	POPUNJAVA NASTAVNIK
MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	Broj ↓ bodova
IME I PREZIME: ANĐELO UGRINIĆ BROJ INDEKSA: 55581 - 2008	

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. ~~20~~

2. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2},0)$, $B(6,\frac{7}{2})$ i $C(\frac{6}{2},\frac{5}{2})$. Izračunati dvostruki integral ~~20~~

$$\iint\limits_X x^3 \mathrm{d}x\mathrm{d}y$$

3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. 20

4. Izračunati 20

$$\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$$

5. Po definiciji izračunati cirkulaciju ravninskog vektorskog polja $w(x,y) = (-x^2y, xy^2)$ po skupu $\Gamma = \{(x,y) \mid x^2+y^2=9\}$. 20

Ukupno: 40

(1) $2x'''(t) + 5x'(t) = t$, $x(0)=1$, $x'(0)=x''(0)=0$

$2\left(s^3X(s) - s^2x(0) - sx'(0) - x''(0)\right) + 5\left(sX(s) - x(0)\right) = \frac{1}{s^2}$

$2s^3X(s) - 2s^2x(0) - 2sx'(0) - 2x''(0) + 5sX(s) - 5x(0) = \frac{1}{s^2}$

$2s^3X(s) - 2s^2 + 5sX(s) - 5 = \frac{1}{s^2}$

$X(s)(2s^3+5s) - 2s^2 - 5 = \frac{1}{s^2}$

$X(s)(2s^3+5s) = \frac{1}{s^2} + 2s^2 + 5 = \frac{1+2s^4+5s^2}{s^2}$

$X(s) = \frac{2s^4+5s^2+1}{s^2} \quad / : (2s^3+5s) = \frac{2s^4+5s^2+1}{s^2(s(2s^2+5s))}$

$\frac{2s^4+5s^2+1}{s^3(2s^2+5s)}$

$$\frac{2s^2+1}{s^2(2s^2+5)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{2s^2+5}$$

$$2s^2+1 = A(2s^2+5) + B([illegible]) + C([illegible]) + D(s^2)$$

$$2s^2+1 = 2As^2 + 5A + [illegible]$$

$2 = B + C \Rightarrow$ [illegible] $\frac{1}{5} + C$ [illegible] $\frac{9}{5}$

$0 = 2A + D$ [illegible] $\frac{1}{5} + D$ [illegible] $\Rightarrow D = 0 - \frac{2}{5} = -\frac{2}{5}$

$0 = 5B \Rightarrow B = \frac{1}{5}$ [illegible]

$1 = 5A \Rightarrow A = \frac{1}{5}$ $\qquad D = -\frac{2}{5}$

$Y_{(s)} = $ [illegible]

$x(t) = $ [illegible] $\frac{1}{2s^2+5}$

$x(t) = \frac{1}{5}t +$ [illegible]

① $2x'''(t)+5x'(t)=t$, $x(0)=1$; $x'(0)=x''(0)=0$

$$2\left(s^3X(s)-s^2x(0)-sx'(0)-x''(0)\right)+5\left(sX(s)-x(0)\right)=\frac{1}{s^2}$$

$$2s^3X(s)-2s^2+5sX(s)-5=\frac{1}{s^2}$$

$$X(s)(2s^3+5s)-2s^2-5=\frac{1}{s^2}$$

$$X(s)(2s^3+5s)=\frac{1}{s^2}+2s^2+5=\frac{1+2s^4+5s^2}{s^2}$$

$$X(s)=\frac{2s^4+5s^2+1}{s^2(2s^3+5s)}=\frac{2s^4+5s^2+1}{s^2(s(2s^2+5))}=\frac{2s^4+5s^2+1}{s^3(2s^2+5)}$$

$$\frac{2s^4+5s^2+1}{s^3(2s^2+5)}=\frac{A}{s^3}+\frac{B}{s^2}+\frac{C}{s}+\frac{Ds+E}{2s^2+5}$$

$$2s^4+5s^2+1=A(2s^2+5)+Bs(2s^2+5)+Cs^2(2s^2+5)+Ds(s^3)+Es^3$$

$$2s^4+5s+1=2As^2+5A+2Bs^3+5Bs+2Cs^4+5Cs^2+Ds^4+Es^3$$

$2=2C+D$

$0=2B+E$

$0=2A+5C$

$5=5B \Rightarrow \boxed{B=1}$

$1=5A \Rightarrow \boxed{A=\frac{1}{5}}$

$0=2\cdot\frac{1}{5}+5C$

$0=\frac{2}{5}+5C$

$5C=-\frac{2}{5}$

$\boxed{C=-\frac{2}{5}=-\frac{25}{2}}$

$0=2\cdot 1+E$

$0=2+E$

$\boxed{E=-2}$

$2=2C+D$

$2=2\cdot\left(-\frac{25}{2}\right)+D$

$2=-25+D$

$D=2-25$

$\boxed{D=-23}$

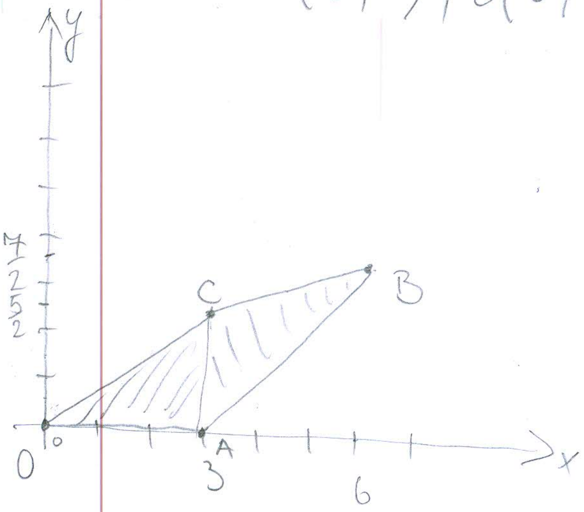
$$X(s)=\frac{1}{5}s^3+\frac{1}{s^2}+\frac{25}{2}\frac{1}{s}+\frac{-23s}{2s^2+5}-\frac{2}{2s^2+5}$$

$$x(t)=\frac{1}{5}\cdot t^2+t-\frac{25}{2}-\frac{23}{2}\cos\sqrt{5}t-2$$

0

VIDI RJEŠENJE 1 OD 2012-09-24

2) $O(0,0)$, $A(3,0)$, $B(6,\frac{7}{2})$, $C(3,\frac{5}{2})$ ANĐELO UGRINIĆ 55581-2008



$\overline{OA}$: $y = 0$

$\overline{AB}$: $(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$$(6-3)(y-0) = (\tfrac{7}{2}-0)(x-3)$$

$$3y = \frac{7}{2}(x-3)$$

$$3y = \frac{7}{2}x - \frac{21}{2}$$

$$y = \frac{6}{7}x - \frac{6}{21}$$

$$\int_{0}^{3}\int_{\overline{OA}}^{\overline{OC}} x^3\,dx\,dy + \int_{3}^{6}\int_{\overline{AB}}^{\overline{CB}} x^3\,dx\,dy = \times$$

$$\int_{0}^{\frac{6}{5}x} x^3\,dx\,dy + \int_{\frac{6}{7}x-\frac{6}{21}}^{\frac{x}{3}+\frac{2}{3}} x^3\,dx\,dy = \quad \times$$

$\overline{CB}$: $(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$$(6-3)(y-\tfrac{5}{2}) = (\tfrac{7}{2}-\tfrac{5}{2})(x-3)$$

$$3(y-\tfrac{5}{2}) = 1(x-3)$$

$$3y - \frac{15}{2} = x - 3$$

$$3y = x - 3 + \frac{15}{2}$$

$$3y = x + \frac{9}{2}$$

$$y = \frac{x + \frac{9}{2}}{3}$$

$$y = \frac{x}{3} + \frac{2}{3}$$

$\overline{OC}$: $(x_2 - x_1)(y - y_1) = (y_2 - y_1)(x - x_1)$

$$(3-0)(y-0) = (\tfrac{5}{2}-0)(x-0)$$

$$3y = \frac{5}{2}x$$

$$y = \frac{\frac{5}{2}}{3}x = \frac{6}{5}x \quad \times$$

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3. $x^2+y^2+z^2=4$

$z \geq 1$

$x^2+y^2+z^2=R^2$

$R^2=4$

$R=\sqrt{4}=2$

$r^2+z^2=4$

$r^2=4-z^2$

$r=\sqrt{4-z^2}$

$z \in [1,2]$

$\varphi \in [0,2\pi]$

$r \in [0,\sqrt{4-z^2}]$

$$V=\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\varphi = 2\pi\int_1^2 \frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}} dz =$$

$$=2\pi\int_1^2 \frac{(\cancel{\sqrt{4-z^2}})^{\cancel{2}}}{2}\,dz = \cancel{2}\pi\int_1^2 \frac{4-z^2}{\cancel{2}}\,dz$$

$$=\pi\left(4z-\frac{z^3}{3}\right)\Big|_1^2=\pi\left(4\cdot2-\frac{2^3}{3}\right)-\left(4\cdot1-\frac{1^3}{3}\right)$$

$$=\pi\left(8-\frac{8}{3}-4+\frac{1}{3}\right)=\pi\left(\frac{24-8-12+1}{3}\right)$$

$$=\pi\,\frac{5}{3}=5\frac{\pi}{3}$$

4. $\int_{(3,2)}^{(5,5)} x\,dy + y\,dx \qquad \int_{(3,2)}^{(5,5)} y\,dx + x\,dy$

$$\begin{bmatrix} y \\ x \end{bmatrix} = -\text{grad}\, f = \begin{bmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -y = f = \int -y\,dx = -yx + C(y) \Rightarrow f = -xy + C(y)$$

$$\frac{\partial f}{\partial y} = -x \Rightarrow \cancel{-x} + \frac{\partial C(y)}{\partial y} = \cancel{-x} \Rightarrow \frac{\partial C(y)}{\partial y} = 0 \Rightarrow$$

$$\Rightarrow C(y) = \int 0\,dy = 0 + C$$

$$f = -xy \checkmark$$

$$f(3,2) - f(5,5) = -3\cdot 2 + 5\cdot 5 = -6 + 25 = 19 \checkmark$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: MARKO VULELIJA BROJ INDEKSA: 57660

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 20

(1.) $2(s^3F(s) - s^2f(0) - sf'(0) - f''(0)) + 5(sF(s) - f(0)) = \frac{1}{s^2}$

$\frac{1}{s^{n+1}} \to t^n$

$\frac{1}{s^2}$

$2s^3F(s) - 2s^2 + 5sF(s) - 1 = \frac{1}{s^2}$

$2s^3F(s) + 5sF(s) = \frac{1}{s^2} + 2s^2 - 1$

$F(s)(2s^3+5s) = \frac{1}{s^2} + 2s^2 - 1$

$$F(s) = \frac{\frac{1}{s^2}+2s^2-1}{(2s^3+5s)} = \frac{\frac{1}{s^2}+2s^2-1}{s(2s^2+5)} = \frac{1+2s^4-s^2}{s^3(2s^2+5)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{2s^2+5} \Big/ s^3(2s^2+5)$$

$= A(2s^2+5) + Bs(2s^2+5) + Cs^2(2s^2+5) + (Ds+E)\cdot s^3$

$5A = 1$ $\quad 5B = 0$

$\boxed{A = \frac{1}{5}}$ $\quad \boxed{B = 0}$

$1+2s^4-s^2 = 2As^2 + 5A + 2Bs^3 + 5Bs + 2Cs^4 + 5Cs^2 + Ds^4 + Es^3$

$= s^4(2C+D) + s^3(2B+E) + s^2(2A+5C) + 5Bs + 5A$

$2C + D = 2$

$2B + E = 0 \Rightarrow \boxed{E = 0}$

$2A + 5C = -1 \Rightarrow \frac{2}{5} + 5C = -1$

$5C = -\frac{7}{5}$

$\boxed{C = -\frac{7}{25}}$

$-\frac{14}{25} + D = 2$

$\boxed{D = \frac{64}{25}}$

$F(s) = \frac{1}{5}\frac{1}{s^3} + \frac{-\frac{7}{25}}{s} + \frac{\frac{64}{25}s}{2s^2+5}$

$F(s) = \frac{1}{5} \quad + -\frac{7}{25} + \frac{64}{25}\frac{s}{2s^2+5}$

$F(s) = \frac{1}{5} \quad -\frac{7}{25} + \frac{64}{25}\frac{s}{2s^2+(\sqrt{5})^2}$

$F(s) = \frac{1}{5} \quad -\frac{7}{25} + \frac{64}{25}\frac{1}{2}\frac{s}{s^2+\sqrt{\frac{3}{2}}^2}$

$F(s) = \frac{1}{5}\frac{1}{2}t^2 - \frac{7}{25}\ \frac{32}{25}\cos\left(\frac{\sqrt{3}}{2}\right)^2 t$

$F(s) = \frac{1}{10}t^2 - \frac{7}{25} + \frac{32}{25}\cos\left(\frac{\sqrt{3}}{2}\right)^2 t$ ✗

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④ $\int_{(3,2)}^{(5,5)} x\,dy + y\,dx$

$d_x f = -y \quad /\int$

$f = -yx + C(y)$

$d_y f = -x$

$d_y(-yx + C(y)) = -x$

$d_y(-x + C(y)) = -x$

$d_y + C(y) = 0$

$C(y) = 0$

$f(x,y) = -yx - (-yx)$ ✗

$= (-2\cdot 3) - (-5\cdot 5)$

$= -6 - (-25)$

$= \underline{\underline{19}}$

③ $x^2 + y^2 + z^2 = 4$

$z \geq 1$

$z \in [1, 2]$ ✓

$\varphi \in [2\pi, 0]$ ✓

$r \in [0, \sqrt{4-z^2}]$ ✓

$x^2 + y^2 = 4 - z^2$

$r^2 = 4 - z^2 \quad /\sqrt{\ }$

$r = \sqrt{4-z^2}$

$$\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r\,dr\,d\varphi\,dz = \int_0^{2\pi} d\varphi \int_1^2 dz \int_0^{\sqrt{4-z^2}} r\,dr = \int_0^{2\pi} d\varphi \int_1^2 dz \left.\frac{r^2}{2}\right|_0^{\sqrt{4-z^2}}$$

$$= \int_0^{2\pi} d\varphi \int_1^2 dz \left(\frac{(\sqrt{4-z^2})^2}{2} - \frac{0}{2}\right)$$

$$= \int_0^{2\pi} d\varphi \int_1^2 dz\, \frac{4-z^2}{2} = \int_0^{2\pi} d\varphi \int_1^2 \left(2 - \tfrac{1}{2}z^2\right) dz$$

$\frac{2^3}{3} - \frac{1}{3}$

$\frac{2^3}{3} - \frac{z^3}{3}$

$$= \int_0^{2\pi} d\varphi\; 2\int_1^2 dz - \frac{1}{2}\int_1^2 z^2\,dz = \int_0^{2\pi} d\varphi\; 2(2-1) - \frac{1}{2}\left(\frac{8}{3} - \frac{1}{3}\right)$$

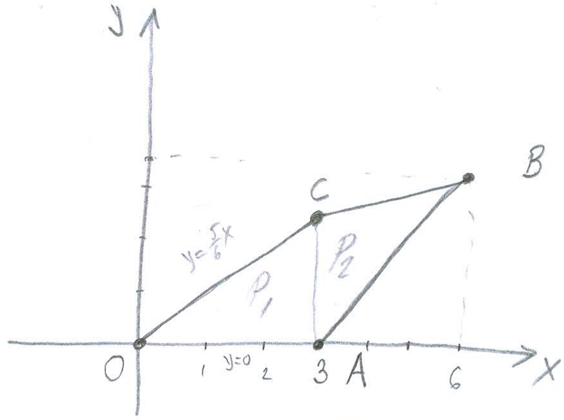
$$\int_0^{2\pi} d\varphi\; 2 - \frac{7}{6} = \int_0^{2\pi} \frac{5}{6}\,d\varphi = \frac{5}{6}\int_0^{2\pi} d\varphi = \frac{5}{6}(2\pi - 0)$$

$= \frac{5}{3}\pi$ ✓

$O(0,0)$, $A(\frac{6}{2},0)$, $B(6,\frac{7}{2})$, $C(\frac{6}{2},\frac{5}{2})$

$$\iint_X x^3\,dx\,dy$$

(2.)



$$\int_0^3\int_{OA}^{CO} x^3\,dx\,dy + \int_3^6\int_{AB}^{CB} x^3\,dx\,dy$$

$$\int_0^3\int_0^{\frac{5}{6}x} x^3\,dx\,dy + \int_3^6\int_{\frac{7}{6}x-\frac{7}{2}}^{-x+\frac{19}{2}} x^3\,dx\,dy$$

$$\int_0^3 x^3dx\int_0^{\frac{5}{6}x}dy + \int_3^6 x^3\int_{\frac{7}{6}x-\frac{7}{2}}^{-x+\frac{19}{2}}dy$$

$$\int_0^3 x^3dx\left(\frac{5}{6}x-0\right)+\int_3^6 x^3\left(-x+\frac{19}{2}-\left(\frac{7}{6}x-\frac{7}{2}\right)\right)dx$$

$$\int_0^3\frac{5}{6}x^4dx+\int_3^6 x^3\left(-x+\frac{19}{2}-\frac{7}{6}x+\frac{7}{2}\right)dx$$

13, $-\frac{13}{6}$

$$\int_0^3\frac{5}{6}x^4dx+\int_3^6 x^3\left(-\frac{13}{6}x+13\right)dx$$

$$\frac{5}{6}\int_0^3 x^4dx+\int_3^6\left(-\frac{13}{6}x^4+13x^3\right)dx$$

$\frac{x^5}{5}+\frac{x^4}{4}$

$$\frac{5}{6}\frac{x^5}{5}\Big|_0^3+\left(-\frac{13}{6}\int_3^6 x^4dx+13\int_3^6 x^3dx\right)$$

$$\frac{5}{6}\left(\frac{243}{5}-\frac{0}{5}\right)-\frac{13}{6}\left(\frac{7776}{5}-\frac{243}{5}\right)+13\left(\frac{1296}{4}-\frac{81}{4}\right)$$

$$\frac{81}{2}-\frac{32643}{10}+\frac{15795}{5}=\boxed{\frac{14499}{20}}\approx 724.95$$

OC

$$y-0=\frac{\frac{5}{2}-0}{3-0}(x-0)$$

$$y-0=\frac{5}{6}x$$

$$\boxed{y=\frac{5}{6}x}$$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

OA

$$y-0=\frac{0-0}{3-0}(x-0)$$

$$\boxed{y=0}$$

CB

$$y-\frac{7}{2}=\frac{\frac{5}{2}-\frac{7}{2}}{3-6}(x-6)$$

-1, -3

$$y-\frac{7}{2}=-x+6 \quad \times$$

$$\boxed{y=-x+\frac{19}{2}} \quad \times$$

$$y-\frac{7}{2}=\frac{1}{3}x-2$$

AB

$$y-0=\frac{\frac{7}{2}-0}{6-3}(x-3)$$

$$\boxed{y=\frac{7}{6}x-\frac{7}{2}}$$

$\omega(x,y) = (-x^2 y, xy^2)$ $\Gamma = \{(x,y) \mid x^2+y^2=9\}$

$$\omega = \begin{pmatrix} -x^2y \\ xy^2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2xy \\ 2yx \\ 0 \end{pmatrix} \cancel{\emptyset}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: MARIN MARAŠ

BROJ INDEKSA: 57651

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. ~~20~~

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$$\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$$

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Ukupno: 0

1. $2x'''(t) + 5x'(t) = t$

$x(0) = 1$

$x'(0) = 0$

$x''(0) = 0$

$s^3X(s) - s^2 \overset{1}{x(0)} - \cancel{s x'(0)} - \cancel{x''(0)} + 5(sX(s) - \overset{1}{x(0)}) = \frac{1}{s^2}$

$s^3X(s) - s^2 + 5sX(s) - 5 = \frac{1}{s^2}$

$s^3X(s) + 5sX(s) = \frac{1}{s^2} + s^2 + 5$

$X(s)(s^3 + 5s) = \frac{s^2 + s^4 + 5s^2}{s^2}$

$X(s) = \frac{s^4 + 6s^2}{(s^3+5s)\cdot s^2}$

$X(s) = \frac{s^4 + 6s^2}{(s^2+5)\cdot s^3}$

1) $$\frac{s^4+6s^2}{s^3(s^2+5)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{s^2+5}$$

MARAŠ MARIN

$$s^4+6s^2 = A(s^2+5) + Bs(s^2+5) + Cs^2(s^2+5) + s^3(Ds+E)$$

$$s^4+6s^2 = As^2+5A+Bs^4+5Bs+Cs^4+5Cs^2+Ds^4+Es^3$$

$1 = B+C+D \longrightarrow 1 = 0 + \frac{6}{5} + D$

$\boxed{0=E}$

$6 = A+5C \longrightarrow 6 = 5C$

$0 = 5B \rightarrow \boxed{B=0}$ $\quad 5C = 6 \mid \cdot \frac{1}{5}$ $\quad 1 - \frac{6}{5} = D$

$0 = 5A \rightarrow \boxed{A=0}$ $\quad \boxed{C=\frac{6}{5}}$ $\quad \boxed{D=-\frac{1}{5}}$

$$= \cancel{\frac{0}{s^3}} + \cancel{\frac{0}{s^2}} + \frac{6}{5}\frac{1}{s} + \frac{-\frac{1}{5}s}{s^2+5}$$

$$= \frac{6}{5}\cdot 1 + \frac{-\frac{1}{5}s}{s^2+5}$$

$$= \frac{6}{5} - \frac{\frac{1}{5}s}{s^2+5}$$

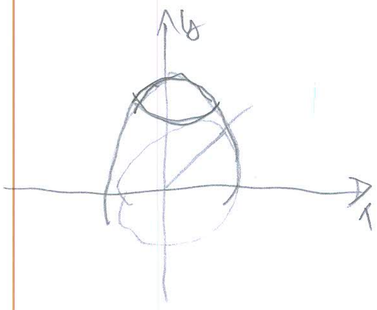
$$= \frac{6}{5} - \frac{1}{5}\frac{s}{s^2+5}$$

VIDI RJEŠENJE 1 OD 2012-09-24

$$= \frac{6}{5} - \frac{1}{5}\cos(\sqrt{5}t) \;//$$

×

3. $x^2+y^2+z^2=4$, r^2 $\quad z \geq 1$



$r=2$

$x = r\cos\varphi$
$y = r\sin\varphi$

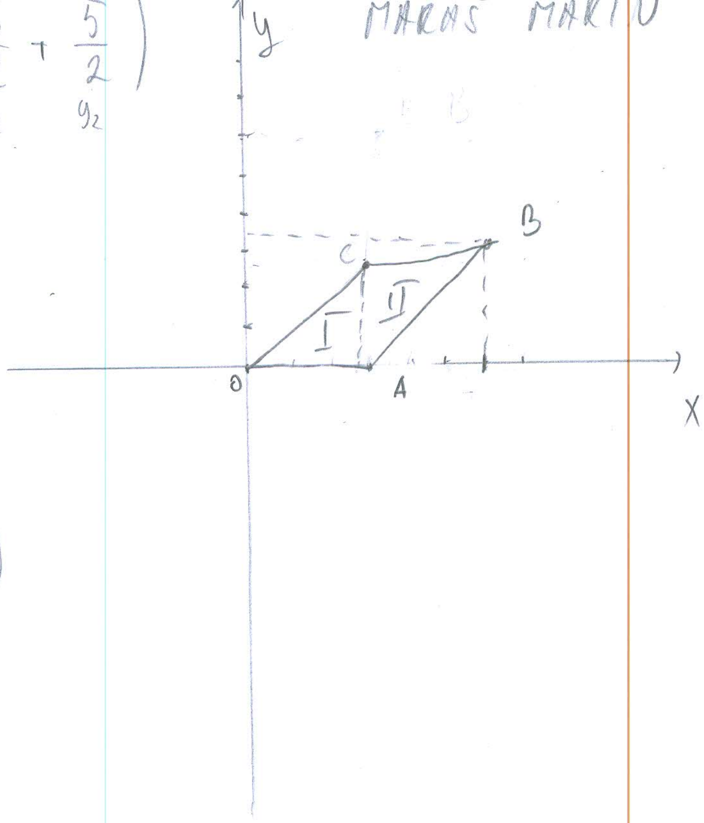
$\varphi \in [0, 2\pi]$ ✓
$r \in [0, 2]$ ✗
$z \in [1, \sqrt{r-4}]$ ✓

$r + z^2 = \sqrt{4}$
$z^2 = 2 - r$
$z = \sqrt{2-r}$

$$\int_0^{2\pi}\int_0^{2}\int_1^{\sqrt{2-r}} dx\,dy\,dz$$

$\cancel{0}$

MARAS MARIO



2. $O(\underset{x_1}{0}, \underset{y_1}{0})$, $A(\underset{x_1}{\tfrac{6}{2}}, \underset{y_1}{0})$, $B(\underset{x_1}{6}, \underset{y_1}{\tfrac{7}{2}})$, $C(\underset{x_2}{\tfrac{6}{2}}, \underset{y_2}{\tfrac{5}{2}})$

OA $(y-y_1)(x_2-x_1) = (x-x_1)(y_2-y_1)$

$$(y-0)\left(\frac{6}{2}-0\right) = (x-0)(0-0)$$

$$\frac{6}{2}y = 0$$

$$\boxed{y=0}$$

AB $(y-0)\left(6-\frac{6}{2}\right) = \left(x-\frac{6}{2}\right)\left(\frac{7}{2}-0\right)$

$$3y = \frac{7}{2}x - \frac{21}{2}$$

$$\boxed{y = \frac{7}{6}x - \frac{7}{2}}$$

OC $(y-0)\left(\frac{6}{2}-0\right) = (x-0)\left(\frac{5}{2}-0\right)$

$$\frac{6}{2}y = \frac{5}{2}x \quad \Big| \cdot \left(\frac{2}{6}\right)$$

$$\boxed{y = \frac{5}{6}x}$$

BC $\left(y-\frac{7}{2}\right)\left(\frac{6}{2}-6\right) = (x-6)\left(\frac{5}{2}-\frac{7}{2}\right)$

$$-3y\overset{+}{-}\frac{21}{2} = -x+6$$

$$3y = -x + 6 + \frac{21}{2}$$

$$3y = -x + \frac{33}{2} \quad \Big| \cdot \frac{1}{3}$$

$$\boxed{y = -x + \frac{11}{2}}$$

$$\iint x^3\,dx\,dy = \underset{\mathrm{I}}{\int_0^{\frac{6}{2}}\int_0^{\frac{5}{6}x} x^3\,dx\,dy} + \underset{\mathrm{II}}{\int_{\frac{6}{2}}^{6}\int_{\frac{7}{6}x-\frac{7}{2}}^{-x+\frac{11}{2}} x^3\,dx\,dy} = \cancel{\emptyset}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	POPUNJAVA NASTAVNIK
IME I PREZIME: DANIJEL KAPOVIĆ **BROJ INDEKSA:** 52590-2005	Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. ~~20~~

2. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{6}{2},0)$, $B(6,\frac{7}{2})$ i $C(\frac{6}{2},\frac{5}{2})$. Izračunati dvostruki integral 20

$$\iint_X x^3 \mathrm{d}x\mathrm{d}y$$

3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. 20

4. Izračunati 20

$$\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$$

5. Po definiciji izračunati cirkulaciju ravninskog vektorskog polja $w(x,y) = (-x^2y, xy^2)$ po skupu $\Gamma = \{(x,y) \mid x^2+y^2=9\}$. 20

Ukupno: ~~0~~

① $2x'''(t) + 5x'(t) = t \qquad x(0)=1 \qquad x'(0)=x''(0)=0$

$$2(s^3X(s) - s^2x(0) - s\cancel{x'(0)}^{0} - \cancel{x''(0)}^{0}) + 5(sX(s) - x(0)) = \frac{1}{s^2}$$

$$2(s^3X(s) - s^2) + 5(sX(s) - 1) = \frac{1}{s^2}$$

$$2s^3X(s) - 2s^2 + 5sX(s) - 5 = \frac{1}{s^2}$$

$$2s^3X(s) + 5sX(s) = \frac{1}{s^2} + 2s^2 + 5$$

$$X(s)(2s^3+5s) = \frac{1+2s^4+5s^2}{s^2}$$

$$X(s) = \frac{2s^4+5s^2+1}{s^2(2s^3+5s)} = \frac{2s^4+5s^2+1}{2s^5+5s^3} = \frac{2s^4+5s^2+1}{s^3(2s^2+5)}$$

$$(Ds+E)\cdot s^3(2s^2+5)$$
$$(Ds^4+Es^3)(2s^2+5)$$
$$2Ds^6+5Ds^4+2Es^5+5Es^3$$

$$2s^4+5s^2+1 = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{2s^2+5}$$

$$2s^4+5s^2+1 = A(2s^2+5) + Bs(2s^2+5) + Cs^2(2s^2+5) + 2Ds^6 + 5Ds^4 + 2Es^5 + 5Es^3$$

$$2s^4+5s^2+1 = 2As^2 + 5A + 2Bs^3 + 5Bs + \underline{2Cs^4} + 5Cs^2 + \underline{2Ds^6} + \underline{5Ds^4} + 2Es^5 + 5Es^3$$

[illegible] = 2D
[illegible] = 2E

DALJE ... ?