21. 2. 2013.

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ £2

IME I PREZIME: Filip Bačinić **BROJ INDEKSA:** 3718

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

F. B...

POPUNJAVA NASTAVNIK
Broj ↓ bodova

~~72~~

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)

~~17~~

5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

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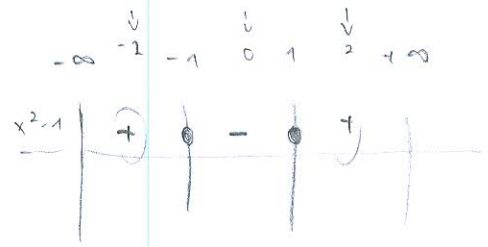
② $f(x) = x - \sqrt{x^2-1}$

$x^2 - 1 \geq 0$

$x^2 - 1 = 0$

$x^2 = 1 \quad x_1 = 1 \quad x_2 = -1$



$D_f: \langle -\infty, -1] \cup [1, +\infty \rangle$ ✓

$\lim_{x \to -1^-} x - \sqrt{x^2-1} = \lim_{x \to -1^-} -1^- - \sqrt{(-1)^2-1} = \lim_{x \to -1^-} -1^- - \sqrt{1^+-1} = \lim_{x \to -1^-} -1^- - 0^+$

$\lim_{x \to +\infty} x - \sqrt{x^2-1} \overset{\infty-\infty}{=} \Big/ \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \frac{1}{x+\sqrt{x^2-1}} = \frac{1}{+\infty} = 0$ DESNA HORIZ. ASIMPTOTA ✓

PROVJERA $k = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x-\sqrt{x^2-1}}{x} \Big/ \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \frac{x^2-(x^2-1)}{x^2+x\sqrt{x^2-1}} = \frac{1}{+\infty} = 0$ ✓

$\lim_{x \to -\infty} x - \sqrt{x^2-1} = \begin{cases} -\infty \to +\infty \\ x \to -x \end{cases} = \lim_{x \to +\infty} -x - \sqrt{(-x)^2-1} = \lim_{x \to +\infty} -x - \sqrt{x^2-1} = -\infty - \infty \Big/ \cdot \frac{-x+\sqrt{x^2-1}}{-x+\sqrt{x^2-1}} =$

$= \frac{(-x)^2-(x^2-1)}{-x+\sqrt{x^2-1}} = \frac{x^2-x^2+1}{-x+\sqrt{x^2-1}} = \frac{1}{\infty} = 0$

③ $g(x) = \ln(x^2+1)$ $\quad x^2+1 > 1$

$x^2 > 0$

ZA SVAKI BROJ $\quad \boxed{D_g : \mathbb{R}}$

$g(-x) = \ln((-x)^2+1)$

$g(x) = \ln(x^2+1) = g(x)$ FUNKCIJA JE PARNA

$$g'(x) = \frac{1}{x^2+1} \cdot (x^2+1)' = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

$$g''(x) = \frac{(2x)'\cdot(x^2+1) - (2x)(x^2+1)'}{(x^2+1)^2} = \frac{2\cdot(x^2+1) - 2x\cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$$

⑥ $\lim\limits_{x\to -2} \dfrac{x^2-4}{x^3+2x^2+x+2} = \left[\dfrac{0}{0}\right]$

$$\lim_{x\to -2^-} \frac{x^2-4}{x^3+2x^2+x+2} = \lim_{x\to -2^-} \frac{4^+-4}{-8^-+8^++(-2)^-+2} = \frac{0^+}{0^-}$$

$$\lim_{x\to -2^+} \frac{x^2-4}{x^3+2x^2+x+2} = \lim_{x\to -2^+} \frac{4^- - 4}{-8^+ + 8^- + (-2)^+ + 2} = \frac{0^-}{0^+}$$

⑤
$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{matrix} \\ \\ \\ \text{IV}-\text{II} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \text{II}-2\text{I} \\ \text{III}-4\text{I} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \\ \text{III}-\text{II} \\ \\ \end{matrix} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & -3 & 3 & 6 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \\ 1:(-1) \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \\ \\ \\ \text{IV}-\text{III} \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} \\ \\ \text{II}+4\text{III} \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & 0 & -9 & -12 \\ 0 & 0 & 1 & -1 & -2 \\ & & & & \end{array}\right] 1:3 \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \text{I}+\text{II} \\ \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -2 & -1 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{matrix} \text{I}-2\text{III} \\ \\ \\ \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right]$$

$a = 3$

$b - 3d = -4 \quad b = -4 + 3\lambda$

$c - d = -2 \quad c = -2 + \lambda$

$d = \lambda$

$4a - b + c + 2d = 14$

$12 - (-4+3\lambda) + (-2+\lambda) + 2\lambda = 14$

$12 + 4 - 3\lambda - 2 + \lambda + 2\lambda = 14$

$14 - 3\lambda + 3\lambda = 14$

$14 = 14$ ✓

Filip Bačinić 3718

④

1) $h(x) = \frac{x^2-1}{x^2+1}$ $\quad x^2+1 \neq 0$

$x^2 \neq -1$ $\quad D_h : \mathbb{R}$

$h(-x) = \frac{(-x)^2-1}{(-x)^2+1} = \frac{x^2-1}{x^2+1} = h(x)$ FUNKCIJA JE PARNA

2) VERT. ASIMPTOTA NEMA JER NEMA TOČKA PREKIDA

$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = \left[\frac{\infty}{\infty}\right] = \frac{\frac{x^2}{x^2} - \left(\frac{1}{x^2}\right)\cdot c}{\frac{x^2}{x^2} + \left(\frac{1}{x^2}\right)\cdot c} = 1$ D.H.A.

$k = \lim_{x\to\infty} \frac{\frac{x^2-1}{x^2+1}}{x} = \frac{x^2-1}{(x^2+1)\cdot x} = \frac{x^2-1 \,/:x^3}{x^3+x \,/:x^3} = \frac{\infty}{\infty} = 0$

FUNKC. JE PARNA $\Rightarrow$ D.H.A = L H.A

KOSIH ASIMPTOTA NEMA

3) FUNKCIJA NIJE PERIODIČNA, OMEĐENA JE

4) $\frac{x^2-1}{x^2+1} = 0$ $\quad x^2-1=0$

$x^2 = 1$ $\quad x_1 = 1, x_2 = -1$ NULTOČKE

SJECIŠTE S OSI y $\quad \frac{0^2-1}{0^2+1} = \frac{-1}{1} = -1$

5) $h'(x) = \frac{(x^2-1)'(x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{2x\cdot(x^2+1) - (x^2-1)\cdot 2x}{(x^2+1)^2} = \frac{2x^3+2x-(2x^3+2x)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

$h''(x) = \frac{(4x)'(x^2+1)^2 - (4x)\left((x^2+1)^2\right)'}{(x^2+1)^4} = \frac{4(x^4+2x^2+1) - 4x(4x^3+4x^2)}{(x^2+1)^4} =$

$= \frac{4x^4+8x^2+4-16x^4-16x^3}{(x^2+1)^4} = \frac{-12x^4-16x^3+8x^2+4}{(x^2+1)^4} =$

$= \frac{4(-3x^4-4x^3+2x^2+1)}{(x^2+1)^4}$

$\left((x^2+1)^2\right)' = 2(x^2+1)\cdot(x^2+1)'$

$= (2x^2+2)\cdot 2x = 4x^3+4x^2$

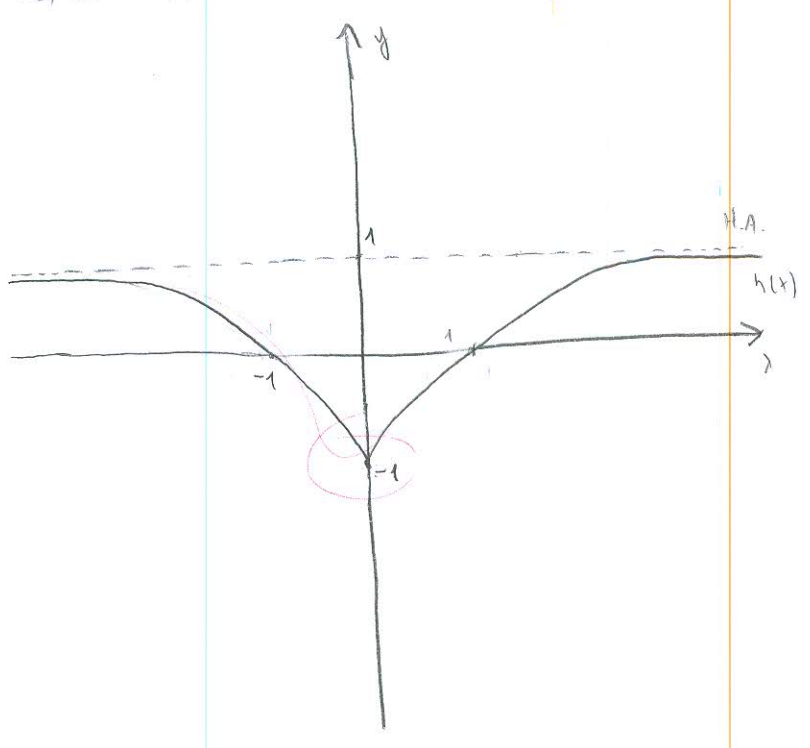
6) $\frac{4x}{(x^2+1)^2} = 0$ $\quad x = 0$

	$-\infty$	$\downarrow -1$		0		$\downarrow 1$	$+\infty$
$h'(x)$		$-$		$+$			
$h(x)$		$\searrow$		$\nearrow$			

LOKALNI GLOB. MIN

	$-\infty$		0		$+\infty$
$h''(x)$		$-$		$-$	
$h(x)$		$\cap$		$\cap$	

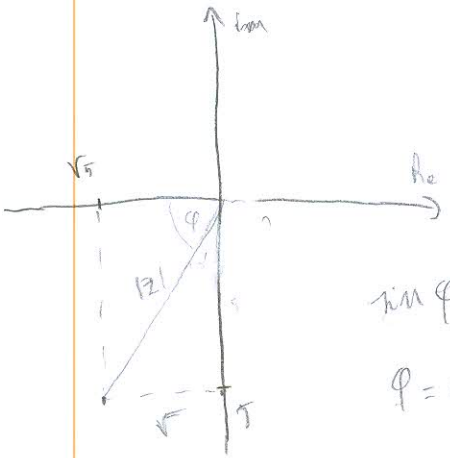
10) SKICA GRAFA



① $z^3 + |3+4i| = \frac{5}{i}$ $\quad z^3 = \frac{5}{i} - |3+4i|$ $\quad z^3 = \frac{5}{i} - \sqrt{3^2+(4i)^2}$ $\quad z^3 = \frac{5}{i} - \sqrt{9-4}$

$z^3 = \frac{5}{i} - \sqrt{5}$ $\quad z^3 = \frac{5 - \sqrt{5}\,i}{i} \Big/ \frac{-i}{-i} = -5i + \sqrt{5}\,i^2 = -5i - \sqrt{5}$

$z^3 = -\sqrt{5} - 5i$ $\quad \varphi = \tan =$



$|z| = \sqrt{30}$

$\sin\varphi = \frac{y}{|z|} = \frac{\sqrt{5}}{\sqrt{30}} = \frac{\sqrt{30}}{6}$

$\varphi = 65.9$

1. SLOŽKA $k=0$

$z_1 = \sqrt{30}\left(\cos\frac{65.9+0}{3} + i\sin\frac{65.9}{3}\right) = \sqrt{30}(\cos 21.9 + i\sin 21.9)$

$= \sqrt{30}(0.928 + i\,0.373) = 5.08 + 2.043\,i$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ Ł2

IME I PREZIME: BRANIMIR PUNCA **BROJ INDEKSA:** MI-2-0086-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

55

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3
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5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

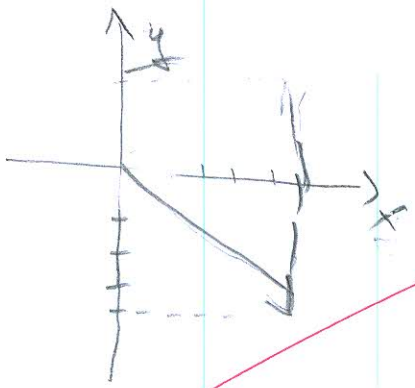
① $|3+4i| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\frac{5}{i} \cdot \frac{-i}{-i} = \frac{-5i}{(-i)^2} = \frac{-5i}{-1} = 5i$

$z^3 = 5+5i$

$\varphi = \frac{y}{x} = \frac{+5}{5} = -1 \quad 45^\circ$

$\varphi = 360 - 45 = 315^\circ$



$|5-5i| = \sqrt{5^2+(5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$k = 0,1,2$

$k=0 \quad \sqrt[6]{50}\left(\cos\frac{315^\circ+0\cdot360^\circ}{3} + i\sin\frac{315^\circ+0\cdot360^\circ}{3}\right)$

$k=1 \quad \sqrt[6]{50}\left(\cos\frac{315^\circ+1\cdot360^\circ}{3} + i\sin\frac{315^\circ+1\cdot360^\circ}{3}\right)$

$k=2 \quad \sqrt[6]{50}\left(\cos\frac{315^\circ+2\cdot360^\circ}{3} + i\sin\frac{315^\circ+2\cdot360^\circ}{3}\right)$

② $x - \sqrt{x^2-1}$ domena asimptote

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq \pm 1$

$D f(x) ; [1, +\infty\rangle$

V.A. $\lim_{x \to 1} x - \sqrt{x^2-1} = 1$ Nema vertikalnih

H.A. $\lim_{x \to +\infty} x - \sqrt{x^2-1} = +\infty$

$\lim_{x \to -\infty} x - \sqrt{x^2-1}$ $\left[\begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix}\right]$

$\lim_{x \to +\infty} (-x) - \sqrt{(-x)^2-1} = \lim_{x \to +\infty} -x - \sqrt{x-1} =$

$-x - \sqrt{x-1} \cdot \frac{-x + \sqrt{x-1}}{-x + \sqrt{x-1}}$

$\lim_{x \to +\infty} \frac{(-x)^2 - (\sqrt{x-1})^2}{-x + \sqrt{x-1}} = \frac{x - x + 1}{-x + \sqrt{x-1}}$

$= \frac{1}{-x + \sqrt{x-1}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{0}{-1 + 0}$

$= 0$ lijeva horizontalna $y = 0$

kosa $k = \frac{f(x)}{x} = \frac{x - \sqrt{x^2-1}}{x} \quad / : x$

$\lim_{x \to +\infty} \frac{1 - \sqrt{\frac{x^2}{x^2} - \frac{1}{x^2}}}{1} = \frac{1-1}{1} = \frac{0}{1} = 0$

$l = f(x) - k \cdot (x)$

$\lim_{x \to +\infty} x - \sqrt{x^2-1} \cdot 0 = 0$

Nema kosih

$y = kx + l$

$y = 0 + 0$

$y = 0$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -4 & -5 & -4 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & -4 & 4 & 8 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{c} 14 \\ 2 \\ 3 \end{array}\right]$$

$$\left[\begin{array}{ll} a) & 3 \\ b) & -4 \\ c) & -2 \end{array}\right]$$

$4 \cdot 3 - 1 \cdot (-4) \cdot 1 \cdot (-2) = 14$ ✓

$2 \cdot 3 + 1 \cdot (-4) + 0 \cdot (-2) + 0 = 2$ ✓

$1 \cdot 3 + (-1) \cdot (-4) + 2 \cdot (-2) = 3$ ✓

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right]$$

3r ↔ 1r

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & -5 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{array}\right]$$

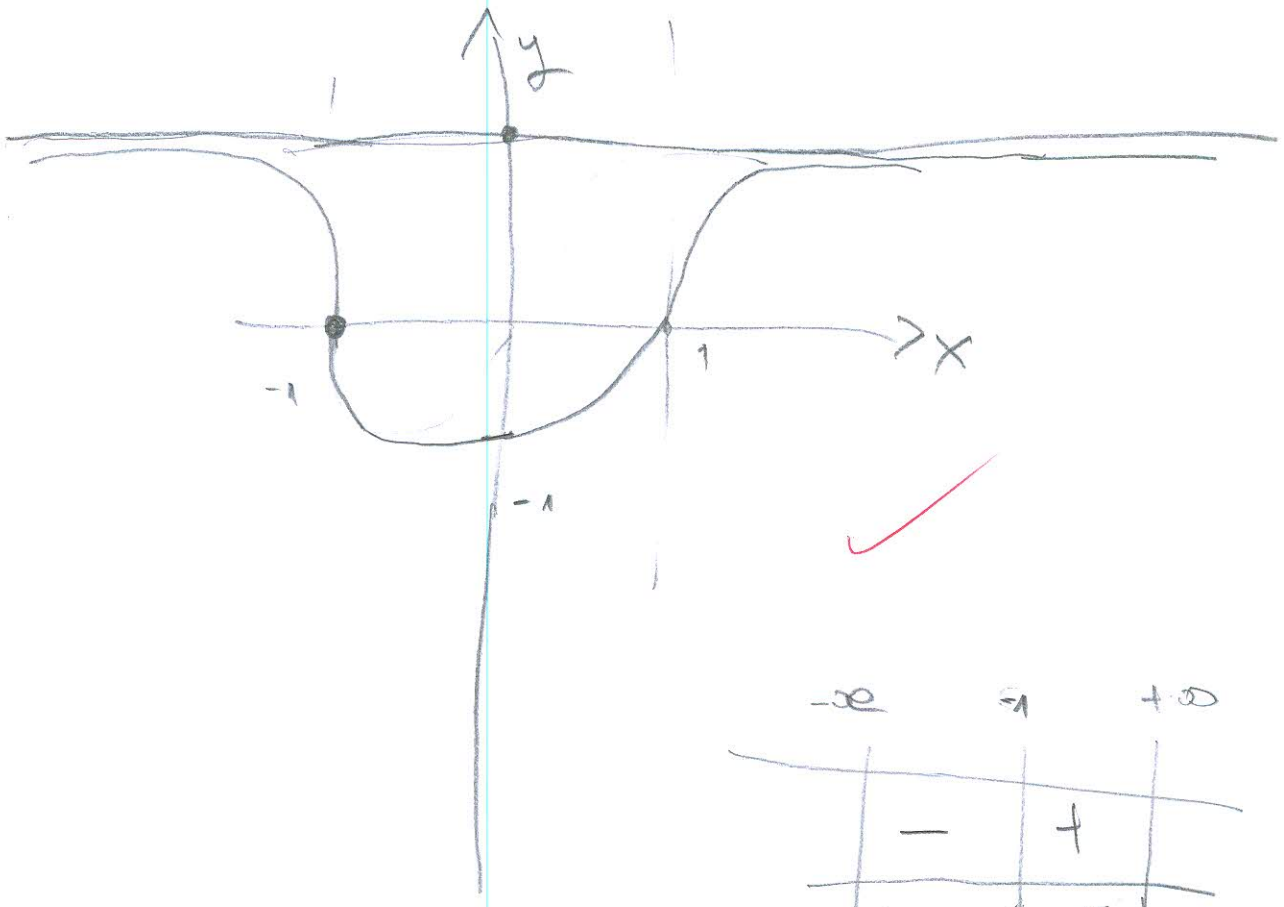
1r·(-2)+2r
1r·(-4)+3r
1r·(-2)+4r

2r·5+3r
2r·(-3)+4r
2r·(1)+1r

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -4 & -1 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & -27 & -17 & -18 \\ 0 & 0 & 9 & 9 & 6 \end{array}\right] \left[\begin{array}{cccc|c} 1 & 0 & -2 & -4 & 1 \\ 0 & 1 & -4 & -5 & -4 \\ 0 & 0 & 1 & 1 & 18/27 \\ 0 & 0 & 9 & 9 & 6 \end{array}\right]$$

3r·(-9)+4r
3r·(-4)+2r

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 7/3 \\ 0 & 1 & 0 & -1 & -4/3 \\ 0 & 0 & 1 & 1 & 18/27 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$



$$\lim_{x\to -2} \frac{x^2-4}{x^3+2x^2+x+2} = \frac{(-2)^2-4}{(-2)^3+2\cdot(-2)^2-2+2}$$

neurčitý obtík
→ l'Hospitalovo pravidlo

$$\lim_{x\to -2} \frac{2x}{3x+4x+1} = \frac{-4}{-13} = \frac{4}{13} \;//$$

③ $g(x) = \ln(x^2+1)$

$g(x) \ln(x^2+1)$

$D(g) = x^2+1 > 0$

$x^2 > -1$

$g'(x) \quad \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

$D(g) = \mathbb{R}$ ✓

$g''(x) = \frac{(2x)' \cdot (x^2+1) - 2x \cdot (x^2+1)'}{(x^2+1)^2}$

$f(x) = f(-x)$

$\ln(x^2+1) = \ln((-x)^2+1)$

$\ln(x^2+1) = \ln(x^2+1)$

funkcija je parna ✓

$= \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$ ✓

$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2}$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right]$$

1k · (-2) + 2k
1k · (-4) + 3k
1k · (-2) + 4k

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 3 & -7 & -2 & -2 \\ 0 & 3 & -4 & -5 & -4 \end{array}\right]$$

2k · (-3) + 3

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & -4 & -4 & -4 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right]$$

3k

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1/3 \\ 0 & 0 & 1 & 0 & 2/3 \\ 0 & 0 & 0 & 1 & 2/3 \end{array}\right]$$

4) $h(x) = \frac{x^2-1}{x^2+1}$

♣9 $D\ h(x)$ $\mathbb{R}$ ✓ $x^2+1 \neq 0$ $x^2 \neq -1$

Ničle: $x^2-1=0$ $x^2=1$ $x=-1$ $x_2=1$

Asimptote

Nema vertikalne

Horizontalna $\lim_{x\to\infty} \frac{x^2-1 \,/x^2}{x^2+1 \,/x^2} = \frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \frac{1}{1} = 1$ ✓

$\lim_{x\to-\infty} \frac{x^2-1}{x^2+1} = \lim_{x\to\infty} \frac{(-x)^2-1}{(-x)^2+1} = \frac{x^2-1}{x^2+1} = \frac{1}{1} = 1$

$y=1$ horizontalna asimptota ✓

koja nema ima obostrano horizontalno

$h'(x) \quad \frac{(x^2-1)' \cdot (x^2+1) - (x^2-1)\cdot(x^2+1)'}{(x^2+1)^2}$

$= \frac{2x\cdot(x^2+1) - (x^2-1)\cdot 2x}{(x^2+1)^2}$

$= \frac{2x^3+2x-(2x^3-2x)}{(x^2+1)^2}$

$f(x)' = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$ ✓

$f(0) = \frac{0^2-1}{0+1} = \frac{-1}{1} = -1$

Stacionarna točka $4x=0 \,/4$

$x=\frac{0}{4}=0$

$[0,0]$ stacionarna točka

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ £2

IME I PREZIME: NIKOLINA KOMLJENOVIĆ **BROJ INDEKSA:** 17-2-0114-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

Komljenović

POPUNJAVA NASTAVNIK Broj ↓ bodova

45

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

(2.) $f(x) = x - \sqrt{x^2-1}$ $\quad x^2-1 \geq 0$

$x^2 \geq 1$

$x = \pm 1$

$-\infty$	-1	1	$+\infty$
+	−	+	

$D\ f(x)\ x \in \langle -\infty, -1] \cup [1, +\infty \rangle$ ✓

V.A.

$\lim_{x \to 1} x - \sqrt{x^2-1} = \lim_{x \to 1} 1 - \quad$ = NEMA D.V.A. PA NEMA NI L.V.A.

H.A.

$\lim_{x \to +\infty} x - \sqrt{x^2-1} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \lim_{x \to +\infty} \frac{(x+\sqrt{x^2-1})(x-\sqrt{x^2-1}) \cdot (x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}} =$

$\lim_{x \to \infty} = \frac{(x+\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \frac{x+x^2-1}{x+\sqrt{x^2-1}} \,/\cdot x^2 = \frac{0+1-0}{0+\sqrt{\frac{x^2}{x^4}-0}} = \infty$

nema H.A.

K.A.

$k \ldots \frac{f(x)}{x} = \frac{x-\sqrt{x^2-1}}{x} \,/\cdot x = \frac{1-\sqrt{1-0}}{1} = \frac{0}{1} = 0$

$y = kx + l$

$l \ldots f(x) - kx \quad x - \sqrt{x^2-1} - x \cdot \frac{x+\sqrt{x^2-1}-x}{x+\sqrt{x^2-1}-x} = \frac{(x-\sqrt{x^2-1}-x)^2}{x+\sqrt{x^2-1}-x} =$

$= \frac{x-x^2-1-x}{x+\sqrt{x^2-1}-x} = \frac{-x^2+2x-1}{x+\sqrt{x^2-1}-x} \,/\cdot x^2 = \frac{-1-0-0}{\frac{x}{x^2}+\sqrt{\frac{x^2}{x^4}-\frac{1}{x^2}}-\frac{x}{x^2}} = \frac{-1}{0} = \infty$

$y = x + l$

3. $g(x) = \ln(x^2+1)$

$x^2+1 > 0$

$x^2 > -1$ nije definirano $D\,f(x)\ x \in \mathbb{R}$

$g(-x) = \ln((-x)^2+1)$

$= \ln(x^2+1)$ $\quad f(-x) = f(x)$ parna funkcija

$f'(x) = \frac{1}{x^2+1} \cdot (x^2+1)'$

$f'(x) = \frac{1}{x^2+1} \cdot 2x$

$f'(x) = \frac{2x}{x^2+1}$

$f''(x) = \frac{(2x)'(x^2+1) - 2x \cdot (x^2+1)'}{(x^2+1)^2}$

$f''(x) = \frac{2x^2+2-2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$f''(x) = \frac{-2x^2+2}{(x^2+1)^2}$

4. $h(x) = \frac{x^2-1}{x^2+1}$

$x^2+1 \neq 0$

$x^2 \neq -1$ nije def.

Domena je $\mathbb{R}$

$f(-x) = f(x)$

$f(0) = \frac{-1}{1} = -1$

$x^2-1 = 0$

$x^2 = 1 \mid \sqrt{}$

$x_1 = 1 \quad (1,0)$

$x_2 = -1 \quad (-1,0)$

V.A. nema jer nema točaka prekida

H.A. $\lim_{x \to +\infty} \frac{x^2-1}{x^2+1} \overset{/:x^2}{=} \frac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}} = \frac{1-0}{1+0} = 1$

$\lim_{x \to -\infty} \frac{x^2-1}{x^2+1} \overset{/:-x^2}{=} \frac{-1+\frac{1}{x^2}}{-1} \to 1$ je G.H.A što znači da kose nema

$f'(x) = \frac{(x^2-1)'(x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{2x(x^2+1) - (x^2-1) \cdot 2x}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$

$= \frac{4x}{(x^2+1)^2}$

$4x = 0 \mid :4$

$x = 0$

	$-\infty$		0		$+\infty$
$f'(x)$	$-$		$+$		
$f(x)$	$\searrow$		min	$\nearrow$	

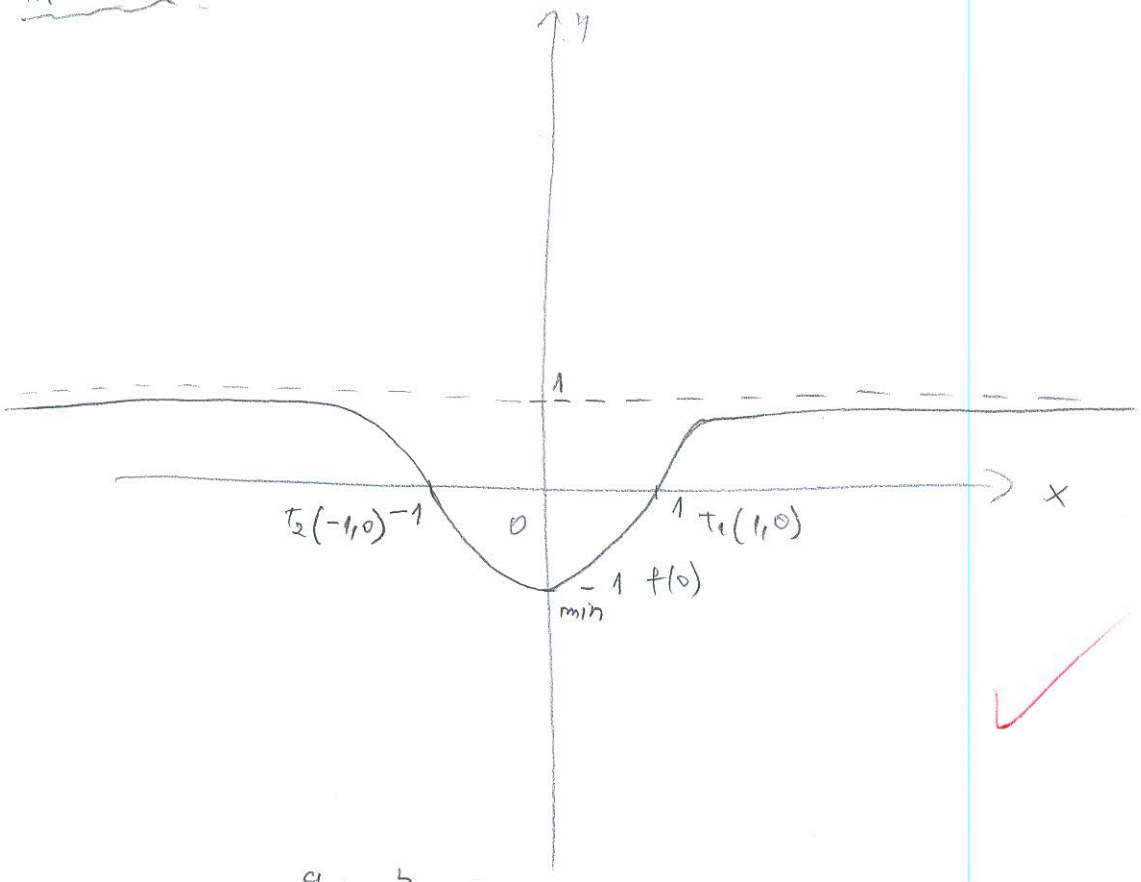
$f''(x) = \frac{4x}{(x^2+1)^2} = \frac{(4x)'(x^2+1)^2 - 4x\left((x^2+1)^2\right)'}{(x^2+1)^4} = \frac{4(x^2+1)^2 - 4x\left(2(x^2+1) \cdot (x^2+1)'\right)}{(x^2+1)^4}$

$= \frac{4\left((x^2)^2 + 2 \cdot x^2 \cdot 1 + 1\right) - \left(4x(2x^2+2) - 2x\right)}{(x^2+1)^4} = \frac{4\left(x^4+2x+1\right) - \left(6x^2(2x^2+2)\right)}{(x^2+1)^4}$

$= \frac{4x^4+8x+4-\left(12x^4+12x^2\right)}{(x^2+1)^4} = \frac{4x^4+8x+4-12x^4-12x^2}{(x^2+1)^4} = \frac{-8+8x+4-12x^2}{(x^2+1)^4} = \frac{-12x^2+8x^4}{(x^2+1)^4}$

NIKOLINA KOMLJENOVIĆ

4. zadatak



$f''(x) = 0 \qquad \underbrace{-12x^2 + 8x - 4 = 0}_{a \quad b \quad c}$

$$x_{1,2} = \frac{-8 \pm \sqrt{64 - 4 \cdot (-12) \cdot (-4)}}{-24}$$

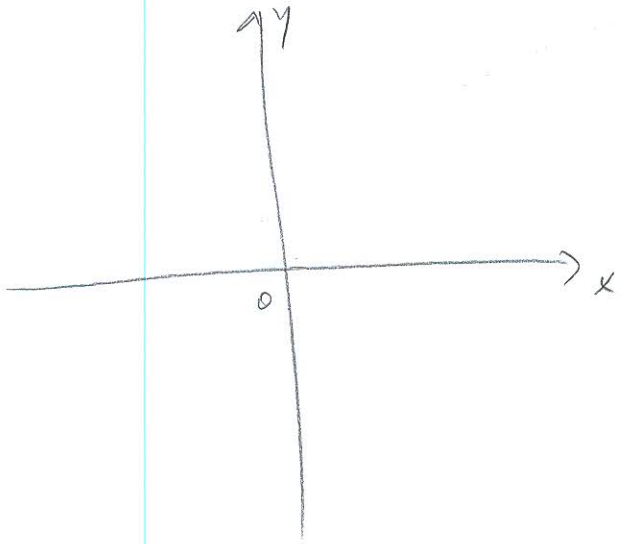
$$x_{1,2} = \frac{1 \pm \sqrt{-128}}{3}$$

nije definirano
nema točki infleksije

(1.) $z^3 + |3+4i| = \frac{5}{i}$

$z^3 + \sqrt{9+16} = \frac{5}{i}$

$z^3 + 5 = \frac{5}{i} \quad / : i$



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IME I PREZIME: JOSIP PREDOVAN **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

30

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6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

(5)

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \leftarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{array}{l} \text{II}-2\cdot\text{I} \\ \text{III}-4\cdot\text{I} \\ \text{IV}-2\cdot\text{I} \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{array}\right] \leftarrow$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -3 & -6 & -6 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -4 & -5 & -4 \end{array}\right] \begin{array}{l} \\ /:3 \\ \\ \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -4 & -5 & -4 \end{array}\right] \begin{array}{l} \text{I}+\text{II} \\ \text{III}-3\cdot\text{II} \\ \text{IV}-3\cdot\text{II} \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & -4 & 4 & 8 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right] /:(-4) \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right] \begin{array}{l} \text{I}-\text{III} \\ \text{II}+\text{III} \\ \text{IV}+\text{III} \end{array}$$

*

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$a = 3$

$b - 3\cdot d = -4$

$c - d = -2$

$d = \lambda$

$c - \lambda = -2$

$c = -2 + \lambda$

$b - 3\cdot\lambda = -4$

$b = -4 + 3\lambda$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ -2 \\ 0 \end{bmatrix} \cdot \lambda \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

MATRIČNO MNOŽENJE

$4\cdot 3 - 1\cdot(-4) + 1\cdot(-2) + 2\cdot 0 = 14$ ✓

$2\cdot 3 + 1\cdot(-4) + 0\cdot(-2) + 2\cdot 0 = 2$ ✓

$1\cdot 3 - 1\cdot(-4) + 2\cdot(-2) + 1\cdot(0) = 3$ ✓

$2\cdot(3) + 1\cdot(-4) + 1\cdot(-2) - 4\cdot(0) = 0$ ✓

② $f(x) = x - \sqrt{x^2-1}$ $\quad D_f \langle -\infty, +\infty \rangle$

$\sqrt{x^2-1} \geq 0$

$x^2 - 1 \geq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$y_{1,2} = \frac{-0 \pm \sqrt{0^2-4\cdot 1\cdot(-1)}}{2\cdot 1}$

$x_{1,2} = \frac{-0 \pm 2}{2}$

$x_1 = 1$

$x_2 = -1$

$f(-x) = -x - \sqrt{(-x^2-1)}$

$= -x - \sqrt{x^2-1}$

- NEPARNA
- NIJE PERIODIČNA
- NEOMEĐENA

V. A

$\lim_{x\to 1^-} x - \sqrt{x^2-1} = 1^- - \sqrt{(1^-)^2-1} = 1 - 0^- = 1$

$\lim_{x\to 1^+} x - \sqrt{x^2-1} = 1^+ - \sqrt{(1^+)^2-1} = 1^+ - 0^+ = 1$

$\lim_{x\to -1^+} (-1^+) - \sqrt{(-1^+)^2-1} = -1 \pm \sqrt{0} = -1 - 0^+ = -1$

$\lim_{x\to -1^-} (-1^-) - \sqrt{(-1^-)^2-1} = -1 - \sqrt{0} = -1 - 0^- = -1$

NEMA V.A

H. A

$\lim_{x\to+\infty} x - \sqrt{x^2-1} = \lim_{x\to+\infty} x - \sqrt{x^2-1} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \frac{x^2-x^2+1}{x+\sqrt{x^2-1}} = \frac{1\ /:x}{x+\sqrt{x^2-1}\ /:x}$

$\frac{\frac{1}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}\cdot\frac{1}{x^2}}} = \frac{0}{1+\sqrt{1}} = \frac{0}{2} = 0$

$\lim_{x\to-\infty} = -x - \sqrt{(-x^2)-1} = \lim_{x\to\infty} -x - \sqrt{x^2-1} \cdot \frac{-x+\sqrt{x^2-1}}{-x+\sqrt{x^2-1}} = \frac{(-x)^2-(x^2-1)}{-x+\sqrt{x^2-1}} = \frac{x^2-x^2+1}{-x+\sqrt{x^2-1}}$

$\frac{1\ /:x}{-x+\sqrt{x^2-1}\ /:x} \quad \frac{\frac{1}{x}}{-\frac{x}{x}+\sqrt{\frac{x^2}{x^2}-\frac{1}{x^2}}} = \frac{0}{-1+1} = \frac{0}{0}$

KOSE: NEMA

JOSIP PREDOVAN

③ $g(x) = \ln(x^2+1)$ DOMENA

$x^2+1 > 0$

$D_f = \langle -\infty, +\infty \rangle$

PARNOST NEPARNOST

$f(-x) \ln((-x)^2+1)$

$= \ln(x^2+1)$

FUNKCIJA JE PARNA

DERIVACIJE

$g(x) \ln(x^2+1)$

$g(x)' = \frac{1}{x^2+1} \cdot (x^2+1)'$

$g(x)' = \frac{1}{x^2+1} \cdot 2x$

$g(x)' = \frac{2x}{x^2+1}$

$g(x)'' = \frac{(2x)' \cdot (x^2+1) - (2x) \cdot (x^2+1)'}{(x^2+1)^2}$

$g''(x) = \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$

$g''(x) = \frac{2x^2+2-2x^2}{(x^2+1)^2} = \frac{2}{(x^2+1)^2}$

⑥ $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} = \frac{(-2)^2-4}{(-2)^3+2\cdot(-2)^2-2+2} = \frac{0}{0}$

$\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} \overset{L'H}{=} \frac{(x^2)'-(4)'}{(x^3)'+(2x^2)'+(x)'+(2)'} \lim_{x \to -2} \frac{2x}{3x^2+4x+1+0} = \frac{2x \quad /:x^2}{3x^2+4x+1 \quad /:x^2}$

$= \frac{\frac{2x}{x^2}}{\frac{3x^2}{x^2}+\frac{4x}{x^2}+\frac{1}{x^2}} = \frac{0}{3} = 0$

① $z^3 + |3+4i| = \frac{5}{i}$

$z^3 + 3 - 4i = \frac{5}{i}$

$z^3 = \frac{5}{i} - 3 + 4i \;/ : i$

$z^3 = 5 - 3 + 4$

$z^3 = 6$

$z = \sqrt[3]{6}$

$z = 1{,}81$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ £2

IME I PREZIME: Anamarija Jozić **BROJ INDEKSA:** 17-2-0104-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

25

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6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} \overset{/:x^3}{\overset{/:x^3}{=}}$ 10

Ukupno:

3. $g(x) = \ln(x^2+1)$

a) DOMENA

$x^2+1 > 0$

$g(x) = x^2+1$

$a = 1 > 0$, ∪

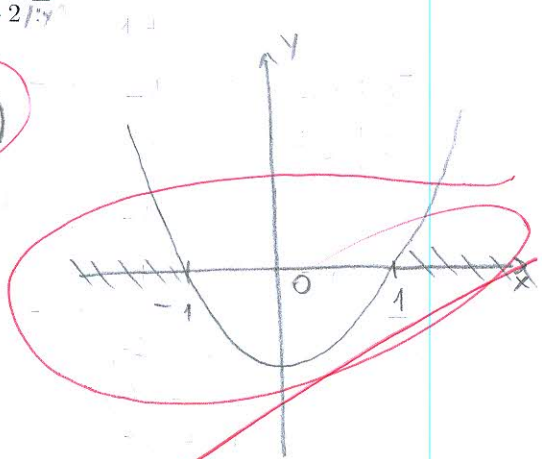
$x^2+1 \neq 0$

$x^2 = -1$

$x = \pm\sqrt{1}$

$x_1 = 1$

$x_2 = -1$



$D(f) = \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle$

b) ne/parnost

$g(x) = g(-x) =$

funkcija je parna ✓

) 2. derivacija

$$g(x) = \ln(x^2+1)$$

$$g'(x) = \frac{1}{x^2+1} \cdot (x^2+1)'$$

$$g'(x) = \frac{1}{x^2+1} \cdot 2x$$

$$g'(x) = \frac{2x}{x^2+1}$$

$$g''(x) = \frac{(2x)' \cdot (x^2+1) - (2x) \cdot (x^2+1)'}{(x^2+1)^2}$$

$$= \frac{2 \cdot (x^2+1) - 2x \cdot (2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$$g''(x) = \frac{-2x^2+2}{(x^2+1)^2}$$

K.A.

$$y = kx + l$$

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{x - \sqrt{x^2-1}}{x}$$

(2.) $f(x) = x - \sqrt{x^2-1}$

$$x^2 - 1 \geq 0$$

$f(x) = x^2 - 1$;

$a = 1 > 0$

$$x^2 - 1 \neq 0$$

$$x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x = 1$$

$$x = -1$$

D: $\langle -\infty, 1 \rangle \cup \langle 1, +\infty \rangle$

V.A.

$$\lim_{x \to 1^+} (x - \sqrt{x^2-1}) = 1 - \sqrt{1^2-1} = 1$$

$$\lim_{x \to -1^-} (x - \sqrt{x^2-1}) = -1 - \sqrt{(-1)^2-1} = -1$$

NEMA V.A.

H.A.

$$\lim_{x \to +\infty} (x - \sqrt{x^2-1}) \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}}$$

$$= \lim_{x \to \infty} \frac{x^2 + (\sqrt{x^2-1})^2}{x + \sqrt{x^2-1}} = \frac{x^2 + x^2 - 1}{x + \sqrt{x^2-1}}$$

$$= \frac{2x^2-1 \;/:x^2}{x + \sqrt{x^2-1}\;/x^2} = \frac{2-1}{\frac{1^0}{x} + \sqrt{1-\frac{1}{x^2}}} = \frac{1}{1} = 1$$

K.A.

5)
$$\begin{bmatrix} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{bmatrix} \sim$$

$3r \leftrightarrow 1r$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{bmatrix} \sim$$

$1r \cdot (-2) + 2r$
$1r \cdot (-4) + 3r$
$1r \cdot (-2) + 4r$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{bmatrix} \sim$$

$2r : 3$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -4/3 & -5/3 & -4/3 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{bmatrix} \sim$$

$2r \cdot (-3) + 3r$
$2r \cdot (-3) + 4r$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -4/3 & -5/3 & -4/3 \\ 0 & 0 & -3 & 3 & 6 \\ 0 & 0 & 1 & -1 & -2 \end{bmatrix} \sim$$

$4r \leftrightarrow 3r$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -4/3 & -5/3 & -4/3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & -3 & 3 & 6 \end{bmatrix} \sim$$

$3r \cdot 3 + 4r$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -4/3 & -5/3 & -4/3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rješenje je parametarsko

Sustav je neodređen, ima beskonačno rješenja

PROVJERA:

$d = \lambda, \quad \lambda \in \mathbb{R}$ — λ - proizvoljno

$c + d = -2$

$c + \lambda = -2$

$c = -2 - \lambda$

$b + c = -\frac{4}{3}$

$b + (-2) - \lambda = -\frac{4}{3}$

$b = -\frac{4}{3} + 2 + \lambda$

$b = \frac{2}{3} + \lambda$

λ - proizvoljno

$\lambda = \frac{1}{3} \qquad d = \frac{1}{3}$

$b = \frac{2}{3} + \frac{1}{3} = 1$

$c = -2 - \frac{1}{3} = -\frac{7}{3}$

$a + b = 3$

$a + \frac{2}{3} + \lambda = 3$

$a = 3 - \frac{2}{3} - \lambda$

$a = \frac{7}{3} - \lambda$

$a = \frac{7}{3} - \frac{1}{3} = 2$

Anamarija Jozić

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ £2

IME I PREZIME: LUKA ĐEKOVIĆ **BROJ INDEKSA:** 17-2-0079-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

15

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

(1.) $z^3 + |3+4i| = \frac{5}{i}$

$z^3 + \sqrt{3^2+4^2} = \frac{5}{i} \cdot \frac{i}{i}$

$z^3 + \sqrt{9+16} = \frac{5i}{i^2}$

$z^3 + 5 = \frac{5i}{-1}$

$z^3 + 5 + 5i = 0$

$z^3 = -5 - 5i$

(2.) $f(x) = x - \sqrt{x^2-1}$

DOMENA

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq \pm 1$

$D_f: [-1,1] \cup [1,+\infty\rangle$

V.A.

$\lim_{x \to 1^+} x - \sqrt{x^2-1} = \lim_{x \to 1^+} 1 - \sqrt{(1^+)^2-1} = 1^+ - 0^+ = 1^+$

$\lim_{x \to 1^-} x - \sqrt{x^2-1} = \lim_{x \to 1^-} 1^- - 0^- = 1^-$

NEMA V.A.

H.A. $\lim_{x \to \infty} x - \sqrt{x^2-1} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \frac{\lim_{x \to \infty} x^2 + x^2 - 1}{\lim_{x \to \infty} x + \sqrt{x^2-1}} = \frac{\lim_{x \to \infty} 2x^2 - 1}{\lim_{x \to \infty} x + \sqrt{x^2-1}}$

$\lim_{x \to \infty} \frac{2x^2-1 \,/:x^2}{x+\sqrt{x^2-1} \,/:x^2} = \frac{2 - \frac{1}{x^2}}{\frac{x}{x^2} + \frac{\sqrt{x^2-1}}{x^2}} = \frac{1}{0} = \infty$

NEMA H.A.

K.A. $(-x^2+1) : (x) = -x$

$\pm x^2 \mp 1$

V.A. $= -x$

3. $g(x) = \ln(x^2+1)$

° DOMENA

$x^2 + 1 > 0$

$x^2 > -1$

$x > \sqrt{-1}$

$D_f = \mathbb{R}$

PARNOST / NEPARNOST

$g(-x) = \ln((-x)^2 + 1)$

$= \ln(x^2+1)$

FUNKCIJA NIJE NITI PARNA NITI NEPARNA

$g(x) = \ln(x^2+1)$

$g'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

$g''(x) = \frac{(2x)' \cdot (x^2+1) - (2x) \cdot (x^2+1)'}{(x^2+1)^2} = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2} = -\frac{2x^2+2}{(x^2+1)}$

6. $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} = \lim_{x \to -2} \frac{(-2)^2-4}{(-2)^3+2(-2)^2-2+2} = \frac{4-4}{-8+8} = \frac{0}{0} = \infty$

LUKA DEKOVIĆ
21.3.2013.

5.

$$\begin{bmatrix} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{bmatrix} \begin{matrix} \\ /\cdot(-2) \\ \\ /\cdot(-4) \\ /\cdot(-2) \end{matrix} \sim \begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{bmatrix} /:3$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{bmatrix} /\cdot 1 \; /\cdot(-3) \sim \begin{bmatrix} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 0 & -4 & 7 & 11 \\ 0 & 0 & -1 & 4 & 5 \end{bmatrix} /\cdot(-1)$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & -4 & 7 & 11 \end{bmatrix} /(-1) \; /\cdot 1 \; /(4) \sim \begin{bmatrix} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & -7 & -8 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & -9 & -9 \end{bmatrix} /:(-9) \sim \begin{bmatrix} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & -7 & -8 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} /\cdot 4 \; /\cdot 7 \; /\cdot(-2)$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$4a - b + c + 2d = 14 \Rightarrow (4\cdot 3) - (-1) + (-1) + (2\cdot 1) = 14$

$12 + 1 - 1 + 2 = 14$ ✓

$2a + b - 3d = 2$ $\quad (2\cdot 3) - 1 - (3\cdot 1) = 2$ ✓

$a - b + 2c + d = 3$ $\quad 3 + 1 - 2 + 1 = 3$ ✓

$2a + b + c - 4d = 0$ $\quad (2\cdot 3) - 1 - 1 + 4 = 0$ ✓

4. $h(x) = \frac{x^2-1}{x^2+1}$

① DOMENA

$x^2+1 \neq 0$

$x^2 \neq -1$

$x \neq \sqrt{-1}$ $\quad D_f = \mathbb{R}$

② NULTOČKE

$h(x) = 0$

$\frac{x^2-1}{x^2+1} = 0 \quad / \cdot (x^2+1)$

$x^2 - 1 = 0 \qquad N(1,0)$

$x^2 = 1$

$x = \pm 1$

③ ASIMPTOTE

V.A.

$\lim_{x \to 1^+} \frac{x^2-1}{x^2+1} = \lim_{x \to 1^+} \frac{(1^+)^2-1}{(1^+)^2+1} = \frac{0^+}{2} = +\infty$

$\lim_{x \to 1^-} \frac{x^2-1}{x^2+1} = \frac{0^-}{2} = -\infty$

} V.A. $x = 1$

H.A.

$\lim_{x \to \infty} \frac{x^2-1 \,/:x^2}{x^2+1 \,/:x^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$ $\quad$ H.A. $\Rightarrow$ NEMA.

K.A.

$y = kx + l$

$k = \frac{f(x)}{x} = \left(\frac{\frac{x^2-1}{x^2+1}}{x} \right) = \frac{x^2+1 \,/:x^2}{x^2-1 \,/:x^2} = \frac{1}{-1} = -1 \Rightarrow k = -1$

$l = f(x) - kx = \frac{x^2-1}{x^2+1} \cdot (-x) = \frac{-x^3+x}{x^2+1} = -x^2 + x$

$y = -x - x^2 + x$

x	-1	0	1
y	1	0	-1

④ Ekstrem.

$h(x) = \frac{x^2-1}{x^2+1} = \frac{(x^2-1)' \cdot (x^2+1) - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$

$h'(x) = \frac{4x}{(x^2+1)^2}$

$h''(x) = 4\left(\frac{x}{(x^2+1)^2}\right)' = 4\left(\frac{x'(x^2+1)^2 - x((x^2+1)^2)'}{(x^2+1)^4}\right) = 4\left(\frac{(x^2+1)^2 - x(2(x^2+1)\cdot 2x)}{(x^2+1)^4}\right)$

$= 4\left(\frac{-(2x^2+1)\cdot 2x}{(x^2+1)^2}\right) = 4\left(\frac{-12x^3+2x}{(x^2+1)^2}\right) = \frac{4(-2x^3-2x)}{(x^2+1)^2}$

$= \frac{-8x^3-8x}{(x^2+1)^2} \quad \times \Rightarrow h''(x) = \frac{-8x^3-8x}{(x^2+1)^2}$

LUKA DEKOVIĆ

21-2-2013.

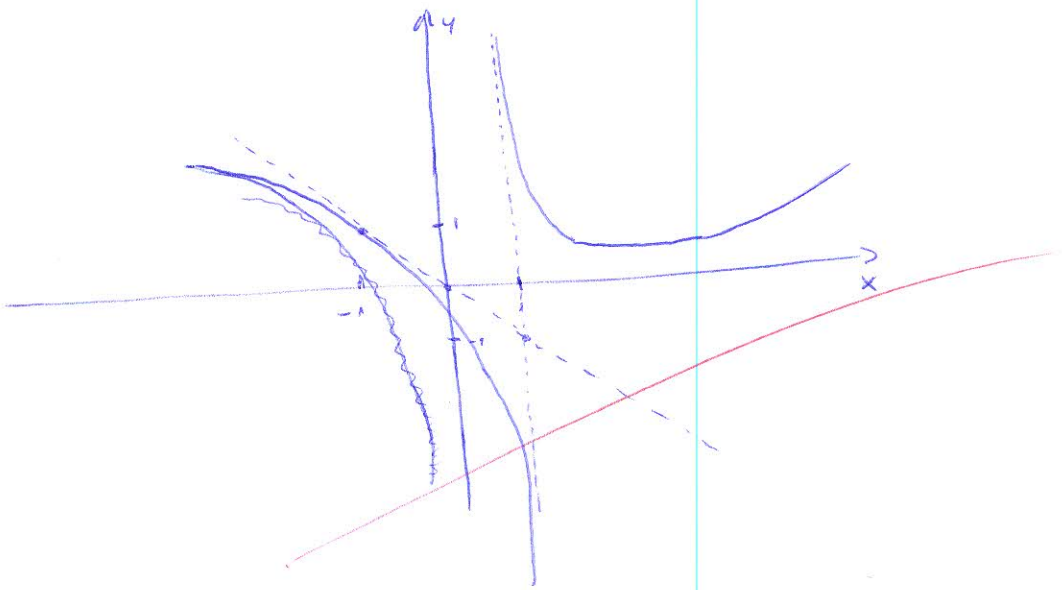
$h(x) = 0$

$$\frac{x^2-1}{x^2+1} = 0$$

$$x^2 = 1$$

$$x_{1,2} = \pm 1$$

	$-\infty$	-1		1		$+\infty$
$f'(x)$		$-$	n/d		$+$	
$f(x)$		↘	-1		↗	



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ Ł2

IME I PREZIME: MARINO Šojić **BROJ INDEKSA:** 0269059771

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

6

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3

2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15

3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10

4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)

5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{L}2$

IME I PREZIME: TONI LULIĆ **BROJ INDEKSA:** 5796

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

10

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3

2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15

3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10

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$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

6. $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} = \left[\frac{0}{0}\right]$

$\lim_{x \to -2} \frac{2x}{3x^2+4x+1} = \frac{2 \cdot (-2)}{3 \cdot 4 + 4 \cdot (-2) + 1} = \frac{-4}{5}$

3. $g(x) = \ln(x^2+1)$

$\ln(x^2+1)$

$x^2+1 \neq 0$

$Df: \langle 0, +\infty \rangle$

$f'(x) = \frac{1}{x^2+1} \cdot (x^2+1)'$

$f'(x) = \frac{1}{x^2+1} \cdot 2x + 0$

$f'(x) = \frac{2x}{x^2+1}$

$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$f''(x) = \frac{(2x)' \cdot (x^2+1) - 2x \cdot (x^2+1)'}{(x^2+1)^2}$

$= \frac{2 \cdot (x^2+1) - 2x \cdot (2x)}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$= \frac{2(x^2+1-2x^2)}{(x^2+1)^2}$

4. $f(x)=\frac{x^2-1}{x^2+1}$

1. Domena

$x^2+1=0$

$x^2\neq 1$

$x\neq -1 \quad i \quad x\neq 1$

$Df: \langle -\infty,-1\rangle \cup \langle -1,1\rangle \cup \langle 1,+\infty\rangle$

2. ASIMPTOTE:

V.A. $\lim_{x\to 1^-} \frac{x^2+1}{x^2-1} = \frac{2}{0^+} = +\infty$

$\lim_{x\to 1^+} \frac{x^2+1}{x^2-1} = \frac{2}{0^-} = -\infty$

$\lim_{x\to -1^-} \frac{x^2+1}{x^2-1} = \frac{2}{0^-} = -\infty$

$\lim_{x\to -1^+} \frac{x^2+1}{x^2-1} = \frac{2}{0^+} = +\infty$

3. GLOBALNA SVOJSTVA

- nije periodična

$f(x)=\frac{(-x)^2+1}{(-x)^2-1}=\frac{x^2+1}{x^2-1}$

funkcija je parna

4. NULTOČKE

$f(x)=0$

$\frac{x^2+1}{x^2-1}=0$

$x^2+1=0$

$x=\sqrt{-1}$

$f(0)=\frac{0^2+1}{0^2-1}=-1$

$T(0,-1)$

5. DERIVACIJE

$f'(x)=\frac{-4x}{(x^2-1)}$

$f''(x)=\frac{4(3x^2+1)}{(x^2-1)^3}$

b) H.A.

$\lim_{x\to\infty} \frac{x^2+1}{x^2-1} \;|:x^2 = \frac{1}{1}=1$

$\lim_{x\to\infty} \frac{x^2+1}{x^2-1} = \begin{cases} x\to(-x) \\ -\infty\to+\infty=\infty \end{cases}$

$\lim_{x\to\infty} \frac{(-x)^2+1}{(-x^2)-1}=1$

$y=1$ KOSE AS. NEMA!!

6. KRITIČNE TOČKE

$f'(x)=0 \qquad f(0)=1$

$-\frac{4x}{(x^2-1)}=0 \qquad T(0,-1)$

$-4x=0$

$x=0$

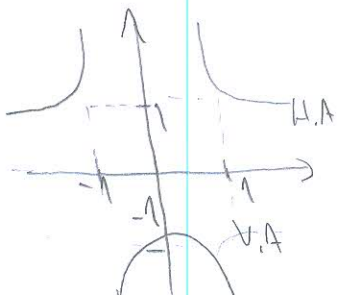
7. MONOTONOST

$-\frac{4x}{(x-1)^2}<0$

	$-\infty$		-1		0		1		$+\infty$
f'		+		+		−		−	
f		↗		↗		↘		↘	

10. ZAKRIVLJENOST

	$-\infty$		-1		1		$+\infty$
$f''(x)$		+		−		+	
$F(x)$		∪		∩		∪	



TONI ĆULIĆ
5796.
21.2.2013.

2. $f(x)=x-\sqrt{x^2-1}$

$x \neq 1$

$\sqrt{x^2-1} \geq 0$

$Df: [0, +\infty\rangle$

5. $\begin{bmatrix} 4 & -1 & 1 & 2 & | & 14 \\ 2 & 1 & 0 & -3 & | & 2 \\ 1 & -1 & 2 & 1 & | & 3 \\ 2 & 1 & 1 & -4 & | & 0 \end{bmatrix} \begin{matrix} I-III \\ \\ \\ IV-II \end{matrix} \sim \begin{bmatrix} 3 & 0 & -1 & 1 & | & 11 \\ 2 & 1 & 0 & -3 & | & 2 \\ 1 & -1 & 2 & 1 & | & 3 \\ 0 & 0 & 1 & 1 & | & 0 \end{bmatrix} \begin{matrix} I-II \\ \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & -1 & -1 & 4 & | & 9 \\ 2 & 1 & 0 & -3 & | & 2 \\ 1 & -1 & 2 & 1 & | & 3 \\ 0 & 0 & 1 & 1 & | & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ II-III \end{matrix}$

$\begin{bmatrix} 1 & -1 & -1 & 4 & | & 9 \\ 1 & 2 & -2 & -4 & | & -1 \\ 1 & -1 & 2 & 1 & | & 3 \\ 0 & 0 & 1 & 1 & | & 0 \end{bmatrix} \begin{matrix} III-I \\ \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & -1 & -1 & 4 & | & 9 \\ 1 & 2 & -2 & -4 & | & -1 \\ 0 & 0 & 3 & -3 & | & -6 \\ 0 & 0 & 1 & 1 & | & 0 \end{bmatrix} \begin{matrix} II-I \\ III-2IV \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & -1 & -1 & 4 & | & 9 \\ 0 & 3 & 1 & -8 & | & -10 \\ 0 & 0 & 1 & -5 & | & -6 \\ 0 & 0 & 1 & 1 & | & 0 \end{bmatrix}$

21.2.2013.

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ £2

IME I PREZIME: MIŠEL GOBIN **BROJ INDEKSA:** 17-1-0034-2010

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

0

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3

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6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

5. $\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ Ł2

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ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK
Broj ↓ bodova

0

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. *Prikaži ih u kompleksnoj ravnini!* 12+3

2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15

3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10

4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)

5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

6. Izračunati: $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2} =$ 10

Ukupno:

1. $z^3 + |3+4i| = \frac{5}{i}$

$z^3 = \frac{5}{i} + 3 + 4i \quad /\cdot i$

$z^3 = 5 + 3i - 4$

$z^3 = 1 + 3i$

$z^3 = r(\cos\alpha + i\sin\alpha)$

$z^3 = 2 \cdot (\cos\frac{3\pi}{3} + \frac{2k\pi}{3} + i\sin\frac{3\pi}{3} + \frac{2k\pi}{3}) \quad /\sqrt[3]{}$

$z = \sqrt[3]{2}\,(\cos\frac{\pi + 2k\pi}{3} + i\sin\frac{\pi + 2k\pi}{3})$

$k = 0, 1, 2$

$\operatorname{tg}\alpha = \frac{3}{1} = 3$

$\alpha = 3\pi$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{1+3}$

$r = 2$

5.
$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{matrix} \text{I} - 3\text{III} \\ \\ \\ \\ \end{matrix} = \left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & 5 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{matrix} \text{II} - 2\text{I} \\ \text{III} - \text{I} \\ \text{IV} - 2\text{I} \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & 5 \\ 0 & -3 & 10 & -1 & -8 \\ 0 & -3 & 7 & 2 & -2 \\ 0 & -4 & 11 & -2 & -10 \end{array}\right] \begin{matrix} \\ \\ \\ \text{IV} - \text{II} \end{matrix} = \left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & 5 \\ 0 & -3 & 10 & -1 & -8 \\ 0 & -3 & 7 & 2 & -2 \\ 0 & -4 & 11 & -2 & -10 \end{array}\right] \begin{matrix} \\ \text{III} + 3\text{II} \\ \text{IV} + 4\text{II} \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & -5 & -1 & 5 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & 4 & 5 & 4 \\ 0 & 0 & 7 & 2 & -2 \end{array}\right] \begin{matrix} \text{I} - 2\text{II} \\ \\ \\ \\ \end{matrix} = \left[\begin{array}{cccc|c} 1 & 0 & -3 & -3 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & \frac{4}{5} & 4 \\ 0 & 0 & 7 & 2 & -2 \end{array}\right] \begin{matrix} \text{I} + 3\text{III} \\ \text{II} + \text{III} \\ \\ \text{IV} - 7\text{III} \end{matrix}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{5} & 13 \\ 0 & 1 & 0 & \frac{9}{5} & 6 \\ 0 & 0 & 1 & \frac{4}{5} & 4 \\ 0 & 0 & 0 & -\frac{18}{5} & -30 \end{array}\right] \cdot\left(-\frac{18}{5}\right) = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{5} & 13 \\ 0 & 1 & 0 & \frac{9}{5} & 6 \\ 0 & 0 & 1 & \frac{4}{5} & 4 \\ 0 & 0 & 0 & 1 & 108 \end{array}\right] \begin{matrix} \text{I} + \frac{2}{5}\text{IV} \\ \text{II} - \frac{9}{5}\text{IV} \\ \text{III} - \frac{4}{5}\text{IV} \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{287}{5} \\ 0 & 1 & 0 & 0 & -\frac{394}{5} \\ 0 & 0 & 1 & 0 & \frac{412}{5} \\ 0 & 0 & 0 & 1 & 108 \end{array}\right]$$

6. $\lim_{x \to -2} \frac{x^2 - 4}{x^3 + 2x^2 + x + 2} = \frac{4 - 4}{-8 + 8 - 2 + 2} = \frac{0}{0}$