

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: LOJANO FUZUL

Broj indeksa: 171-0070-204

Vrijeme: od 08-00 do 11-00 ♣C

Broj bodova:

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (17.5) Odredi inverz matrice:

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix}$$

2. (17.5) Riješi u skupu \mathbb{C} jednadžbu:

$$z = i - \left(\frac{1-i}{1+i}\right)^{32}$$

3. (15) Odredi asimptote sljedeće funkcije:

$$f(x) = \frac{3 - 2x^2}{x - 1}$$

4. (12.5+12.5)

- a) Deriviraj funkciju:

$$f(x) = 3 \ln \frac{x-1}{x+1}$$

- b) Odredi domenu funkcije:

$$f(x) = \frac{x^2}{\sqrt{x^2-1}}$$

5. (25) Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \ln(x + \sqrt{x^2+1})$$

~~g.) a) $\ln \frac{x-1}{x+1}$~~

~~$f'(x) = (3) \cdot \left(\ln \frac{x-1}{x+1}\right)' + (3) \cdot \left(\ln \frac{x-1}{x+1}\right)'$~~

~~$= (0) \cdot \left(\ln \frac{x-1}{x+1}\right) + 3 \cdot \left(\frac{1}{\frac{x-1}{x+1}}\right) \cdot \left(\frac{x-1}{x+1}\right)'$~~

~~$= 3 \cdot \left(\frac{x+1}{x-1}\right) \cdot \frac{1}{(x+1)^2} \cdot ((x-1)(x+1) - (x-1)(x+1))$~~

5

~~$= 3 \cdot \left(\frac{x+1}{x-1}\right) \cdot \left(\frac{x+1 - x-1}{(x+1)^2}\right)$~~

~~$= 3 \cdot \left(\frac{x+1}{(x-1) \cdot (x+1)^2}\right)$~~

~~$= 3 \cdot \frac{1}{(x-1)(x+1)}$~~

~~$= \frac{3}{(x-1)(x+1)}$~~

$$3.) f(x) = \frac{3-2x^2}{x-1}$$

$$D_f = \langle -\infty, 1 \rangle \cup \langle 1, +\infty \rangle$$

$$x-1 \neq 0 \\ x \neq 1$$

VERTIKALNA ASIMPTOTA

$$f(x) = \lim_{x \rightarrow 1} \frac{3-2x^2}{x-1} = \frac{3-2 \cdot 1^2}{1-1} = \frac{1}{0} = \infty \quad \checkmark$$

HORIZONTALNA ASIMPTOTA

$$f(x) = \lim_{x \rightarrow \infty} \frac{3-2x^2}{x-1} = \frac{-2}{0} = \infty \quad \text{--- nema horizontalne a.}$$

KOSA ASIMPTOTA

$$y = kx + c$$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{3-2x^2}{x-1} \cdot \frac{1}{x} = \lim_{x \rightarrow \infty} \frac{3-2x^2}{x^2-x} = \frac{-2}{1} = -2 \quad \checkmark$$

$$c = \lim_{x \rightarrow \infty} (f(x) - kx) = \lim_{x \rightarrow \infty} \left(\frac{3-2x^2}{x-1} - (-2)x \right) = \lim_{x \rightarrow \infty} \frac{3-2x^2 + 2x(x-1)}{x-1} = \lim_{x \rightarrow \infty} \frac{3-2x^2 + 2x^2 - 2x}{x-1} = \lim_{x \rightarrow \infty} \frac{-2x+3}{x-1}$$

$$= \lim_{x \rightarrow \infty} \frac{-2x+3}{x-1} = \lim_{x \rightarrow \infty} \frac{-2x+3}{x-1} = \lim_{x \rightarrow \infty} \frac{-2x+3}{x-1} = \lim_{x \rightarrow \infty} \frac{-2x+3}{x-1}$$

$$= \frac{-2}{1} = -2 \quad \checkmark$$

15

$$y = -2x - 2$$

a.) b) $f(x) = \frac{x^2}{\sqrt{x^2-1}}$

$D_f = \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle$

• c) 1^o Nazivnik

$x^2 - 1 > 0$

$|\sqrt{x^2-1} \neq 0|^2$

$x^2 > 1$

$x^2 - 1 \neq 0$

$x^2 > \sqrt{1}$

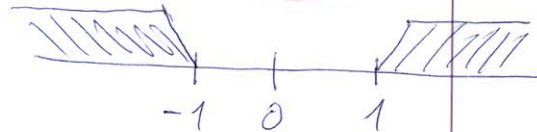
$x^2 \neq 1$

$x > \pm 1$

$x \neq \sqrt{1}$

$x = \pm 1$

12.5



~~Handwritten scribbles and crossed-out work.~~

~~Handwritten scribbles.~~

2.) $z = i - \left(\frac{1-i}{1+i}\right)^{32}$

$z = i - (-i)^{32}$

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i-i+i^2}{1+i-i-i^2}$

$z = i - (-i)$

5

$= -\frac{2i}{2} = -i$

$z = 2i$

$|0+2i| = \sqrt{0^2+2^2} = 2$

$i^{32} = i^{4 \cdot 8} = 1$

$\varphi = 180^\circ - 90^\circ = 90^\circ$

a.) $f(x) = 3 \ln \frac{x-1}{x+1}$

$f'(x) = (3) \cdot \left(\ln \frac{x-1}{x+1}\right)' + 3 \cdot \left(\ln \frac{x-1}{x+1}\right)'$

$= 3 \cdot \left(\frac{1}{\frac{x-1}{x+1}}\right) \cdot \left(\frac{x-1}{x+1}\right)'$

12.5

$= 3 \cdot \frac{x+1}{x-1} \cdot \left(\frac{(x-1)(x+1) - (x-1)(x+1)'}{(x+1)^2}\right) = 3 \cdot \frac{x+1}{x-1} \cdot \frac{x+1-x-1}{(x+1)^2}$

$= 3 \cdot \left(\frac{x+1}{x-1}\right) \cdot \left(\frac{2}{(x+1)^2}\right) = \frac{6(x+1)}{(x-1)(x+1)^2}$

$$\begin{aligned}
 & \left[\begin{array}{cccc|cccc} 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cccc|cccc} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2-2R_1 \\ R_3-3R_1 \\ R_4-2R_1}} \left[\begin{array}{cccc|cccc} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \left[\begin{array}{cccc|cccc} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{R_1-R_2 \\ R_3-2R_2 \\ R_4-2R_2}} \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 0 \end{array} \right] \xrightarrow{\substack{R_1+\frac{1}{2}R_3 \\ R_2-\frac{1}{2}R_3 \\ R_4+\frac{1}{2}R_3}} \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 0 \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & -2 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & -2 & 0 \end{array} \right] \xrightarrow{\substack{R_1-R_4 \\ R_2+R_4 \\ R_3-R_4}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & -2 & 0 \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \left[\begin{array}{cccc} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{array} \right] \cdot \left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 2 & -2 & 0 \end{array} \right] = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]
 \end{aligned}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 2 & -2 & 0 \end{bmatrix}$$

