MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	POPUNJAVA NASTAVNIK
IME I PREZIME: MATE BALJAK	BROJ INDEKSA: 57115

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~ 5

$$y'''(t) - y(t) = t, \quad y(0) = 1,\ y''(0) = 2,\ y'(0) = 1.$$

2. Izračunati integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $z = x^2$, $z = x$, $y = -5$ i $y = 6$. 20

3. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint\limits_K (2x + 3)\,dxdy$? 20

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\,dxdy$? 20

5. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint\limits_S 3dxdy$? (pomoć: rot $(3x\mathbf{j}) = 3\mathbf{k}$) 20

Ukupno: 45

3.

$$\boxed{\begin{aligned} x &= r\cos\varphi + 2 \\ y &= r\sin\varphi + 1 \\ dxdy &= r\,dr\,d\varphi \end{aligned}}$$

$$\int_0^{2\pi} d\varphi \int_0^1 2(r\cos\varphi + 2) + 3 \ r\,dr \quad ✓$$

$$\int_0^{2\pi} d\varphi \int_0^1 (2r\cos\varphi + 4 + 3)\ r\,dr$$

$$\cancel{2\int_0^{2\pi} d\varphi \int_0^1 r\cos\varphi\ r\,dr + 7\int_0^{2\pi} d\varphi \int_0^1}$$

$$2\int_0^{2\pi} d\varphi \int_0^1 r^2\cos\varphi\,dr + \int_0^{2\pi} d\varphi \int_0^1 7r\,dr$$

$$2\int_0^{2\pi} d\varphi\ \frac{r^3}{3}\cdot\cos\varphi\Big|_0^1 + 7\int_0^{2\pi} d\varphi\ \frac{r^2}{2}\Big|_0^1$$

$$2\int_0^{2\pi} d\varphi\ \frac{1}{3}\cdot\cos\varphi + 7\int_0^{2\pi} d\varphi\ \frac{1}{2}$$

$$\frac{2}{3}\int_0^{2\pi} d\varphi\cos\varphi + \frac{7}{2}\int_0^{2\pi} d\varphi$$

$$\frac{2}{3}\cdot\left((\overset{0}{-\sin 2\pi}) - (\overset{0}{-\sin 0})\right) + \frac{7}{2}\cdot 2\pi$$

$$0 + \frac{14}{2}\pi = 7\pi \ // \quad ✓$$

$Y'''(x) - Y(x) = x$ $\qquad Y(0)=1, \quad Y''(0)=2, \quad Y'(0)=1$ $\qquad$ 1.

$$s^3 Y(s) - s^2 \overset{1}{Y(0)} - s\overset{1}{Y'(0)} - \overset{2}{Y''(0)} - \frac{1}{1} Y\left(\frac{s}{1}\right) = \frac{1}{s^2}$$

$$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$$

$$Y(s)\,(s^3-1) = \frac{1}{s^2} + s^2 + s + 2$$

$$Y(s)\,(s^3-1) = \frac{1+s^4+s^3+2s^2}{s^2} \quad \Big| \cdot \frac{1}{s^3-1}$$

$$\frac{s^4+s^3+2s^2+1}{s^2(s^2+s+1)(s-1)} = \frac{A}{s^2}$$

$$\cancel{\frac{s^4+s^3+2s^2+1}{s(s-1)(s+1)(s^2+s+1)}}$$

$$\frac{s^4+s^3+2s^2+1}{s^2(s-1)(s^2+s+1)} = \frac{A}{s^2} + \frac{B}{s-1} + \frac{Cs+D}{s^2+s+1}$$

$$s^4+s^3+2s^2+1 = A(s-1)(s^2+s+1) + Bs^2(s^2+s+1) + (Cs+D)s^2(s-1)$$

$$= As^3 \cancel{+As^2} \cancel{+As} \cancel{-As^2} \cancel{-As} - A + Bs^4 + Bs^3 + Bs^2 + Cs^4 - Cs^3 + Ds^3 - Ds^2$$

$$= As^3 - A + Bs^4 + Bs^3 + Bs^2 + Cs^4 - Cs^3 + Ds^3 - Ds^2$$

$s^4 \to 1 = B + C$

$s^3 \to 1 = A + B - C + D$

$s^2 \to 2 = B - D$

$1 = -A$

$\boxed{A = -1}$ ✓

$s_2 = 1 \to 5 = 3B$

$3B = 5$

$\boxed{B = \frac{5}{3}}$ ✓

$1 = \frac{5}{3} + C$

$-C = \frac{5}{3} - 1$

$-C = \frac{5-3}{3}$

$-C = \frac{2}{3}$

$\boxed{C = -\frac{2}{3}}$ ✓

$2 = \frac{5}{3} - D$

$D = \frac{5}{3} - 2$

$= \frac{5-6}{3}$

$\boxed{D = -\frac{1}{3}}$ ✓

$1 = -1 + \frac{5}{3} + \frac{2}{3} - \frac{1}{3}$

$1 = \frac{-3+5+2-1}{3}$

$1 = 1$

$s_2 = 1$

$5 = 3B$

$3B = 5$

$B = \frac{5}{3}$

MATE BALJAK

4.) $w = \begin{bmatrix} 0 \\ 0 \\ 2x+3 \end{bmatrix}$ $w' = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \operatorname{div} w = 0$

$$\iint_{\partial K} (2x+3)\,dx\,dy = \iiint w'\,dx\,dy\,dz$$

$$= \iiint 0\,dx\,dy\,dz$$

$$= 0$$

NASTAVAK 1. ZADATKA

5

$$-\frac{1}{s^2} + \frac{\frac{5}{3}}{s-1} + \frac{-\frac{2}{3}s - \frac{1}{3}}{s^2+s+1}$$

$$-t + \frac{5}{3} \cdot e^t + \text{?}$$

$$-\frac{1}{3} \cdot \frac{2s+1}{s^2+s+1}$$

$$-\frac{1}{3} \cdot \frac{-2s-1}{s^2+s+1}$$

$$-\frac{1}{3} \cdot \frac{+2s+1}{s^2+s+1}$$

$$\frac{1}{3} \cdot \frac{2}{2}$$

$(-2s-1)^2$

$4s^2 +$

$\frac{1}{6}$

$$-\frac{\frac{2}{3}s}{s^2+s+1} = -\frac{2}{3}\frac{s}{s^2+s+1}$$

$s^2 - 2s + 1^2$

$$\frac{2}{6} \cdot \frac{-s-1}{s^2+s+1}$$

$(-2s-1)^2$

$(s-1)^2$

$s^2 - 2s + 1$ $\left(s - \frac{1}{2}\right)^2$

$$\frac{1}{3} \cdot \frac{-2s-1}{s^2+s+1}$$

$4s^2 + 2 \cdot 2s \cdot 1 + 1^2$

$4s^2 + 4s + 1$

$+2s^2$

$s^2 - s + \frac{1}{4}$

$(s^2-1)^2 = s^4 - 2s^2 + 1$

$(s^2-s)^2$

$s^4 - 2s^3 -$

$(s-1)^2$

$s^2 - 2s + 1$

$\frac{1}{3} \cdot x^2$

$\frac{2}{s^2+1}$

VIDI ZADAĆU COVRE COVRIĆ 2013-02-01

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: TONI ŠESTAN **BROJ INDEKSA:** 55283 - 2007

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$y'''(t) - y(t) = t, \quad y(0) = 1,\ y''(0) = 2,\ y'(0) = 1.$$

2. Izračunati integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $z = x^2$, $z = x$, $y = -5$ i $y = 6$. 20

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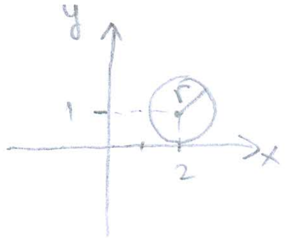
Ukupno: 40

3.

$r = 1$

$T(2,1)$



$r \in [0,1]$

$\varphi \in [0, 2\pi]$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$$\iint\limits_K (2x+3)\, dx dy$$

$$\int_0^{2\pi}\int_0^1 (2(r\cos\varphi + 2) + 3)\, r\, dr\, d\varphi$$

$$\int_0^{2\pi}\int_0^1 (2r\cos\varphi + 4 + 3)\, r\, dr\, d\varphi$$

$$\underbrace{2\int_0^{2\pi}\int_0^1 r^2\cos\varphi\, dr\, d\varphi}_{\mathrm{I}} + \underbrace{7\iint r\, dr\, d\varphi}_{\mathrm{II}}$$

$$\mathrm{I}: 2\int_0^{2\pi}\cos\varphi\, d\varphi \int_0^1 r^2 dr = 2\int_0^{2\pi}\cos\varphi\, d\varphi \left(\frac{r^3}{3}\right)\Big|_0^1 = \frac{2}{3}\int_0^{2\pi}\cos\varphi\, d\varphi$$

$$\frac{2}{3}\sin\varphi\Big|_0^{2\pi} = \frac{2}{3}(\sin 2\pi - \sin 0) = \emptyset$$

$$\mathrm{II}: 7\int_0^{2\pi} d\varphi \int_0^1 r\, dr = 7\int_0^{2\pi} d\varphi \left(\frac{r^2}{2}\right)\Big|_0^1 = \frac{7}{2}\int_0^{2\pi} d\varphi = \frac{7}{2}\varphi\Big|_0^{2\pi} = \frac{7}{2}2\pi = 7\pi$$

$$\mathrm{I} + \mathrm{II} = 7\pi$$

4. $a=2$

$$\iint_{\partial K} (2x+3)\,dx\,dy$$

$$\iint 2x\,dx\,dy + 3\,dx\,dy$$

$$W=\begin{bmatrix} 0 \\ 0 \\ 2x+3 \end{bmatrix} \qquad \operatorname{div} W = 0+0+0 = \phi$$

$$\iiint_K \operatorname{div} W\,dx\,dy = \phi$$ ✓

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: DOMAGOJ KNEŽEVIĆ

BROJ INDEKSA:

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$y'''(t) - y(t) = t, \quad y(0) = 1,\ y''(0) = 2,\ y'(0) = 1.$$

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Ukupno: 20

1. $y'''(t) - y(t) = t \qquad y(0)=1,\ y''(0)=2,\ y'(0)=1$

$$s^3 \cdot Y(s) - s^2 y(0) - s\cdot y'(0) - y''(0) - Y(s) = \frac{1}{s^2}$$

$$Y(s)\left[s^3-1\right] - s^2 - s - 2 = \frac{1}{s^2}$$

$$Y(s)\left[s^3-1\right] = \frac{1}{s^2} + s^2 + s + 2 = \frac{1+s^4+s^3+2s^2}{s^2}$$

$$Y(s) = \frac{s^4+s^3+2s^2+1}{s^2(s^3-1)} = \frac{s^4+s^3+2s^2+1}{s^2(s-1)(s^2+s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds+E}{s^2+s+1} \Big/ \cdot s^2(s-1)(s^2+s+1)$$

$$s^4+s^3+2s^2+1 = As(s-1)(s^2+s+1) + B(s-1)(s^2+s+1) + Cs^2(s^2+s+1) + (Ds+E)\cdot(s^3-s^2)$$

$$s^4+s^3+2s^2+1 = As(s^3+\cancel{s^2}+\cancel{s}-\cancel{s^2}-\cancel{s}-1) + B(s^3-1) + Cs^4+Cs^3+Cs^2+Ds^4-Ds^3+Es^3-Es^2$$

$$s^4+s^3+2s^2+1 = As^4 - As + Bs^3 - B + Cs^4 + Cs^3 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$$

(0) $\boxed{B=-1}$

(s) $-A=0$ $\boxed{A=0}$

(s^2) $C-E=2$

(s^3) $B+C-D+E=1$

(s^4) $A+C+D-E=1$

$\cancel{-1}+C-D+E=\cancel{1}$

$C-D+E=0$

$C+D-E=1$ $+$

$2C=1$

$\boxed{C=\frac{1}{2}}$

$C-D+E=0$

$C+E=D \Rightarrow D=C+E$

$D = \frac{1}{2}-\frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

$\boxed{D=\frac{1}{4}}$

$\frac{1}{2}-E=2$

$\frac{1}{2}-2=E$

$E=\frac{1}{2}-2$

$\boxed{E=-\frac{3}{4}}$

$$Y(s) = -\frac{1}{s^2} + \frac{1}{2}\cdot\frac{1}{s-1} + \left(\frac{1}{4}s - \frac{3}{4}\right)\frac{1}{s^2+s+1}$$

$(s^2+s+1) = (s-1)^2 + s^2$

$$Y(s) = -t + \frac{1}{2}e^t$$

3. $\underline{r=1}$ $T(2,1)$

$$x-2=r\cos\varphi \Rightarrow x=r\cos\varphi+2 \qquad \varphi\in[0,2\pi]$$
$$y-1=r\sin\varphi \qquad r\in[0,1]$$

$$\underline{I}=\int_0^{2\pi}d\varphi\int_0^1\left[2(r\cos\varphi+2)+3\right]r\,dr=\int_0^{2\pi}d\varphi\int_0^1(2r^2\cos\varphi+7r)\,dr$$

$$=\int_0^{2\pi}d\varphi\left[\int_0^1 2r^2\cos\varphi\,dr+\int_0^1 7r\,dr\right]=$$

$$=\int_0^{2\pi}d\varphi\left[r^2\cos\varphi\Big|_0^1+\frac{7}{2}r^2\Big|_0^1\right]=$$

$$=\int_0^{2\pi}\left(\cos\varphi+\frac{7}{2}\right)d\varphi=\int_0^{2\pi}\cos\varphi\,d\varphi+\frac{7}{2}\int_0^{2\pi}d\varphi=\sin\varphi\Big|_0^{2\pi}+\frac{7}{2}\varphi\Big|_0^{2\pi}$$

$$=\sin 2\pi-\sin 0+\frac{7}{\cancel{2}}\cancel{2}\pi=0-0+7\pi=\boxed{7\pi}\ \checkmark$$

2. $z=x^2 \quad y=-5$

$z=x \quad y=6$

$I=\iiint x\,dx\,dy\,dz$

$x=r\cos\varphi$

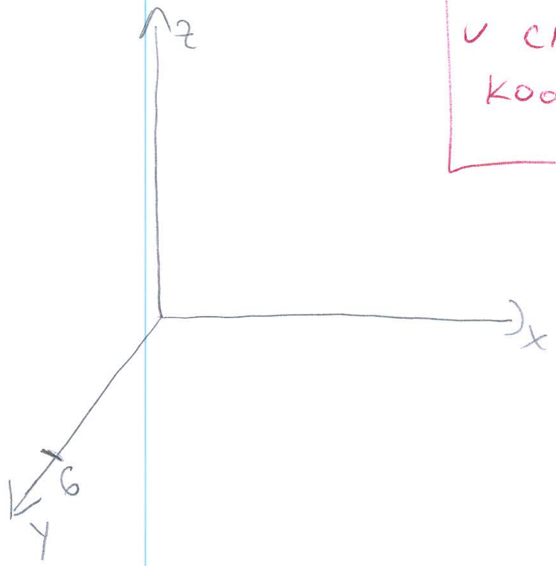
$y=r\sin\varphi$

$z=z$

$x^2=z \,/\sqrt{\ }$

$x=\sqrt{z}$

BEZ PRELASKA U CILINDRIČNE KOORDINATE



$\varphi\in[0,2\pi]$

$r\in[0,6]$

$z\in[r\cos\varphi, r^2\cos\varphi]$

$z=r^2\cos^2\varphi$

$z=r\cos\varphi$

$$I=\int_0^{2\pi}d\varphi\int_0^6 r\,dr\int_{r\cos\varphi}^{r^2\cos^2\varphi}dz=\int_0^{2\pi}d\varphi\int_0^6 r\,dr\cdot z\Big/_{r\cos\varphi}^{r^2\cos^2\varphi}=\int_0^{2\pi}d\varphi\int_0^6 r\,dr\cdot(r^2\cos^2\varphi-r\cos\varphi)$$

$$=\int_0^{2\pi}d\varphi\int_0^6\left[r^3\left(\frac{1+\cos(2\varphi)}{2}\right)-r^2\cos\varphi\right]dr$$

$$=\int_0^{2\pi}d\varphi\cdot\left[\frac{r^4}{4}\left(\frac{1+\cos(2\varphi)}{2}\right)-\frac{r^3}{3}\cos\varphi\right]\Big/_0^6$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	POPUNJAVA NASTAVNIK
IME I PREZIME: DUJE KRALJEV **BROJ INDEKSA:** 17-2-0015-2016	Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~
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Ukupno: 20

1) $s^3 Y(s) - s^2 \underset{1}{y(0)} - s\underset{1}{y'(0)} - \underset{2}{y''(0)} - Y(s) = \frac{1}{s^2}$

$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$

$s^3 Y(s) - Y(s) = \frac{1}{s^2} + s^2 + s + 2$

$Y(s)(s^3 - 1) = \frac{1 + s\cdot 1 + s\cdot s^2 + 2\cdot s^2}{s^2} = \frac{1 + s + s^3 + 2s^2}{s^2} = \frac{s^3 + 2s^2 + s + 1}{s^2}$

$Y(s) = \frac{\frac{s^3 + 2s^2 + s + 1}{s^2}}{s^3 - 1} = \frac{s^3 + 2s^2 + s + 1}{(s^3 - 1)\cdot s^2} = \frac{s^3 + 2s^2 + s + 1}{(s^2 + s + 1)(s - 1)s^2}$

$\frac{s^3 + 2s^2 + s + 1}{(s^2 + s + 1)(s - 1)s^2} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s - 1} + \frac{Ds + E}{s^2 + s + 1} \Big/ (s^2 + s + 1)(s - 1)s^2$

$s^3 + 2s^2 + s + 1 = A\underbrace{(s^2 + s + 1)(s - 1)}_{s^3 + s^2 + s - s^2 - s - 1} + B\underbrace{s(s^2 + s + 1)(s - 1)}_{s^4 - s} + C\underbrace{s^2(s^2 + s + 1)}_{s^4 + s^3 + s^2} + (Ds + E)\underbrace{s^2(s - 1)}_{s^3 - s^2}$

$s = 0$

$1 = A\cdot 1(-1)$

$1 = -A$

$A = -1$

$s - 1 = 0$

$s = 1$

$1 + 2 + 1 + 1 = C \cdot 1 \cdot 3$

$5 = 3C$

$C = \frac{5}{3}$

$s^2 = 0$

$s = 0$

$B = 0$

$$s^3+2s^2+s+1 = \cancel{As^3} - A + \cancel{Bs^4} - \cancel{Bs} + Cs^4 + \cancel{Cs^3} + \cancel{Cs^2} + \cancel{Ds^4} - \cancel{Ds^2} + \cancel{Es^3} - \cancel{Es^2}$$

$$s^3+2s^2+s+1 = (B+C+D)s^4 + (A+C+E)s^3 + (C-E-D)s^2 - Bs - A$$

$-A = 1$

$A = -1$

$B+C+D=0$

$A+C+E=1$

$C-E-D=2$

$-1+\frac{5}{3}+D=0$

$D = 1-\frac{5}{3}$

$D = -\frac{2}{3}$

$-1+\frac{5}{3}+E=1$

$C = \frac{5}{3}$

$E = 1+1-\frac{5}{3}$

$E = \frac{1}{3}$

$$Y(s) = \frac{-1}{s^2} + \frac{0}{s} + \frac{\frac{5}{3}}{s-1} + \frac{-\frac{2}{3}s + \frac{5}{3}}{}$$

$(s-a)^2+b^2 = s^2+s+1$

$s^2 - 2sA + A^2 + B^2 = s^2 + s + 1$

$s^2 = 1 \Rightarrow s = 1$

$2A = 1 \Rightarrow A = \frac{1}{2}$

$\cancel{A^2 = 1 \Rightarrow A = 1}$

$B^2 = 1 \Rightarrow B = 1$

$(-s - \frac{1}{2})^2 + 1^2$

$s^2 + 2s\frac{1}{2} + 1^2$

$s^2 + s + 1$

$s^2 -$

$A^2 + B^2 = 1$

$(s-1)^2 + 1^2$

$s^2 - 2s + 1 + 1$

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{9}{16}$

$(s - \frac{1}{2})^2 + \frac{3}{4}$

$s^2: \quad 1 = 1$

$s: \quad -2A = 1 \Rightarrow A = -\frac{1}{2}$

bez s: $A^2 + B^2 = 1 \Rightarrow B = \sqrt{1 - \frac{1}{4}}$

$\Rightarrow B = \frac{\sqrt{3}}{2}$

VIDI ZADAĆU LOVRE LOVRIĆ 2013-02-01

$\frac{1}{4} + \frac{9}{16}$

$\frac{4}{16} + \frac{9}{16}$ 13

DUJO KRALJOV

5) $x^2+y^2=r^2$

$x^2+y^2=1$

$x=r\cos\varphi$

$y=r\sin\varphi$

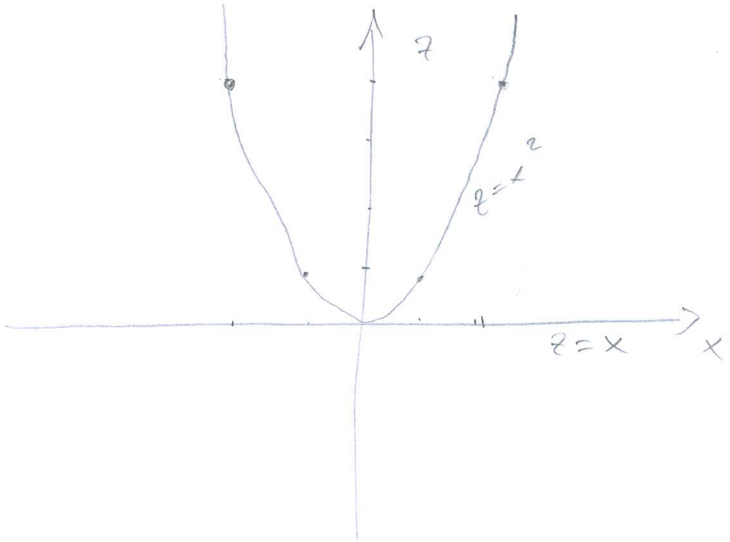
$dx\,dy=r\,dr\,d\varphi$

$\emptyset$

2) $z=x^2$

$z=x$

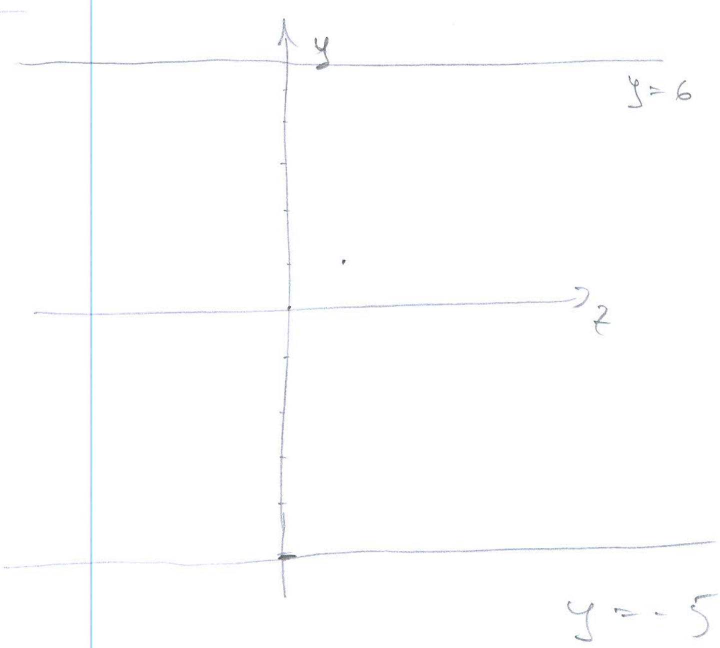
x	-1	1	0	-2	2
$z=x^2$	1	1	0	4	4



$z=x$

$x=z$

z	1	0
$x=z$	1	0



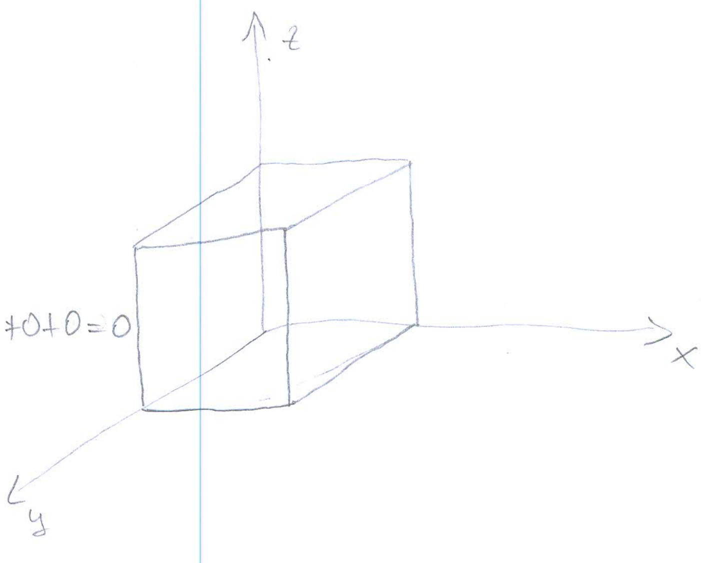
?

4) $a=2$

DU 5 KRALSOV

K.. kocka $T(0,0,0)$

$$\iint_{\partial K} w_x\,dy\,dz + w_y\,dx\,dz + w_z\,dx\,dy$$



$$w=\begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} \qquad w\begin{pmatrix} 0 \\ 0 \\ 2x+3 \end{pmatrix}$$

$$\operatorname{div} w = \frac{\partial(0)}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial(2x+3)}{\partial z} = 0+0+0=0$$

$$\iint_{\partial K} (2x+3)\,dx\,dy = \iint_{\partial K} (w\,|\,ds) = \iiint_K \operatorname{div} w\,dx\,dy\,dz$$

$$= \iiint_K 0\,dx\,dy\,dz = 0 \quad \checkmark$$

3)

$x^2 + y^2 = r^2$

$x^2 + y^2 = 1$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$dx\,dy = r\,dr\,d\varphi$

$\varphi = [0, 2\pi]$

$r = [0, 1]$

$$\iint_K (2x+3)\,dx\,dy = \int_0^{2\pi}\int_0^1 \left(2\underbrace{(r\cos\varphi+1)}_{=0} + 3\right) r\,dr\,d\varphi$$

X

∅

$$\int_0^{2\pi}\int_0^1 3r\,dr\,d\varphi = \int_0^{2\pi} 3\,\frac{r^2}{2}\Big|_0^1\,d\varphi = \int_0^{2\pi} 3\,\frac{1^2}{2}\,d\varphi$$

$$= \int_0^{2\pi} \frac{3}{2}\,d\varphi = \frac{3}{2}\varphi\Big|_0^{2\pi} = \frac{3}{2}\,2\pi = 3\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: DINO KURIĆ BROJ INDEKSA: 56192-2008

	POPUNJAVA NASTAVNIK Broj ↓ bodova
1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $$y'''(t) - y(t) = t, \quad y(0) = 1, y''(0) = 2, y'(0) = 1.$$	~~20~~
2. Izračunati integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $z = x^2$, $z = x$, $y = -5$ i $y = 6$.	20
3. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint\limits_K (2x + 3)\,dxdy$?	~~20~~
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\,dxdy$?	20
5. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint\limits_S 3dxdy$? (pomoć: rot $(3x\mathbf{j}) = 3\mathbf{k}$)	20

Ukupno: 20

3. $r=1 \quad T(2,1) \qquad \iint\limits_K (2x+3)$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 2$

$\boxed{\begin{array}{l} \varphi \in [0, 2\pi] \\ r \in [0, 1] \end{array}}$

$\boxed{\begin{array}{l} x = 1\cdot\cos\varphi + 2 \\ y = 1\cdot\sin\varphi + 1 \end{array}}$

$$\int_0^{2\pi}\left(\int_0^1 (2\cos\varphi + 3)\cdot 1\, dr\right) d\varphi = \times$$

$(r\cos\varphi + 2)$

$$\int_0^{2\pi}\left(\int_0^1 2\cos\varphi\, dr + \int_0^1 3\, dr\right) d\varphi$$

$$\int_0^{2\pi}\left(2\int_0^1 \cos\varphi\, dr + 3\int_0^1 dr\right) d\varphi =$$

$$\int_0^{2\pi}\left(2\cdot\sin\varphi\Big|_0^1 + 3\int_0^1 r\right) d\varphi =$$

$$\int_0^{2\pi}\left(2\,\cancel{\sin 1}^{\,0} + 3\right) d\varphi$$

$$\int_0^{2\pi} (5)\, d\varphi = \int_0^{2\pi} 5\, d\varphi$$

$$= 5\int d\varphi$$

$$= 5\cdot\varphi = 5\cdot 2\pi = 10\pi$$

1. $y'''(t) - y(t) = t$ $y(0) = 1$ $y''(0) = 2$ $y'(0) = 1$

$$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) - Y(s) = \frac{1}{s^2}$$

$$s^3 Y(s) - s^2 - s - 2 - Ys = \frac{1}{s^2}$$

$$s^3 Y(s) - Y(s) = \frac{1}{s^2} + s^2 + s + 2$$

$$s^3 Y(s) - Y(s) = \frac{1 + s^4 + s^3 + 2s^2}{s^2}$$

$$Y(s)(s^3 - 1) = \frac{1 + s^4 + s^3 + 2s^2}{s^2} \quad / \; s^3 - 1$$

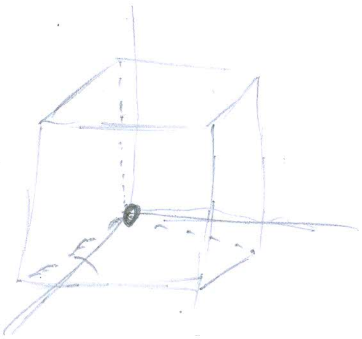
$$Y(s) = \frac{\frac{1 + s^4 + s^3 + 2s^2}{s^2}}{\frac{s^3 \cdot (s^3 - 1)}{1}}$$

$$Y(s) = \frac{1 + s^4 + s^3 + 2s^2}{s^5 \cdot (s^5 - s^2)}$$

$$Y(s) = \frac{1 + s^4 + s^3 + 2s^2}{s^5 \cdot s^2 \cdot (s^3 - 1)}$$

$$Y(s) = \frac{1 + s^4 + s^3 + 2s^2}{s^7 \cdot (s^2 + s + 1) \cdot (s - 1)}$$

4) $a=2$ centrirana u ishodistu $\iint_{\partial K} (2x+3)\,dx\,dy$



$x \in (-1,1)$

$y \in [-1,1]$

$z \in [-1,1]$

$\iint F(x) = \iiint \operatorname{div} w \, dx\,dy\,dz$

$$F \begin{bmatrix} 0 \\ 0 \\ 2x+3 \end{bmatrix} \begin{matrix} \to \partial x = 0 \\ \to \partial y = 0 \\ \to \partial z = 0 \end{matrix} \Bigg\} \operatorname{div} w = 0$$

$$\iint_{\partial K} (2x+3)\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 0 \, dx\,dy\,dz = 0$$