MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ P4

IME I PREZIME: ANĐELO ŽMIRE **BROJ INDEKSA:** 0269065578

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

21.2.2013.

POPUNJAVA NASTAVNIK Broj ↓ bodova

65

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} x + 2y + 3z &= 3 \\ -2x \quad + z &= -2 \\ x + 2y - z &= 3 \\ -x + 2y + 12z &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x + 1} - \log_{10}(x^2 - x)$. 15
4. Odrediti tok funkcije $f(x) = \frac{x^2 - 7}{x^2 + 5}$ 20(graf)
5. Odrediti i provjeriti:
   (a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3}$. 4+1
   (b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}}$. 8+2
6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

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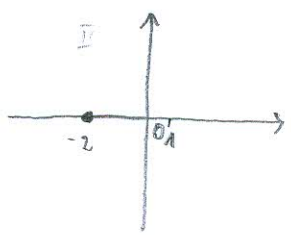
21 : 4 = 5
1



(1) $z^3 - \frac{(1-i)^6}{i^{21}} = 0$

$z^3 - \frac{[(1-i)^2]^3}{i^1} = 0$

$z^3 = \frac{[(1^2 - 2\cdot 1(-i) + i^2)]^3}{i}$

$z^3 = \frac{[(1+2i-1]^3}{i}$

$z^3 = \frac{(2i)^3}{i}$

$z^3 = \frac{-8i}{i}$

$z^3 = -8 \mid \sqrt[3]{\ }$

$z = \sqrt[3]{-8}$

$\boxed{z = -2}$

$\varphi_1 = tg^{-1}\frac{y}{x}$

$\varphi_1 = tg^{-1}\frac{0}{-2}$

$\varphi_1 = 0$

$\varphi = \pi - \varphi_1$

$\varphi = \pi$

$z_n = \sqrt[n]{r}\left(\cos\frac{\varphi + k2\pi}{n} + i\sin\frac{\varphi + k2\pi}{n}\right) \quad (k = 0,1,2)$

$z_1 = -2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= -2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -1 - \sqrt{3}i$

BRAVO

$z_2 = -2\left(\cos\frac{\pi + 2\pi}{3} + i\sin\frac{\pi + 2\pi}{3}\right)$

$= -2\left(\cos\frac{2\pi}{3} + i\sin\frac{3\pi}{3}\right)$

$= -2(\cos\pi + i\sin\pi)$

$= -2(-1 + 0) = 2$

$z_3 = -2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$= -2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1 + \sqrt{3}i$

✓

KOMPL. RAVNINA

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 3 & \\ -2 & 0 & 1 & -2 & \\ 1 & 2 & -1 & 3 & \\ -1 & 2 & 12 & 1 & \end{array}\right] \begin{array}{l} \text{I}-\text{III} \\ \text{II}+\text{III} \\ \text{III}+\text{IV} \\ \\ \end{array} \left[\begin{array}{cccc|c} 0 & 0 & 4 & 0 & \\ -1 & 2 & 0 & 1 & \\ 0 & 4 & 11 & 4 & \\ -1 & 2 & 12 & 1 & \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ -1 & 2 & 0 & 1 & \\ 0 & 4 & 11 & 4 & \\ 0 & 0 & 4 & 0 & \end{array}\right] \begin{array}{l} \text{II}+\text{I} \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & -12 & 0 & \\ 0 & 4 & 11 & 4 & \\ 0 & 0 & 4 & 0 & \end{array}\right] \begin{array}{l} \text{III}+\text{IV} \\ \\ \\ \cdot 3 \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & -12 & 0 & \\ 0 & 4 & -1 & 4 & \\ 0 & 0 & 12 & 0 & \end{array}\right] \begin{array}{l} \text{II}+\text{IV} \\ \\ \\ :12 \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & 0 & 0 & \\ 0 & 4 & -1 & 4 & \\ 0 & 0 & 1 & 0 & \end{array}\right] \begin{array}{l} \text{III}+\text{IV} \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & -12 & -1 & \\ 0 & 0 & 0 & 0 & \\ 0 & 4 & 0 & 4 & \\ 0 & 0 & 1 & 0 & \end{array}\right] \begin{array}{l} \text{I}+12\text{III} \\ \\ :4 \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & -1 & \\ 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 1 & \\ 0 & 0 & 1 & 0 & \end{array}\right] \begin{array}{l} \text{I}+2\text{III} \\ \\ \\ \\ \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} x=1 \\ y=1 \\ z=0 \end{array}$$

$x+2y+3z=3$

$1+2\cdot 1+3\cdot 0=3$

$1+2+0=3$

$3=3$ ✓

$-2x+z=-2$

$-2\cdot 1+0=-2$

$-2=-2$

$x+2y-z=3$

$1+2-0=3$

$3=3$

③ $g(x)=\sqrt{x^2+x+1}-\log_{10}(x^2-x)$

① $\sqrt{x^2+x+1} \geq 0$

$x^2+x+1 \geq 0$

$x \in \langle -\infty, +\infty \rangle$

$D(f)=\langle -\infty, 0\rangle \cup \langle 1, +\infty\rangle$

② $x^2-x>0$

$x(x-1)>0$

$x=0$ $\quad x-1=0 \quad x=1$

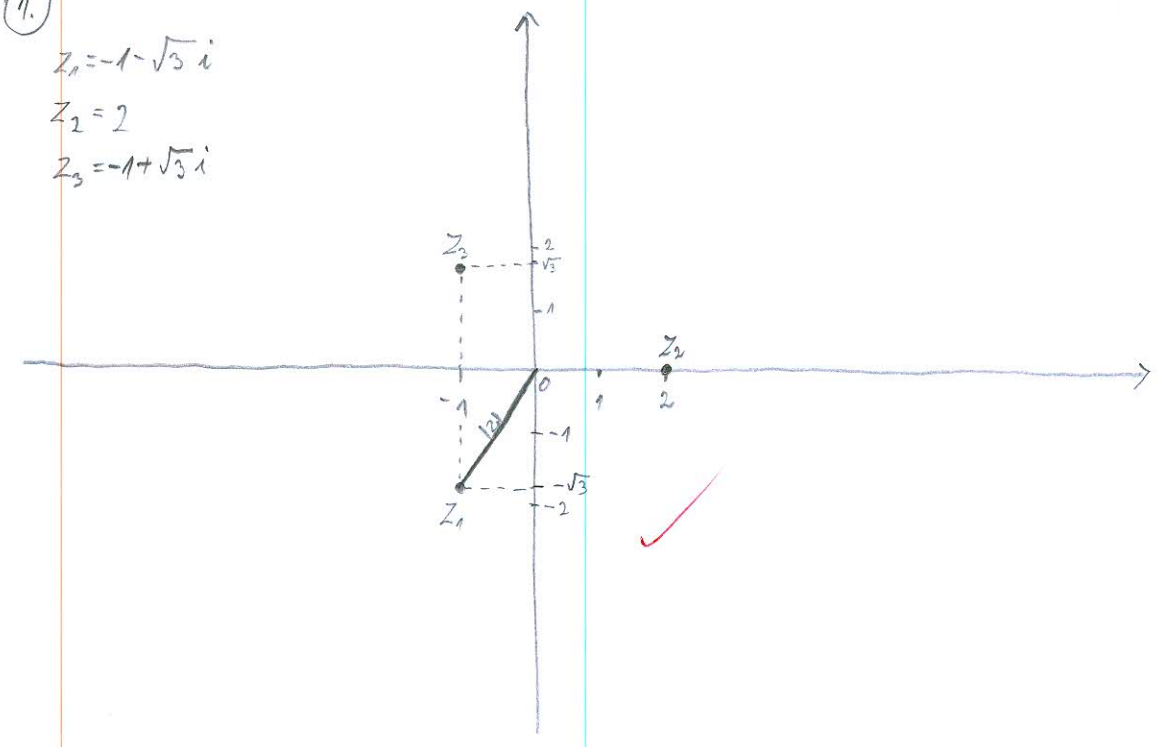
	$-\infty$		-1		0		$\frac{1}{2}$		1		2		$+\infty$
x		$-$		$-$	0	$+$		$+$		$+$		$+$	
$(x-1)$		$-$		$-$		$-$		$-$	0	$+$		$+$	
		$+$		$-$		$+$		$-$		$+$		$+$	

$x \in \langle -\infty, 0\rangle \cup \langle 1, +\infty\rangle$ ✓

① $z_1=-1-\sqrt{3}\,i$

$z_2=2$

$z_3=-1+\sqrt{3}\,i$



✓

ANDEZO OMIRE

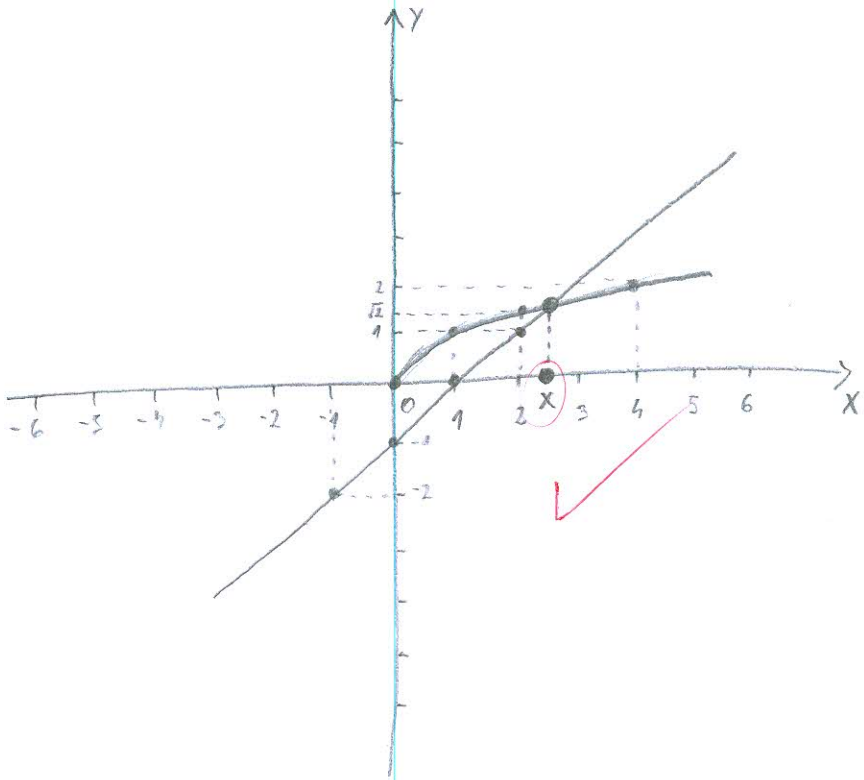
6. $x-1=\sqrt{x}$

$f(x)=x-1$

$g(x)=\sqrt{x}$

x	f(x)
1	0
-1	-2
0	-1
2	1

x	g(x)
1	1
2	≈1.41
4	2
0	0



$x=2.6$

$2.6-1\approx\sqrt{2.6}$

$1.6\approx1.6$

5. a) $\lim_{n\to\infty}\frac{\sin n / n^2}{1-n^3 / n^3}=\lim_{n\to\infty}\frac{0}{-1}=0$

b) $\lim_{n\to\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\lim_{n\to\infty}\frac{x^{\frac{1}{2}}+x^{\frac{1}{3}}+x^{\frac{1}{4}}}{\sqrt{2x+1}}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ P4

IME I PREZIME: IVAN KELAVA **BROJ INDEKSA:** 17-1-0084-11

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova
50

1. Riješiti jednadžbu: $z^3 - \frac{(1-i)^6}{i^{21}} = 0$. *Prikaži ih u kompleksnoj ravnini!* 12+3

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6. Grafičkom metodom riješiti jednadžbu: $x - 1 = \sqrt{x}$. *Provjeri uvrštavanjem!* 15+5

Ukupno:

3. 1. $x^2 + x + 1 \geq 0$

$x^2 + x + 1 = 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2}$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2}$

$x_{1,2}$ = NEMA RJ.

⇊

$D(f) = \mathbb{R}$

2. $x^2 - x > 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ $\quad x - 1 = 0$

$x = 1$

$D(f) = \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$

✓

$D(f) = \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$

(5) a) $\lim_{n\to\infty} \frac{\sin n}{1-n^3} : n^3 = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{\frac{\sin n}{n^3}}{\frac{1}{n^3} - \frac{n^3}{n^3}} = \frac{0}{1} = 0 \Rightarrow \frac{\sin \infty}{1-\infty}$

b) $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}} : x^{\frac{1}{2}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\frac{\sqrt{x}}{x^{\frac{1}{2}}} + \frac{\sqrt[3]{x}}{x^{\frac{1}{2}}} + \frac{\sqrt[4]{x}}{x^{\frac{1}{2}}}}{\frac{\sqrt{2x+1}}{x^{\frac{1}{2}}}} = \lim_{x\to\infty} \frac{\sqrt{\frac{x}{x}} + \sqrt[3]{\frac{x}{x^3}} + \sqrt[4]{\frac{x}{x^2}}}{\sqrt{\frac{2x+1}{x}}} =$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{x}{x}}^{1} + \sqrt[3]{\frac{x}{x^3}}^{0} + \sqrt[4]{\frac{x}{x^2}}^{0}}{\sqrt{\frac{2x}{x} + \frac{1}{x^0}}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

(2)
$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 3 \\ -2 & 0 & 1 & 1 & -2 \\ 1 & 2 & -1 & 1 & 3 \\ -1 & 2 & 12 & 1 & 1 \end{array}\right] \begin{array}{l} \\ +2R1 \\ -R1 \\ +R1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 3 \\ 0 & 4 & 7 & 1 & 4 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 4 & 15 & 1 & 4 \end{array}\right] :4 \simeq \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 3 \\ 0 & 1 & \frac{7}{4} & 1 & 1 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 4 & 15 & 1 & 4 \end{array}\right] \begin{array}{l} -2R2 \\ \\ \\ -4R2 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 1 & \frac{7}{4} & 1 & 1 \\ 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 8 & 1 & 0 \end{array}\right] :-4 \sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 1 & \frac{7}{4} & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 8 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R4-8R3 \\ R2-\frac{7}{4}R3 \\ R1+\frac{1}{2}R3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} x \\ y \\ z \end{array} \quad = \text{RANG MATRICE } \underline{\underline{3}}$$

✓

$x + 2y + 3z = 3 \Rightarrow 1 + 2\cdot 1 + 3\cdot 0 = 3$ ✓

$-2x + z = -2 \Rightarrow -2\cdot 1 + 0 = -2$ ✓

$x + 2y + z = 3 \Rightarrow 1 + 2\cdot 1 - 0 = 3$ ✓

$-x + 2y + 12z = 1 \Rightarrow -1 + 2\cdot 1 + 12\cdot 0 = 1$ ✓