

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

03

NASTAVNIK

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Broj ↓
bodova

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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1. Odrediti kompleksne brojeve z koji zadovoljava jednačbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 0 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 7 \\ & & - & 7x_2 & + & 3x_3 & + & x_4 & = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$.

5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$.

15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$.

2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

(a) koji su lokalni ekstremi?

10

(b) koji su globalni ekstremi?

5

Ukupno:

1.

$$\frac{|z|}{2(z+i)} = 3i$$

$$z = x + yi$$

$$\frac{|x+yi|}{2(x+yi+i)} = 3i$$

$$\frac{\sqrt{x^2+y^2}}{2(x+(y+1)i)} = 3i \quad \left[2(x+(y+1)i) \right]$$

$$\sqrt{x^2+y^2} = 3i(2x+2y+2i)$$

$$\sqrt{x^2+y^2} = 6xi + 6yi - 6$$

$$\sqrt{x^2+y^2} = 6xi - 6y - 6$$

$$\sqrt{x^2+y^2} = -6y - 6$$

$$6x = 0 \quad \left| :6 \right.$$

$$x = 0$$

$$\sqrt{y^2} = -6y - 6$$

$$|y| = -6y - 6$$

$$1^\circ \quad y > 0 = -y - 6$$

$$-y + 6y = -6$$

$$7y = -6 \quad \left| :7 \right.$$

$$y = -\frac{6}{7}$$

$$2^\circ \quad y < 0$$

$$y + 6y = -6$$

$$7y = -6 \quad \left| :7 \right.$$

$$y = -\frac{6}{7}$$

$$z = x + yi$$

$$z = -\frac{6}{7}i$$

2) Rješiti sustav Gaussovom metodom i obratno provjeriti rješenje

$$x_1 - 2x_2 + 3x_3 - 4x_4 = 0$$

$$x_2 - x_3 + x_4 = 1$$

$$x_1 + 3x_2 - 3x_4 = 7$$

$$-7x_2 + 3x_3 + x_4 = -15$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{bmatrix} \begin{array}{l} /: (-1) \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -5 & 3 & -1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{bmatrix} \begin{array}{l} \\ /: 5 /: 7 \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 & 2 \end{bmatrix} \begin{array}{l} \\ \\ /: (-1) \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \\ \\ \\ 0 \neq 1 \end{array}$$

SUSTAV NEMA RJEŠENJA

$$\begin{bmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 7 \\ -15 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 7 \\ -15 \end{bmatrix}$$

$$(4, 4), (4, 1) = (4, 4)$$

$$\begin{bmatrix} 0 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$$

$$\begin{array}{cccc|c} -1 & 2 & -3 & 4 & 0 \\ 1 & 3 & 0 & -3 & 7 \end{array}$$

$$\begin{array}{cccc|c} 0 & 5 & -3 & 1 & 7 \\ 0 & -5 & 3 & -1 & -7 \end{array} \begin{array}{l} /: (-1) \\ \\ \end{array}$$

$$\begin{array}{cccc|c} 0 & 5 & -5 & 5 & 5 \\ 0 & -5 & 3 & -1 & -7 \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 1 & -2 & 1 \end{array} \begin{array}{l} /: (-2) \\ \\ \end{array}$$

$$\begin{array}{cccc|c} 0 & 7 & -7 & 7 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & -4 & 8 & -8 \\ 0 & 0 & 1 & -2 & 2 \end{array} \begin{array}{l} /: (-4) \\ \\ \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & -2 & 2 \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & -1 \end{array}$$

3. Ispitati domen i sve asimptote funkcije

$$g(x) = \sqrt{x^2 - 5x + 1} - x$$

DOMENA:

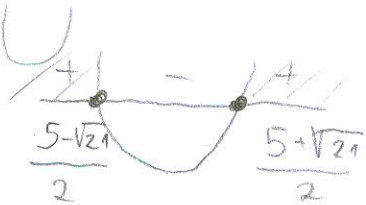
$$U: x^2 - 5x + 1 \geq 0$$

$$x^2 - 5x + 1 = 0$$

$$x_{1,2} = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{5 \pm \sqrt{25 - 4}}{2} = \frac{5 \pm \sqrt{21}}{2}$$

$x_1 = \frac{5 - \sqrt{21}}{2} \approx 0,209$
 $x_2 = \frac{5 + \sqrt{21}}{2} \approx 4,791$

$a > 0$



$$x \in \left(-\infty, \frac{5 - \sqrt{21}}{2}\right] \cup \left[\frac{5 + \sqrt{21}}{2}, +\infty\right)$$

$$Dg = \left(-\infty, \frac{5 - \sqrt{21}}{2}\right] \cup \left[\frac{5 + \sqrt{21}}{2}, +\infty\right) \checkmark$$

VERTIKALNU NEMA KER NEMA TOČKE PREKIDA

HORIZONTALNA:

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 - 5x + 1} - x \right) = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 5x + 1} + x}{\sqrt{x^2 - 5x + 1} + x} \cdot \frac{\sqrt{x^2 - 5x + 1} - x}{\sqrt{x^2 - 5x + 1} - x} = \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 - 5x + 1})^2 - x^2}{\sqrt{x^2 - 5x + 1} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 - 5x + 1 - x^2}{\sqrt{x^2 - 5x + 1} + x} = \lim_{x \rightarrow \infty} \frac{-5x + 1}{\sqrt{x^2 - 5x + 1} + x} \stackrel{\frac{-\infty}{\infty}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 5}{\sqrt{\frac{x^2 - 5x + 1}{x^2}} + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{-5}{\sqrt{1 - \frac{5}{x} + \frac{1}{x^2}} + 1} = \lim_{x \rightarrow \infty} \frac{-5}{1 + 1} = \frac{-5}{2} \text{ D.H.A.}$$

NEMA L.H.A

$$\lim_{x \rightarrow -\infty} \left(\sqrt{x^2 - 5x + 1} - x \right) = \left. \begin{matrix} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{matrix} \right\} = \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 5x + 1} + x \right) = \infty$$

$$l = \lim_{x \rightarrow -\infty} \left(\frac{f(x)}{x} \right) = \left. \begin{matrix} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{matrix} \right\} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 5x + 1} + x}{-x} \stackrel{\frac{\infty}{-\infty}}{=} \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2 + 5x + 1}{x^2}} + 1}{-1}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{5}{x} + \frac{1}{x^2}} + 1}{-1} = \frac{2}{-1} = -2$$

KAKO MOŽETE
DETALJI

$$l = \lim_{x \rightarrow -\infty} \left(f(x) - kx \right) = \left. \begin{matrix} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{matrix} \right\} = \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 5x + 1} + x - 2(-x) \right) = \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 5x + 1} - x \right) \cdot \frac{\sqrt{x^2 + 5x + 1} + x}{\sqrt{x^2 + 5x + 1} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 5x + 1})^2 - x^2}{\sqrt{x^2 + 5x + 1} + x} = \lim_{x \rightarrow \infty} \frac{x^2 + 5x + 1 - x^2}{\sqrt{x^2 + 5x + 1} + x} = \lim_{x \rightarrow \infty} \frac{5x + 1}{\sqrt{x^2 + 5x + 1} + x} \stackrel{\frac{\infty}{\infty}}{=} \lim_{x \rightarrow \infty} \frac{5x + 1}{x \left(\sqrt{1 + \frac{5}{x} + \frac{1}{x^2}} + 1 \right)}$$

KOSU

KAD

$$= \lim_{x \rightarrow \infty} \frac{1 + \frac{1}{x}}{\sqrt{1 + \frac{5}{x} + \frac{1}{x^2}} + 1} = \lim_{x \rightarrow \infty} \frac{1}{1 + 1} = \frac{1}{2}$$

SVE NAŠLI
HORI ENTITIV!

$$y = kx + l, \quad y = -2x + \frac{1}{2}$$

$$4) f(x) = \frac{5-x}{x-3}$$

1° a) DOMENA $D_f = \mathbb{R} \setminus \{3\}$

$$U: x-3 \neq 0 \\ x \neq 3$$

b) $f(0) = \frac{5}{-3} = -\frac{5}{3} \quad (0, -\frac{5}{3})$

$$f(x) = 0$$

$$\frac{5-x}{x-3} = 0 / \cdot (x-3)$$

$$5-x=0$$

$$-x = -5 / \cdot (-1)$$

$$x=5 \quad (5, 0)$$

c) $f(-x) = \frac{5+x}{-x-3} = \frac{-(-5-x)}{-(x+3)} = \frac{-5-x}{x+3} \neq f(x)$ NIT PARNA
 $\neq -f(x)$ NIT NEPARNA
 NIT PERIODIČNA

d) ASIMPTOTE

$$\lim_{x \rightarrow 3^-} \frac{5-x}{x-3} = \frac{2}{0} = -\infty \quad \text{L.V.A}$$

$$\boxed{x=3}$$

$$\lim_{x \rightarrow 3^+} \frac{5-x}{x-3} = \frac{2}{0^+} = +\infty \quad \text{D.V.A}$$

$$\lim_{x \rightarrow \infty} \frac{5-x}{x-3} \stackrel{0}{\neq} \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x} - 1}{1 - \frac{3}{x}} = \frac{-1}{1} = -1 \quad \text{D.H.A}$$

$$\boxed{y=-1}$$

$$\lim_{x \rightarrow -\infty} \frac{5-x}{x-3} = \left| \begin{array}{l} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{array} \right| = \lim_{x \rightarrow \infty} \frac{5+x}{-x-3} \stackrel{0}{\neq} \frac{\infty}{-\infty} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x} + 1}{-1 - \frac{3}{x}} = \frac{1}{-1} = -1 \quad \text{L.H.A}$$

2° $f'(x) = \frac{(5-x)' \cdot (x-3) - (5-x) \cdot (x-3)'}{(x-3)^2} = \frac{-1(x-3) - (5-x) \cdot 1}{(x-3)^2} = \frac{-x+3-5+x}{(x-3)^2} = \frac{-2}{(x-3)^2}$

$$f'(x) = 0$$

$$\frac{-2}{(x-3)^2} = 0 / \cdot (x-3)^2$$

$-2 \neq 0$ NEMA STACIONARNIH TOČKA PA NEMA NI EKSTREMA

	$-\infty$	0	3	4	$+\infty$
$f'(x)$		-		-	
$f(x)$		↘		↘	

$$f'(0) = \frac{-}{+} = -$$

$$f'(4) = \frac{-}{+} = -$$

interval konkvavnosti:

$$\langle -\infty, 3 \rangle \cup \langle 3, +\infty \rangle$$

