prof. Mute Kosor

97.5/100

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: Ivan Colić **Broj indeksa:** 17-2-0152-2011

Vrijeme: od_____ **do**_____ **♣D** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Izračunaj determinantu matrice:

$$A = \begin{bmatrix} 2 & 4 & -6 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 2 \\ 0 & -4 & 1 & -2 \end{bmatrix}.$$

2. *(17.5)* Riješi u skupu $\mathbb{C}$ jednadžbu:

$$z^3 = \left(\frac{\sqrt{3} - i}{\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12})}\right)^2$$

3. *(15)* Odredi asimptote sljedeće funkcije:

$$f(x) = \sqrt{x^2 - x}$$

4. *(12.5+12.5)*

a) Deriviraj funkciju:

$$f(x) = x \cdot \arcsin x + \sqrt{1 - x^2}$$

b) Odredi domenu funkcije:

$$f(x) = \ln(x^2 - 3x + 5)$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \frac{5 - x}{9 - x^2}$$

3.

$f(x)=\sqrt{x^2-x}$

$x^2-x \geq 0$

$x(x-1) \geq 0$

$x=0 \quad x-1=0$

$x=1$

1. Pomenka

	$-\infty$		0		1		$+\infty$
$x=0$	$-$		0	$+$		$+$	
$x-1=0$	$-$			$-$	0	$+$	
	$\oplus$			$-$		$\oplus$	

$Df: \langle -\infty, 0] \cup [1, +\infty \rangle$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2-x}}{x} \cdot \frac{\sqrt{x^2-x}}{\sqrt{x^2-x}} = \lim_{x\to\infty} \frac{x^2-x \;/:x^2}{x\sqrt{x^2-x}\;/:x^2} = \frac{1-0}{0} = +\infty$$

Zaključek: NEMA Asimptota !!!

2. Asimptote

$\lim_{x\to 0^-} \sqrt{x^2-x} = 0$

$\lim_{x\to 1^+} \sqrt{x^2+x} = 0$

V. A. Nema

H. A

$$\lim_{x\to\infty} \sqrt{\frac{x^2-x\;/:x^2}{x^2-x\;/:x^2}} =$$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{x}}{\sqrt{\frac{1}{x^2}-\frac{1}{x^2}}} = \frac{1}{0} = +\infty$$

H.A. Nema

♣D

1. Determinanten

$$\begin{bmatrix} 2 & 4 & 6 & 6 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 2 \\ 0 & -4 & 1 & -2 \end{bmatrix} = 2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 1 & 3 & -1 & 1 & 3 \\ 0 & 2 & 2 & 0 & 2 \\ -4 & 1 & -2 & -4 & 1 \end{vmatrix} = 2 \cdot \left(1 \cdot 2 \cdot (-2) + 3 \cdot 2 \cdot (-4) + 0 - 0 - 1 \cdot 2 \cdot 1 - (-1) \cdot 2 \cdot (-4)\right)$$

$$= 2 \cdot \left(\left(1 \cdot 2 \cdot (-2)\right) + \left(3 \cdot 2 \cdot (-4)\right) + 0 - 0 - \left(1 \cdot 2 \cdot 1\right) - \left(-1 \cdot 2 \cdot -4\right)\right)$$

$$= 2 \cdot \left(-4 - 24 - 2 - 8\right) = 2 \cdot (-38) = -76$$

✓ (17.5)

4. a) $f(x) = x \cdot \arcsin x + \sqrt{1-x^2}$

$\arcsin x - \frac{1}{\sqrt{1-x^2}}$

$$f'(x) = x' \cdot \arcsin x + x \cdot (\arcsin x)' + \frac{1}{2\sqrt{1-x^2}} \cdot (1-x^2)'$$

$$= 1 \cdot \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}} + \left(\frac{-2x}{2\sqrt{1-x^2}}\right)$$

$$= \arcsin x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \arcsin x$$

✓ (12.5)

b) $\ln(x^2 - 3x + 5)$

$x^2 - 3x + 5 > 0$ $\quad x^2 - 3x + 5 \neq 0$

Koji god $x \in \mathbb{R}$ uvrstimo

$$x_{1,2} = \frac{-(-3) \mp \sqrt{9 - 4 \cdot 1 \cdot 5}}{2a}$$

$$= \frac{3 \mp \sqrt{-11}}{2}$$

$$= \frac{3 \mp i\sqrt{11}}{2}$$

$D(f) = \mathbb{R}$

✓ (12.5)

5.

$f(x)=\frac{5-x}{9-x^2}$

1. Područje

$9-x^2\neq 0$

$x^2=9$

$x_1=-3$

$x_2=3$

$x=-3$

$x=3$ su V.A. ove funkcije ✓

$\lim_{x\to\infty}\frac{5-x\,/:x^2}{9-x^2\,/:x^2}=\frac{\frac{5}{x^2}-\frac{x}{x^2}}{\frac{9}{x^2}-\frac{x^2}{x^2}}=\frac{0}{-1}\Rightarrow$ nul'

Nema H. A.

K. A.

$y=kx+l$

$k=\lim_{x\to\infty}\frac{\frac{5-x}{9-x^2}}{x}=\lim_{x\to\infty}\frac{5-x}{x(9-x^2)}=\lim_{x\to\infty}\frac{5-x\,/:x^3}{9x-x^3\,/:x^3}=\frac{\frac{5}{x^3}-\frac{x}{x^3}}{\frac{9x}{x^3}-\frac{x^3}{x^3}}=\frac{0}{-1}$

Nema K. A.

4. Nul točke

$f(x)=0$

$0=\frac{5-x}{9-x^2}$

$0=5-x$

$x=5$

$T(5,0)$

$x=0$

$y=\frac{5-0}{9-0^2}$

$y=\frac{5}{9}$

$T(0,\frac{5}{9})$

2. $\lim_{x\to -3^-}\frac{5-x}{9-x^2}=-\infty$

$\lim_{x\to -3^+}\frac{5-x}{9-x^2}=-\infty$

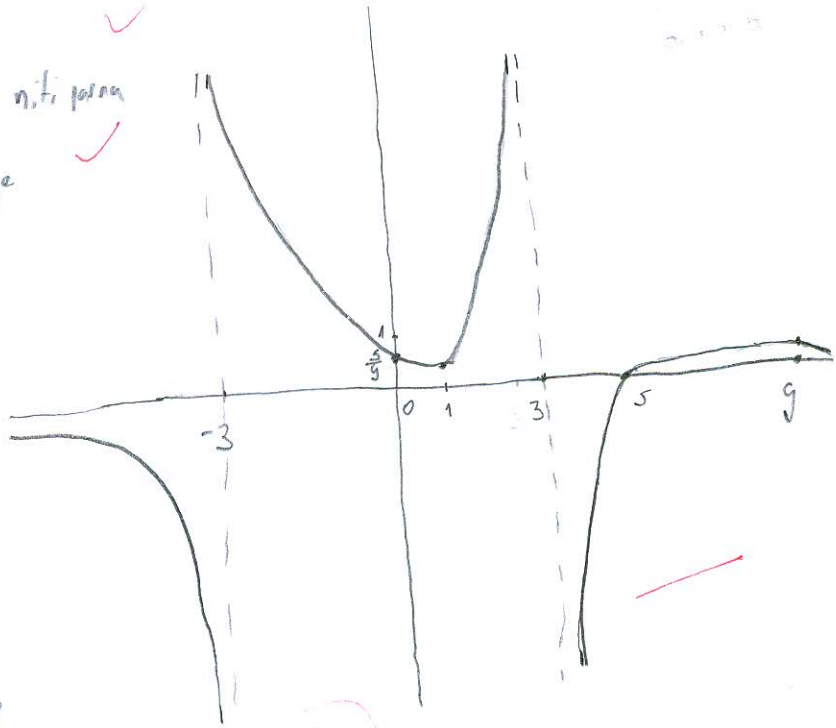
$\lim_{x\to 3^-}\frac{5-x}{9-x^2}=+\infty$

$\lim_{x\to 3^+}\frac{5-x}{9-x^2}=-\infty$

3. Parnost i neparnost

$f(x)=\frac{5+x}{9-x^2}$ ✓

- Funkcija je niti parna niti neparna ✓
- Funkcija $f(x)$ nije periodična



25

$-x^2+10x-9=0$

$x_{1,2}=\frac{-10\mp\sqrt{10^2-4\cdot(-1)\cdot(9)}}{-2}$

$x_{1,2}=\frac{-10\mp\sqrt{100-36}}{-2}$

$=\frac{-10\mp\sqrt{64}}{-2}$

$=\frac{-10\mp 8}{-2}$

$x_1=9$

$x_2=\frac{-1}{-2}$

$x_2=1$

$f(9)=\frac{5-9}{9-9^2}=\frac{1}{18}$

$f(1)=\frac{5-1}{9-1^2}=\frac{4}{8}=\frac{1}{2}$ $T(9,\frac{1}{18}),T(1,\frac{1}{2})$

5. Derivacije

$f'(x)=\frac{(5-x)'(9-x^2)-(5-x)(9-x^2)'}{(9-x^2)^2}$

$=\frac{-1\cdot(9-x^2)-[(5-x)\cdot(-2x)]}{(9-x^2)^2}$

$=\frac{-9+x^2-(10x+2x^2)}{(9-x^2)^2}$

$=\frac{-9+x^2+10x-2x^2}{(9-x^2)^2}$

$=\frac{-x^2+10x-9}{(9-x^2)^2}$

$f''(x)=\frac{(-x^2-10x+9)'\cdot[(9-x^2)^2]-(-x^2-10x+9)\cdot(9-x^2)^{2'}}{[(9-x^2)^2]^2}$

$=\frac{(-2x-10)(9-x^2)^2-(-x^2-10x+9)\cdot 2(9-x^2)^1}{[9-x^2]^2}$

Složeno

$-\infty$	-3		1		3		9	$+\infty$
	-4	-2		2		5		10
$f(x)$	$-$	$-$	0	$+$	$\nexists$	$+$	0	$-$
	↘	↘	lok Minimum	↗		↗	lok Max	↘

Lokalni min

$T(1,\frac{1}{2})$ ✓

Lok max

$T(9,\frac{1}{18})$ ✓

Funkcija nije gornjeomeđena odozdo niti odozgo

USMENI - Nate Kosor

30/100

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: DUJE SURAĆ **Broj indeksa:** 17-1-0118-2012

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a) Deriviraj funkciju:

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1) $\det A = \begin{vmatrix} 2 & 4 & -6 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 2 \\ 0 & -4 & 1 & -2 \end{vmatrix} = 2 \cdot (-1)^{1+1} \underbrace{\begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & 2 \\ -4 & 1 & -2 \end{vmatrix}}_{D_1} +$

$0 \cdot (-1)^{2+1} \underbrace{\begin{vmatrix} 4 & -6 & 9 \\ 0 & 2 & 2 \\ -4 & 1 & -2 \end{vmatrix}}_{D_2} + 0 \cdot (-1)^{3+1} \underbrace{\begin{vmatrix} 4 & -6 & 9 \\ 1 & 3 & -1 \\ -4 & 1 & -2 \end{vmatrix}}_{D_3} + 0 \cdot (-1)$

$\underbrace{\begin{vmatrix} 4 & -6 & 9 \\ 1 & 3 & -1 \\ 0 & 2 & 2 \end{vmatrix}}_{D_4} = 2 \cdot D_1 - 0 \cdot D_2 + 0 \cdot D_3 - 0 \cdot D_4$

4.)

a) $f(x) = x \cdot \arcsin x + \sqrt{1-x^2}$

$f'(x) =$

b) $f(x) = \ln(x^2 - 3x + 5)$

UVJET: $x^2 - 3x + 5 > 0$

$x =$

1.) $D_1 = \begin{vmatrix} 1 & 3 & -1 & 1 & 3 \\ 0 & 2 & 2 & 0 & 2 \\ -4 & 1 & -2 & -4 & 1 \end{vmatrix} = -4 + (-24) + 0 - 8 - 2 - 0 =$

♣D $D_1 = -4 - 24 - 8 - 2 = -38$

$D_1 = -38$

$D_2 = \begin{vmatrix} 4 & -6 & 9 & 4 & -6 \\ 0 & 2 & 2 & 0 & 2 \\ -4 & 1 & -2 & -4 & 1 \end{vmatrix} = -16 + 48 + 0 - (-72) - 8 - 0 =$

$D_2 = -16 + 48 + 72 - 8 = 96$

$D_2 = 96$

$D_3 = \begin{vmatrix} 4 & -6 & 9 & 4 & -6 \\ 1 & 3 & -1 & 1 & 3 \\ -4 & 1 & -2 & -4 & 1 \end{vmatrix} = -24 + (-24) + 9 - (-108) - (-4) - 12$

$D_3 = -24 - 24 + 9 + 108 + 4 - 12$

$D_3 = 61$

$D_4 = \begin{vmatrix} 4 & -6 & 9 & 4 & -6 \\ 1 & 3 & -1 & 1 & 3 \\ 0 & 2 & 2 & 0 & 2 \end{vmatrix} = 24 + 0 + 18 - 0 - 8 - (-12)$

$D_4 = 24 + 18 - 8 + 12$

$D_4 = 46$

$\det A = 2 \cdot D_1 - 0 \cdot D_2 + 0 \cdot D_3 - 0 \cdot D_4 = 2 \cdot (-38) - 0 \cdot 96 + 0 \cdot 61 - 0 \cdot 46$

$\det A = -76 - 0 + 0 - 0 = -76$ ✓ (17.5)

$\det A = -76$

3.) $f(x) = \sqrt{x^2 - x}$

DOMENA:

$x^2 - x \geq 0$

$x(x-1) \geq 0$

$x_1 \geq 0, \quad x_2 \geq 1$

V. A.

$\lim_{x \to 0} \sqrt{x^2+x} = \sqrt{0} = 0$

$\lim_{x \to 1} \sqrt{x^2-x} = \sqrt{1-1} = \sqrt{0} = 0$

H. A.

$\lim_{x \to \infty} \sqrt{x^2-x} = +\infty$

$\lim_{x \to -\infty} \sqrt{x^2-x} = -\infty$

- nemo ni horizontalnih, ni vertikalnih asimptota.

K. A.

$k_1 = \lim_{x \to \infty} \frac{\sqrt{x^2-x}\,/:x}{x\,/:x} = \frac{1}{1} = 1$

$k_1 = \lim_{x \to \infty} \sqrt{x^2-x} \cdot x = \lim_{x \to \infty} \sqrt{x^2+3x}$

$-x \cdot \frac{\sqrt{x^2-x}}{\sqrt{x^2-x}} = \frac{x}{\sqrt{x^2-x}} = \frac{1}{2}$

$y = x + \frac{1}{2}$ ✓ (12.5)

$k_2 = \lim_{x \to -\infty} \frac{\sqrt{x^2-x}}{x} \left\{ \begin{matrix} x \to -x \\ -\infty \to +\infty \end{matrix} \right\}$

$= \lim - \frac{\sqrt{x^2-x}\,/:x}{x\,/:x} = \frac{-1}{1} = -1$

$k_1 = \lim_{x \to -\infty} \left(\sqrt{x^2} - x + x\right)$

$= \lim \sqrt{x^2-x} - x \cdot \frac{\sqrt{x^2-x}+x}{\sqrt{x^2-x}+x}$

$= \lim \frac{x^2 - x - x^2\,/:x}{\sqrt{x^2-x}+x\,/:x} = \frac{-x}{\sqrt{1-\frac{x}{x}}+\frac{x}{x}}$

$= \frac{-1}{1-1+1} = \frac{-1}{1} = -1$

$y = -x - 1$