MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ Ø3

IME I PREZIME: LUKA MILIN **BROJ INDEKSA:** 17-2-0177-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

45

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3
2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)
5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9
6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:
   (a) koji su lokalni ekstremi? 10
   (b) koji su globalni ekstremi? 5

Ukupno:

1. $z = x + yi$

$$\frac{|z|}{2(z+i)} = 3i$$

$$\frac{|x+yi|}{2(x+yi+i)} = 3i$$

$$\frac{\sqrt{x^2+y^2}}{2(x+(y+1)i)} = 3i \quad / \cdot [2(x+(y+1)\cdot i]$$

$$\sqrt{x^2+y^2} = 3i(2x+2yi+2i)$$

$$\sqrt{x^2+y^2} = 6xi + 6yi^2 + 6i^2$$

$$\sqrt{x^2+y^2} = 6xi - 6y - 6$$

$$\sqrt{x^2+y^2} = -6y - 6 \qquad \sqrt{y^2} = |y|$$

$$6x = 0 \quad / \cdot \tfrac{1}{6}$$

$$x = 0$$

$$\sqrt{y^2} = -6y - 6$$

$$|y| = -6y - 6$$

1° $y > 0$ = $-y$

$$-y + 6y = -6$$

$$7y = -6 \quad / \cdot (\tfrac{1}{7})$$

$$y = -\frac{6}{7}$$

2° $y < 0$

$$y + 6y = -6$$

$$7y = -6 \quad / \cdot \tfrac{1}{7}$$

$$y = -\frac{6}{7}$$

$z = x + yi$; $z = -\frac{6}{7}i$

2) Riješiti sustav Gaussovom metodom i obvezno pregnuti rješenje

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 0 \\ x_2 - x_3 + x_4 &= 1 \\ x_1 + 3x_2 - 3x_4 &= 7 \\ -7x_2 + 3x_3 + x_4 &= -15 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} /\cdot(-1) \\ \\ + \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -5 & 3 & -1 & -7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} \\ \\ /\cdot 5 \; /\cdot 7 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 & 2 \end{array}\right] \begin{array}{l} \\ \\ \\ /\cdot(-1) \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad 0 \neq 1$$

SUSTAV NEMA RJEŠENJA

$$\left[\begin{array}{cccc|c} -1 & 2 & -3 & 4 & 0 \\ 1 & 3 & 0 & -3 & 7 \\ \hline 0 & 5 & -3 & 1 & 7 \\ 0 & -5 & 3 & -1 & -7 \end{array}\right] \begin{array}{l} \\ \\ /\cdot(-1) \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & 5 & -5 & 5 & 5 \\ 0 & -5 & 3 & -1 & -7 \\ \hline 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} \\ \\ /:(-2) \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & 7 & -7 & 7 & 7 \\ 0 & -7 & 3 & 1 & -15 \\ \hline 0 & 0 & -4 & 8 & -8 \\ 0 & 0 & 1 & -2 & 2 \end{array}\right] \begin{array}{l} \\ \\ /:(-4) \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & -2 & 2 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 7 \\ -15 \end{bmatrix}$$

$$(4,4),((4,1) = (4,1)$$

$$\begin{bmatrix} 0-2+3-4 \\ 0+1-1+1 \\ 0+3+0-3 \\ 0-7+3-1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ -3 \end{bmatrix}$$

MATEMATIKA 1

Luka Milun

BROJ INDEKSA: 17-2-0177-2012

3. Ispitati domenu i sve asimptote funkcije

$g(x) = \sqrt{x^2-5x+1} - x$

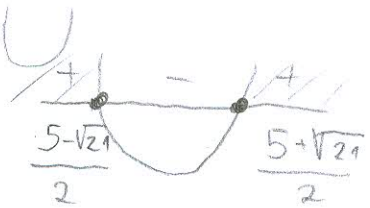
DOMENA:

U: $x^2 - 5x + 1 \geq 0$

$x^2 - 5x + 1 = 0$

$x_{1,2} = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{5 \pm \sqrt{25-4}}{2} = \frac{5 \pm \sqrt{21}}{2} =$ ↗ $x_1 = \frac{5-\sqrt{21}}{2} \approx 0.209$ ↘ $x_2 = \frac{5+\sqrt{21}}{2} \approx 4,791$

$a > 0$ ∪



$x \in \left\langle -\infty, \frac{5-\sqrt{21}}{2} \right] \cup \left[\frac{5+\sqrt{21}}{2}, +\infty \right\rangle$

$Dg = \left\langle -\infty, \frac{5-\sqrt{21}}{2} \right] \cup \left[\frac{5+\sqrt{21}}{2}, +\infty \right\rangle$ ✓

VERTIKALNU NEMA JER NEMA TOČKE PREKIDA

HORIZONTALNA:

$\lim_{x\to\infty} \left(\sqrt{x^2-5x+1} - x \right)_{\infty-\infty} \cdot \frac{\sqrt{x^2-5x+1}+x}{\sqrt{x^2-5x+1}+x} = \lim_{x\to\infty} \frac{\left(\sqrt{x^2-5x+1}\right)^2 - x^2}{\sqrt{x^2-5x+1}+x}$

$= \lim_{x\to\infty} \frac{x^2-5x+1-x^2}{\sqrt{x^2-5x+1}+x} = \lim_{x\to\infty} \frac{1-5x \,/:x}{\sqrt{x^2-5x+1}+x \,/:x} = \left[\frac{-\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\frac{1}{x}^{\to 0} - 5}{\sqrt{\frac{x^2-5x+1}{x^2}} + 1}$

$= \lim_{x\to\infty} \frac{-5}{\sqrt{1-\frac{5}{x}+\frac{1}{x^2}}+1} = \lim_{x\to\infty} \frac{-5}{1+1} = -\frac{5}{2}$ D.H.A.

NEMA L.H.A

$\lim_{x\to-\infty} \left(\sqrt{x^2-5x+1} - x \right) = \left| \begin{matrix} x\to -x \\ -\infty \to +\infty \end{matrix} \right| = \lim_{x\to\infty} \left(\sqrt{x^2+5x+1} + x \right)_{\infty+\infty} = \infty$

$k = \lim_{x\to-\infty} \left(\frac{f(x)}{x} \right) = \left| \begin{matrix} x\to -x \\ -\infty \to +\infty \end{matrix} \right| = \lim_{x\to\infty} \frac{\sqrt{x^2+5x+1}+x \,/:x}{-x \,/:x} = \left[\frac{\infty}{-\infty}\right] = \lim_{x\to\infty} \frac{\sqrt{\frac{x^2+5x-1}{x^2}} + 1}{-1}$

$= \lim_{x\to\infty} \frac{\sqrt{1+\frac{5}{x}+\frac{1}{x^2}}+1}{-1} = \frac{2}{-1} = -2$

KAKO MOŽETE DETALJNIJE?

$l = \lim_{x\to-\infty} \left(f(x) - kx \right) = \left| \begin{matrix} x\to -x \\ -\infty \to \infty \end{matrix} \right| = \lim_{x\to\infty} \left(\sqrt{x^2+5x+1} + x + 2(+x) \right) = \lim_{x\to\infty} \left(\sqrt{x^2+5x+1} - x \right)_{\infty\cdot\infty} \cdot \frac{\sqrt{x^2+5x+1}+x}{\sqrt{x^2+5x+1}+x}$

$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+5x+1}\right)^2 - x^2}{\sqrt{x^2+5x+1}+x} = \lim_{x\to\infty} \frac{x^2+5x+1-x^2}{\sqrt{x^2+5x+1}+x} = \lim_{x\to\infty} \frac{5x+1 \,/:x}{\sqrt{x^2+5x+1}+x \,/:x} = \left[\frac{\infty}{\infty}\right]$

KOSI KAD STE NAŠLI ASIMPTOTU!!

$= \lim_{x\to\infty} \frac{1+\frac{1}{x}}{\sqrt{\frac{x^2+5x+1}{x^2}}+1} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{5}{x}+\frac{1}{x^2}}+1} = \frac{1}{1+1} = \frac{1}{2}$

$y = kx + l$, $\boxed{y = -2x + \frac{1}{2}}$

4) $f(x)=\frac{5-x}{x-3}$

1° a) DOMENA $\quad D_f=\mathbb{R}\setminus\{3\}$

U: $x-3\neq 0$

$x\neq 3$

b) $f(0)=\frac{5}{-3}=-\frac{5}{3}\quad\left(0,-\frac{5}{3}\right)$

$f(x)=0$

$\frac{5-x}{x-3}=0 \;/\cdot(x-3)$

$5-x=0$

$-x=-5\;/\cdot(-1)$

$x=5 \qquad (5,0)$

c) $f(-x)=\frac{5+x}{-x-3}=\frac{-(-5-x)}{-(x+3)}=\frac{-5-x}{x+3}\neq f(x)$ NIT PARNA

$\neq -f(x)$ NIT NEPARNA

NIJE PERIODIČNA

d) ASIMPTOTE

$\lim\limits_{x\to 3^-}\frac{5-3}{3-3}=\frac{2}{0}=-\infty$ L.V.A $\quad \boxed{x=3}$

$\lim\limits_{x\to 3^+}\frac{5-3}{3-3}=\frac{2}{0^+}=+\infty$ D.V.A

$\lim\limits_{x\to\infty}\frac{5-x}{x-3}\overset{/:x}{\underset{/:x}{\left[\frac{\infty}{\infty}\right]}}=\lim\limits_{x\to+\infty}\frac{\frac{5}{x}-1}{1-\frac{3}{x}}=\frac{-1}{1}=-1$ D.H.A $\quad \boxed{y=-1}$

$\lim\limits_{x\to-\infty}\frac{5-x}{x-3}=\left|\begin{matrix}x\to-x\\ -\infty\to+\infty\end{matrix}\right|=\lim\limits_{x\to\infty}\frac{5+x}{-x-3}\overset{/:x}{\underset{/:x}{\left[\frac{\infty}{-\infty}\right]}}=\lim\limits_{x\to\infty}\frac{\frac{5}{x}+1}{-1-\frac{3}{x}}=\frac{1}{-1}=-1$ L.H.A

($\frac{5}{x}\to 0$, $\frac{3}{x}\to 0$)

2° $f'(x)=\frac{(5-x)'\cdot(x-3)-(5-x)\cdot(x-3)'}{(x-3)^2}=\frac{-1(x-3)-(5-x)\cdot 1}{(x-3)^2}=\frac{-x+3-5+x}{(x-3)^2}$

$=\frac{-2}{(x-3)^2}$

$f'(x)=0$

$\frac{-2}{(x-3)^2}=0\;/\cdot(x-3)^2$

$-2\neq 0$ NEMA STACIONARNIH TOČAKA PA NEMA NI EKSTREMA

	$-\infty$		0		3		4		$+\infty$
$f'(x)$		−			∄		−		
$f(x)$		↘			∄		↘		

$f'(0)=\frac{-}{+}=-$

$f'(4)=\frac{-}{+}=-$

intervali konkavnosti:

$\langle -\infty, 3\rangle \cup \langle 3, +\infty\rangle$

$$f''(x) = \frac{(-2)' \cdot (x-3)^2 - (-2) \cdot [(x-3)^2]'}{(x-3)^4} = \frac{0 \cdot (x-3)^2 - (-2) \cdot 2(x-3) \cdot \overbrace{(x-3)'}^{1}}{(x-3)^4}$$

$$f'(x) = \frac{-(-2) \cdot (2x-6)}{(x-3)^4} = \frac{4x-12}{(x-3)^4}$$

$f''(x) = 0$

$$\frac{4x-12}{(x-3)^4} = 0 \;/\cdot (x-3)^4$$

$4x - 12 = 0$

$4x = 12 \;/\cdot \frac{1}{4}$

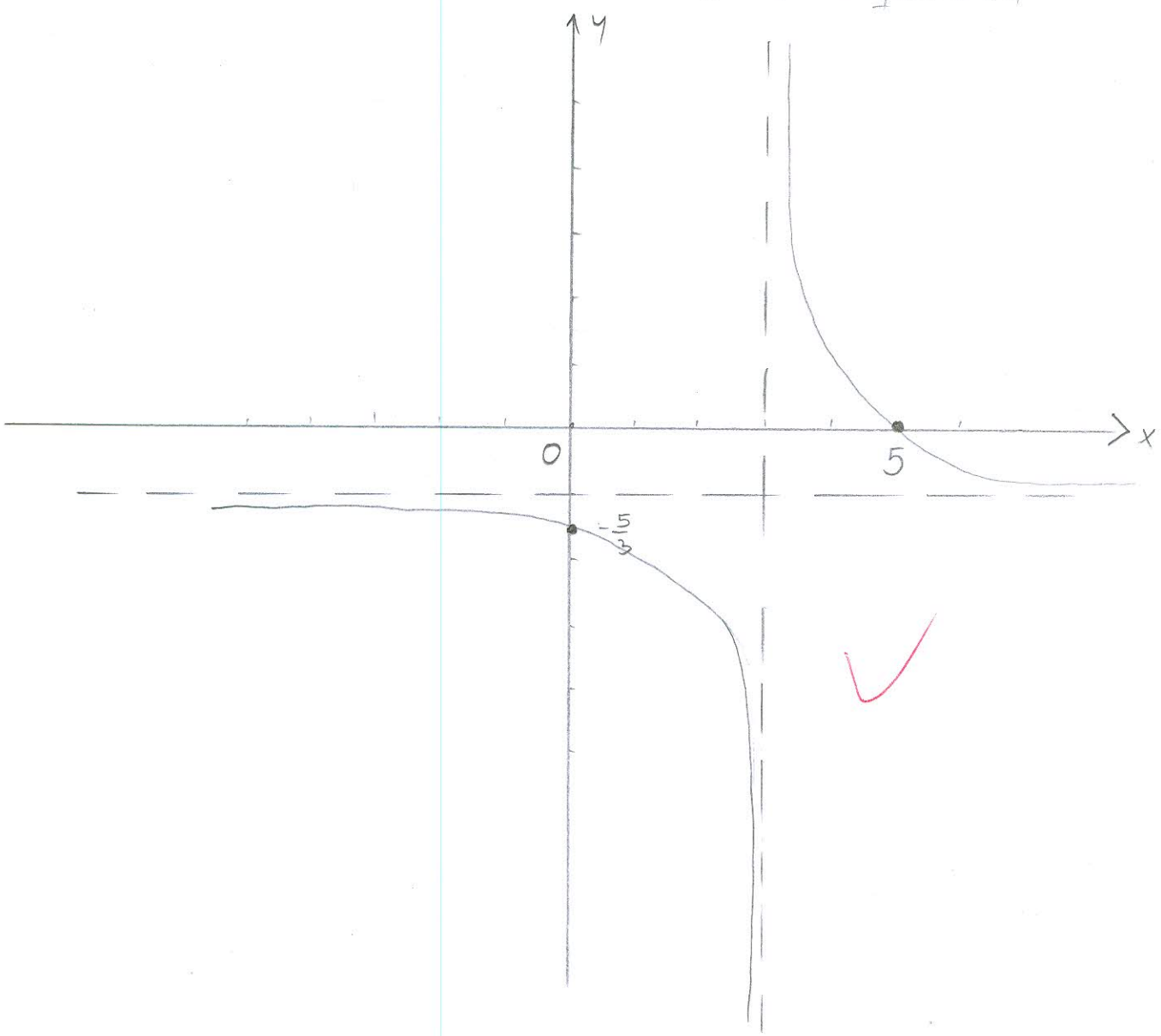
$x = 3$ N.T. $f''(x)$ mje u D_f

	$-\infty$		0		3		5		$+\infty$
$f''(x)$		−		+		+		+	
$f(x)$		∩		∪		∪		∪	

$$f''(0) = \frac{-}{+} = -$$

$$f''(4) = \frac{+}{+} = +$$

$$f''(6) = \frac{+}{+} = +$$



5) Domena, period (ako postoji), parnost, $g''(x)$

$h(x) = \cos(4x+1)$

$D_g = \mathbb{R}$

$P = \frac{2k\pi}{4}$

$k = 1 \quad P_0 = \frac{2\pi}{4}$

$h(-x) = \cos(4\cdot(-x)+1)$

$h(-x) = \cos(-4x+1)$

$(\cos(-x)) = \cos x$ P

$h(-x) = \cos(4x+1)$ funkcija je parna

$g'(x) = -\sin(4x+1)\cdot(\overbrace{4x+1}^{4})'$

$g'(x) = -\sin(4x+1)\cdot 4$

$= -4\sin(4x+1)$

$g''(x) = -4\cos(4x+1)\cdot(\overbrace{4x+1}^{4})'$

$g''(x) = -16\cos(4x+1)$

---

6) $g(x) = \sqrt{2x-x^2}$

U: $2x-x^2 \geq 0 \quad a<0$ ∩

$2x-x^2 = 0$

$x(2-x) = 0$

$x_1 = 0$

$2-x = 0$

$-x = -2 /\cdot(-1)$

$x = 2$

$x \in [0,2] = D_g$

$g'(x) = \frac{1}{2\sqrt{2x-x^2}}\cdot(2x-x^2)'$

$g'(x) = \frac{1}{2\sqrt{2x-x^2}}\cdot(2-2x)$

$g'(x) = \frac{2-2x}{2\sqrt{2x-x^2}}$

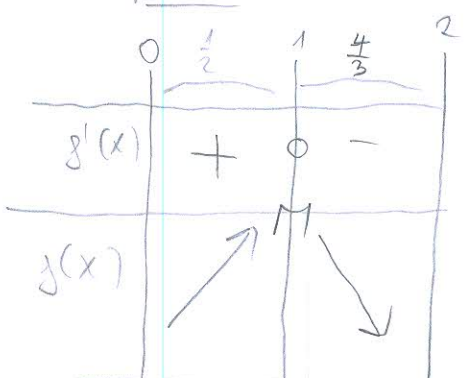
$g'(x) = 0$

$\frac{2-2x}{2\sqrt{2x-x^2}} = 0 /\cdot(2\sqrt{2x-x^2})$

$2-2x = 0$

$-2x = -2 /\cdot(-\frac{1}{2})$

$x = 1$



$g'(\frac{1}{2}) = \frac{2-2\cdot\frac{1}{2}}{2\sqrt{2\cdot\frac{1}{2}-(\frac{1}{2})^2}}$

$g'(\frac{1}{2}) = \frac{+}{+} = +$

$g'(\frac{4}{3}) = \frac{2-2\cdot\frac{4}{3}}{2\sqrt{2\cdot\frac{4}{3}-(\frac{4}{3})^2}}$

$g'(\frac{4}{3}) =$

$T_{MAX}(1, g(1)) = (1,1)$

$g(1) =$

$\langle 0,1 \rangle$ interval rasta

$\langle 1,2 \rangle$ interval pada

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IME I PREZIME: RANKO BRKIĆ **BROJ INDEKSA:** 17-1-0091-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

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$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15
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   (a) koji su lokalni ekstremi? 10
   (b) koji su globalni ekstremi? 5

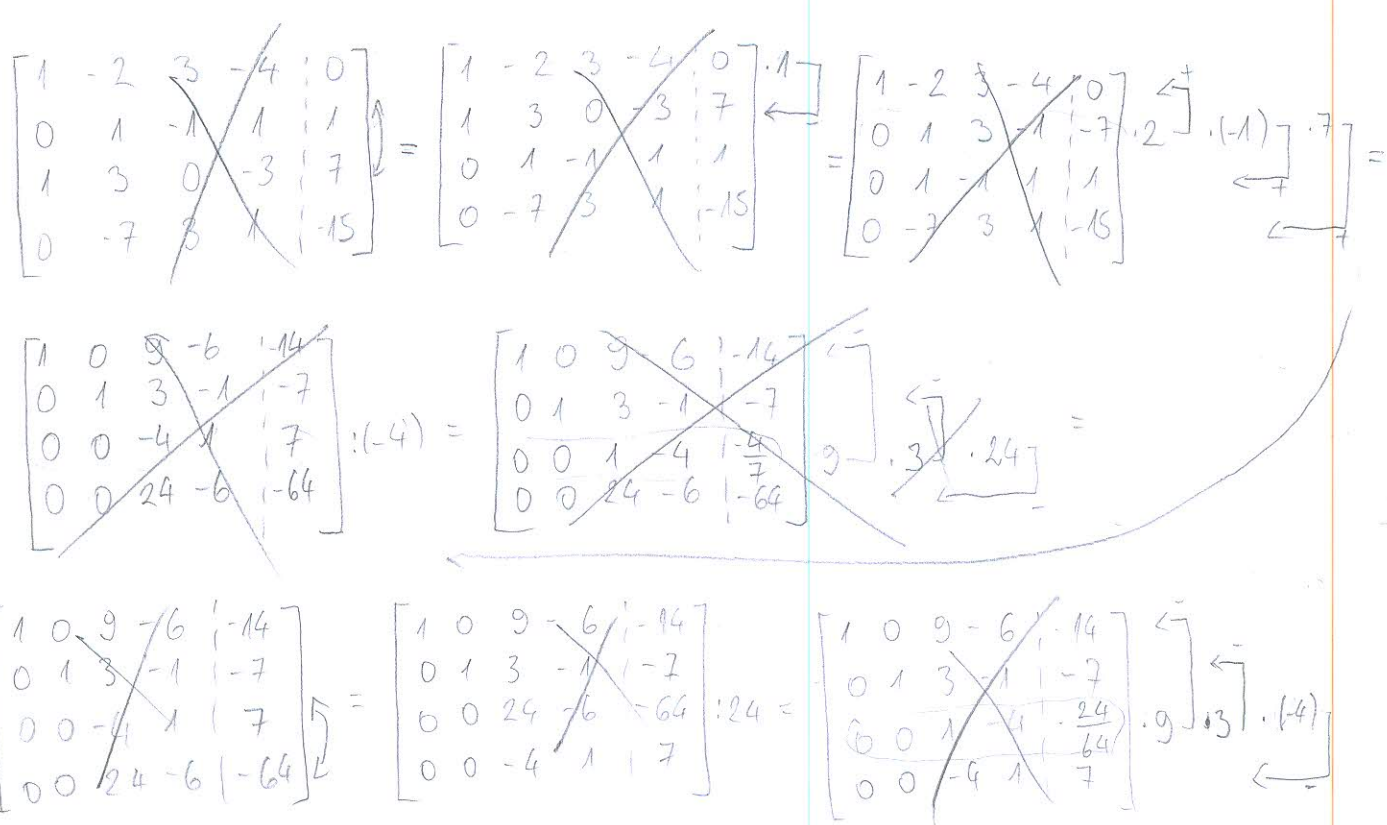
Ukupno:

(2.) $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$

$x_2 - x_3 + x_4 = 1$

$x_1 + 3x_2 - 3x_4 = 7$

$-7x_2 + 3x_3 + x_4 = -15$



③ $g(x) = \sqrt{x^2-5x+1} - x$

$x^2 - 5x + 1 \geq 0$

$x_{1,2} = \frac{5 \pm \sqrt{5^2-4\cdot1\cdot1}}{2\cdot1}$

$x_{1,2} = \frac{5 \pm \sqrt{25-4}}{2}$

$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$

$x_{1,2} = \frac{5 \pm 4.58}{2}$

$x_1 = 0{,}21 \qquad x_2 = 4{,}79$

$D(g) = \langle -\infty, 0{,}21] \cup [4{,}79, +\infty \rangle$ ✓

$\lim_{+\infty} \sqrt{x^2-5x+1} - x = -\infty$ NEMA!

$\lim_{-\infty} \sqrt{x^2-5x+1} - x = +\infty$ NEMA!

$\lim_{+\infty} \frac{f(x)}{x} = \lim_{+\infty} \frac{\sqrt{x^2-5x+1}-x}{x} = \lim_{+\infty} \frac{\sqrt{x^2-5x+1}}{x} - 1 =$

$= \lim_{+\infty} \frac{\sqrt{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{1}{x^2}}}{\sqrt{\frac{x^2}{x^2}}} - 1 = \frac{1}{1} - 1 = 1 - 1 = 0$

$a = 0$

$\lim_{+\infty} f(x) - ax = \sqrt{x^2-5x+1} - x \cdot \frac{\sqrt{x^2-5x+1}+x}{\sqrt{x^2-5x+1}+x} =$

$= \lim_{+\infty} \frac{x^2-5x+1-x^2}{\sqrt{x^2-5x+1}+x} = \lim_{+\infty} \frac{-5x+1}{\sqrt{x^2-5x+1}+x} : x =$

$= \frac{-5}{1+1} = -\frac{5}{2} \qquad b = -\frac{5}{2}$

$y = -\frac{5}{2}$ KOSA ASIMPTOTA

④ $f(x) = \frac{5-x}{x-3}$

$x - 3 \neq 0$

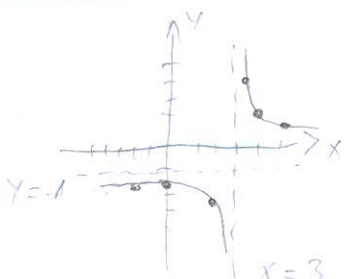
$x \neq 3$

$D(f) \langle -\infty, 3 \rangle \cup \langle 3, +\infty \rangle$ ✓

↓

TOČKA PREKIDA

UDE DVIJU I V.A



x	y
0	$-\frac{5}{3}$
2	-3
4	1
-2	$-\frac{7}{5}$
6	$-\frac{1}{3}$
$\frac{7}{2}$	$-\frac{3}{1}$
$\frac{5}{2}$	3

$\lim_{x \to +\infty} \frac{5-x}{x-3} = \lim_{x \to +\infty} \frac{\frac{5}{x} - 1}{1 - \frac{3}{x}} = \frac{-1}{1} = -1$

$y = -1$

$\lim_{x \to -\infty} \frac{5-x}{x-3} = \lim_{x \to -\infty} \frac{\frac{5}{x} \cdot 1}{-1 - \frac{3}{x}} = \frac{1}{1} = -1$ ✓ H.A.

$\lim_{x \to 3} \frac{5-x}{x-3} = \frac{5-3}{3-3} = \frac{2}{0} = +\infty$ $\quad x = 3 \to$ V.A ✓

$\frac{5-\frac{7}{2}}{\frac{7}{2}-3} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$

① $\frac{|z|}{2(z+i)} = 3i$

$\frac{|z|}{2z+2i} = 3i \mid \cdot 2z+2i$

$|z| = 6zi - 6$

$\sqrt{x^2+y^2} = 6i\cdot(x+iy) - 6$

$\sqrt{x^2+y^2} = 6ix - 6y - 6$

$\sqrt{x^2+y^2} = 6ix - 6y - 6$

$Re \rightarrow \sqrt{x^2+y^2} + 6y = -6$

$Im \Rightarrow 6x = 0$

$x = 0$

Provjera

$\frac{\sqrt{x^2+y^2}}{2\cdot(x+iy+i)} = 3i$

$\frac{\sqrt{0+\left(-\frac{6}{7}\right)^2}}{2\cdot\left(0+i\cdot\left(-\frac{6}{7}\right)+i\right)}$

$\frac{\frac{6}{7}}{2\cdot\frac{1}{7}i} = 3i$

$\frac{\frac{6}{7}}{\frac{2}{7}i} = 3i$

$3i = 3i$

$|z| = \sqrt{x^2+y^2}$

$z = x+iy$

$\sqrt{0^2+y^2} + 6y = -6$

$y + 6y = -6$

$7y = -6$

$y = -\frac{6}{7}$

② $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$

$x_2 - x_3 + x_4 = 1$

$x_1 + 3x_2 - 3x_4 = 7$

$-7x_2 + 3x_3 + x_4 = -15$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] R3-R1 \approx$$

$$\approx \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} R1+2R2 \\ R3-5R2 \\ R4+7R2 \end{array} \approx$$

$$\approx \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & 34 \end{array}\right] :2 \approx \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & 34 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & 34 \end{array}\right] \begin{array}{l} R1-R3 \\ R2+R3 \\ R4+4R3 \end{array} \approx \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 38 \end{array}\right]$$

NULE U RETKU

PA MATRICA NEMA RJEŠENJE!

(2)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -30 & \frac{85}{8} \\ 0 & 1 & 0 & 11 & \frac{47}{8} \\ 0 & 0 & 1 & -4 & -\frac{24}{64} \\ 0 & 0 & 0 & 15 & -\frac{11}{2} \end{array}\right] :15 = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -30 & \frac{85}{8} \\ 0 & 1 & 0 & -11 & \frac{47}{8} \\ 0 & 0 & 1 & 4 & -\frac{24}{64} \\ 0 & 0 & 0 & 1 & -\frac{11}{30} \end{array}\right] \begin{array}{l} \leftarrow +\\ \leftarrow +\\ \leftarrow +\\ \end{array} \begin{array}{l} \\ \\ \\ \cdot 30 \\ \cdot 11 \\ \cdot 4 \end{array} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{3}{8} \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & & \end{array}\right]$$

(6) $f(x) = \sqrt{2x - x^2}$:

a) koji su lokalni ekstremi ?

b) koji su globalni ekstremi

$$x_{1,2} = \frac{-2 \pm \sqrt{4-0}}{2 \cdot (-1)}$$

$$x_{1,2} = \frac{-2 \pm 2}{-2}$$

$$x_1 = \frac{-2+2}{-2}$$

$$x_1 = \frac{0}{-2}$$

$$x_1 = 0$$

$$x_2 = 2$$

$$D(f) \langle -\infty, 0] \cup [2, +\infty \rangle$$

Globalni ekstrem

funkcija je neomeđena odozgo i odozdo

$$f'(x) = \frac{1}{2\sqrt{2x - x^2}} \cdot (x - 2x)$$

$$f'(x) = \frac{-2x + x}{2\sqrt{2x - x^2}}$$

$$-2x + x = 0$$

$$-x = 0$$

Nema lokalnog ekstrema

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 03

IME I PREZIME: Lovre Kerež **BROJ INDEKSA:** 57933

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

36

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

1. $\frac{|z|}{2(z+i)} = 3i \mid \cdot 2(z+i)$

$|z| = \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = 3i \cdot 2(x+yi+i)$

$\sqrt{x^2+y^2} = 3i \cdot (2x+2yi+2i)$

$\sqrt{x^2+y^2} = 6xi + 6yi - 6$

$x+y = 6xi \cdot 6y_i - 6$

$x+y+6 = 6xi + 6y$

2. $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$

$x_2 - x_3 + x_4 = 1$

$x_1 + 3x_2 - 3x_4 = 7$

$-7x_2 + 3x_3 + x_4 = -15$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] R3-R1 \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} R1+2R2 \\ \\ R3-5R2 \\ R4+7R2 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] ;2R3 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \begin{array}{l} R1-R3 \\ R2+R3 \\ \\ R4+4R3 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -4 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right]$$

$x_1\ x_2\ x_3\ x_4$

nema rješenja!

a

3. $g(x) = \sqrt{x^2 - 5x + 1} = x$

• Domena $\sqrt{\ } \geq 0$

$x^2 - 5x + 1 = 0$

$$x_{1,2} = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{5 \pm \sqrt{25-4}}{2} = \frac{5 \pm \sqrt{21}}{2}$$

$$x_1 = \frac{5+\sqrt{21}}{2}$$

$$x_2 = \frac{5-\sqrt{21}}{2}$$

$$D_{(g)} = \left\langle -\infty, \frac{5-\sqrt{21}}{2} \right] \cup \left[\frac{5-\sqrt{21}}{2}, \frac{5+\sqrt{21}}{2} \right] \cup \left[\frac{5+\sqrt{21}}{2}, +\infty \right\rangle$$

H.A V.A V.A H.A

• Asimptote

$$\lim_{x \to \frac{5-\sqrt{21}}{2}} \sqrt{x^2 - 5x + 1} - 1 =$$

$$\lim_{x \to \frac{5-\sqrt{21}}{2}} \sqrt{x^2 - 5x + 1} - 1 =$$

4. $f(x) = \frac{5-x}{x-3}$

$D(f) = \mathbb{R} \setminus \{3\}$

- $x - 3 = 0$

  $x = 3$

- Ničle

$5 - x = 0$

$-x = -5 \quad (\cdot(-1))$

$\boxed{x = 5}$

- Asimptote

$\lim_{x \to 3^+} \frac{5-x}{x-3} = +\infty$

$\lim_{x \to 3^-} \frac{5-x}{x-3} = -\infty$

} V.A. $x = 3$ ✓

$\lim_{x \to \pm\infty} \frac{5-x}{x-3} \; / : x = \frac{\frac{5}{x} - \frac{x}{x}}{\frac{x}{x} - \frac{3}{x}} = \frac{-1}{1} = -1$

($5/x \to 0$, $x/x \to 1$, $x/x \to 1$, $3/x \to 0$)

H.A. $y = -1$ ✓

$\lim_{x \to -\infty} \frac{5-x}{x-3} = |x \to -x| = \lim_{x \to +\infty} \frac{5+x}{-x-3} \; / : x = \frac{\frac{5}{x} + \frac{x}{x}}{-\frac{x}{x} - \frac{3}{x}} = \frac{1}{-1} = -1$

($5/x \to 0$, $x/x \to 1$, $x/x \to 1$, $3/x \to 0$)

- Ničle odvoda (vast. pad)

$f(x) = \frac{5-x}{x-3}$

$f'(x) = \frac{(5-x)' \cdot (x-3) - (5-x)(x-3)'}{(x-3)^2} = \frac{1 \cdot (x-3) - (5-x) \cdot 1}{(x-3)^2} = \frac{x-3-5+x}{(x-3)^2}$

$= \frac{-8}{(x-3)^2}$

$x^2 - 6x + 9 = 0$

$x_{1,2} = \frac{6 \pm \sqrt{36-36}}{2} = \frac{6}{2} = 3$

- Zaključek ničel

$f'(x) = \frac{-8}{(x-3)^2} \neq 0$

$f''(x) = \frac{(-8)' \cdot (x^2-6x+9) - (-8) \cdot (x^2-6x+9)'}{((x-3)^2)^2}$

$f''(x) = \frac{8 \cdot (2x-6)}{(x-3)^4} = \frac{16x-48}{(x-3)^4}$

$16x - 48 = 0$

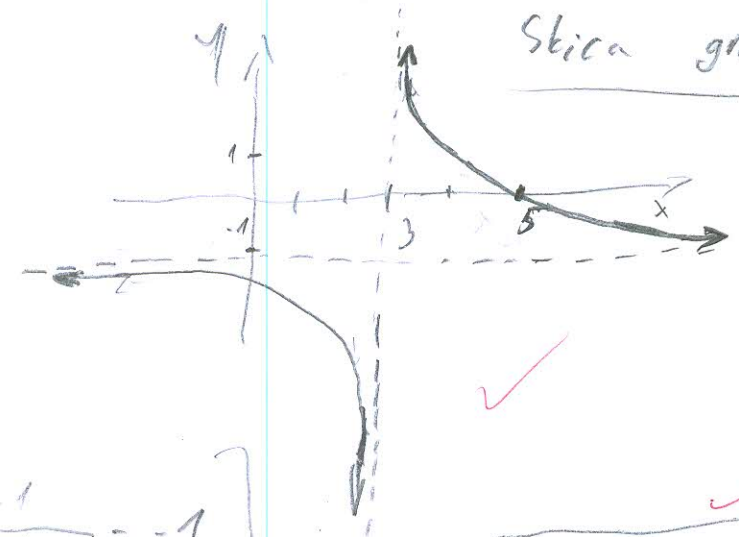
$16x = 48$

$x = \frac{48}{16} = 3$

	$-\infty$	2	3	4	$+\infty$
$f'(x)$					
$f(x)$		⌒		⌣	

	$-\infty$	3	$+\infty$
$f'(x)$	−	+	
	↘	→	

Skica grafa !!!



5. $h(x) = \cos(4x+1)$

- Domena $D_{(h)} = \mathbb{R}$ ✓

- Period – Funkcija je periodična jer ima v sebi trigonometrijskih radnji

- Parost

Funkcija je parna!!! ✓

- Derivacija

$h(x) = \cos(4x+1)$

$h'(x) = -\sin \cdot (4x+1)' = -\sin \cdot 4$

$h''(x) = -\cos(4)' = -\cos 0$

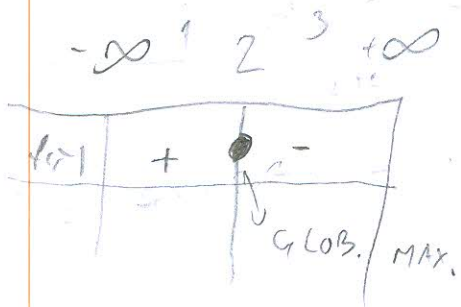
6. $f(x) = \sqrt{2x - x^2}$ $D_{(f)} = \mathbb{R} \setminus \{2\}$

$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0 \quad 2-x = 0$

$x = 2$



$f'(x) = \frac{1}{2x - x^2} \cdot (2x - x^2)'$

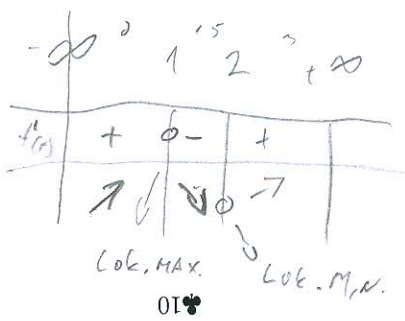
$f'(x) = \frac{2 - 2x}{2x - x^2}$

Nul točke

$2 - 2x = 0$

$2x = 2$

$\boxed{x = 1}$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 03

IME I PREZIME: Ivan Rubelj **BROJ INDEKSA:** 17-2-0085-2017

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

36

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

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   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

(2.)

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \sim$$

$1R \times (-1) + 3R$

$2R \times 2 + 1R$
$2R \times (-5) + 3R$
$2R \times 7 + 4R$

$3R : 2$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right] \Rightarrow 0+0+0+0 \neq -4$$

$3R \times (-1) + 1R$
$3R \times 1 + 2R$
$3R \times 4 + 4R$

Sustav nema rješenja.

PROVJERA:

$a+b+c+d=c$
$1+0+0+0=1$ ✓
$0+1+0-1 \neq 2$ —
$0+0+1-2 \neq 1$ —
$0+0+0+0 \neq -4$ —

$a+b+c+d=0$
$1+2+1-4=0$
$0=0$ ✓

5.) $h(x) = \cos(4x+1)$

① DOMENA

$D(h) = \mathbb{R}$ ✓ - domena su svi realni brojevi

② Funkcija nije periodična jer nije trigonometrijska.

③

a) $h(-x) = \cos(4x+1)$

$\cos(4\cdot(-x)+1) = \cos(4x+1)$

$\cos(-4x+1) = \cos(4x+1)$ $\Rightarrow$ Funkcija nije parna.

b) $-h(x) = \cos(4x+1)$

$-\cos(4x+1) = \cos(4x+1)$ $\Rightarrow$ Funkcija nije neparna.

ZAKLJUČAK: Funkcija nije niti parna niti neparna.

④ $h(x) = \cos(4x+1)$

$h'(x) = -\sin(4x+1)\cdot(4x+1)'$

$h'(x) = -\sin(4x+1)\cdot 4$ $= -4\sin(4x+1)$ //

$h''(x) = \left[-4\sin(4x+1)\right]'$

$h''(x) = (-4)'\cdot\sin(4x+1) + (-4)\cdot\left[\sin(4x+1)\right]'$

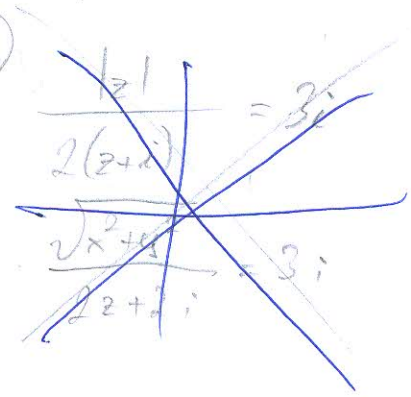
$h''(x) = -4\cdot\left[\cos(4x+1)\cdot(4x+1)'\right]$

$h''(x) = -4\cdot\cos(4x+1)\cdot 4$

$h''(x) = -16\cos(4x+1)$ ✓ //

Ivan Rubeli NAUTIKA

4.



4.

1. DOMENA

$$f(x)=\frac{5-x}{x-3}$$

$$x-3\neq 0$$

$$x\neq 3$$

$$D(f)=\mathbb{R}\setminus\{3\}$$

2. NULTOČKE

$$5-x=0$$

$$-x=-5$$

$$x=5$$

$$T(5,0)$$

3. PERIODIČNOST

Funkcija nije periodična jer nije trigonometrijska.

4. PARNOST / NEPARNOST

$$f(x)=\frac{5-x}{x-3}$$

$$f(-x)=\frac{5+x}{-x-3} \Rightarrow \text{FUNKCIJA NIJE PARNA}$$

$$-f(x)=-\frac{5-x}{x-3}$$

$$-f(x)=\frac{-5+x}{x+3} \Rightarrow \text{FUNKCIJA NIJE NEPARNA}$$

5. GLOBALNI, SVOJSTVA, TJ. EKSTREMI

$$f'(x)=\frac{5-x}{x-3}$$

$$f'(x)=\frac{(5-x)'\cdot(x-3)-(5-x)\cdot(x-3)'}{(x-3)^2}$$

$$f'(x)=\frac{-x+3-(5-x)\cdot 1}{(x-3)^2}$$

$$f'(x)=\frac{-x+3-5+x}{(x-3)^2}$$

$$f'(x)=\frac{-2}{(x-3)^2}$$

$$-2\neq 0$$

Nemaju ekstremnih točaka.

~~(5.) Ispitati domenu i vrste asimptota funkcije $g(x) = \sqrt{x^2-5x+1} - x$~~

(6.) $f(x) = \sqrt{2x-x^2}$

(1.) DOMENA

$\sqrt{2x-x^2} \geq 0$

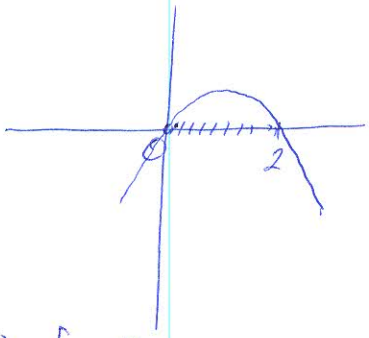
$2x-x^2 = 0$

$x(2-x) = 0$

$x = 0 \quad 2-x = 0$

$-x = -2$

$x = 2$



$D(f) = [0, 2]$

(2.) NULTOCKE

$\sqrt{2x-x^2} = 0$

$2x-x^2 = 0$

$x(2-x) = 0$

$x = 0 \quad 2-x = 0$

$-x = -2$

$x = 2$

$T_1(0,0)$

$T_2(2,0)$

(3.) $f'(x) = (\sqrt{2x-x^2})'$

$f'(x) = \frac{1}{2\sqrt{2x-x^2}} \cdot (2x-x^2)'$

$f'(x) = \frac{1}{2\sqrt{2x-x^2}} \cdot (2-2x)$

$f'(x) = \frac{2-2x}{2\sqrt{2x-x^2}} = \frac{2(1-x)}{2\sqrt{2x-x^2}} = \frac{1-x}{\sqrt{2x-x^2}}$

$1-x = 0$

$-x = -1$

$x = 1$

$\in (1,0) \Rightarrow$ LOKALNI EKSTREM

$f''(x) = \left(\frac{1-x}{\sqrt{2x-x^2}}\right)'$

$f''(x) = \frac{(1-x)' \cdot \sqrt{2x-x^2} - (1-x) \cdot (\sqrt{2x-x^2})'}{(\sqrt{2x-x^2})^2}$

$f''(x) = \frac{-1\sqrt{2x-x^2} - (1-x) \cdot \frac{1}{2\sqrt{2x-x^2}} \cdot (2x-x^2)'}{2x-x^2}$

$f''(x) = \frac{-\sqrt{2x-x^2} - (1-x) \cdot \frac{1}{2\sqrt{2x-x^2}} \cdot (2-2x)}{x(2-x)}$

$f''(x) = \frac{-\sqrt{2x-x^2} - \cdots}{x(2-x)}$ ...

Ivan Ruželj NAUTIKA

♣1

4. NASTAVAK: ASIMPTOTE.

V.A.

$\lim_{x \to 3} \frac{5-x}{x-3} = \frac{5-3}{3-3} = \frac{2}{0} = \infty$ V.A $x=3$

H.A.

$\lim_{x \to \infty} \frac{5-x}{x-3} \,/:x = \frac{\frac{5}{x} - \frac{x}{x}}{\frac{x}{x} - \frac{3}{x}} = \frac{-1}{1} = -1$

$\lim_{x \to -\infty} \frac{5-x}{x-3} = \frac{5+x}{-x-3} = \frac{x}{-x} = \frac{1}{-1} = -1$ NEMA H.A.

KOSA A.

$y = kx + l$

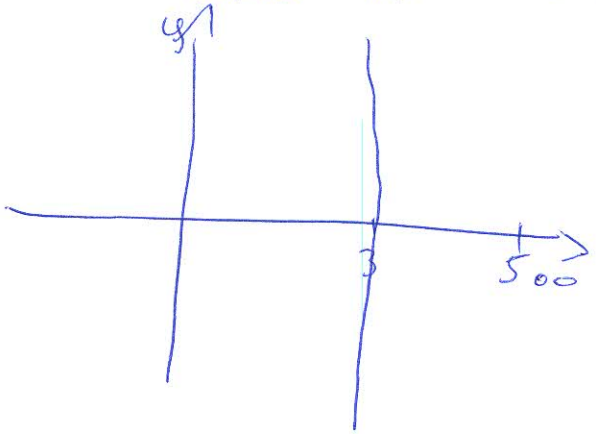
$k = \frac{f(x)}{x} = \frac{\frac{5-x}{x-3}}{\frac{x}{1}} = \frac{5-x}{x^2-3x} \,/:x^2 = \frac{\frac{5}{x^2} - \frac{x}{x^2}}{1-0} = 0$ $k=0$

$l = f(x) - kx = \frac{5-x}{x-3} \,/:x = \frac{-1}{1} = -1$ $y=-1$ kosa

$f(x)' = \frac{5-x}{x-3} = \frac{-1(x-3)-(5-x)\cdot 1}{(x-3)^2} = \frac{-x+3-5+x}{(x-3)^2} = \frac{-2}{(x-3)^2}$ Nema kosih

STACIONARNE TOČKE:

$-2 \neq 0$ nema stacionarnih točaka



	$y=-1$	0	
x		1	2

Ivan Rubelj

NAUTILA

♣10

1.

$$\frac{|z|}{2(z+i)} = 3i$$

$z = x + yi$

$\bar{z} = x - yi$

$|z| = \sqrt{x^2+y^2}$

$$\frac{\sqrt{x^2+y^2}}{2\cdot(x+yi+i)} = \frac{3i}{1}$$

$$\frac{\sqrt{x^2+y^2}}{2x+2yi+2i} \times \frac{3i}{1}$$

$$\sqrt{x^2+y^2} = (3i)\cdot(2x+2yi+2i)$$

$$\sqrt{x^2+y^2} = 6xi+6yi^2+6i^2$$

$$\sqrt{x^2+y^2} = 6xi-6y-6$$

$$\sqrt{x^2+y^2} = 6(xi-y-1) \quad /^2$$

$$x^2+y^2 = 36\cdot\left[(xi-y)^2-1^2\right]$$

$$x^2+y^2 = 36\cdot\left[x^2i^2-2\cdot xi\cdot y+y^2-1\right]$$

$$x^2+y^2 = 36\cdot\left[-x^2-2xyi+y^2-1\right]$$

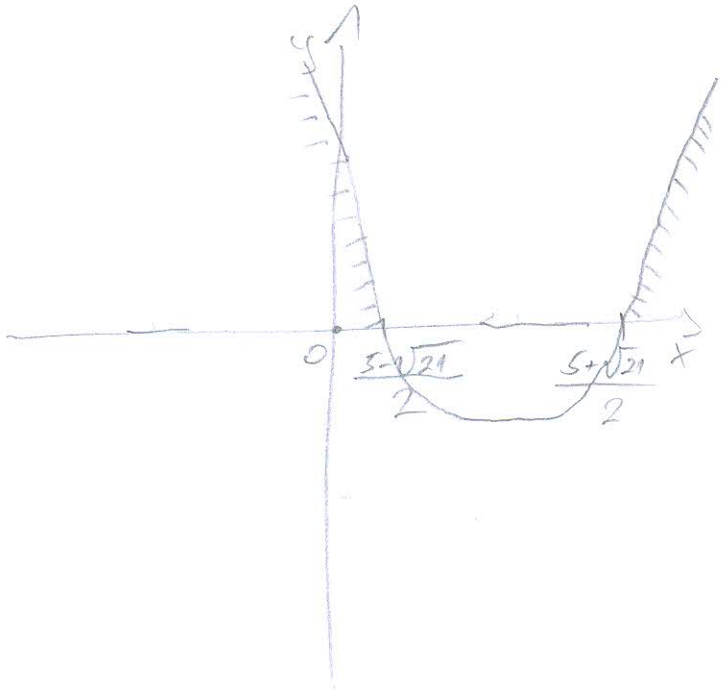
$$x^2+y^2 = -36x^2-72xyi+36y^2-36$$

$$x^2+y^2-36x^2+36y^2 = -72xyi-36$$

$$-35x^2+37y^2 = -72xyi-36$$

3.

$$y(x) = \sqrt{x^2 - 5x + 1} - x$$



① DOMENA

$x^2 - 5x + 1 \geq 0$

$x^2 - 5x + 1 = 0$

$a = 1, b = -5, c = 1$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 1}}{2}$

$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$

$x_1 = \frac{5 + \sqrt{21}}{2} \qquad x_2 = \frac{5 - \sqrt{21}}{2}$

$$D(g) = \mathbb{R} \setminus \left\{ \frac{5-\sqrt{21}}{2}, \frac{5+\sqrt{21}}{2} \right\}$$

$$(g) = \left\langle -\infty, \frac{5-\sqrt{21}}{2} \right] \cup \left[\frac{5+\sqrt{21}}{2}, +\infty \right\rangle$$ ✓

① VERTIKALNA ASIMPTOTA

$$\lim_{x \to \frac{5-\sqrt{21}}{2}} \sqrt{x^2 - 5x + 1} - x = \lim_{x \to \frac{5-\sqrt{21}}{2}} \sqrt{\left(\frac{5-\sqrt{21}}{2}\right)^2 - 5 \cdot \left(\frac{5-\sqrt{21}}{2}\right) + 1} - \frac{5-\sqrt{21}}{2} =$$

$$= \lim_{x \to \frac{5-\sqrt{21}}{2}} \sqrt{\frac{25 - 10\sqrt{21} + 21}{4} - \frac{25 - 5\sqrt{21}}{2} + 1} - \frac{5-\sqrt{21}}{2}$$ ✗ – nemo rješenja

NEMA VERTIKALNIH ASIMPTOTA. ✓

② HORIZONTALNA ASIMPTOTA

$$\lim_{x \to \infty} \sqrt{x^2 - 5x + 1} - x =$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ O3

IME I PREZIME: MAURO MIŠLOV **BROJ INDEKSA:** 0269065450

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

31

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = 0 \\ & & x_2 & - & x_3 & + & x_4 = 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

3) VERTIKALNE ASIMPTOTE NEMA

HORIZONTALNA ASIMPTOTA

$$\lim_{x\to\infty} g(x) \lim_{x\to\infty} \left(\sqrt{x^2-5x+1} - x\right) =$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2-5x+1} - x\right)\left(\sqrt{x^2-5x+1} + x\right)}{\sqrt{x^2-5x+1} + x} = \lim_{x\to\infty} \frac{x^2-5x+1-x^2}{\sqrt{x^2-5x+1} + x}$$

$$= \lim_{x\to\infty} \frac{-5x+1}{\sqrt{x^2-5x-1} + x} : x = \frac{-5}{2}$$ D. H. A $y = \frac{-5}{2}$ //

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} g(-x) = \lim_{x\to\infty} \left(\sqrt{x^2+5x+1} + x\right) = +\infty$$ //

KOSE: $$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2-5x+1} - x}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2+5x+1} + x}{x} =$$

$$= \lim_{x\to+\infty} \frac{\sqrt{x^2+5x+1} : x}{x : x} + 1 = 2 \text{ //} \rightarrow k = 2$$

$$l = \lim_{x\to-\infty} \left(f(x) - kx\right) = \lim_{x\to-\infty} \left(\sqrt{x^2-5x+1} - x - 2x\right) = \lim_{x\to-\infty} \left(\sqrt{x^2-5x+1} - 3x\right)$$

$$= \lim_{x\to+\infty} \left(\sqrt{x^2+5x+1} + 3x\right) = +\infty \rightarrow$$ NEMA LIJEVU KOSU ASIMPTOTU

$$= \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2-5x+1} - x}{x} = 1 - 1 = 0$$ NEMA DESNU KOSU ASIMPTOTU

ASIMPTOTE: DESNA HORIZONTALNA ASIMPTOTA $y = \frac{-5}{2}$

5) $h(x) = \cos(4x+1)$

a) $D(h(x)) = \mathbb{R}$ ✓

b) $T = ?$

$w = 4x \quad w = \frac{2\pi}{T}$

$T = \frac{1}{2}\pi$ ✓

c) PARNOST ?

PARNA: $f(-x) = f(x)$

NEPARNA $f(-x) = -f(x)$

$h(-x) = \cos(-4x+1)$ -> NIJE NI PARNA NI NEPARNA

d) $\frac{d^2}{dx^2} h = ?$ $\quad h'(x) = -\sin(-4x+1)\cdot 4 = -4\sin(4x+1) =$

$h''(x) = -4 \cdot \cos(4x+1)\cdot 4 = -16\cos(4x+1)$ ✓

6) $f(x) = \sqrt{2x-x^2} \qquad 2x - x^2 = 0 \quad x_1 = 0 \quad x_2 = 2$

$D(f(x)) = [0, 2]$

$f'(x) = \frac{1}{2\sqrt{2x-x^2}}(-2x+2) = \frac{1-x}{\sqrt{2x-x^2}}$

$f'(x) = 0 \rightarrow \frac{1-x}{\sqrt{2x-x^2}} = 0 \rightarrow x = 1$ -> STACIONARNA TOČKA

$f''(x) = \frac{-1\sqrt{2x-x^2} + (1-x)\cdot\frac{1-x}{\sqrt{2x-x^2}}}{(\sqrt{2x-x^2})^2} = \frac{\sqrt{2x-x^2} + \frac{(1-x)^2}{\sqrt{2x-x^2}}}{2x-x^2}$

$\frac{2x - x^2 + 1 - 2x + x^2}{(2x-x^2)^{\frac{3}{2}}} \Rightarrow f''(x) = \frac{1}{(2x-x^2)^{\frac{3}{2}}}$

$f'''(x) = \frac{-3(x-1)\sqrt{2x-x^2}}{(2x-x^2)^3} = \frac{3-3x}{(2x-x^2)^{\frac{5}{2}}}$

$f'''(x) = 0 \rightarrow \frac{3-3x}{(2x-x^2)^{\frac{5}{2}}} = 0 \rightarrow x = 1$

TOČKA NIJE EKSTREMA JER JE DERIVACIJA NEPARNOG REDA

a) FUNKCIJA NEMA LOKALNI MAKSIMUM

b) FUNKCIJA NEMA GLOBALNI MAKSIMUM

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 03

IME I PREZIME: FRANE MAHAOIĆ **BROJ INDEKSA:** 17-1-0077-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova
25

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9 ~~/5~~ −cos x

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

2) $\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \cdot(-1) \downarrow + \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \cdot(-5) \cdot 7 \downarrow + \downarrow +$

$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] :2 \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \cdot 4 \cdot 2 \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \cdot 4 \downarrow +$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right] \leftarrow + \leftarrow + \cdot(1) \cdot(-1) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right]$ ✓

SUSTAV NEMA RJEŠENJA.

3) $g(x) = \sqrt{x^2 - 5x + 1} - x$

1° domena funkcije

$x^2 - 5x + 1 \geq 0$

$x^2 - 5x + 1 = 0$

$a = 1$

$b = -5$

$c = 1$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$$

$$x_{1,2} = \frac{5 \pm 4{,}6}{2}$$

$x_1 = 4{,}8$

$x_2 = 0.2$

$$D_f = \langle -\infty, 0{,}2] \cup [4{,}8, +\infty \rangle$$

5) $h(x) = \cos(4x+1)$

$h'(x) = -\sin(4x+1) \cdot (4x+1)'$

$h'(x) = -\sin(4x+1) \cdot 4 + 0$

$h'(x) = -\sin(4x+1) \cdot 4 = -4\sin(4x+1)$

$h''(x) = (-4\sin(4x+1))'$

$h''(x) = -4 \cdot \cos(4x+1) \cdot (4x+1)'$

$h''(x) = -4\cos(4x+1) \cdot 4$

$h''(x) = -8\cos(4x+1)$

$\frac{a}{b} \Rightarrow b \neq 0$

$\sqrt{b} \Rightarrow b \geq 0$

4) $f(x) = \frac{5-x}{x-3}$

1° domena

$x - 3 \neq 0$

$x \neq 3$

$Df = \mathbb{R} \setminus \{3\}$

2° HORIZONTALNA ASIMPTOTA

$\lim_{x \to 3} \frac{5-x}{x-3} = \lim_{x \to 3} \frac{5-3}{3-3} = \frac{-2}{0} = \infty$

VERTIKALNA ASIMPTOTA

$\lim_{x \to \infty} \frac{5-x \,/:x}{x-3 \,/:x} \Rightarrow \lim_{x \to \infty} \frac{\frac{5}{x} - \frac{x}{x}}{\frac{x}{x} - \frac{3}{x}} = \frac{-1}{1}$

KOSE ASIMPTOTE NEMA.

6) $f(x) = \sqrt{2x - x^2}$

1° domena

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & 1 & -2 & 2 \\ & & & 1 & \end{array}\right] \begin{array}{l} \\ \\ \\ :1.2 \\ :1.4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ & & & & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot(-1) \\ \end{array}$$

$$\left[\begin{array}{cccc|c} & & & & \\ & & & & \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{cc} 1 & 0 \\ 0 & \\ 0 & \\ 0 & \end{array}\right]$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ Ø3

IME I PREZIME: MARTIN ŠOŠA **BROJ INDEKSA:** 17-1-0097-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova
15

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right]$$

$R1 \cdot (-1) + R3$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] :2$$

$R2 \cdot 2 + R1$
$R2 \cdot (-5) + R3$
$R2 \cdot 7 + R4$

→

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -4 & 8 & -8 \\ 0 & 0 & 1 & -2 & 1 \end{array}\right] \sim$$

$(:(-4))$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

$R3 \cdot (-1) + R1$

$R3 + R2$

$R3 \cdot (-1) + R4$

(2)

♣A

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right]$$

$R3 \cdot (1) + R1$
$R3 + R2$
$R3 \cdot 4 + R4$

$x_1 = 1$

$x_2 - x_4 = 2$

$x_3 - 2x_4 = 1 \Rightarrow x_3 = 2x_4 + 1$

$x_2 = 2 + x_4$

$1 - 2x_2 + 3x_3 - 4x_4 = 0$

$-2x_2 + 3x_3 - 4x_4 = -1 / \cdot (-1)$

$2x_2 - 3x_3 + 4x_4 = 1$

$2(2 + x_4) - 3x_3 + 4x_4 = 1$

$4 + 2x_4 - 3x_3 + 4x_4 = 1$

$6x_4 - 3x_3 = 1 - 4$

$6x_4 - 3(2x_4 + 1) = -3$

$6x_4 - 6x_4 - 3 = -3$

$1 + 3x_2 - 3x_4 = 7$

$3x_2 - 3x_4 = 6$

$3 \cdot (2 + x_4) - 3x_4 = 6$

$6 + 3x_4 - 3x_4 = 6$

Sustav nema rješenja

(5) $h(x) = \cos(4x+1)$

$h'(x) = -\sin(4x+1) \cdot$

$h(x) = -4\sin(4x+1)$

$h''(x) = -4\cos(4x+1) \cdot (4x+1)'$

$= -4\cos(4x+1) \cdot 4$

$h''(x) = -8\cos(4x+1)$

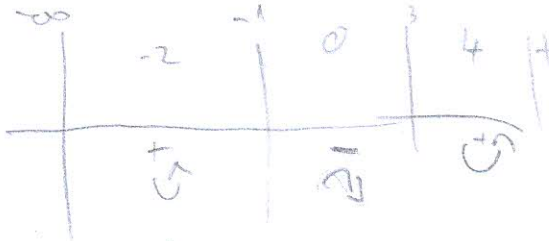
$f''(x) = \frac{-2}{(x-3)^2} \Rightarrow \frac{(-2)' \cdot (x-3)^2 - (-2) \cdot ((x-3)^2)'}{((x-3)^2)^2}$

$\Rightarrow \frac{1+2x \cdot 2 \cancel{x-3}}{(x-3)^4} = \frac{x+1}{(x-3)^3}$

$\Rightarrow D_f = [0,2 ; 48] \cup [48, +\infty)$

$x+1=0$

$x=-1$



(3) $\frac{5-x}{x-3}$

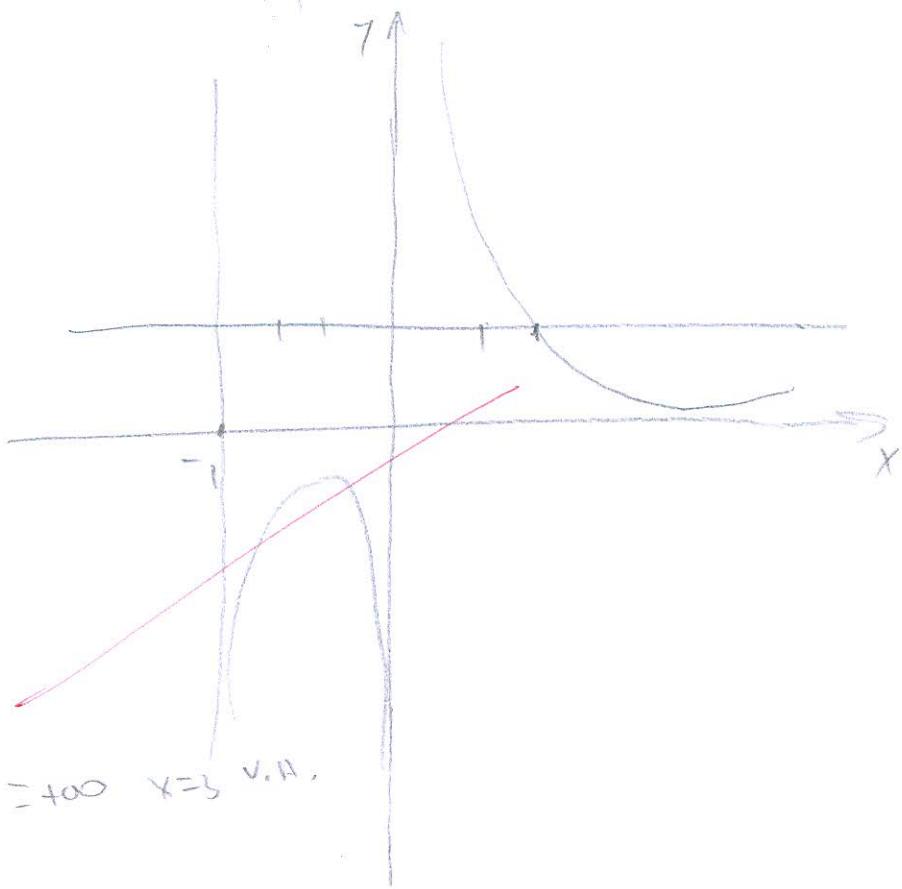
$x=3 \quad D_f = \mathbb{R} \setminus \{3\}$

Max locu

$5-x=0$

$x=-5$

$x=5$



Asimptote

V.A. $\lim_{x \to 3} \frac{5-x}{x-3} = \frac{-3-3}{3-3} = \frac{-6}{0} = +\infty$ $x=3$ V.A.

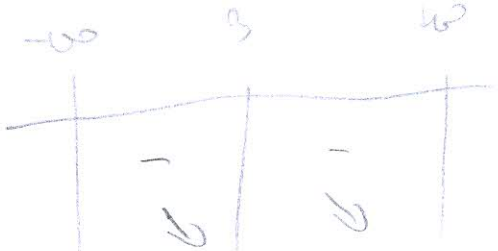
H.A.

$\lim_{x \to +\infty} \frac{5-x}{x-3} = \frac{\frac{5}{x} - \frac{x}{x}}{\frac{x}{x} - \frac{3}{x}} = \frac{-1}{1} = -1$ H.A. $y=-1$

$(5-x)' \cdot (x-3) - (5-x) \cdot (x-3)'$

$f'(x) = \frac{5-x}{x-3} =)$

$f'(x) = \frac{-2}{(x-5)^2} = -2 = 0$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ 03

IME I PREZIME: DOMENIKO MIOČEV **BROJ INDEKSA:** 0269071897

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

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6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

2) a)

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \overline{III} - \overline{I} \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & 3 & 1 & -7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} \overline{I} + 2\overline{II} \\ \overline{III} - 5\overline{II} \\ \overline{IV} + 7\overline{II} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 8 & -4 & -2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \overline{III} + \overline{IV} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 4 & -10 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \begin{array}{l} \sim \\ 1:4 \\ 1:-4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & -\frac{10}{4} \\ 0 & 0 & 1 & -2 & 2 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{10}{4} \end{array}\right] \begin{array}{l} \overline{I} - \overline{III} \\ \overline{II} + \overline{III} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -4 & 4 \\ 0 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{10}{4} \end{array}\right] \begin{array}{l} \overline{I} + 4\overline{IV} \\ \overline{II} - 3\overline{IV} \\ \overline{III} - 2\overline{IV} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 & -\frac{13}{2} \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -\frac{10}{4} \end{array}\right] \quad X = \begin{bmatrix} -6 \\ -\frac{13}{2} \\ 3 \\ -\frac{10}{4} \end{bmatrix}$$

?

e

$$-6 - 2 \cdot \left(-\frac{13}{2}\right) + 3 \cdot 3 - 4 \cdot \frac{-10}{4}$$

$$= -6 + \frac{9}{2} + 9 + \frac{8}{4}$$

$$= \frac{-12 + 9 + 18 + 3}{2} = \frac{18}{2} = 9$$

$$-2 - 2 \cdot \frac{-10}{4}$$

$$-2 + \frac{20}{4} = \frac{-8+20}{4} = 3$$

$$-1 - 3 \cdot \frac{-10}{4} \quad -1 + \frac{30}{4} = \frac{-4+30}{4} = \frac{26}{4} = \frac{13}{2}$$

$$4 + 4 \cdot \frac{-10}{4} = 4 - \frac{40}{4} = 4 - 10 = -6$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ 03

IME I PREZIME: Marko Mustać **BROJ INDEKSA:** 17-2-0135-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

0

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

2.)

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] /\cdot(-1) \quad \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 5 & -3 & -12 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] /\cdot 2 \; /\cdot(-5) \; /\cdot 7 \quad \left[\begin{array}{cccc|c} 1 & 0 & 5 & -8 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -8 & -17 & -12 \\ 0 & 0 & 10 & 8 & -8 \end{array}\right]$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ 03

IME I PREZIME: LUKA KNEŽEVIĆ **BROJ INDEKSA:** 0269061313

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

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$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

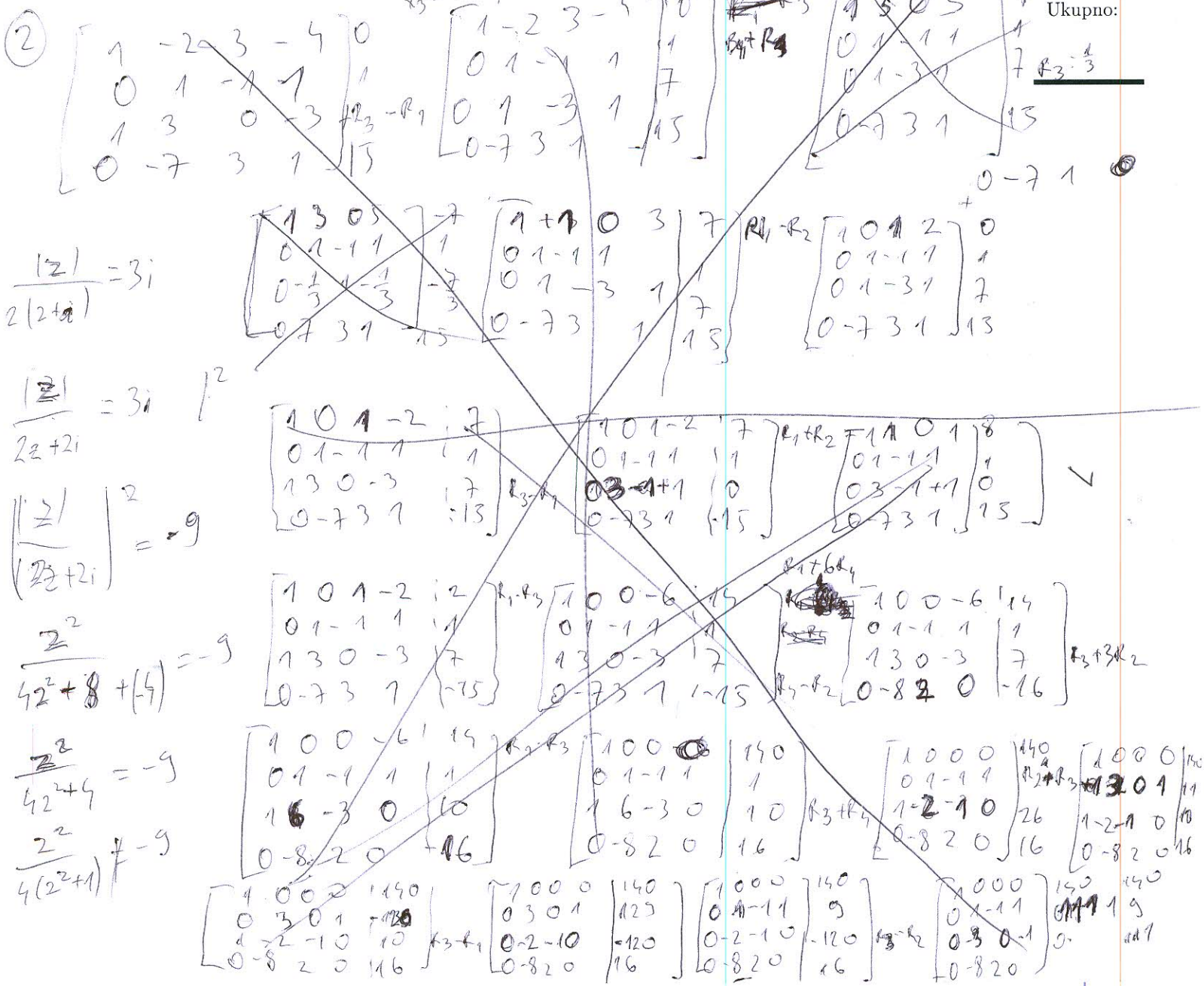
5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$: 2 $x_1 - 5x_2 + 5x_3 - 5x_4 = -7$

   (a) koji su lokalni ekstremi? $x_1 = 2$ 10

   (b) koji su globalni ekstremi? $x_2 = 5$ $x_3 = 1$ $x_4 = 0$ 5

Ukupno:



(6) $f(x) = \sqrt{2x - x^2}$

$\sqrt{2x - x^2} = 0$

$2x^{\frac{1}{2}} - x^{\frac{2}{2}} = 0$

$2x^{\frac{1}{2}} - x^1 = 0$

$x_1 = 2$ $\quad x_2 = -2$

$\lim_{x \to +\infty} \sqrt{2x - x^2} = \lim 2x^{\frac{1}{2}} - x^{\frac{2}{2}} = 2x^{\frac{1}{2}\cdot\frac{1}{1}} - x^1 = 2x^{\frac{1}{2}} - x^1 = 2x - 1 = 1$

~~$D_f \langle -2, 2 \rangle$~~ $D_f \langle 2, -2 \rangle$

(5) $k(x) = \cos(4x+1)$

$D_f \langle -\infty, +\frac{1}{4} \rangle$

$\cos(1+4x+1) = \cos(4x+1)$

$\cos(4x+2) \neq \cos(4x+1)$

NIJE PERIODIČNO

$-f(x) = \cos(-4x+1) = \cos(4x+1)$

~~SAMO PARNE~~ NIJE NITI PARNA NITI NEPARNA

$f(x)' = \cos(4x+1)$

$f(x) = -\sin x(4x+1) + \cos(4) \cdot (4)$

$= -\sin x(4x+1) + \cos x(16)$

$f(x)'' = -\cos x(4x+1) - \cos 4(-\sin x \, 16) + \sin x * 16$

$= -\cos x4 - 2\sin x16$

(3) $\sqrt{x^2 - 5x + 1} - x$ $\qquad \sqrt{\frac{1}{25} - 1 + 1} - \frac{1}{5}$

$x^2 - 5x + 1 - x = 0$

$x - \sqrt{5x+1} - x = 0$

$x^2 - 6x + 1 = 0$

$-\sqrt{5x+1} = 0$

$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \mp \sqrt{36 - 4}}{2}$

$x = \frac{6 \mp \sqrt{32}}{2}$

$x_1 = \frac{6 \mp 2\sqrt{8}}{2} = \frac{6 - 2\sqrt{8}}{2} \approx 4.58$

$\frac{6+2\sqrt{8}}{2} = 7.41$

$x_2 =$

$x = \frac{1}{5}$

$D_f \langle \frac{1}{5} \rangle$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 3 & -1 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \xrightarrow{R_1+2R_2} \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 3 & -1 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \xrightarrow{R_3-R_1} \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -1 & -6 & 14 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \xrightarrow{R_1+R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -8 & 16 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -1 & -6 & 14 \\ 0 & -7 & 3 & 1 & 15 \end{array}\right] \xrightarrow{R_2-R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -8 & 16 \\ 0 & 1 & 0 & 7 & -13 \\ 0 & 0 & -1 & -6 & 14 \\ 0 & -7 & 3 & 1 & 15 \end{array}\right] \xrightarrow{R_1-R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -8 & 16 \\ 0 & 1 & 0 & 7 & -13 \\ 0 & 0 & -1 & -6 & 14 \\ 0 & 0 & 0 & -8 & 240 \end{array}\right] \xrightarrow{R_1-R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -224 \\ 0 & 1 & 0 & 7 & -13 \\ 0 & 0 & -1 & -6 & 14 \\ 0 & 0 & 0 & -8 & 240 \end{array}\right]$$

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & -8 \end{array}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ ∅3

IME I PREZIME: IVAN BAGIĆ **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

0

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

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6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

2. $A = \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right]$ $\begin{array}{l} I\,R + 2\cdot II\,R \\ \\ III\,R + (-1)\cdot I\,R \\ III\,R + 7\cdot II\,R \end{array}$ $= \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right]$ $III\,R + (-5)\cdot II\,R$ $1:4$

$A = 0$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -1 & 2 & -2 \end{array}\right]$ $1:2$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 2 & -2 \end{array}\right]$ $\begin{array}{l} I\,R + II\,R \\ \\ \\ III\,R + IV \end{array}$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$ $I\,R + (-1)\cdot III$ $= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & A \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$

NEMA RJEŠENJA !!

PROVJERA: $2 - 2(3) + 3 - 4(-1) = 0$

$2 - 6 + 3 + 4 = 0$

$3 \neq 0$

(5) $h(x) = \cos(4x+1)$

$Df: \in \mathbb{R} \setminus \{-\frac{1}{4}\}$

$4x + 1 = 0$

$4x = -1$

$x = -\frac{1}{4}$

$h'(x) = -\sin(4x+1) \cdot (4x+1)'$

$h'(x) = -\sin(4x+1) \cdot 4$

$h''(x) = [\text{illegible}] \; (-\sin(4x+1))' \cdot 4 + [-\sin(4x+1)] \cdot (4)'$

$h''(x) = ($

(4) $f(x) = \frac{5-x}{x-3}$

$x - 3 \neq 0$

$x \neq 3$

$Df: \in \mathbb{R} \setminus \{3\}$ ✓

NULTOČKE

$5 - x = 0$

$-x = -5$

$x = 5$

$T(5, 0)$

IVAN BABIĆ
NAUTIKA

3. $g(x) = \sqrt{x^2 - 5x + 1} - x$

$x^2 - 5x + 1 \geq 0 \qquad -x \neq 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4}}{2} = \frac{5 \pm \sqrt{21}}{2}$$

$$x_1 = \frac{5 + \sqrt{21}}{2}$$

$$x_2 = \frac{5 - \sqrt{21}}{2}$$

$$D_f : \in \mathbb{R} \setminus \left\{ \frac{5 + \sqrt{21}}{2}, \frac{5 - \sqrt{21}}{2} \right\}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ 03

IME I PREZIME: Jelena Maleš **BROJ INDEKSA:** 17-2-0103-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

(3) $g(x) = \sqrt{x^2 - 5x + 1}$

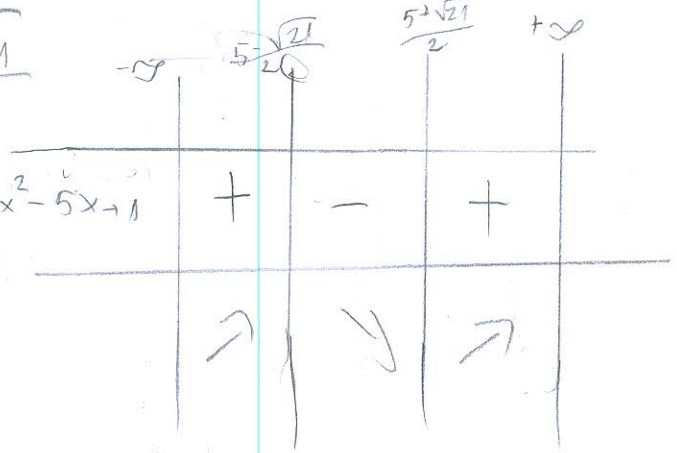
$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$

$x_1 = \frac{5 + \sqrt{21}}{2}$

$x_2 = \frac{5 - \sqrt{21}}{2}$



$\text{Dff} = \left\langle -\infty, \frac{5-\sqrt{21}}{2} \right\rangle \cup \left\langle \frac{5+\sqrt{21}}{2}, +\infty \right\rangle$

VERTIKALNA ASIMPTOTA

$\lim_{x \to \frac{5-\sqrt{21}}{2}} \sqrt{x^2 - 5x + 1} - x = \lim_{x \to \frac{5+\sqrt{21}}{2}} \sqrt{\left(\frac{5\sqrt{21}}{2}\right)^2 - 5\left(\frac{5\sqrt{21}}{2}\right) + 1} =$

=

Petrović Marko

③

$$\cos(4x+1) \overset{f'}{=} \cos' \cdot (4x+1) + \cos(4x+1)'$$

$$= -\sin x(4x+1) + \cos 4$$

$$f''(x) = -\sin x(4x+1) + \cos 4 =$$

Periodičnost

2. Riješi sustav Gaussovom metodom. Tekuća Tales

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 6 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 3 & -5 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \sim$$

$1r \circ (-1) + 3r$

$2r \circ (-6) + 3r$

$2r \circ (7) + 4r$

$3r : 3$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -5/3 & 1/3 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -5/3 & 1/3 \\ 0 & 0 & 0 & 4/3 & 28/3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -5/3 & 1/3 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right] \sim$$

$3r \circ (4) + 4r$

$4r : 4/3$

$4r \circ (5/3) + 3r$

$4r \circ (-1) + 2r$

$4r \circ (4) + 1r$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & 0 & 28 \\ 0 & 1 & -1 & 0 & -6 \\ 0 & 0 & 1 & 0 & 12 \\ 0 & 0 & 0 & 1 & -7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & -8 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 12 \\ 0 & 0 & 0 & 1 & -7 \end{array}\right]$$

$3r \circ (1) + 2r$

$3r \circ (-3) + 1r$

$2r \circ (2) + 1r$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 12 \\ 0 & 0 & 0 & 1 & -7 \end{array}\right] \begin{array}{l} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}$$

$x_1 = -4$

$x_2 = 6$

$x_3 = 12$

$x_4 = -7$

Provjera:

$x_1 - 2x_2 + 3x_3 - 4x_4 = 0$

$4 - (2 \cdot 6) + 3 \cdot (12) - 4 \cdot (7) = 0$

$0 = 0$

$x_2 - x_3 + x_4 = 1$

$6 - 12 + 7 = 1$

$1 = 1$

(3) $g(x) =$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ 03

IME I PREZIME: Goran Nojanović **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

kosor

POPUNJAVA NASTAVNIK
Broj ↓ bodova

0

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

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Ukupno:

2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 6 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \cdot 2; \cdot(-5); \cdot 7 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \cdot(-4) \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & -2 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 3 & 4 & 1 \end{array}\right] \cdot(-1); \cdot(-3)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 10 & -5 \end{array}\right] \cdot 1/10 \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} \end{array}\right] \cdot 2; \cdot 1 \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{2} \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

$$\begin{array}{rcrcrcrcl} {\scriptstyle 0} & & {\scriptstyle 5/2} & & {\scriptstyle 1} & & {\scriptstyle -1/2} & & \\ x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 0 \checkmark \\ & & {\scriptstyle 5/2} & & {\scriptstyle 1} & & {\scriptstyle -1/2} & & \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \checkmark \\ {\scriptstyle 0} & & {\scriptstyle 5/2} & & & & {\scriptstyle -1/2} & & \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 7 \times \\ & & {\scriptstyle 5/2} & & {\scriptstyle 1} & & {\scriptstyle -1/2} & & \\ & - & 7x_2 & + & 3x_3 & + & x_4 & = & -15 \checkmark \end{array}$$

4. $f(x) = \frac{5-x}{x-3}$

DOMENA

$x - 3 \neq 0$

$x \neq 3$

$D_f / R\{3\}$

PARNOST, NEPARNOST

$f(-x) = \frac{5-(-x)}{-x-3} = \frac{5+x}{-x-3}$

Ni parna, ni ne parna!

ASIMPTOTE

H. A.

$x = 3$

VERTIK. A.

$\lim_{x \to \infty} \frac{5-x \,/\, x}{x-3 \,/\, x} = \lim \frac{\frac{5}{x} - \frac{x}{x}}{\frac{x}{x} - \frac{3}{x}} = \frac{-1}{1} = -1$

SSEČIŠTA

$x = 0$

$f(0) = \frac{5-0}{0-3} = \frac{5}{-3}$

$y = 0$

$\frac{5-x}{x-3} = 0 \;/\cdot(x-3)$

$5 - x = 0$

$-x = -5$

$x = 5$

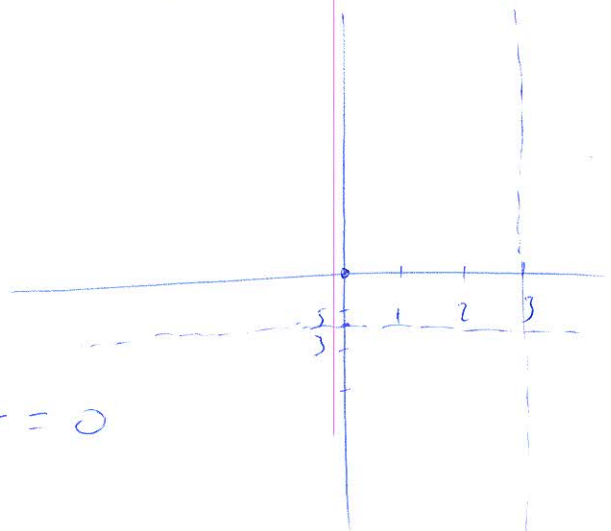
$T\left(5, \frac{5}{-3}\right)$

EKSTREMI

$f'(x) = \frac{5-x}{x-3} = \frac{(5-x)'(x-3) - (5-x)(x-3)'}{(x-3)^2}$

$= \frac{-1(x-3) - (5-x)1}{(x-3)^2}$

$= \frac{-x+3-5+x}{(x-3)^2} = \frac{-2}{(x-3)^2}$



$f''(x) = \frac{-2}{(x-3)^2} = \frac{(-2)'(x-6) - (-2)(2x-6)'}{(2x-6)^2} = \frac{0}{(2x-6)^3} = 0$

$(x-3)^2 = 2(x-3)(x-3)'$

$= (2x-6)(A)$

$= 2x - 6$

KONV. KONK.

$\frac{-2}{(x-3)^3} = 0 \;/\cdot(x-3)^2$

$-2 = 0$?

$-\infty$		$-\frac{5}{3}$		-1		0		$+\infty$
$f'(x) = 0$								
$\frac{-2}{(x-3)^3}$		−		−		−		
		↘		↘		↘		

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ 03

IME I PREZIME: MARIJA ERSLAN **BROJ INDEKSA:** 5953

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

0

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. Na kraju provjeriti rješenja. 12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \sqrt{x^2 - 5x + 1} - x$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{x-3}$. 15(graf)

5. Odrediti domenu, period (ako postoji), (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x + 1)$. 2+5+4+9

6. Zadana je funkcija $f(x) = \sqrt{2x - x^2}$:

   (a) koji su lokalni ekstremi? 10

   (b) koji su globalni ekstremi? 5

Ukupno:

2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & 3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \text{III} - \text{II} \approx \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 6 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \text{III} - \text{I} \approx$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 4 & -2 & 6 & 6 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \text{III} - 4\text{I} \approx \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -14 & 22 & 6 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} :2 \\ \text{IV} + 7\text{II} \end{array} \approx$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -7 & 11 & 3 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \text{I} + 2\text{II} \approx \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -7 & 11 & 3 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right]$$