

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: ANĐELO UGRINIĆ

BROJ INDEKSA: 55581 - 2008

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. 20

2. X je zadan kao četverokut s vrhovima $O(0, 0)$, $A(\frac{6}{2}, 0)$, $B(6, \frac{7}{2})$ i $C(\frac{6}{2}, \frac{5}{2})$. Izračunati dvostruki integral 20

$$\iint_X x^3 dx dy$$

3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. 20

4. Izračunati

$$\int_{(3,2)}^{(5,5)} x dy + y dx$$

5. Po definiciji izračunati cirkulaciju ravninskog vektorskog polja $w(x, y) = (-x^2y, xy^2)$ po skupu $\Gamma = \{(x, y) | x^2 + y^2 = 9\}$. 20

20

Ukupno:

40

① $2x'''(t) + 5x'(t) = t$, $x(0) = 1$, $x'(0) = x''(0) = 0$

~~$$2(0^3 X(0) - 0^2 x(0) - 0 x'(0) - x''(0)) + 5(0 X(0) - x(0)) = \frac{1}{0^2}$$~~

~~$$2 \cdot 0^3 X(0) - 2 \cdot 0^2 x(0) - 2 \cdot 0 x'(0) - 2 x''(0) + 5 \cdot 0 X(0) - 5 x(0) = \frac{1}{0^2}$$~~

~~$$2 \cdot 0^3 X(0) - 2 \cdot 0^2 + 5 \cdot 0 X(0) - 5 = \frac{1}{0^2}$$~~

~~$$X(0) (2 \cdot 0^3 + 5) - 2 \cdot 0^2 - 5 = \frac{1}{0^2}$$~~

~~$$X(0) (2 \cdot 0^3 + 5) = \frac{1}{0^2} + 2 \cdot 0^2 + 5 = \frac{1 + 2 \cdot 0^4 + 5 \cdot 0^2}{0^2}$$~~

~~$$X(0) = \frac{2 \cdot 0^4 + 5 \cdot 0^2 + 1}{0^2} \quad / : (2 \cdot 0^3 + 5) \equiv \frac{2 \cdot 0^4 + 5 \cdot 0^2 + 1}{0^2 (2 \cdot 0^2 + 5)}$$~~

$$\frac{2 \cdot 0^4 + 5 \cdot 0^2 + 1}{0^2 (2 \cdot 0^2 + 5)}$$

$$\frac{2s^3 + 1}{s^2(s^2 + 5)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C(s + D)}{s^2 + 5}$$

$$2s^3 + 1 = A(2s^2 + 5) + B(s^2 + 5) + C(s^2 + 5) + D(s^2)$$

$$2s^3 = 2As^2 + Bs^2 + Cs^2 + Ds^2 + 5A + 5B + 5C + 5D$$

$$\begin{aligned} 2 &= 5A + C \Rightarrow 1/5 + C = 2 \Rightarrow C = 9/5 \\ 0 &= 2A + D \Rightarrow 1/5 + D = 0 \Rightarrow D = -2/5 \\ 0 &= 5B \Rightarrow B = 0 \\ 1 &= 5A \Rightarrow A = 1/5 \end{aligned}$$

$$Y(s) = \frac{1}{5s^2} + \frac{1}{5s} + \frac{9}{5(s^2 + 5)}$$

$$x(t) = \left\{ \frac{1}{5} \int_0^t \tau d\tau + \frac{1}{5} + \frac{9}{5} \frac{1}{\omega^2 + 5} \right\}$$

$$x(t) = \frac{1}{10} t^2 + \frac{1}{5} + \frac{9}{5} \frac{1}{\omega^2 + 5}$$

$$\textcircled{1} \quad 2x'''(t) + 5x'(t) = 7, \quad x(0) = 1; \quad x'(0) = x''(0) = 0$$

$$2(\partial^3 X(s) - \partial^2 x(0) - \partial x'(0) - x''(0)) + 5(\partial X(s) - x(0)) = \frac{1}{\partial^2}$$

$$2\partial^3 X(s) - 2\partial^2 + 5\partial X(s) - 5 = \frac{1}{\partial^2}$$

$$X(s)(2\partial^3 + 5\partial) - 2\partial^2 - 5 = \frac{1}{\partial^2}$$

$$X(s)(2\partial^3 + 5\partial) = \frac{1}{\partial^2} + 2\partial^2 + 5 = \frac{1 + 2\partial^4 + 5\partial^2}{\partial^2}$$

$$X(s) = \frac{2\partial^4 + 5\partial^2 + 1}{\partial^2(2\partial^3 + 5\partial)} = \frac{2\partial^4 + 5\partial^2 + 1}{\partial^2(\partial(2\partial^2 + 5))} = \frac{2\partial^4 + 5\partial^2 + 1}{\partial^3(2\partial^2 + 5)}$$

$$\frac{2\partial^4 + 5\partial^2 + 1}{\partial^3(2\partial^2 + 5)} = \frac{A}{\partial^3} + \frac{B}{\partial^2} + \frac{C}{\partial} + \frac{D\partial + E}{2\partial^2 + 5}$$

$$2\partial^4 + 5\partial^2 + 1 = A(2\partial^2 + 5) + B\partial(2\partial^2 + 5) + C\partial^2(2\partial^2 + 5) + D\partial(\partial^3) + E\partial^3$$

$$2\partial^4 + 5\partial^2 + 1 = 2A\partial^2 + 5A + 2B\partial^3 + 5B\partial + 2C\partial^4 + 5C\partial^2 + D\partial^4 + E\partial^3$$

$$2 = 2C + D$$

$$0 = 2 \cdot 1 + E$$

$$0 = 2B + E$$

$$0 = 2 + E$$

$$0 = 2A + 5C$$

$$E = -2$$

$$5 = 5B \Rightarrow B = 1$$

$$1 = 5A \Rightarrow A = \frac{1}{5}$$

$$2 = 2C + D$$

$$2 = 2 \cdot \left(\frac{25}{2}\right) + D$$

$$0 = 2 \cdot \frac{1}{5} + 5C$$

$$0 = \frac{2}{5} + 5C$$

$$2 = -25 + D$$

$$5C = -\frac{2}{5}$$

$$D = 2 - 25$$

$$C = -\frac{2}{5} = -\frac{25}{2}$$

$$D = -23$$

$$X(s) = \frac{1}{5} \frac{1}{\partial^3} + \frac{1}{\partial^2} + \frac{25}{2} \frac{1}{\partial} + \frac{-23\partial}{2\partial^2 + 5} + \frac{2}{2\partial^2 + 5}$$

$$x(t) = \frac{1}{5} \cdot \frac{t^2}{2} + t - \frac{25}{2} + \frac{23}{2} \cos\sqrt{5}t + \frac{2}{2\sqrt{5}}$$

VIDI REŠENJE 1 OD 2012-09-24

