MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20 5

$$y'''(t) - y(t) = t, \quad y(0) = 1, y''(0) = 2, y'(0) = 1.$$

2. Izračunati integral funkcije $f(x, y, z) = x$ u dijelu prostora omeđenog plohama $z = x^2$, $z = x$, $y = -5$ i $y = 6$. 20

3. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint_K (2x + 3)\,dxdy$? 20

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x + 3)\,dxdy$? 20

5. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Izračunati $\iint_S 3dxdy$? (pomoć: rot $(3x\mathbf{j}) = 3\mathbf{k}$) 20

Ukupno: 45

3.

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$dxdy = r\,dr\,d\varphi$

$$\int_0^{2\pi} d\varphi \int_0^1 2(r\cos\varphi + 2) + 3\; r\,dr \quad \checkmark$$

$$\int_0^{2\pi} d\varphi \int_0^1 (2r\cos\varphi + 4 + 3)\; r\,dr$$

$$2\int_0^{2\pi} d\varphi \int_0^1 r^2\cos\varphi\,dr + \int_0^{2\pi} d\varphi \int_0^1 7r\,dr$$

$$2\int_0^{2\pi} d\varphi \left[\frac{r^3}{3}\cdot\cos\varphi\right]_0^1 + 7\int_0^{2\pi} d\varphi \left[\frac{r^2}{2}\right]_0^1$$

$$2\int_0^{2\pi} d\varphi\, \frac{1}{3}\cdot\cos\varphi + 7\int_0^{2\pi} d\varphi\, \frac{1}{2}$$

$$\frac{2}{3}\int_0^{2\pi} d\varphi\cos\varphi + \frac{7}{2}\int_0^{2\pi} d\varphi$$

$$\frac{2}{3}\cdot\left((-\sin 2\pi) - (-\sin 0)\right) + \frac{7}{2}\cdot 2\pi$$

$$0 + \frac{14}{2}\pi = 7\pi \quad \checkmark$$

$Y(0) = 1 \quad , \quad Y'(0) = 2 \quad , \quad Y''(0) = 1$ (1.)

$$s^3 Y(s) - s^2 Y(0) - s Y'(0) - Y''(0) - \frac{1}{1} Y\left(\frac{s}{1}\right) = \frac{1}{s^2}$$

$$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^3 - 1) = \frac{1}{s^2} + s^2 + s + 2$$

$$Y(s)(s^3 - 1) = \frac{1 + s^4 + s^3 + 2s^2}{s^2} \quad / \cdot \frac{1}{s^3 - 1}$$

$$\frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^2 + s + 1)(s - 1)} = \frac{A}{s}$$

$$\frac{s^4 + s^3 + 2s^2 + 1}{s^2(s-1)(s+1)(s^2+s+1)}$$

$$\frac{s^4 + s^3 + 2s^2 + 1}{s^2(s-1)(s^2+s+1)} = \frac{A}{s^2} + \frac{B}{s-1} + \frac{Cs + D}{s^2 + s + 1}$$

$$s^4 + s^3 + 2s^2 + 1 = A(s-1)(s^2+s+1) + B s^2(s^2+s+1) + (Cs + D)s^2(s-1)$$

$$= As^3 + As^2 + As - As^2 - As - A + Bs^4 + Bs^3 + Bs^2 + Cs^4 - Cs^3 + Ds^3 - Ds^2$$

$$= As^3 - A + Bs^4 + Bs^3 + Bs^2 + Cs^4 - Cs^3 + Ds^3 - Ds^2$$

$s^4 \rightarrow 1 = B + C$

$s^3 \rightarrow 1 = A + B - C + D$

$s^2 \rightarrow 2 = B - D$

$1 = -A$

$\boxed{A = -1}$ ✓

$s_2 = 1 \rightarrow 5 = 3B$

$3B = 5$

$\boxed{B = \frac{5}{3}}$ ✓

$1 = \frac{5}{3} + C$

$-C = \frac{5}{3} - 1$

$-C = \frac{5-3}{3}$

$-C = \frac{2}{3}$

$\boxed{C = -\frac{2}{3}}$ ✓

$2 = \frac{5}{3} - D$

$D = \frac{5}{3} - 2$

$= \frac{5-6}{3}$

$\boxed{D = -\frac{1}{3}}$ ✓

$1 = -1 + \frac{5}{3} + \frac{2}{3} - \frac{1}{3}$

$1 = \frac{-3+5+2-1}{3}$

$1 = 1$