MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: STIPE VULIĆ

BROJ INDEKSA: 57663

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je $K$ krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,ds$? 20
2. Neka je $K$ krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orijentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,dy$? 20 15
3. Neka je $K$ kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,dxdydz$? 20
4. Neka je $K$ kugla radijusa $r = 1$ sa centrom u ishodištu. Izračunati preko definicije plošnog integrala $\iint\limits_{\partial K} 3\,dS$ 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

Ukupno: 35

①

$r=2 \qquad T(0,0)$

$$\int\limits_{\partial K} (2x+3)\,ds =$$

$x = r\cos t \Rightarrow x = 2\cos t$

$y = r\sin t \Rightarrow x = 2\sin t$

PARAMETRIZACIJA

$$= \int_0^{2\pi} (2\cdot(2\cos t)+3\cdot 1\,dt \quad \times$$

$$= \int_0^{2\pi} 4\cos t + 3\,dt$$

$$= \int_0^{2\pi} 4\cos t\,dt + \int_0^{2\pi} 3\,dt$$

$$= 4\int_0^{2\pi}\cos t\,dt + 3\int_0^{2\pi} dt$$

$$= 4\sin t\Big|_0^{2\pi} + 3t\Big|_0^{2\pi}$$

$$= 4(\sin 2\pi - \sin 0) + 3(2\pi - 0)$$

$$= 4\cdot 0 + 3\cdot 2\pi$$

$$= 6\pi$$

$r(t) = (2\cos t, 2\sin t)$

$r'(t) = (-\sin t, \cos t)$ $\quad r'(t) = (-2\sin t, 2\cos t)$

$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2}$ $\quad \|r'\| = 2$

$\|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t}$

$\|r'(t)\| = 1 \quad \times$

② $r=1$ $T(0,-1)$ SUPROTNO OD KAZAJKE NA SATU

$$\int_{\partial K}(2x+3)\,dy = ?$$

PARAMETRIZACIJA $W=\begin{pmatrix}0\\2x+3\end{pmatrix}$

$r(t) = (\overset{x}{\cos t}, \overset{y}{\sin t} - 1)$

$r'(t) = (-\sin t, \cos t)$ ✓

$$= \int_0^{2\pi} \begin{pmatrix}0\\2\cdot\cos t+3\end{pmatrix} \cdot \begin{pmatrix}-\sin t\\ \cos t\end{pmatrix}$$

✓

$$= \int_0^{2\pi} 2\cos^2 t\,dt + 3\cos t\,dt = 2\int_0^{2\pi}\cos^2 t\,dt + \overset{0}{\int_0^{2\pi}\cos t\,dt} =$$

$0$ ✗

15

$= \cancel{\int_0^{2\pi} \cos t\,dt}$ ✗

$\iint\limits_{\partial K} 3\,ds$

$\uparrow$

PLOŠNI INTEGRAL

DIVERGENCIJA?

$W = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$ ✗

PARAMETRIZACIJA S DISKA NA POLUSFERU

$r(u,v) = \left(u, v, \sqrt{1-u^2-v^2}\right)$ ✓

$\frac{\partial r}{\partial u} = \begin{pmatrix} 1 \\ 0 \\ \frac{u}{\sqrt{1-u^2-v^2}} \end{pmatrix}$ $\quad \frac{\partial r}{\partial v} = \begin{pmatrix} 0 \\ 1 \\ \frac{v}{\sqrt{1-u^2-v^2}} \end{pmatrix}$

$\vec{u} = \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = \begin{pmatrix} \frac{+u}{\sqrt{1-u^2-v^2}} \\ \frac{v}{\sqrt{1-u^2-v^2}} \\ 1 \end{pmatrix}$ ✓

$= \int_0^{2\pi}\int_0^1 \cancel{F \cdot ds \cdot \vec{u}} = \int_0^{2\pi}\int_0^1 \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \times \vec{u}$ ✗

$= \int_0^{2\pi}\int_0^1 3 \cdot \underbrace{P(D)}_{r^2\pi} = 3\pi$

$\iint 3\,ds = \int_0^{2\pi}\int_0^1 3 \cdot \|\vec{n}(r,\varphi)\|\,dr\,d\varphi$

OK

PLACEOVA TRANSFORMACIJA

57663

$$) + x'(t) = 0 \qquad x(0) = x''(0) = 1 \qquad x'(0) = 0$$

$$(s) - s^2 \cancel{x(0)}^{1} - s\cancel{x'(0)}^{0} - \cancel{x''(0)}^{1} + sF(s) - \cancel{x(0)}^{1} = 0$$

$$(s) - s^2 - 1 + sF(s) - 1 = 0$$

$$(s) + sX(s) = s^2 + 1 + 1$$

$$(s) + sx(s) = s^2 + 2$$

$$(s) = \frac{s^2+2}{s^3+1}$$

$$(s) = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad \Big/ \cdot \text{NAZIVNIK}$$

$$s^2+2 = A(s^2+1) + Bs + C(s)$$

$$s^2+2 = As^2 + A + Bs^2 + Cs$$

$$\text{uz } s^2 = A + B = 1 \quad \Rightarrow 2 + B = 1$$

$$\text{uz } s^1 = A = 2 \qquad B = -1$$

$$\text{uz } s^0 = C = 0$$

$A = 2$

$B = 1$

$C = 0$

$$f(s) = \frac{2}{s} + \frac{-1s + 0}{s^2+1}$$

$$f(s) = 2 - \frac{s}{s^2+1}$$

$$f(s) = 2 - \cos t \;//$$

| | |
|---|---|
| **MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.<br>**IME I PREZIME:** ANTE BOTICA **BROJ INDEKSA:** | POPUNJAVA NASTAVNIK<br>Broj ↓ bodova |

1. Neka je $K$ krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$? ~~20~~

2. Neka je $K$ krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$? ~~20~~

3. Neka je $K$ kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? ~~20~~

4. Neka je $K$ kugla radijusa $r = 1$ sa centrom u ishodištu. Izračunati preko definicije plošnog integrala $\iint\limits_{\partial K} 3\mathrm{d}S$ 20

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

Ukupno: 20

$X(0)=1$

$X'(0)=0$

$X''(0)=1$

(5.) $L(x'''(t)) + L(x'(t)) = 0$

$s^3 X(s) - s^2 \overset{1}{x(0)} - s\overset{0}{X'(0)} - \overset{1}{x''(0)} + sX(s) - x(0) = 0$

$s^3X(s) - s^2 - 1 + sX(s) - 1 = 0$

$s^3X(s) + sX(s) = s^2+2$

$X(s)(s^3+s) = s^2+2$

$X(s) = \dfrac{s^2+2}{s^3+s}$ $\quad s^3+s=0 \;;\; s(s^2+1) \;;\; s^2+1=0$

$s=0 \;;\; A \qquad s^2=-1$

$s=\sqrt{-1} - Bs+C$

$\dfrac{s^2+2}{s^3+s} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+1} \quad / \, s(s^2+1)$

$s^2+2 = A(s^2+1) + (Bs+C)s$

1° za $s=0$

$2 = A + 0$

$A=2$

2° za $s=1$

$3 = 2A + B + C$

$B+C+2A=3$

$B+C+4=3$

$B+C=-1$

3° za $s=2$

$6 = 2(4+1) + 2(2B+C)$

$6 = 10 + 4B + 2C \quad /:2$

$3 = 5 + 2B + C$

$-2 = 2B+C$

$2B+C=-2$

$C=-2-2B$

$B + (-2-2B) = -1$

$B - 2 - 2B = -1$

$-B = -1+2$

$-B = 1$

$B=-1$

$C = -2-2B$

$C = -2-2(-1)$

$C = -2+2 = 0$

$C=0$

5.

$2\left(\frac{1}{s}\right) = 2 \cdot 1$

$$\frac{s^2+2}{s^3+s} = \frac{2}{s} - \frac{s}{s^2+1}$$

$$X(s) = 2\left(\frac{1}{s}\right) - \cos t$$

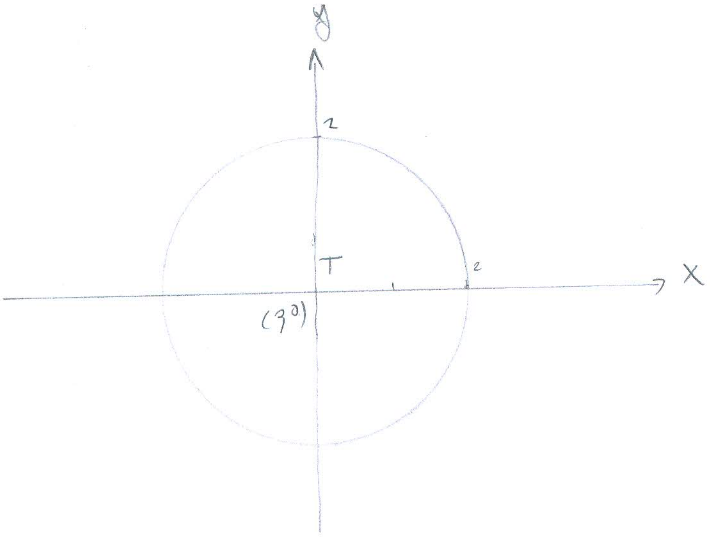
$$\boxed{X(s) = 2 - \cos t}$$

✓

$$\frac{p}{p^2+a^2} = \cos at = \cos t$$

$a=1$

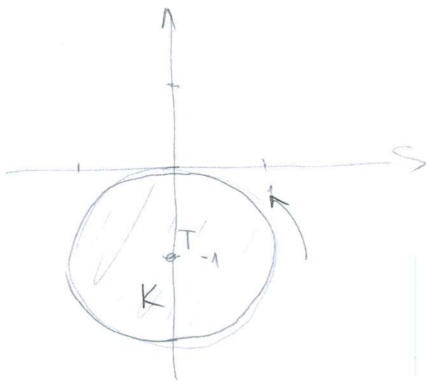
(1.)




$$\int_{\partial K} (2x+3)\,ds$$

?

(2.)




$x = r\cos\varphi \Rightarrow 2r\cos\varphi$

$y = r\sin\varphi \Rightarrow r\sin\varphi$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

$$\int_0^{\pi} (2r\cos\varphi + 3)\,d\varphi = 2\int_0^{\pi} r\cos\varphi + 3\int_0^{\pi} d\varphi$$

0

$$\|r'\| = \sqrt{\left(\frac{x}{t}\right)^2 + \left(\frac{y}{t}\right)^2 + \left(\frac{z}{t}\right)^2}$$

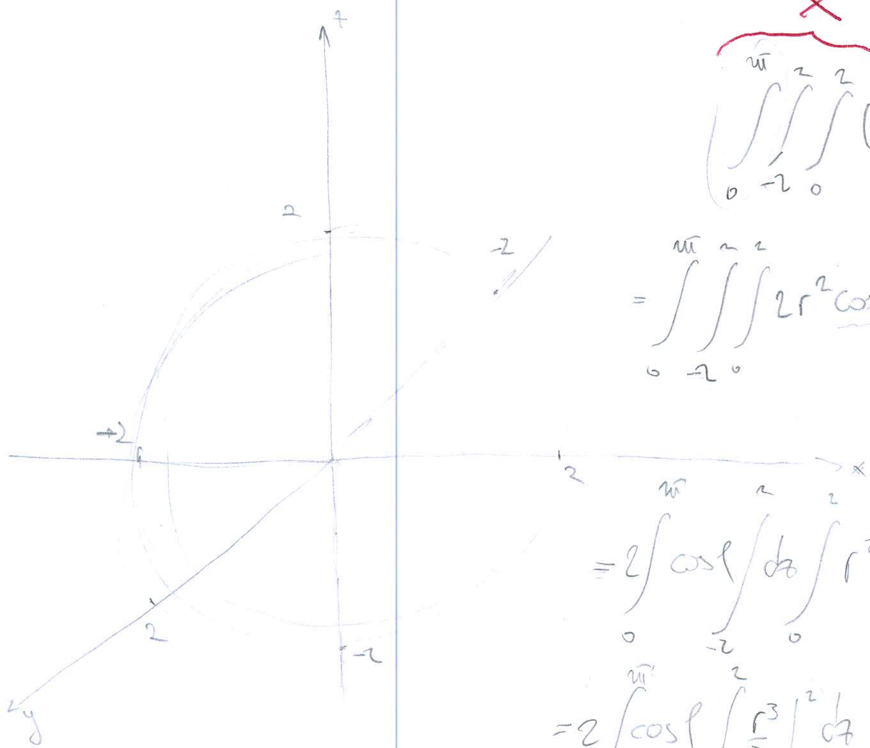
?

(3) K, kugla ; $r=2$

$$\iiint_K (2x+3)\,dx\,dy\,dz$$

$x = r\cos\varphi \qquad y = r\sin\varphi \qquad z = z$

$x \Rightarrow 2r\cos\varphi \qquad y \Rightarrow 0$



$$\int_0^{2\pi}\int_{-2}^{2}\int_0^{2} (2r\cos\varphi+3)\,r\,dr\,d\varphi\,dz =$$

$$= \int_0^{2\pi}\int_{-2}^{2}\int_0^{2} 2r^2\cos\varphi + 3r\,dr\,d\varphi\,dz$$

$$= 2\int_0^{2\pi}\cos\varphi\int_{-2}^{2}dz\int_0^{2} r^2\,dr + 3\int_0^{2\pi}d\varphi\int_{-2}^{2}dz\int_0^{2} r\,dr$$

$$= 2\int_0^{2\pi}\cos\varphi\int_{-2}^{2}\frac{r^3}{3}\Big|_0^2 dz + 3\int_0^{2\pi}d\varphi\int_{-2}^{2}dz\,\frac{r^2}{2}\Big|_0^2 \qquad \frac{4}{2}=2$$

$$= 2\int_0^{2\pi}\cos\varphi\int_{-2}^{2}\frac{8}{3}\,dz + 3\int_0^{2\pi}d\varphi\int_{-2}^{2}2\,dz$$

$$= \frac{16}{3}\int_0^{2\pi}\cos\varphi\; z\Big|_{-2}^{2} + 6\int_0^{2\pi}d\varphi\; z\Big|_{-2}^{2}$$

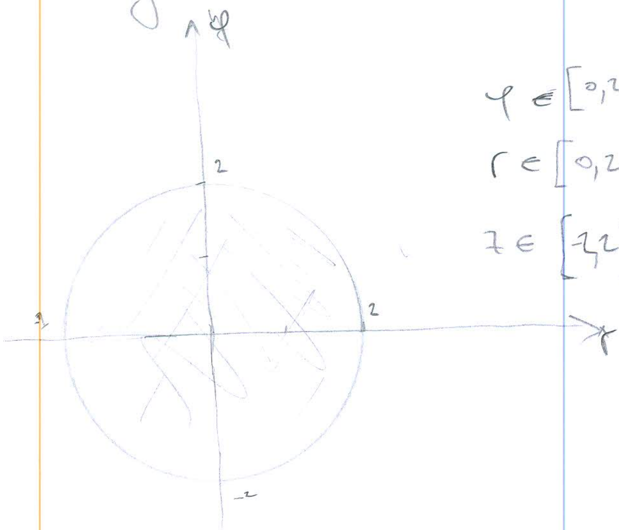
$$= \frac{16}{3}\int_0^{2\pi}\cos\varphi\,(2+2) + 6\int_0^{2\pi}d\varphi\,(2+2)$$

$$= \frac{16}{3}\cdot 4\int_0^{2\pi}\cos\varphi\,d\varphi + 6\cdot 4\int_0^{2\pi}d\varphi$$

$$= \frac{64}{3}\sin\varphi\Big|_0^{2\pi} + 24\,\varphi\Big|_0^{2\pi} = \frac{64}{3}(\sin 2\pi - \sin 0) + 24(2\pi - 0)$$

$$= 48\pi$$

x-y k.s.



$\varphi \in [0, 2\pi]$

$r \in [0, 2]$

$z \in [-2, 2]$

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|---|---|
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1. Neka je $K$ krug radijusa $r=2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K}(2x+3)\,\mathrm{d}s$? ~~20~~
2. Neka je $K$ krug radijusa $r=1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}}(2x+3)\,\mathrm{d}y$? ~~20~~
3. Neka je $K$ kugla radijusa $r=2$ sa centrom u ishodištu. Izračunati $\iiint\limits_{K}(2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? ~~20~~
4. Neka je $K$ kugla radijusa $r=1$ sa centrom u ishodištu. Izračunati preko definicije plošnog integrala $\iint\limits_{\partial K}3\mathrm{d}S$ ~~20~~ 15
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t)+x'(t)=0,\quad x(0)=x''(0)=1,\ x'(0)=0.$$

Ukupno: 15

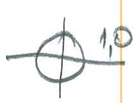
1) $\int\limits_{\partial K}(2x+3)\,ds$   $r=2$   $T(0,0)$

$$\int_C (w|ds) = \iint_S ((f\circ r)\ \|r'\|)(t)\,dt$$

$$r=\begin{pmatrix}2\cos p\\ 2\sin p\end{pmatrix} \qquad r'=\begin{pmatrix}-2\sin p\\ 2\cos p\end{pmatrix}$$

$$\|r'\| = \sqrt{(-2\sin p)^2+(2\cos p)^2} = \sqrt{4\sin^2 p+4\cos^2 p} = \sqrt{4(\underbrace{\sin^2 p+\cos^2 p}_{1})} = \sqrt{4} = 2$$

$$\int_0^{2\pi}\int_0^2 (2\cdot(2\cos p)+3)\cdot 2\,dr\,dp = \int_0^{2\pi}\int_0^2 (4\cos p+3)\cdot 2\,dr\,dp$$

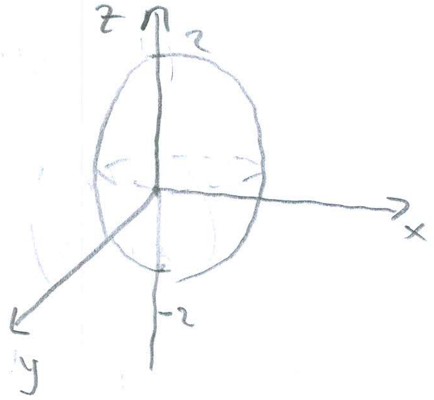


$$= \int_0^{2\pi}\int_0^2 8\cos p+6\,dr\,dp = 8\int_0^{2\pi}\int_0^2 \cos p\,dr\,dp + 6\int_0^{2\pi}\int_0^2 dr\,dp = 0+24\pi = 24\pi$$

$$8\int_0^{2\pi}\int_0^2 \cos p\,dr\,dp = 8\int_0^{2\pi}\cos p\,[r]_0^2\,dp = 8\int_0^{2\pi} 2\cdot\cos p\,dp = 16[\sin p]_0^{2\pi} = 0$$

$$6\int_0^{2\pi}\int_0^2 dr\,dp = 6\int_0^{2\pi}[r]_0^2 = 6\int_0^{2\pi} 2\,dp = 12[p]_0^{2\pi} = 24\pi$$

3) $\iiint_K (2x+3)\,dx\,dy\,dz$




$\varphi \in [0, 2\pi]$

$z \in [-2, 2]$

$r \in [0, 4]$

?  $\varnothing$

4.) $\iint_{\partial K} 3\,dS \qquad r=1$

$\iint_S f\,dS = \iint_S (f\circ r)\cdot \|n\|\,dx\,dy$

Parametrizacija $\boxed{\text{GORNJE POLOVICE}}$

$r(u,v) = \begin{pmatrix} u \\ v \\ \sqrt{1-u^2-v^2} \end{pmatrix}$

$\vec{n} = \frac{dr}{du} \times \frac{dr}{dv} = \begin{pmatrix} 1 \\ 0 \\ \frac{u}{\sqrt{1-u^2-v^2}} \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{v}{\sqrt{1-u^2-v^2}} \end{pmatrix} = \begin{pmatrix} \frac{u}{\sqrt{1-u^2-v^2}} \\ \frac{v}{\sqrt{1-u^2-v^2}} \\ 1 \end{pmatrix}$

$\|n\| = \sqrt{\left(\frac{u}{\sqrt{1-u^2-v^2}}\right)^2 + \left(\frac{v}{\sqrt{1-u^2-v^2}}\right)^2 + 1} = \sqrt{\frac{u^2+v^2}{1-u^2-v^2} + 1}$

$= \sqrt{\frac{u^2+v^2+1-u^2-v^2}{1-u^2-v^2}} = \sqrt{\frac{1}{1-u^2-v^2}} = \frac{1}{\sqrt{1-u^2-v^2}}$

$= \iint_{\partial K} 3 \cdot \frac{1}{\sqrt{1-u^2-v^2}}\,du\,dv$ ✓ 1.5

$u = r\cos\varphi$
$v = r\sin\varphi$

$= 3\int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{1-(r\cos\varphi)^2-(r\sin\varphi)^2}}\, r\,dr\,d\varphi$ ✓ $= 3\int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{1-r^2(\cos^2\varphi+\sin^2\varphi)}}\,d\varphi$ ✗ $= 3\int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{1-r^2}}\,d\varphi$

5) $x'''(t) + x'(t) = 0$

$x(0) = 1$

$x''(0) = 1$

$x'(0) = 0$

$$s^3 X(s) - s^2 \underbrace{x(0)}_{1} \cdot \underbrace{s x'(0)}_{0} - \underbrace{x''(0)}_{1} + s X(s) - \underbrace{x(0)}_{1} = 0$$

$$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$$

$$s^3 X(s) + s X(s) = s^2 - s + 2$$

$$X s \left( s^3 + s \right) = s^2 - s + 2$$

$$X s = \frac{s^2 - s + 2}{\underbrace{s^3 + s}_{s^2(s+1)}}$$

$$\frac{s^2 - s + 2}{s^2 (s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs + D}{s+1} \quad / \cdot s^2(s+1)$$

$$s^2 - s + 2 = A(s+1) + B\left[\underbrace{s(s+1)}_{s^2+s}\right] + (Cs + D) \cdot s^2$$

$$s^2 - s + 2 = As + A + Bs^2 + Bs + Cs^3 + Ds^2$$

$2 = A \Rightarrow \boxed{A = 2}$

$-1 = A + B \Rightarrow B = -1 - 2 \Rightarrow \boxed{B = -3}$

$1 = B + D \Rightarrow \boxed{D = 4}$

$0 = C \Rightarrow \boxed{C = 0}$

$$X s = 2 \cdot \frac{1}{s^2} - 3 \cdot \frac{1}{s} + \frac{0 + 4}{s+1}$$

$$X s = 2 \cdot \frac{1}{s^2} - 3 \cdot \frac{1}{s} + 4 \cdot \frac{1}{s+1}$$

$$\mathcal{L}^{-1}(t) = 2t^2 - 3 + 4e^{-t}$$

2.) $\oint_{\partial k} (2x+3)\,dy \qquad r=1 \quad T(0,-1)$

$w = \begin{pmatrix} 0 \\ 2x+3 \end{pmatrix}$ ✓ $\quad r = \begin{pmatrix} \cos p \\ \sin p - 1 \end{pmatrix} \quad r' = \begin{pmatrix} -\sin p \\ \cos p \end{pmatrix}$ ✓

$\int_0^{2\pi}\int_0^1 \begin{pmatrix} 0 \\ 2\cos p + 3 \end{pmatrix} \cdot \begin{pmatrix} -\sin p \\ \cos p \end{pmatrix} dr\,dp = \int_0^{2\pi}\int_0^1 2\cos^2 p + 3\cos p \,dr\,dp$ ✓

$= 0$

$\int_0^{2\pi}\int_0^1 2\cos^2 p \,dr\,dp = 2\int_0^{2\pi} \cos^2 p\,dp = 2 \cdot \left[\frac{1+\cos 2p}{2}\right]_0^{2\pi}$

$2 \cdot \left( \frac{1+\overbrace{\cos 2(2\pi)}^{1}}{2} - \frac{1+\overbrace{\cos 2(0)}^{1}}{2} \right) = 2 \cdot 0 = 0$

$3\int_0^{2\pi}\int_0^1 \cos p \,dr\,dp = 3\int_0^{2\pi} \cos p\,dp = 3\left[\sin p\right]_0^{2\pi} = 3(\underbrace{\sin(2\pi) - \sin 0}_{0})$

$= 0$

4.) NASTAVAK

$3\int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{1}-\sqrt{r^2}}\,dr\,dp = 3\int_0^{2\pi}\int_0^1 \frac{1}{1-r^2}\,dr\,dp$

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|---|---|
| **MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. | POPUNJAVA NASTAVNIK |
| IME I PREZIME: VEDRAN DELAŠ BROJ INDEKSA: 52706 | Broj ↓ bodova |

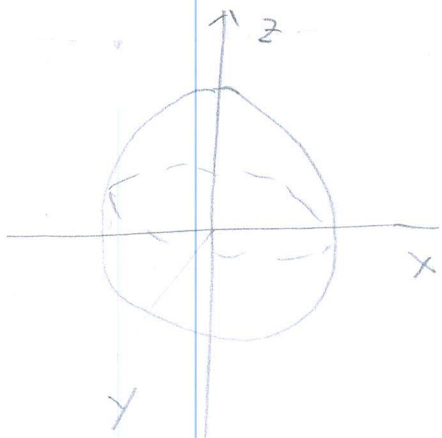
1. Neka je $K$ krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$? 20
2. Neka je $K$ krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$? 20
3. Neka je $K$ kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? 20
4. Neka je $K$ kugla radijusa $r = 1$ sa centrom u ishodištu. Izračunati preko definicije plošnog integrala $\iint\limits_{\partial K} 3\mathrm{d}S$ 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

Ukupno: 0

(3) $r = 2\,cm$

$$\iiint\limits_K (2x+3)\,dx\,dy\,dz$$




$$\iiint (2r\cos\varphi + 3)\,r\,dr\,d\varphi\,dz$$

$$\underbrace{\iiint 2r^2\cos\varphi\,dr\,d\varphi\,dz}_{I} + \underbrace{\iiint 3\,r\,dr\,d\varphi\,dz}_{II}$$

I $\quad 2\int\limits_0^{2\pi}\cos\varphi\,d\varphi\int\limits_{-2}^{2}dz\int\limits_0^2 r^2\,dr$

$$2\int\limits_0^{2\pi}\cos\varphi\,d\varphi\int\limits_{-2}^{2}dz\ \frac{r^3}{3}\Big|_0^2 = 2\int\limits_0^{2\pi}\cos\varphi\,d\varphi\int\limits_{-2}^{2}dz\ \frac{8}{3}$$

$$\frac{16}{3}\int\limits_0^{2\pi}\cos\varphi\,d\varphi\int\limits_{-2}^{2}dz = \frac{16}{3}\int\limits_0^{2\pi}\cos\varphi\,d\varphi\,(2+2) = 21.3\int\limits_0^{2\pi}\cos\varphi\,d\varphi$$

$$21.3\cdot(\underbrace{\sin 2\pi - \sin 0}_{0}) = \emptyset$$

$\varphi \in [0, 2\pi]$

$z \in [-2, 2]$ ✗

$r \in [0, 2]$ ✗

II $\quad 3\int\limits_0^{2\pi}d\varphi\int\limits_{-2}^{2}dz\int\limits_0^2 r\,dr = 3\int\limits_0^{2\pi}d\varphi\int\limits_{-2}^{2}dz\ \frac{r^2}{2}\Big|_0^2 = 3\int\limits_0^{2\pi}d\varphi\int\limits_{-2}^{2}dz\ 2 = 6\int\limits_0^{2\pi}d\varphi\,(2+2) = 24\,\varphi\Big|_0^{2\pi} =$

$24(2\pi - 0) = 48\pi$

$I + II = 48\pi$

⑤ $x'''(t) + x'(t) = 0$

$x(0) = 1$

$x''(0) = 1$

$x'(0) = 0$

$s^3X(s) - s^2x(0) - s\cancel{x'(0)} - x''(0) + sX(s) - x(0) =$

$s^3X(s) - s^2 - 1 + sX(s) - 1 = 0$

$X(s)\,(s^3+s) = s^2+2 \quad /\cdot s^3+1$

$X_s = \frac{s^2+2}{(s^3+s)(s^2+2)} = \frac{s^2+2}{s(s^2+1)(s^2+2)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} + \frac{Ds+E}{s^2+2}$

$(s^2+1)(s^2+2)$

$s^4+s^2+2s^2+2$

$s^2+2 = A(s^2+1)(s^2+2) + (Bs+C)(s^3+2s) + (Ds+E)(s^3+s)$

$s^2+2 = A(s^4+s^2+2s^2+2) + Bs^4+2Bs^2+Cs^3+2Cs+Ds^4+Ds^2+Es^3+Es$

$s^2+2 = As^4+As^2+2As^2+2A+Bs^4+2Bs^2+Cs^3+2Cs+Ds^4+Ds^2+Es^3+Es$

$0 = A+B+D$

$0 = C+E$

$1 = A+2A+2B+D$

$0 = 2C+E$

$2 = 2A$

$A = 1$

$-1 = B+D \quad /\cdot 2$

$-2 = 2B+D$

$2 = -2B-2D$

$-2 = 2B+D$

$0 = -D$

$D = 0$

$0 = C+E \quad /\cdot 2$

$0 = 2C+E$

$0 = -2C-2E$

$0 = 2C+E$

$E = 0$

$C = 0$

$-2 = 2B+0$

$-2B = 2$

$B = -1$

$x(t) = 1\,\frac{1}{s} - 1\,\frac{s}{s^2+1}$

$x(t) = 1 - \cos(t)$

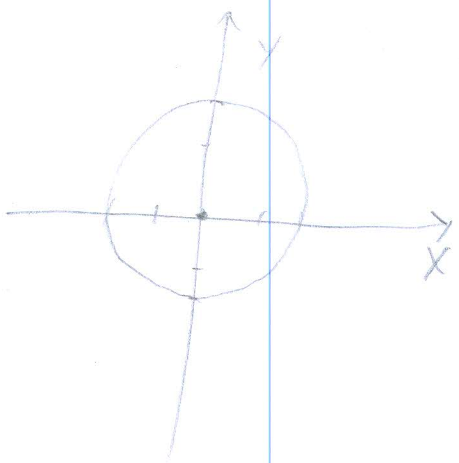
PROVJERA?

Vedran Dreoš

(1) $r=2$

$T(0,0)$

$\int (2x+3)\, ds$



$x^2+y^2=r^2$

$r^2=4$

$$\int_0^{2\pi}\int_0^{2} (2r\cos\varphi+3)\, r\, dr\, d\varphi$$

$$\cancel{\int_0^{2\pi} 2r^2\cos\varphi\, r\, dr\, d\varphi + 3\int_0^{2\pi} r\, dr\, d\varphi}$$

$$\cancel{2\int_0^{2\pi}}$$

$$\cancel{\int_0^{2\pi}\int_0^{2} 2r^2\, dr\, d\varphi}$$

$$\int_0^{2\pi}\int_0^{2} 2r^2\cos\varphi\, dr\, d\varphi + \int_0^{2\pi}\int_0^{2} 3r\, dr\, d\varphi =$$

$$2\int_0^{2\pi}\cos\varphi\, d\varphi \int_0^{2} r^2\, dr + 3\int_0^{2\pi} d\varphi \int_0^{2} r\, dr$$

$$2\int_0^{2\pi}\cos\varphi\, d\varphi\ \frac{r^3}{3}\Big|_0^2 + 3\int_0^{2\pi} d\varphi\ \frac{r^2}{2}\Big|_0^2$$

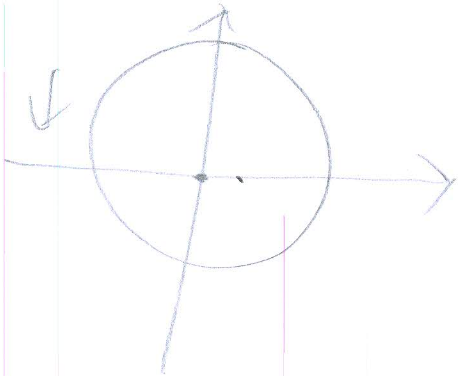
$$2\int_0^{2\pi}\cos\varphi\, d\varphi\ \frac{8}{3} + 3\int_0^{2\pi} d\varphi \cdot 2 =$$

$$\frac{16}{3}\int_0^{2\pi}\cos\varphi\, d\varphi + 6\int_0^{2\pi} d\varphi = \frac{16}{3}(\sin 2\pi - \sin 0) + 6\cdot 2\pi =$$

$$12\pi$$

(2) $r=1$

$T(0,-1)$



$\int (2x+3)\, dy$

| | |
|---|---|
| MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. | POPUNJAVA NASTAVNIK |
| IME I PREZIME: Luka Mardetko BROJ INDEKSA: 55821-2008 | Broj ↓ bodova |

1. Neka je $K$ krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} (2x+3)\,\mathrm{d}s$? — ~~20~~
2. Neka je $K$ krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$? — ~~20~~
3. Neka je $K$ kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? — ~~20~~
4. Neka je $K$ kugla radijusa $r = 1$ sa centrom u ishodištu. Izračunati preko definicije plošnog integrala $\iint_{\partial K} 3\mathrm{d}S$ — 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — ~~20~~

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

Ukupno: ∅

5.

$x'''(t) + x'(t) = 0 \qquad x(0) = x''(0) = 1 \quad x'(0) = 0$

$s^3F(s) - s^2 f(0) - s\cancel{f'(0)} - f''(0) + sF(s) - f(0) = 0$

$s^3F(s) - s^2 - 1 + sF(s) - 1 = 0$

$(s^3 + s) = s^2 + 2$

$\dfrac{s^2+2}{s\cdot(s^2+1)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+1}$

$s^2 + 2 = A\cdot(s^2+1) + (Bs+C)\cdot s$

$= As^2 + A + Bs^2 + Cs$

$1 = (A+B) \qquad 1 = 2 + B$

$0 = C \qquad B = 2 - 1 = 1$ ✗

$A = 2$

$\Rightarrow 2 + \cos(t)$ ✗

PROVJERA?

LUKA MARDEIĆ

(1.) $r=2 \quad T(0,0) \qquad \int (2x+3)\,ds$

$x = \cos t$
$y = \sin t \qquad r(t)\begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$

$r'(t)\begin{bmatrix} 2\sin t \\ -2\cos t \end{bmatrix}$

$$\|r'(t)\| = \sqrt{(2\sin t)^2 + (2\cos t)^2} \checkmark$$
$$= \sqrt{2\sin^2 t + 2\cos^2 t} \times$$
$$= \sqrt{2\cdot(\sin^2 t + \cos^2 t)} = \sqrt{2} \times$$

$$\Rightarrow \int (2x+3)\,dt$$
$$= \sqrt{2}\int_0^{2\pi} (2\cdot\cos t + 3)\,dt$$
$$= \sqrt{2}\,(2\sin t + 3t \Big|_0^{2\pi} = \sqrt{2}\cdot(2\sin 2\pi + 3\cdot 2\pi)$$
$$= \sqrt{2}\cdot(2\sin 2\pi + 6\pi)$$

0

(3.) $r=2 \qquad \iiint (2x+3)\,dx\,dy\,dz$

$$w = \begin{bmatrix} 0 \\ 0 \\ 2x+3 \end{bmatrix} \times \qquad \operatorname{div} w = d_x w_x + d_y w_y + d_z w_z = 0$$

$$\int w\cdot ds = \iiint \operatorname{div} w = \iiint 0 = 0$$

(2.) $r=1 \quad T(\overset{x}{0}, \overset{y}{-1}) \qquad \int (2x+3)\,dy$

$x = \cos t$
$y = \sin t - 1 \qquad r(t)\begin{bmatrix} \cos t \\ \sin t - 1 \end{bmatrix} \; r'(t)\begin{bmatrix} \sin t \\ -\cos t - 1 \end{bmatrix} \times$

$$\|r'(t)\| = \sqrt{(\sin t)^2 + (\cos t - 1)^2}$$
$$= \sqrt{\sin^2 t + \cos^2 t + 1} = 1$$

$\longrightarrow$

$$= \int_0^{2\pi} (2 \cdot \cos t + 3)\, dt$$

**MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: STIPE ŠPANJA  BROJ INDEKSA: 17-2-0018-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je $K$ krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$? 20

2. Neka je $K$ krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$? 20

3. Neka je $K$ kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? 20

4. Neka je $K$ kugla radijusa $r = 1$ sa centrom u ishodištu. Izračunati preko definicije plošnog integrala $\iint\limits_{\partial K} 3\mathrm{d}S$ 20

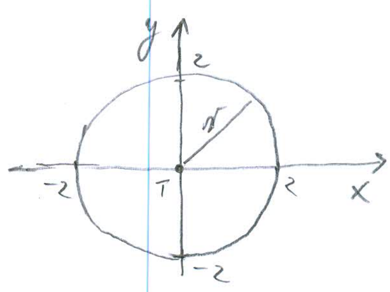
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

Ukupno: 0

1. $r = 2$  $\int\limits_K (2x+3)\,ds$

$T(0,0)$




$x = r\cos\varphi$  $r \in [0, 2]$

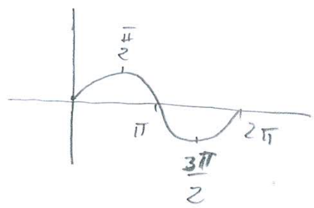
$y = r\sin\varphi$  $\varphi \in [0, 2\pi]$

$$* = \int_0^{2\pi}\int_0^2 (2r\cos\varphi + 3)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^2 (\underbrace{2r^2\cos\varphi}_{I} + \underbrace{3r}_{II})\,dr\,d\varphi$$

$$I : 2\int_0^{2\pi}\cos\varphi\,d\varphi\int_0^2 r^2\,dr = 2\int_0^{2\pi}\cos\varphi\,d\varphi\left(\frac{r^3}{3}\right)\Big|_0^2$$

$$= 2\cdot\frac{1}{3}\cdot 8\int_0^{2\pi}\cos\varphi\,d\varphi = = \frac{16}{3}\int_0^{2\pi}\cos\varphi\,d\varphi = \frac{16}{3}(\sin 2\pi - \sin 0)$$

$$= \frac{16}{3}\cdot 0 = 0$$




$$II : 3\int_0^{2\pi}d\varphi\int_0^2 r\,dr = 3\int_0^{2\pi}d\varphi\left(\frac{r^2}{2}\right)\Big|_0^2 = 3\cdot\cancel{\frac{1}{2}}\cdot\cancel{2}\int_0^{2\pi}d\varphi$$

$$= 3\int_0^{2\pi}d\varphi = 3\cdot(2\pi - 0) = 6\pi$$

$$* \quad I + II = 0 + 6\pi = 6\pi$$

5. $x'''(t) + x'(t) = 0 \quad , \quad x(0) = x''(0) = 1$
$x'(0) = 0$

$s^3 X(s) - s^2 x(0) - \cancel{s x'(0)} - x''(0) + s X(s) - x(0) = 0$

$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$

$s^3 X(s) + s X(s) = s^2 + 1 + 1$

$X(s)(s^3 + s) = s^2 + 2 \quad / : (s^3 + s)$

$X(s) = \frac{s^2 + s}{s^3 + s} = \frac{s^2 + s}{s(s^2+1)}$

$\frac{s^2 + s}{s(s^2+1)} = \frac{A}{s} + \frac{Bs + C}{s^2+1} \quad / \cdot s(s^2+1)$

$s^2 + s = A(s^2+1) + s(Bs + C)$

$s^2 + s = \underline{As^2} + A + \underline{Bs^2} + Cs$

$1 = A + B \quad \boxed{B = 1}$

$\boxed{1 = C}$

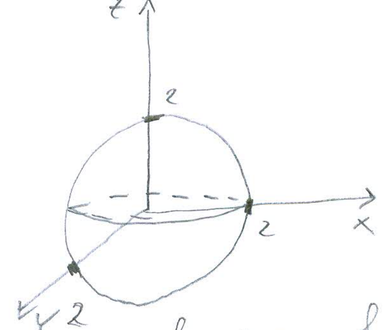
$\boxed{0 = A}$

$X(s) = \frac{s + 1}{s^2 + 1} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$

$\mathcal{L}^{(-1)} = \cos t + \sin t$

3. $r = 2$ $\iiint_K (2x+3)\,dx\,dy\,dz$

$T(0,0)$




$x^2+y^2+z^2=4$

$r^2+z^2=4$

$z^2=4-r^2$

$z=\sqrt{4-r^2}$

$r\in[0,2]$

$\varphi\in[0,2\pi]$

$z\in[0,\sqrt{4-r^2}]$

POLOVINA KUGLE (GORNJA)

$f = 2r\cos\varphi + 3$

$$\underbrace{2\int_0^{2\pi}\cos\varphi\,d\varphi\int_0^{2} r^{2}\,dr\int_0^{\sqrt{4-r^2}} dz}_{\mathrm{I}} + \underbrace{3\int_0^{2\pi}d\varphi\int_0^{2} r\,dr\int_0^{\sqrt{4-r^2}} dz}_{\mathrm{II}}$$

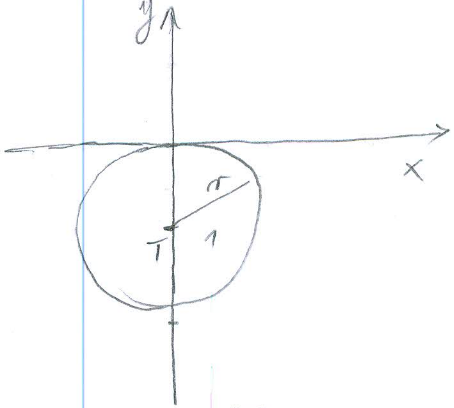
DVIJE GREŠKE

$\cancel{0}$

---

2. $r = 1$ $\int_K (2x+3)\,dy$

$T(0,-1)$

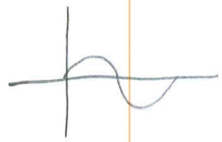



$x = r\cos\varphi$ $r\in[0,1]$

$y = r\sin\varphi - 1$ $\varphi\in[0,2\pi]$

$$\int_0^{2\pi}\int_0^{1}(2r\cos\varphi+3)\,r\,dr\,d\varphi$$

X



$$\mathrm{I}: 2\int_0^{2\pi}\cos\varphi\,d\varphi\int_0^1 r^2\,dr = 2\int_0^{2\pi}\cos\varphi\,d\varphi\left(\frac{r^3}{3}\right)\Big|_0^1 = 2\cdot\frac{1}{3}\int_0^{2\pi}\cos\varphi\,d\varphi$$

$$=\frac{2}{3}(\sin\varphi)\Big|_0^{2\pi} = \frac{2}{3}\cdot(\sin 2\pi - \sin 0) = 0$$

$$\mathrm{II}: 3\int_0^{2\pi}d\varphi\int_0^1 r\,dr = 3\int_0^{2\pi}d\varphi\left(\frac{r^2}{2}\right)\Big|_0^1 = 3\cdot\frac{1}{2}\int_0^{2\pi}d\varphi = \frac{3}{\cancel{2}}\cdot\cancel{2}\pi = 3\pi$$

STIPE ŠPANJA

4. $r = 1$

$T(0,0)$

$\iint\limits_{\partial K} 3\,dS$

$x^2 + y^2 + z^2 = 1$

$r^2 + z^2 = 1$

$z^2 = 1 - r^2$

$z = \sqrt{1-r^2}$

$r \in [0,1]$

$z \in [0, \sqrt{1-r^2}]$

$\varphi \in [0, 2\pi]$

$\emptyset$

3. $I = 2\int_0^{2\pi} \cos\varphi \, d\varphi \int_0^2 r\left(\sqrt{4-r^2}\right) dr$ $\emptyset$

**MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

| | | |
|---|---|---|
| **Ime i prezime:** JOSIP KALEBIĆ | **Broj indeksa:** 56776-2008 | Popunjava nastavnik Broj ↓ bodova |

1. Neka je $K$ krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Izračunati $\int_{\partial K} (2x+3)\,\mathrm{d}s$? 20
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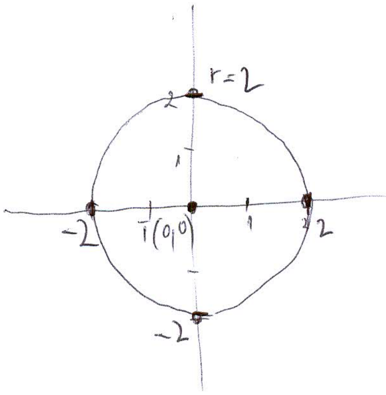
$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

Ukupno: $\emptyset$

1.)




$$I = \int_{\partial K} (2x+3)\,ds$$

$$= 2\int_{\partial K} \left(x + \frac{3}{2}\right) ds$$

$\emptyset$

(5.) $x'''(t) + x'(t) = 0$ $\quad x(0) = x''(0) = 1$

$x'(0) = 0$

$\cancel{s^3 F(s) - s^2 f(0)}$ $\quad s^3 X(s) - s^2 \cdot X(0) - s \cdot X'(0) - X''(0) + s \cdot X(s) - X(0) = 0$

$s^3 \cdot X(s) - s^2 \cdot 1 - \cancel{s} \cdot 0 - 1 + s \cdot X(s) - 1 = 0$

$s^3 \cdot X(s) - s^2 + s \cdot X(s) - 2 = 0$

---

$s^3 \cdot \left(\frac{1}{s^2}\right) - s^2 \cdot 1 - s \cdot 0 - 1 + s \cdot \left(\frac{1}{s^2}\right) - 1 = 0$

$s - s^2 + \frac{1}{s} - 2 = 0$

$s - s^2 + \frac{1}{s} = 2 \;/ \cdot s$

$s^2 - s^3 + 1 = 2s$

$-s^3 + s^2 - 2s + 1 = 0$

DALJE ... ?

$\varnothing$