21. 2. 2013.

**MATEMATIKA 1**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ £2

**IME I PREZIME:** Filip Bačinić **BROJ INDEKSA:** 3718

**ZAOKRUŽITI AKO ŽELITE:** ustmeni kod prof. Uglešića

F. B...

POPUNJAVA NASTAVNIK
Broj ↓ bodova

~~72~~

1. Pronaći sve kompleksne brojeve $z$ takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)

~~17~~

5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

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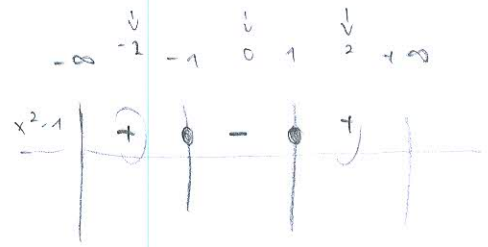
② $f(x) = x - \sqrt{x^2-1}$

$x^2 - 1 \geq 0$

$x^2 - 1 = 0$

$x^2 = 1 \quad x_1 = 1 \quad x_2 = -1$




$D_f: \langle -\infty, -1] \cup [1, +\infty \rangle$ ✓

$\lim_{x \to -1^-} x - \sqrt{x^2-1} = \lim_{x \to -1^-} -1^- - \sqrt{(-1)^2-1} = \lim_{x \to -1^-} -1^- - \sqrt{1^+-1} = \lim_{x \to -1^-} -1^- - 0^+$

$\lim_{x \to +\infty} x - \sqrt{x^2-1} \overset{\infty-\infty}{=} \Big/ \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \frac{1}{x+\sqrt{x^2-1}} = \frac{1}{+\infty} = 0$ DESNA HORIZ. ASIMPTOTA ✓

PROVJERA $k = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x-\sqrt{x^2-1}}{x} \Big/ \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \frac{x^2-(x^2-1)}{x^2+x\sqrt{x^2-1}} = \frac{1}{+\infty} = 0$ ✓

$\lim_{x \to -\infty} x - \sqrt{x^2-1} = \begin{cases} -\infty \to +\infty \\ x \to -x \end{cases} = \lim_{x \to +\infty} -x - \sqrt{(-x)^2-1} = \lim_{x \to +\infty} -x - \sqrt{x^2-1} = -\infty - \infty \Big/ \cdot \frac{-x+\sqrt{x^2-1}}{-x+\sqrt{x^2-1}} =$

$= \frac{(-x)^2-(x^2-1)}{-x+\sqrt{x^2-1}} = \frac{x^2-x^2+1}{-x+\sqrt{x^2-1}} = \frac{1}{\infty} = 0$

③ $g(x) = \ln(x^2+1)$ $\quad x^2+1 > 1$

$x^2 > 0$

ZA SVAKI BROJ $\quad \boxed{D_g : \mathbb{R}}$

$g(-x) = \ln((-x)^2+1)$

$g(x) = \ln(x^2+1) = g(x)$ FUNKCIJA JE PARNA

$$g'(x) = \frac{1}{x^2+1} \cdot (x^2+1)' = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

$$g''(x) = \frac{(2x)'\cdot(x^2+1) - (2x)(x^2+1)'}{(x^2+1)^2} = \frac{2\cdot(x^2+1) - 2x\cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$$

⑥ $\lim\limits_{x\to -2} \dfrac{x^2-4}{x^3+2x^2+x+2} = \left[\dfrac{0}{0}\right]$

$$\lim_{x\to -2^-} \frac{x^2-4}{x^3+2x^2+x+2} = \lim_{x\to -2^-} \frac{4^+-4}{-8^-+8^++(-2)^-+2} = \frac{0^+}{0^-}$$

$$\lim_{x\to -2^+} \frac{x^2-4}{x^3+2x^2+x+2} = \lim_{x\to -2^+} \frac{4^- - 4}{-8^+ + 8^- + (-2)^+ + 2} = \frac{0^-}{0^+}$$

⑤
$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{matrix} \\ \\ \\ \text{IV}-\text{II} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \text{II}-2\text{I} \\ \text{III}-4\text{I} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \\ \text{III}-\text{II} \\ \\ \end{matrix} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & -3 & 3 & 6 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \\ 1:(-1) \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \\ \\ \text{IV}-\text{III} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} \\ \\ \text{II}+4\text{III} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & 0 & -9 & -12 \\ 0 & 0 & 1 & -1 & -2 \\ & & & & \end{array}\right] 1:3 \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \text{I}+\text{II} \\ \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -2 & -1 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \text{I}-2\text{III} \\ \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right]$$

$a = 3$

$b - 3d = -4 \quad b = -4 + 3\lambda$

$c - d = -2 \quad c = -2 + \lambda$

$d = \lambda$

$4a - b + c + 2d = 14$

$12 - (-4+3\lambda) + (-2+\lambda) + 2\lambda = 14$

$12 + 4 - 3\lambda - 2 + \lambda + 2\lambda = 14$

$14 - 3\lambda + 3\lambda = 14$

$14 = 14$ ✓

Filip Bačinić 3718

④

1) $h(x) = \frac{x^2-1}{x^2+1}$ $\quad x^2+1 \neq 0$

$x^2 \neq -1$ $\quad D_h : \mathbb{R}$

$h(-x) = \frac{(-x)^2-1}{(-x)^2+1} = \frac{x^2-1}{x^2+1} = h(x)$ FUNKCIJA JE PARNA

2) VERT. ASIMPTOTA NEMA JER NEMA TOČKA PREKIDA

$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = \left[\frac{\infty}{\infty}\right] = \frac{\frac{x^2}{x^2} - \left(\frac{1}{x^2}\right)\cdot c}{\frac{x^2}{x^2} + \left(\frac{1}{x^2}\right)\cdot c} = 1$ D.H.A.

$k = \lim_{x\to\infty} \frac{\frac{x^2-1}{x^2+1}}{x} = \frac{x^2-1}{(x^2+1)\cdot x} = \frac{x^2-1 \,/:x^3}{x^3+x \,/:x^3} = \frac{\infty}{\infty} = 0$

FUNKC. JE PARNA $\Rightarrow$ D.H.A = L H.A

KOSIH ASIMPTOTA NEMA

3) FUNKCIJA NIJE PERIODIČNA, OMEĐENA JE

4) $\frac{x^2-1}{x^2+1} = 0$ $\quad x^2-1=0$

$x^2 = 1$ $\quad x_1 = 1, x_2 = -1$ NULTOČKE

SJECIŠTE S OSI y $\quad \frac{0^2-1}{0^2+1} = \frac{-1}{1} = -1$

5) $h'(x) = \frac{(x^2-1)'(x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{2x\cdot(x^2+1) - (x^2-1)\cdot 2x}{(x^2+1)^2} = \frac{2x^3+2x-(2x^3+2x)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

$h''(x) = \frac{(4x)'(x^2+1)^2 - (4x)\left((x^2+1)^2\right)'}{(x^2+1)^4} = \frac{4(x^4+2x^2+1) - 4x(4x^3+4x^2)}{(x^2+1)^4} =$

$= \frac{4x^4+8x^2+4-16x^4-16x^3}{(x^2+1)^4} = \frac{-12x^4-16x^3+8x^2+4}{(x^2+1)^4} =$

$= \frac{4(-3x^4-4x^3+2x^2+1)}{(x^2+1)^4}$

$\left((x^2+1)^2\right)' = 2(x^2+1)\cdot(x^2+1)'$

$= (2x^2+2)\cdot 2x = 4x^3+4x^2$

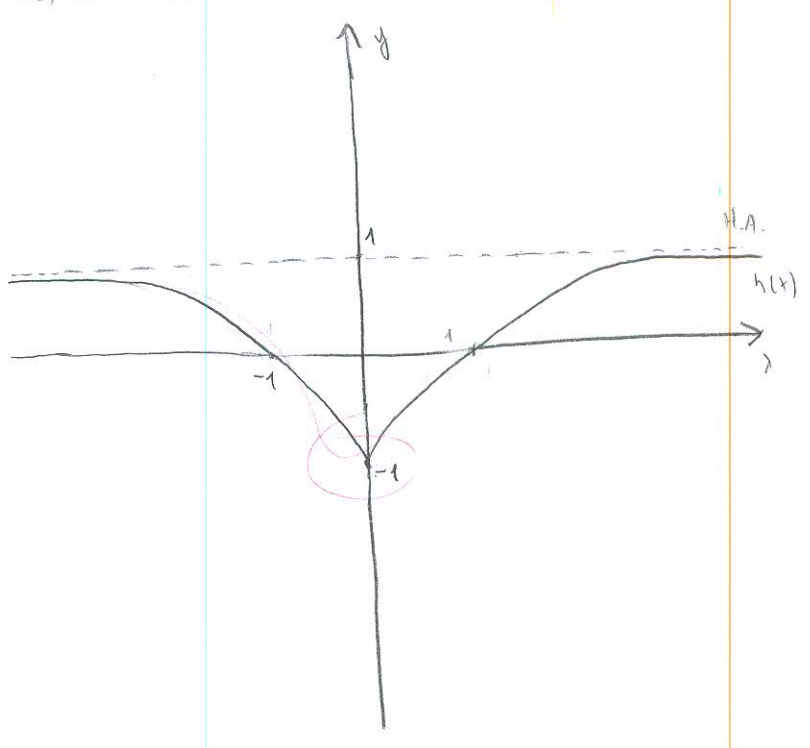
6) $\frac{4x}{(x^2+1)^2} = 0$ $\quad x = 0$

| | $-\infty$ | $\downarrow -1$ | | $0$ | | $\downarrow 1$ | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $h'(x)$ | | $-$ | | $+$ | | | |
| $h(x)$ | | $\searrow$ | | $\nearrow$ | | | |

LOKALNI GLOB. MIN

| | $-\infty$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|
| $h''(x)$ | | $-$ | | $-$ | |
| $h(x)$ | | $\cap$ | | $\cap$ | |

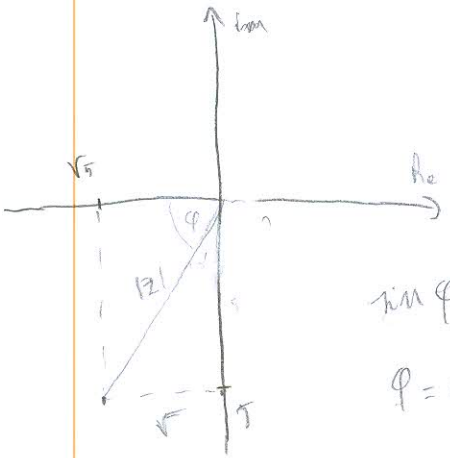
10) SKICA GRAFA



① $z^3 + |3+4i| = \frac{5}{i}$ $\quad z^3 = \frac{5}{i} - |3+4i|$ $\quad z^3 = \frac{5}{i} - \sqrt{3^2+(4i)^2}$ $\quad z^3 = \frac{5}{i} - \sqrt{9-4}$

$z^3 = \frac{5}{i} - \sqrt{5}$ $\quad z^3 = \frac{5 - \sqrt{5}\,i}{i} \Big/ \frac{-i}{-i} = -5i + \sqrt{5}\,i^2 = -5i - \sqrt{5}$

$z^3 = -\sqrt{5} - 5i$ $\quad \varphi = \tan =$




$|z| = \sqrt{30}$

$\sin\varphi = \frac{y}{|z|} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{30}}{6}$

$\varphi = 65.9$

1. SLOŽKA $k=0$

$z_1 = \sqrt{30}\left(\cos\frac{65.9+0}{3} + i\sin\frac{65.9}{3}\right) = \sqrt{30}(\cos 21.9 + i\sin 21.9)$

$= \sqrt{30}(0.928 + i\,0.373) = 5.08 + 2.043\,i$

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**IME I PREZIME:** BRANIMIR PUNCA **BROJ INDEKSA:** MI-2-0086-2011

**ZAOKRUŽITI AKO ŽELITE:** ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

55

1. Pronaći sve kompleksne brojeve $z$ takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3
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5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

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6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

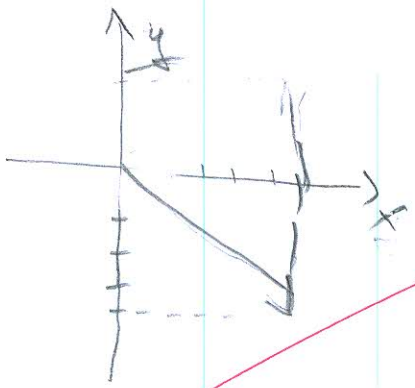
① $|3+4i| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\frac{5}{i} \cdot \frac{-i}{-i} = \frac{-5i}{(-i)^2} = \frac{-5i}{-1} = 5i$

$z^3 = 5+5i$

$\varphi = \frac{y}{x} = \frac{+5}{5} = -1 \quad 45^\circ$

$\varphi = 360 - 45 = 315^\circ$




$|5-5i| = \sqrt{5^2+(5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$k = 0,1,2$

$k=0 \quad \sqrt[6]{50}\left(\cos\frac{315^\circ+0\cdot360^\circ}{3} + i\sin\frac{315^\circ+0\cdot360^\circ}{3}\right)$

$k=1 \quad \sqrt[6]{50}\left(\cos\frac{315^\circ+1\cdot360^\circ}{3} + i\sin\frac{315^\circ+1\cdot360^\circ}{3}\right)$

$k=2 \quad \sqrt[6]{50}\left(\cos\frac{315^\circ+2\cdot360^\circ}{3} + i\sin\frac{315^\circ+2\cdot360^\circ}{3}\right)$