

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

BORIS DURBIĆ

BROJ INDEKSA:

57640

- Izračunati volumen tijela omeđenog plohami $z = x^2 + y^2$, $z = 5$. 20
- Neka je C plašt cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z -osi, visine $v = 2$ s centrom u ishodištu ($z \in [-1, 1]$). Podrazumijeva se orijentacija plašta cilindra prema van. Izračunati $\iint_C 2x + 3dydz$? 20
- Primjenom Greenove formule izračunati integral $\oint_C 2(x^2 + y^2)dx + (x + y)^2dy$, gdje je C kontura trokuta $A(1, 1)$, $B(2, 2)$ i $C(1, 3)$ prijeđena u pozitivnom smislu (suprotno od kazaljke na satu) 20
- Provjeri da li je $g(x, y, z) = (x + y, x + y, 1)$ potencijalno polje? Koja vrsta integrala se lagano rješava u potencijalnom polju? 15-5
- Zadana je kružna uzvojnica (spirala) s jednadžbama $x = 2\cos t$, $y = 2\sin t$ i $z = t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara periodu iskorištenih trigonometrijskih funkcija) 20

Ukupno:

95

$$1. \quad x^2 + y^2 = z \quad z = 5 \quad r \in [0, \sqrt{z}]$$

$$r^2 = z \quad z \in [0, 5]$$

$$r = \sqrt{z}$$

$$\iiint_0^5 \int_0^{\sqrt{z}} \int_0^{\sqrt{z}} r dr dz dy = \int_0^5 \int_0^{\sqrt{z}} \frac{r^2}{2} \Big|_0^{\sqrt{z}} dz dy = 2\pi \cdot \frac{1}{2} \cdot \int_0^5 z dz = \pi \cdot \frac{z^2}{2} \Big|_0^5$$

$$= \frac{25\pi}{2}$$

$$5. \quad x = 2\cos t$$

$$y = 2\sin t$$

$$z = t$$

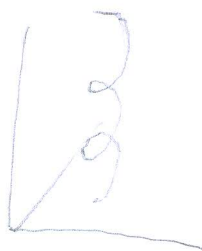
$$r' = \begin{bmatrix} -2\sin t \\ 2\cos t \\ 1 \end{bmatrix}$$

$$|r'| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1^2}$$

$$= \sqrt{4\sin^2 t + 4\cos^2 t + 1}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t) + 1} = \sqrt{5}$$

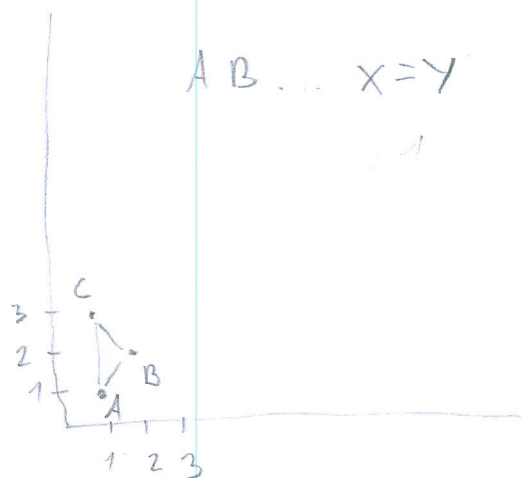
$$\int_0^{6\pi} \sqrt{5} dt = 6\sqrt{5}\pi$$



$$3. \oint_P 2(x^2+y^2)dx + (x+y)^2 dy$$

$$2(x+y) \cdot 1 - 2 \cdot 2y \quad \checkmark$$

$$= 2x + 2y - 4y = 2x - 2y \quad \checkmark$$



$$CB \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 3 = \frac{2 - 3}{2 - 1} (x - 1)$$

$$y - 3 = \frac{-1}{1} (x - 1)$$

$$y - 3 = -x + 1$$

$$y = -x + 4$$

$$\int_1^2 \int_x^{-x+4} (2x - 2y) dy dx = \int_1^2 2xy - 2 \frac{y^2}{2} \Big|_x^{-x+4} dx$$

$$= \int_1^2 (2 \cdot x \cdot (-x + 4) - x^2 + 8x - 16 - 2x^2 + x^2) dx$$

$$= \int_1^2 (-2x^2 + 8x - x^2 + 8x - 16 - 2x^2 + x^2) dx$$

$$= \int_1^2 (-4x^2 + 16x - 16) dx = \left[-4 \frac{x^3}{3} + 16 \frac{x^2}{2} - 16x \right]_1^2$$

$$= -\frac{4 \cdot 8}{3} + 32 - 32 + \frac{4}{3} - 8 + 16 =$$

$$= \frac{-32 + 4 + 24}{3} = -\frac{4}{3}$$

$$2, r=1 \quad v=2 \quad z(-1,1)$$

$$r = (\cos u, \sin u, v) \checkmark$$

$$\frac{\partial r}{\partial u} = \begin{bmatrix} -\sin u \\ \cos u \\ 0 \end{bmatrix} \quad \frac{\partial r}{\partial v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\int_0^1 \int_{-1}^1 \int_0^{2\pi} \begin{bmatrix} 2 \cdot \cos u + 3 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \cos u \\ \sin u \\ 0 \end{bmatrix} dv du \checkmark$$

$$h = \begin{bmatrix} -\sin u & 0 \\ \cos u & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos u \\ \sin u \\ 0 \end{bmatrix} \checkmark$$

$$x = \cos u \quad y = \sin u$$

$$\int_0^1 \int_{-1}^1 (2 \cos^2 u + 3 \cos u) dv du \checkmark$$

$$\int_0^1 \int_{-1}^1 v \cdot 2 \cos^2 u + v \cdot 3 \cos u \Big|_{v=-1}^{v=1} du = 4 \int_0^1 \cos^2 u du + 6 \int_0^1 \cos u du$$

$$= 4 \int_0^1 \frac{1 + \cos(2u)}{2} + 6 \sin u \Big|_0^1$$

$$= 4 \cdot \frac{1}{2} \cdot 2\pi + 4 \cdot \frac{1}{2} \cdot 2 \sin(2u) \Rightarrow 4\pi \checkmark$$

VEKTORSKE FUNKCIJE

4. b) krivoljni integrali u potencijalnom polju

$$f(x,y) = x + y / \sqrt{dx}$$

$$f = \frac{x^2}{2} + yx$$

$$C(z) = 1 / \sqrt{dz}$$

$$Cz = z$$

$$f = \frac{x^2}{2} + yx + \frac{y^2}{2} + z \quad \times$$

funkcija $g(x,y,z)$ je u potencijalnom polju

$$\frac{\partial}{\partial y} (x^2 + yx + C(y,z)) = x + y$$

$$x + \frac{\partial g}{\partial x} = x + y / \sqrt{dx}$$

$$C(y) = y^2$$

$$\frac{\partial}{\partial z} (x^2 + yx + y^2 + C(z)) = 1$$

$$\int_0^{2\pi} \int_{-1}^1 2\cos^2 v \, du \, dv + \int_0^{2\pi} \int_0^1 3\cos v \, du \, dv \quad \underline{15}$$

3. $\oint 2(x^2+y^2) dx + (x+y)^2 dy$
 $\int P dx + Q dy$

$$\int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx \, dy$$

$$\boxed{\frac{\partial Q}{\partial x} = 2x+2y} \quad \frac{\partial P}{\partial y} = 4y$$

$$\iint (2x+2y-4y)$$

$$\boxed{\iint (2x-2y)}$$

$$(y-y_1)(x_2-x_1) = (x-x_1)(y_2-y_1)$$

$$\int_1^2 \int_1^y (2x-2y) dx \, dy \quad \checkmark$$

$$+ \int_2^3 \int_1^{(y+1)} (2x-2y) dx \, dy \quad \checkmark$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$P = 2x^2 - 2y^2$$

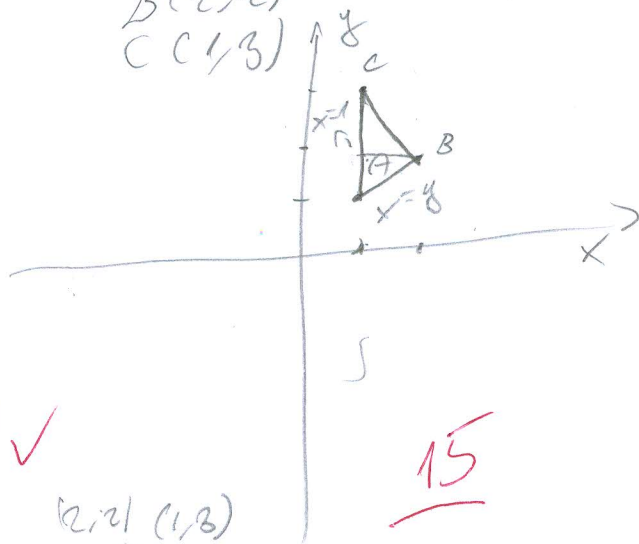
$$Q = (x+y)^2$$

$$Q = x^2 + 2xy + y^2$$

$$A(1,1)$$

$$B(2,2)$$

$$C(1,3)$$



$$2,2 \quad (1,3)$$

$$BC$$

$$(y-2)(1-2) = (x-2)(3-2)$$

$$(y-2)(-1) = (x-2)(1)$$

$$-y+2 = x-2$$

$$-y = x-2-2$$

$$-y = x-4 \quad | \cdot (-1)$$

$$y = -x+4$$

$$-x = y-2-2$$

$$-x = y-4$$

$$x = -y+4$$

15

5. $x = 2 \cos t$
 $y = 2 \sin t$
 $z = t$

$f(x, y, z) = 11x^2 y$
 $t \in [0, 6\pi]$

$r = \begin{pmatrix} 2 \cos t \\ 2 \sin t \\ t \end{pmatrix}$

$r' = \begin{pmatrix} -2 \sin t \\ 2 \cos t \\ 1 \end{pmatrix}$

$\|r'\|$

$= \sqrt{(-2 \sin t)^2 + (2 \cos t)^2 + 1}$

$= \sqrt{4 \sin^2 t + 4 \cos^2 t + 1}$

$= \sqrt{4(\sin^2 t + \cos^2 t) + 1}$

$= \sqrt{4 \cdot (1) + 1}$

$= \sqrt{4 + 1}$

$= \sqrt{5}$

$\int_0^{2\pi} 1 \cdot \sqrt{5} dt$

$\int_0^{6\pi} 1 \cdot \sqrt{5} dt$ ✓

3. $x = 2y^2 - 1$
 $y = 2z^2 - 1$
 $z = 2t^2 - 1$
 $t \in [0, 1]$
 $f(x, y, z) = 2x^2 - 3y^2 + 4z^2$
 $f(2y^2 - 1, 2z^2 - 1, 2t^2 - 1) = 2(2y^2 - 1)^2 - 3(2z^2 - 1)^2 + 4(2t^2 - 1)^2$
 $= 2(4y^4 - 4y^2 + 1) - 3(4z^4 - 4z^2 + 1) + 4(4t^4 - 4t^2 + 1)$
 $= 8y^4 - 8y^2 + 2 - 12z^4 + 12z^2 - 3 + 16t^4 - 16t^2 + 4$
 $= 8y^4 - 12z^4 + 16t^4 - 8y^2 + 12z^2 - 16t^2 + 3$
 $= 8(2z^2 - 1)^4 - 12(2t^2 - 1)^4 + 16(2t^2 - 1)^4 - 8(2z^2 - 1)^2 + 12(2t^2 - 1)^2 - 16(2t^2 - 1)^2 + 3$
 $= 8(4z^4 - 8z^2 + 4) - 12(4t^4 - 8t^2 + 4) + 16(4t^4 - 8t^2 + 4) - 8(4z^2 - 4z + 1) + 12(4t^2 - 4t + 1) - 16(4t^2 - 4t + 1) + 3$
 $= 32z^4 - 64z^2 + 32 - 48t^4 + 96t^2 - 48 + 64t^4 - 128t^2 + 64 - 32z^2 + 32z - 8 + 48t^2 - 48t + 12 - 64t^2 + 64t - 16 + 16 + 3$
 $= 32z^4 - 48t^4 + 64t^4 - 64z^2 + 48t^2 - 64t^2 + 32z - 48t + 12 - 16 + 3$
 $= 32z^4 - 48t^4 + 64t^4 - 64z^2 + 48t^2 - 64t^2 + 32z - 48t + 12 - 16 + 3$

$-\frac{1}{2} - \frac{2}{3} - \frac{1}{2} + 1 =$

$= 12 - \frac{12}{3} - 2 + 3 - \frac{1}{2} - \frac{2}{3} - \frac{1}{2} + 1,$

$= \frac{12 - 4 - 6 + 9}{3} - 1 = \frac{11}{3} - 1 = \frac{8}{3}$

$\frac{12 - 4 - 6 + 9}{3} = \frac{11}{3}$

$\frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$

potencijsno polje:
 $(x+y) dy = 1 dz$

$$\int dx$$

$$= c(y) + \frac{\partial \phi}{\partial y}$$

$$-x-y \int dy$$

$$-\frac{y^2}{2} + c_1$$

$$-\frac{x^2}{2} - yx - \frac{y^2}{2} + c_2$$

$$= -1 \int dz$$

$$z = -z$$

$$-\frac{x^2}{2} - yx - \frac{y^2}{2} - z$$

$\phi(x,y,z)$ je potencijsno polje
a pomoću pot polje a lako riješeno krivolinijski integral

u vrste.

RUČE

Stape gawino

$$\textcircled{1} \int_0^{2\pi} \int_{-\sqrt{5}}^{\sqrt{5}} \int_{r^2}^5 1 \cdot r \, dz \, dr \, d\theta$$

$$\int_0^{2\pi} \int_{-\sqrt{5}}^{\sqrt{5}} \left[\frac{z}{r^2} \right]_{r^2}^5 r \, dr \, d\theta$$

$$\int_0^{2\pi} \int_{-\sqrt{5}}^{\sqrt{5}} (5r - r^2 \cdot r) \, dr \, d\theta$$

$$\int_0^{2\pi} \int_{-\sqrt{5}}^{\sqrt{5}} \left[\frac{5r^2}{2} - \frac{r^3}{3} \right]_{-\sqrt{5}}^{\sqrt{5}} d\theta$$

$$\int_0^{2\pi} \left(\frac{5 \cdot (\sqrt{5})^2}{2} - \frac{(\sqrt{5})^3}{3} \right) d\theta$$

$$\int_0^{2\pi} \left(\frac{25}{2} - \frac{25}{3} \right) d\theta$$

$$\int_0^{2\pi} \frac{50 - 25}{3} d\theta$$

$$\int_0^{2\pi} \frac{25}{3} d\theta$$

$$2\pi \cdot \frac{25}{3} = \frac{25}{2} \pi \quad \checkmark$$

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IME I PREZIME: BRUNO UPOŠIĆA

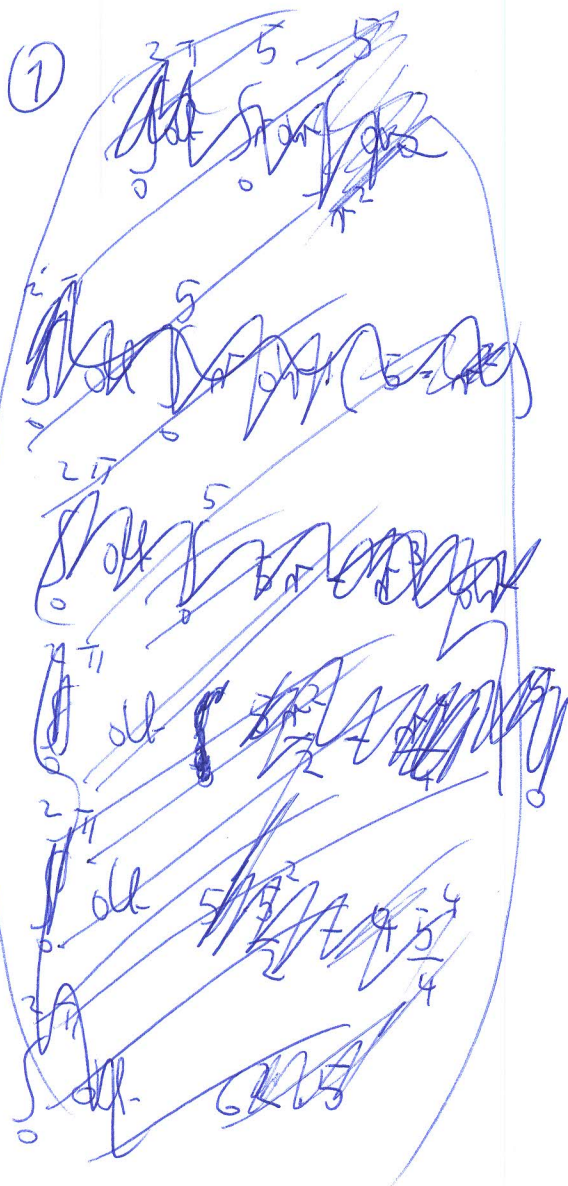
BROJ INDEKSA: 54960

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Ukupno:

20



~~z = x^2 + y^2~~
~~z = 5~~
~~z = 5~~
~~z = 5~~

$$\textcircled{1} \int_0^{2\pi} dt \int_0^5 \int_0^{\pi^2} r dr dz$$

$$\int_0^{2\pi} dt \int_0^5 \pi dr (\pi^2 - 5)$$

$$\neq (0, 2\pi) \checkmark$$

$$\neq (0, 5) \times$$

$$\neq (5, \pi^2) \times$$

$$\int_0^{2\pi} dt \int_0^5 (\pi^3 - 5\pi)$$

$$\int_0^{2\pi} dt \left[\frac{\pi^4}{4} - \frac{5\pi^2}{2} \right]_0^5$$

$$\int_0^{2\pi} \left[\frac{5^4}{4} - 5 \cdot \frac{5^2}{2} \right] dt$$

$$\int_0^{2\pi} \left[\frac{625}{4} - 62.5 \right] dt$$

$$\int_0^{2\pi} 156.25 - 62.5 dt$$

$$\int_0^{2\pi} 93.75 dt$$

$$93.75 \cdot 2\pi$$

⑤

$$x = 2 \cos t$$

$$y = 2 \sin t$$

$$z = t$$

$$\|r'\| = \sqrt{(2 \sin t)^2 + (2 \cos t)^2 + 1^2}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t) + 1}$$

$$= \sqrt{4(1) + 1}$$

$$= \sqrt{5}$$

$$= 3 \int_0^{2\pi} \sqrt{5} \quad \checkmark$$

$$\begin{pmatrix} 2 \cos t & -2 \sin t \\ 2 \sin t & 2 \cos t \\ t & 1 \end{pmatrix}$$

②

$$\int_0^{2\pi} \int_0^1 (2x + 3) \, dy \, dx$$

$$\int_0^{2\pi} \int_0^1 (2r \cos \theta + 3) r \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^1 (2r^2 \cos \theta + 3r) \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^1 2r^2 \cos \theta + 3r \, dr \, d\theta$$

$$\int_0^{2\pi} \left. 2 \frac{r^3}{3} \cos \theta + 3 \frac{r^2}{2} \right|_0^1 d\theta$$

$$x = r \cos \theta$$

$$dy \, dx$$

$$= r \, dr \, d\theta$$

X

$$\int_0^{2\pi} 2 \frac{1^3}{3} \cos y + 3 \frac{1^2}{2}$$

$$\int_0^{2\pi} \frac{2}{3} \cos y + \frac{3}{2}$$

$$\frac{2}{3} \sin y + \frac{3}{2} y \Big|_0^{2\pi}$$

$$\frac{2}{3} \sin 2\pi + \frac{3}{2} 2\pi - \left(\frac{2}{3} \sin 0 + \frac{3}{2} \cdot 0 \right)$$

$$0 + \frac{3}{2} \cdot 2\pi - (0)$$

$$= 3\pi \quad \times$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: LUKA HULJEV

BROJ INDEKSA: 58079

POPUNJAVA
NASTAVNIK
Broj ↓
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Ukupno:

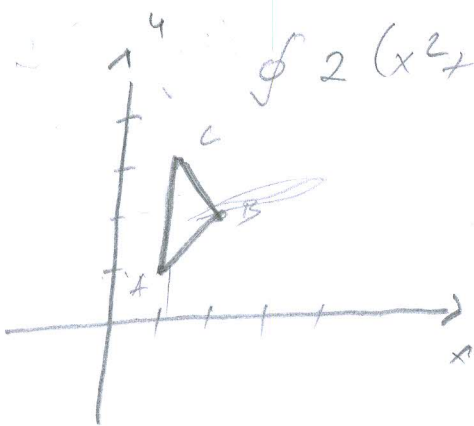
20

21.

3.

$$\oint_C 2(x^2 + y^2) dx + (x + y)^2 dy$$

$$\oint P dx + Q dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$



$$\oint 2(x^2 + y^2) dx + (x + y)^2 dy$$

$$= \iint \left(\frac{\partial (x + y)^2}{\partial x} - \frac{\partial 2(x^2 + y^2)}{\partial y} \right) dx dy$$

$$= \iint \left(\frac{x^2 + 2xy + y^2}{dx} - \frac{2x^2 + 2y^2}{dy} \right) dx dy$$

$$= \iint (2x + 2y - 4y) dx dy = \iint (2x - 2y) dx dy$$

$2xy dx$

$2x^2 dy$
 $2y^2 dy$

$$r(t) = \begin{bmatrix} 2\cos t \\ 2\sin t \\ 1 \end{bmatrix} \quad r'(t) = \begin{bmatrix} -2\sin t \\ 2\cos t \\ 0 \end{bmatrix}$$

$$|r'(t)| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 0^2}$$

$$= \sqrt{4\sin^2 t + 4\cos^2 t}$$

~~$$= \sqrt{4}$$~~

$$= \sqrt{4(\sin^2 t + \cos^2 t)}$$

$$= \sqrt{5} dt$$

3 namotaja · 2π = 6π

~~$$\int_0^{2\pi} \sqrt{5} dt = \sqrt{5} \int_0^{2\pi} dt = \sqrt{5} (t) \Big|_0^{2\pi}$$~~

$$= \sqrt{5} (2\pi - 0)$$

$2\pi\sqrt{5}$



3) NASTAVAK

LUKA HUGOVIĆ

A(0,1) B(2,2) C(1,3)

AB: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$

A(0,1)

B(2,2) $y - 1 = \frac{2 - 1}{2 - 0} (x - 0)$

$y - 1 = 1x - 1$

$y = 1x$

AC: $y - 1 = \frac{3 - 1}{1 - 0} (x - 0)$

A(0,1)

C(1,3)

$y - 1 = 2$

$y = 3$

BC: $y - 2 = \frac{3 - 2}{1 - 2} (x - 2)$

B(2,2)
C(1,3)

$y - 2 = \frac{1}{-1} (x - 2)$

$y - 2 = -1x + 2$

$y = -1x + 4$

~~$\int_1^2 2 \left(\frac{-1x+4}{2} \right)^2 - \left(\frac{-1x}{2} \right)^2 dx$~~

$\int_1^2 \int_{1x}^{-1x+4} (2x - 2y) dx dy$

$\int_1^2 \left(2 \int_{1x}^{-1x+4} x dx - 2 \int_{1x}^{-1x+4} y dx \right) dy$

$\int_1^2 \left(2 \int_{1x}^{-1x+4} \frac{x^2}{2} - 2 \int_{1x}^{-1x+4} x \right) dy$

$\int_1^2 \left(2 \frac{x^3}{6} - 2x^2 \right) dy$

