $\quad i^1$

$z^3 - \frac{(1-i)^6}{i} = 0$

$\sqrt[6]{-2i} = 2^{\frac{1}{12}} \left(\cos \frac{\frac{7\pi}{4} + k2\pi}{6} + i \sin \frac{\frac{7\pi}{4} + 2k\pi}{6} \right)$

$\operatorname{tg} \varphi = \frac{y}{x}$

$\operatorname{tg} \varphi = \frac{-1}{1} =$

$\sqrt[6]{-2i} = \sqrt[12]{2} \left(\cos 0.9163 + i \sin 0.9163 \right)$

$\varphi = 315^\circ = \frac{7\pi}{4}$

$(0.6087 + i0.793)$

$\sqrt{ix}$

$r = \sqrt{1^2 + 1^2}$

$r = \sqrt{2}$

$z^3 = \frac{}{i}$

$z = \sqrt[3]{i}$

$\operatorname{tg} \varphi = \frac{1}{0} = \frac{\pi}{2}$

$r = \sqrt{0^2 + 1^2}$

$r = 1$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ P4

IME I PREZIME: TOMISLAV GLAVAN **BROJ INDEKSA:** 17-0115-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} x + 2y + 3z &= 3 \\ -2x \quad + z &= -2 \\ x + 2y - z &= 3 \\ -x + 2y + 12z &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x + 1} - \log_{10}(x^2 - x)$. 15

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6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

4) D(f) $\frac{x^2-7}{x^2+5} = \mathbb{R}$

PARNOST

$f(-x) = \frac{-x^2-7}{-x+5}$ = NI PARNA NI NEPARNA

IDEE

$f'(x)\ \frac{x^2-7}{x^2+5} = \frac{2x\cdot(x^2-7) - (x^2+5)\cdot 2x}{(x^2+5)^2} = \frac{2x^3 - 14x - 2x^3 + 10x}{(x^2+5)^2} = \frac{4x}{(x^2+5)^2}$

ZIDOVI

$f''(x)\ \frac{4x}{(x^2+5)^2} = \frac{2x^2+2x+25\cdot(4) - 4\cdot(x^2+5)^2}{(x^2+5)^4} = \frac{8x^2 + 8x + 100 - 4x^4 - 100}{(x^2+5)^4}$

$\frac{-4x^4 + 8x^2 + 8x}{(x^2+5)^4}$