

prof. Mate Kosor

57.5/100

MATEMATIKA 1

7. veljače 2013.

Ime i prezime:

Ivan Colić

Broj indeksa:

17-2-0152-2011

Vrijeme: od _____ do _____ ♣D

Broj bodova:

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (17.5) Izračunaj determinantu matrice:

$$A = \begin{bmatrix} 2 & 4 & -6 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 2 \\ 0 & -4 & 1 & -2 \end{bmatrix}.$$

2. (17.5) Riješi u skupu \mathbb{C} jednadžbu:

$$z^3 = \left(\frac{\sqrt{3} - i}{\cos(\frac{\pi}{12}) + i \sin(\frac{\pi}{12})} \right)^2$$

3. (15) Odredi asimptote sljedeće funkcije:

$$f(x) = \sqrt{x^2 - x}$$

4. (12.5+12.5)

- a) Deriviraj funkciju:

$$f(x) = x \cdot \arcsin x + \sqrt{1 - x^2}$$

- b) Odredi domenu funkcije:

$$f(x) = \ln(x^2 - 3x + 5)$$

5. (25) Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \frac{5 - x}{9 - x^2}$$

3.

$$f(x) = \sqrt{x^2 - x}$$

$$x^2 - x \geq 0$$

$$x(x-1) \geq 0$$

$$x = 0 \quad x - 1 = 0$$

$$x = 1$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 - x}}{x} = \frac{\sqrt{x^2 - x}}{\sqrt{x^2 - x}} = \lim_{x \rightarrow +\infty} \frac{x^2 - x \cdot \frac{1}{x^2}}{\sqrt{x^2 - x} \cdot \frac{1}{x^2}} = \frac{1 - 0}{0} = +\infty$$

1. Pomenu

	$-\infty$	0	1	$+\infty$
$x=0$	-	0	+	+
$x=1$	-	-	0	+
	\oplus		\ominus	\oplus

$$D: \langle -\infty, 0 \cup]1, +\infty \rangle$$

2. Asimptote

$$\lim_{x \rightarrow 0^-} \sqrt{x^2 - x} = 0$$

$$\lim_{x \rightarrow 1^+} \sqrt{x^2 - x} = 0$$

V. A. Nema

H. A.

$$\lim_{x \rightarrow \infty} \sqrt{\frac{x^2 - x}{x^2}} =$$

$$= \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x}}{\sqrt{\frac{1}{x^2} - \frac{1}{x^2}}} = \frac{1}{0} = +\infty$$

H. A. Nema

Zaključak: NEMA Asimptota !!!

♣D

1. Determinante

$$\begin{vmatrix} 2 & 4 & 6 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 2 \\ 0 & -4 & 1 & -2 \end{vmatrix} = 2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 1 & 3 & -1 & 1 & 3 \\ 0 & 2 & 2 & 0 & 2 \\ -4 & 1 & -2 & -4 & 1 \end{vmatrix} = 2 \cdot (1 \cdot 2 \cdot (-2) + 3 \cdot 2 \cdot (-4) + 0 - 0 - 12 \cdot 1 - (-1 \cdot 2 \cdot (-4)))$$

$$= 2 \cdot ((1 \cdot 2 \cdot (-2)) + (3 \cdot 2 \cdot (-4)) + 0 - 0 - (1 \cdot 2 \cdot 1) - (-1 \cdot 2 \cdot (-4)))$$

$$= 2 \cdot (-4 - 24 - 2 - 8) = 2 \cdot (-38) = -76 \quad \checkmark \quad 17.5$$

4. a) $f(x) = x \cdot \arcsin x + \sqrt{1-x^2}$

$$\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = x' \cdot \arcsin x + x \cdot (\arcsin x)' + \frac{1}{2\sqrt{1-x^2}} \cdot (1-x^2)'$$

$$= 1 \cdot \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}} + \left(\frac{-2x}{2\sqrt{1-x^2}} \right)$$

$$= \arcsin x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \arcsin x \quad \checkmark \quad 12.5$$

b) $\ln(x^2 - 3x + 5)$

$$x^2 - 3x + 5 > 0$$

$$x^2 - 3x + 5 \neq 0$$

Koji: god $x \in \mathbb{R}$ vrijedi

$$x_{1,2} = \frac{-(-3) \pm \sqrt{9 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$= \frac{3 \pm \sqrt{-11}}{2}$$

$$= \frac{3 \pm i\sqrt{11}}{2}$$

$$D(f) = \mathbb{R}$$

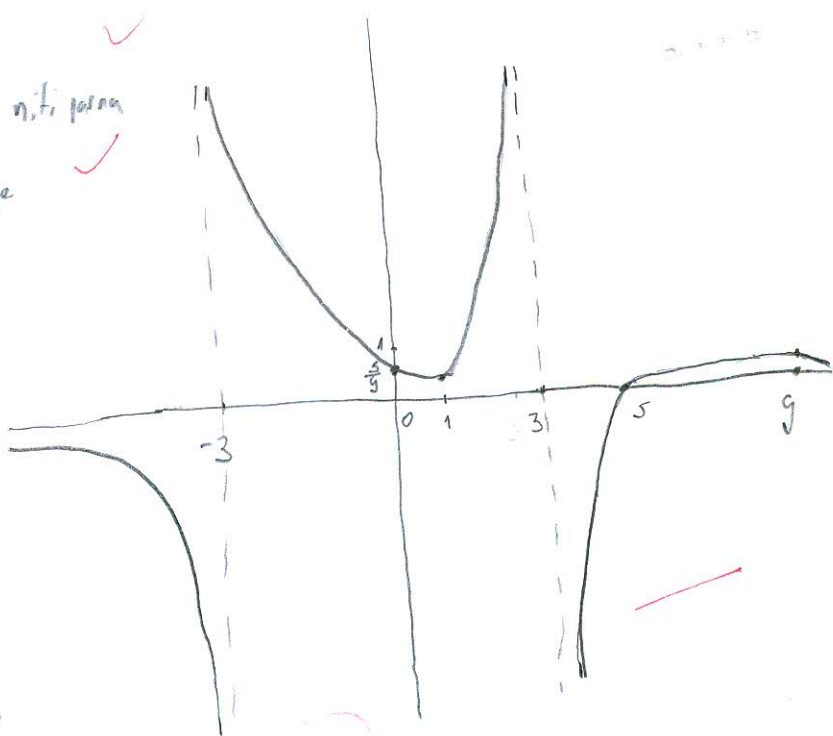
12.5

5. $f(x) = \frac{5-x}{9-x^2}$

1. Pomenu $9-x^2 \neq 0$
 $x^2 = 9$
 $x_1 = -3$
 $x_2 = 3$

2. $\lim_{x \rightarrow 3^-} \frac{5-x}{9-x^2} = -\infty$
 $\lim_{x \rightarrow 3^+} \frac{5-x}{9-x^2} = +\infty$
 $\lim_{x \rightarrow -3^-} \frac{5-x}{9-x^2} = -\infty$
 $\lim_{x \rightarrow -3^+} \frac{5-x}{9-x^2} = +\infty$

3. Parnost i neparnost
 $f(x) = \frac{5-x}{9-x^2}$
 - Funkcija je niti parna niti neparna
 - Funkcija f(x) nije periodična



$x = -3$
 $x = 3$ su v.a. ove funkcije
 $\lim_{x \rightarrow \infty} \frac{5-x/x^2}{9-x^2/x^2} = \frac{0}{-1} = 0 \Rightarrow \text{ml}$
 Nema H.A.

K.A.
 $y = kx + l$
 $k = \lim_{x \rightarrow \infty} \frac{5-x}{9-x^2} = \lim_{x \rightarrow \infty} \frac{5-x/x^2}{9-x^2/x^2} = \lim_{x \rightarrow \infty} \frac{5/x^2 - 1/x}{9 - 1/x^2} = \frac{0}{-1} = 0$

Nema k.A.
 4. Nul tačke
 $f(x) = 0$
 $0 = \frac{5-x}{9-x^2}$
 $0 = 5-x$
 $x = 5$
 $T(5, 0)$
 $T(0, \frac{5}{9})$

5. Derivacije
 $f'(x) = \frac{(5-x)'(9-x^2) - (5-x)(9-x^2)'}{(9-x^2)^2}$
 $= \frac{-1 \cdot (9-x^2) - (5-x) \cdot (-2x)}{(9-x^2)^2}$
 $= \frac{-9 + x^2 + 10x - 2x^2}{(9-x^2)^2}$
 $= \frac{-x^2 + 10x - 9}{(9-x^2)^2}$

$f''(x) = \frac{(-x^2 - 10x + 9)' \cdot (9-x^2)^2 - (-x^2 - 10x + 9) \cdot (9-x^2)'}{[(9-x^2)^2]^2}$
 $= \frac{(-2x - 10)(9-x^2)^2 - (-x^2 - 10x + 9) \cdot 2(9-x^2)}{[9-x^2]^4}$

Složeno

Lokalni min
 $T(1, \frac{1}{4})$
 Lok max
 $T(9, \frac{1}{18})$

$-x^2 + 10x - 9 = 0$
 $x_{1,2} = \frac{-10 \pm \sqrt{10^2 - 4 \cdot (-1) \cdot 9}}{-2}$
 $x_{1,2} = \frac{-10 \pm \sqrt{100 - 36}}{-2}$
 $= \frac{-10 \pm \sqrt{64}}{-2}$
 $= \frac{-10 \pm 8}{-2}$

$-\infty$	-3	1	3	9	$+\infty$
	-4	-2	2	5	10
$f(x)$	\downarrow	\downarrow	\uparrow	\uparrow	\downarrow
			lok Minimum	lok Max	

Funkcija nije granična odozdo niti odozgo

$x_1 = 9$
 $x_2 = -\frac{1}{2}$
 $x_2 = 1$
 $f(9) = \frac{5-9}{9-9^2} = \frac{1}{18}$

$f(1) = \frac{5-1}{9-1^2} = \frac{4}{8} = \frac{1}{2}$
 $T(9, \frac{1}{18}), T(1, \frac{1}{2})$

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1)
$$\det A = \begin{vmatrix} 2 & 4 & -6 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 2 \\ 0 & -4 & 1 & -2 \end{vmatrix} = 2 \cdot (-1)^{1+1} \begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & 2 \\ -4 & 1 & -2 \end{vmatrix} +$$

$$0 \cdot (-1)^{2+1} \begin{vmatrix} 4 & -6 & 9 \\ 0 & 2 & 2 \\ -4 & 1 & -2 \end{vmatrix} + 0 \cdot (-1)^{3+1} \begin{vmatrix} 4 & -6 & 9 \\ 1 & 3 & -1 \\ -4 & 1 & -2 \end{vmatrix} + 0 \cdot (-1)^{4+1}$$

$$\begin{vmatrix} 4 & -6 & 9 \\ 1 & 3 & -1 \\ 0 & 2 & 2 \end{vmatrix} = 2 \cdot D_1 - 0 \cdot D_2 + 0 \cdot D_3 - 0 \cdot D_4$$

4.)

$$a) f(x) = x \cdot \arcsin x + \sqrt{1-x^2}$$

$$f'(x) =$$

$$b) f(x) = \ln(x^2 - 3x + 5)$$

$$\text{UV)ET: } x^2 - 3x + 5 > 0$$

$$x =$$

