MATE KOSOR 79/100

MATEMATIKA 1
7. veljače 2013.

Ime i prezime: LOVRO FUZUL **Broj indeksa:** 171-0070-204

Vrijeme: od 09-00 **do** 11-00 ♣C **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Odredi inverz matrice:

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix}.$$

2. *(17.5)* Riješi u skupu $\mathbb{C}$ jednadžbu:

$$z = i - \left(\frac{1-i}{1+i}\right)^{32}$$

3. *(15)* Odredi asimptote sljedeće funkcije:

$$f(x) = \frac{3-2x^2}{x-1}$$

4. *(12.5+12.5)*

a) Deriviraj funkciju:

$$f(x) = 3\ln\frac{x-1}{x+1}$$

b) Odredi domenu funkcije:

$$f(x) = \frac{x^2}{\sqrt{x^2-1}}$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \ln(x + \sqrt{x^2+1})$$

4.) a) $3\ln\frac{x-1}{x+1}$

$f'(x) = (3)'\cdot\left(\ln\frac{x-1}{x+1}\right) + 3\left(\ln\frac{x-1}{x+1}\right)'$

$= (0)\cdot\left(\ln\frac{x-1}{x+1}\right) + 3\left(\frac{1}{\frac{x-1}{x+1}}\right)\cdot\left(\frac{x-1}{x+1}\right)'$

$= 0$

$= 3\left(\frac{x+1}{x-1}\right)\cdot\left(\frac{(x-1)(x+1)-(x-1)(x+1)}{(x+1)^2}\right)$

$= 3\left(\frac{x+1}{x-1}\right)\cdot\left(\frac{x+1-x+1}{(x+1)^2}\right)$

$= 3\cdot\left(\frac{x+1}{(x-1)\cdot(x+1)^2}\right)$

$= 3\cdot\frac{1}{(x-1)(x+1)}$

$= \frac{3}{(x-1)(x+1)}$

3.) $f(x) = \frac{3-2x^2}{x-1}$ $\quad D_f = \langle -\infty, 1\rangle \cup \langle 1, +\infty\rangle$

$x-1 \neq 0$
$x \neq 1$

VERTIKALNA ASIMPTOTA

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{3-2x^2}{x-1} = \frac{3-2\cdot 1^2}{1-1} = \frac{1}{0} = \infty$$ ✓

HORIZONTALNA ASIMPTOTA

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3-2x^2}{x-1} = \frac{-2}{0} = \infty$$ — nema horizontalne a.

KOSA ASIMPTOTA

$y = kx + l$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{3-2x^2}{x-1}}{\frac{x}{1}} = \lim_{x\to\infty} \frac{3-2x^2}{x^2-x} = \frac{-2}{1} = -2$$ ✓

$$l = \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\frac{3-2x^2}{x-1} - (-2)x\right) = \lim_{x\to\infty} \frac{3-2x^2+2x(x-1)}{x-1} = \lim_{x\to\infty} \frac{3-2x^2+2x^2-2x}{x-1} = \lim_{x\to\infty} \frac{-2x+3}{x-1}$$

$$= \frac{-2}{1} = -2$$ ✓

15

$$\boxed{y = -2x - 2}$$

4.) b) $f(x) = \frac{x^2}{\sqrt{x^2-1}}$

$D_f = \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle$ ✓

♣ 1° Nazivnik

$\sqrt{x^2-1} \neq 0 \quad /^2$

$x^2 - 1 \neq 0$

$x^2 \neq 1$

$x \neq \sqrt{1}$

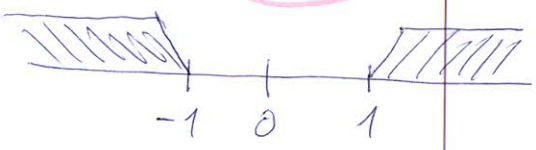
$x = \pm 1$

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq \sqrt{1}$

$x \geq \pm 1$



12.5

2.) $z = i - \left(\frac{1-i}{1+i}\right)^{32}$

$z = i - (-i)^{32}$

$z = i - (-i)$

$z = 2i$

5

$|0 + 2i| = \sqrt{0^2 + 2^2} = 2$

$\varphi = 180° - 90° = 90°$

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{1 - i - i + i^2}{1 + i - i - i^2}$

$= \frac{-2i}{2} = -i$

$i^{32} = i^{4 \cdot 8}$

4.) a) $f(x) = 3 \ln \frac{x-1}{x+1}$

$f'(x) = (3)' \cdot \left(\ln \frac{x-1}{x+1}\right) + 3 \cdot \left(\ln \frac{x-1}{x+1}\right)'$

$= 3 \cdot \left(\frac{1}{\frac{x-1}{x+1}}\right) \cdot \left(\frac{x-1}{x+1}\right)'$

$= 3 \cdot \frac{x+1}{x-1} \cdot \left(\frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2}\right) = 3 \frac{x+1}{x-1} \cdot \frac{x+1-x+1}{(x+1)^2}$

$= 3 \left(\frac{x+1}{x-1}\right) \left(\frac{2}{(x+1)^2}\right) = \frac{6(x+1)}{(x-1)(x+1)^2}$ ✓

12.5

1)

$$\left[\begin{array}{cccc|cccc} 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \frac{1}{2}R1 \sim \left[\begin{array}{cccc|cccc} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R2-2R1 \\ R3-3R1 \\ R4-2R1 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 0 \end{array}\right] (-1)$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & +1 & -1 & 0 & 0 \\ 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R1-R2 \\ \sim \\ R3-2R2 \\ R3-2R2 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 0 \end{array}\right] (-2)$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 0 \end{array}\right] \begin{array}{l} R1+\frac{1}{2}R3 \\ R2-R3 \\ \sim \\ R4+R3 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & -2 & -1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & -2 & 0 \end{array}\right] \begin{array}{l} R1-R4 \\ R2+R4 \\ R3-R4 \end{array} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{array}\right] \cdot \left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right]$$

5.) $f(x)=\ln\left(x+\sqrt{x^2+1}\right)$

1° Domena

$x+\sqrt{x^2+1}>0 \quad /^2$

$x^2+x^2+1>0$

$2x^2>-1$

$x^2>-\frac{1}{2}$

$x>\sqrt{-\frac{1}{2}}$

$\sqrt{x^2+1}\neq 0 \quad /^2$

$x^2+1\neq 0$

$x^2\neq -1$

$x\neq \sqrt{-1}$

$D_f=\mathbb{R}$ ✓

2° ASIMPTOTE

V.A. -> nema

H.A. ->

$$f(A)=\lim_{x\to\infty}\ln\left(x+\sqrt{x^2+1}\right)\cdot\frac{\ln\left(x-\sqrt{x^2+1}\right)}{\ln\left(x-\sqrt{x^2+1}\right)}=\lim_{x\to\infty}\frac{\ln^2\left(x^2-x^2-1\right)}{\ln\left(x-\sqrt{x^2+1}\right)}=\frac{\ln^2(-1)}{\ln\left(x-\sqrt{x^2+1}\right)}$$

$-3.m$
-0.58
0
$\pm$

$=\frac{-2}{\infty}=0$ W

Ekstremi

Nema ekstrema

Sjecište sa osima

osx

$\ln\left(x+\sqrt{x^2+1}\right)=0 \quad /e^x$

$\left(x+\sqrt{x^2+1}\right)=e^0=1$

$\left(x+\sqrt{x^2+1}\right)=1$

$\left(x+\sqrt{x^2+1}\right)-1=0 \quad /^2$

$\left(x^2+x^2+1\right)-1^2=0$

$(x^2+x^2+1)+1=0$

$2x^2+1=0$

$2x^2=-1$

$x^2=-\frac{1}{2}$

$x=\sqrt{-\frac{1}{2}}$

$X=0$ W

osy

$f(0)=\ln\left(0+\sqrt{0^2+1}\right)$

$=\ln 1$

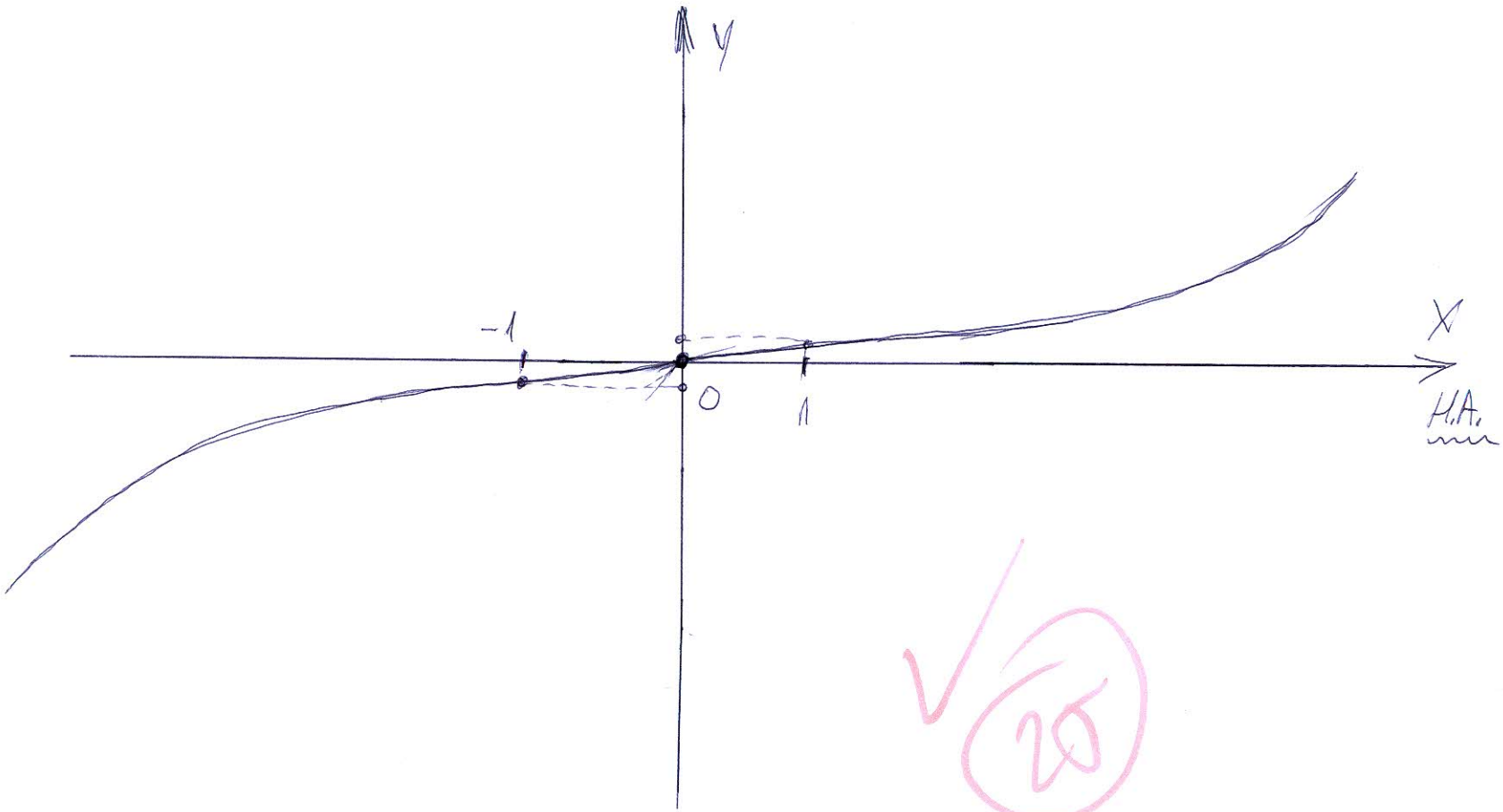
$=0$

W

Dodatne točke

$f(1) = \ln\left(1 + \left(\sqrt{1^2+1}\right)\right) = 0.88$

$f(-1) = \ln\left(-1 + \left(\sqrt{(-1)^2+1}\right)\right) = -0.88$



√ (20)

35/100

M. Kosor

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: Branimir Pijaca **Broj indeksa:** 17-2-0086-2011

Vrijeme: od _____ **do** _____ **♣C** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Odredi inverz matrice:

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix}.$$

2. *(17.5)* Riješi u skupu $\mathbb{C}$ jednadžbu:

$$z = i - (\frac{1-i}{1+i})^{32}$$

3. *(15)* Odredi asimptote sljedeće funkcije:

$$f(x) = \frac{3-2x^2}{x-1}$$

4. *(12.5+12.5)*

a) Deriviraj funkciju:

$$f(x) = 3 \ln \frac{x-1}{x+1}$$

b) Odredi domenu funkcije:

$$f(x) = \frac{x^2}{\sqrt{x^2-1}}$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \ln(x + \sqrt{x^2+1})$$

(7) $z = i - \left(\frac{1-i}{1+i}\right)^{32}$

$z = i - (-i)^{32}$

$z = i - (-i)$

$z = 2i$

5

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} \quad \frac{1-i-i+i^2}{1+i-i-i^2}$

$= \frac{-2i}{2} = -i$

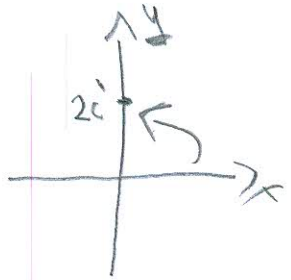
$i^{32} \cdot 0.8$

$i = i$

$|0+2i| = \sqrt{0^2+2^2} = 2$

$\operatorname{tg} = \frac{2}{0}$

$\varphi = 180 - 90 = 90^\circ$



③ $f(x) = \frac{3-2x^2}{x-1}$

$x-1 \neq 0$
$x \neq 1$

Def: $\mathbb{R} \setminus \{1\}$

$x=1$ Vertikalna asimptota

V.A. $\lim_{x\to 1} \frac{3-2x^2}{x-1} = \frac{3-2\cdot 1}{1-1} = \frac{3-2}{0} = \frac{1}{0} = +\infty$

H.A $\lim_{x\to\infty} \frac{3-2x^2}{x-1} \; /:x^2 = \frac{\frac{3}{x^2} - \frac{2\cdot x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \frac{0-2}{0} = \frac{-2}{0} = -\infty$

$\lim_{x\to\infty} \frac{3-2x^2}{x-1} \Rightarrow \begin{bmatrix} x\to -x \\ -\infty \to +\infty \end{bmatrix} = \lim_{x\to+\infty} \frac{3-2\cdot(-x)^2}{(-x)-1}$

$\lim_{x\to+\infty} \frac{3-2x}{x-1} \; /:x \quad \lim_{x\to+\infty} \frac{\frac{3}{x} - \frac{2x}{x}}{\frac{x}{x} - \frac{1}{x}} = \frac{0-2}{1-0} = 2$

lijeva horizontalna asimptota $y=2$
lijeve kose nema

desna kosa

$y = kx + l$

$k = \lim_{x\to\infty} \frac{f(x)}{x} \quad \lim_{x\to\infty} \frac{\frac{3-2x^2}{x-1}}{x} = \lim_{x\to\infty} \frac{3-2x^2}{x^2-x} \; /:x^2 \quad \lim_{x\to\infty} \frac{\frac{3}{x^2} - \frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{x}{x^2}}$

$\lim_{x\to\infty} \frac{0-2}{1-0} = 2 \quad k=2$

$l \quad \lim_{x\to\infty} (f(x) - k\cdot x) \quad \lim_{x\to\infty} \frac{3-2x^2}{x-1} - 2\cdot x$

$\lim_{x\to\infty} \frac{3-2x^2}{x-1} - \frac{2x}{1} \quad \lim_{x\to\infty} \frac{3-2x^2-2x^2+2x}{x-1} = \frac{3-2x-4x^2}{x-1} \; /:x^2$

$\lim_{x\to\infty} \frac{\frac{3}{x^2} - \frac{2x}{x^2} - \frac{4x^2}{x^2}}{\frac{x}{x^2} - \frac{1}{x^2}} = \lim_{x\to\infty} \frac{0-0-4}{0-0} = \frac{-4}{0} = -\infty$

$y = 2x$

desna kosa

Jinca inverz

♣2

$$\begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix} \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

1. sor · (-2)

1. sor · (-3) + 3. sor

1. sor · (-2) + 4. sor

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1/2 & -1/2 & -3/2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -1/2 & -1/2 & -3/2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

2. sor · (-2) + 3. sor

· (-2) + 4. sor

· (-1) + 1. sor

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & 1/2 & -1/2 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -5/2 & -1/2 & -7/2 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & 1/2 & -1/2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 1 \\ 0 & 0 & -5/2 & -1/2 & -7/2 & 2 & 1 & 0 \end{array}\right]$$

4. ↔ 3.

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & 1/2 & -1/2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 & -3 & -2 & 0 & -1 \\ 0 & 0 & -5/2 & -1/2 & -7/2 & 2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1/2 & 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 & -2 & 0 & -1 \\ 0 & 0 & 0 & -1/2 & 4 & -3 & 1 & -5/2 \end{array}\right]$$

3. sor · (5/2) + 4. sor

3. sor · (-1) + 2. sor

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1/2 & 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & -8 & 6 & -2 & 5 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 5 & -3 & 1 & -3 \\ 0 & 1 & 0 & 0 & -2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & -8 & 6 & -2 & 5 \end{array}\right]$$

$$A \begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix}$$

$$A^{-1} \begin{bmatrix} 5 & -3 & 1 & -3 \\ -2 & 1 & 0 & 1 \\ 3 & -2 & 0 & -1 \\ -8 & 6 & -2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix}$$

$\det A = 1 \neq 0$
ima inverz

$$\left[\begin{array}{cccc|cccc} 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

1. · (−2) + 2.
1. · (−3) + 3.
1. · (−2) + 4.

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -2 & -1/2 & -3/2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1/2 & -3/2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

2. : (−1)

2. · (−2) + 3. + 4.
2. · (−1) + 1.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & 1/2 & -1/2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -4 & -1/2 & -7/2 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -3 & -2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & 1/2 & -1/2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/8 & 7/48 & -1/2 & -1/4 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 1 \end{array}\right]$$

3. : (−4)

3. · 1 + 4.

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 7/16 & -41/96 & 5/4 & -1/8 & 0 \\ 0 & 1 & 0 & -1/8 & 41/48 & -1/2 & 1/4 & 0 \\ 0 & 0 & 1 & 1/8 & 7/48 & -1/2 & -1/4 & 0 \\ 0 & 0 & 0 & 1/8 & 137/48 & 3/2 & -1/4 & 1 \end{array}\right]$$

4. : 1/8

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 7/16 & -41/96 & 5/4 & -1/8 & 0 \\ 0 & 1 & 0 & -1/8 & 41/48 & -1/2 & 1/4 & 0 \\ 0 & 0 & 1 & 1/8 & 7/48 & -1/2 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & 137/6 & 12 & -2 & 8 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 7/16 & -41/96 & 5/4 & -1/8 & 0 \\ 0 & 1 & 0 & -1/8 & 41/48 & -1/2 & 1/4 & 0 \\ 0 & 0 & 1 & 1/8 & 7/48 & -1/2 & -1/4 & 0 \\ 0 & 0 & 0 & 1 & 137/6 & 12 & -2 & 8 \end{array}\right]$$

$4r \cdot (-1/8) + 3$
$\cdot (1/8) + 2$
$\cdot (-7/16) + 1)$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -395/16 & -4 & 3/4 & -7/2 \\ 0 & 1 & 0 & 0 & 89/24 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -65/24 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 137/6 & 12 & -2 & 8 \end{array}\right]$$

---

⑤ $f(x) = \ln(x$

$$\begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{bmatrix} A$$

$$A^{-1} \begin{bmatrix} 1/2 & 0 & 1 & -3/2 \\ -2 & 3 & 0 & 1 \\ -3 & -2 & 0 & -1 \\ -8 & 6 & -2 & 5 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

1k : 2

1k · (−2) + 2
1k · (−3) + 3
1k · (−2) + 4k

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1/2 & -1/2 & -3/2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1/2 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & -1/2 & -1/2 & -3/2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

2k · (−1)

2k · (−1) + 1k
2k · (−2) + 3k
2k · (−2) + 4k

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & 1/2 & -1/2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -5/2 & -1/2 & -7/2 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1/2 & \frac{1}{2} & -1/2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/5 & 7/5 & -4/5 & -2/5 & 0 \\ 0 & 0 & -1 & 0 & -3 & 2 & 0 & 1 \end{array}\right]$$

3k : $\left(-\frac{5}{2}\right)$

3k · (1) + 4k
3k · (−1) + 2k
3k · $\left(\frac{1}{2}\right)$ + 1k

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3/10 & -19/10 & 8/5 & 2/5 & 0 \\ 0 & 1 & 0 & -1/5 & -2/5 & 9/5 & 2/5 & 0 \\ 0 & 0 & 1 & 1/5 & 7/5 & -4/5 & -2/5 & 0 \\ 0 & 0 & 0 & 1/5 & -8/5 & 6/5 & -2/5 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3/10 & -19/10 & 8/5 & \frac{2}{5} & 0 \\ 0 & 1 & 0 & -1/5 & -2/5 & 9/5 & 2/5 & 0 \\ 0 & 0 & 1 & 1/5 & 7/5 & -4/5 & -2/5 & 0 \\ 0 & 0 & 0 & 1 & -8 & 6 & -2 & 5 \end{array}\right]$$

4k : 5

4k · (−3/10) + 1k
4k · $\left(\frac{1}{5}\right)$ + 2
4k · (−1/5) + 3

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/2 & 0 & 1 & -3/2 \\ 0 & 1 & 0 & 0 & -2 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 & -3 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & -8 & 6 & -2 & 5 \end{array}\right]$$

④ $f(x) = 3 \ln \frac{x-1}{x+1}$

♣C $3' \cdot \ln \frac{x-1}{x+1} + 3 \cdot \ln\left(\frac{x-1}{x+1}\right)'$

$0 \cdot \ln \frac{x-1}{x+1} + 3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \frac{(x-1)' \cdot (x+1) - (x-1) \cdot (x+1)'}{(x+1)^2}$

$3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \frac{(x-1)' \cdot (x+1) - (x-1) \cdot (x+1)'}{(x+1)^2}$

---

④ $f(x) = 3 \cdot \ln \frac{x-1}{x+1} = 3' \cdot \ln \frac{x-1}{x+1} \cdot 3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \left(\frac{x-1}{x+1}\right)'$

$= 3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \left(\frac{(x-1)' \cdot (x+1) - (x-1)(x+1)'}{(x+1)^2} \right)$

$+ 3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \frac{1 \cdot (x+1) - (x-1) \cdot 1}{(x+1)^2}$

$= 3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \left(\frac{(x+1) - (x-1)}{(x+1)^2} \right)$

$= 3 \cdot \frac{1}{\frac{x-1}{x+1}} \cdot \frac{x+1-x+1}{(x+1)^2}$

$f'(x) = 3 \cdot \left(\frac{x+1}{x-1}\right) \cdot \frac{2}{(x+1)^2}$ ✓ 12.5

---

① ⑥ $\frac{x^2}{\sqrt{x^2-1}}$

$\sqrt{x^2-1} \geq 0$

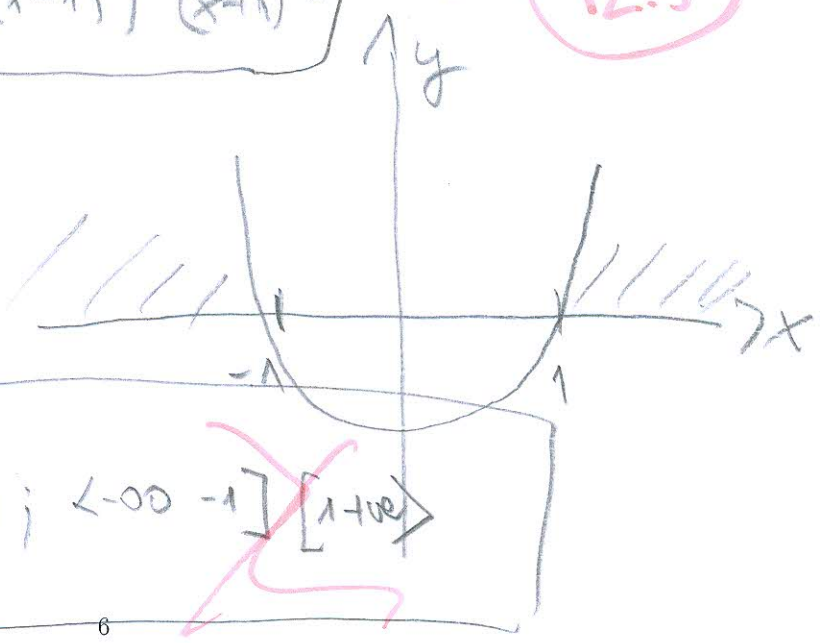
$x^2 - 1 = 0$

$x^2 = 1$

$x = \sqrt{1}$

$x_1 = 1$

$x_2 = -1$



$D(f) : \langle -\infty, -1] \cup [1, +\infty \rangle$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 2 & 1/2 & 0 & 0 & -1 & 3/2 \\ 0 & -3 & -2 & 2 & 0 & 1 & 0 & -1 \\ 0 & -2 & -1 & 0 & 1 & 0 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -3 & -1/2 & 0 & 0 & 2 & -5/2 \\ 0 & 1 & 2 & 1/2 & 0 & 0 & -1 & 3/2 \\ 0 & 0 & 4 & 7/2 & 0 & 1 & -3 & 7/2 \\ 0 & 0 & 3 & 1 & 1 & 0 & -2 & 2 \end{array}\right]$$

$2r\cdot(3)+3r$
$2r\cdot(2)+4r$
$2r\cdot(-2)+1r$

$3r:4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -3 & -1/2 & 0 & 0 & 2 & -5/2 \\ 0 & 1 & 2 & 1/2 & 0 & 0 & -1 & 3/2 \\ 0 & 0 & 1 & 7/8 & 0 & 1/4 & -3/4 & 7/8 \\ 0 & 0 & 3 & 1 & 1 & 0 & -2 & 2 \end{array}\right]$$

$3r\cdot(-3)+4r$
$3r\cdot(-2)+2r$
$3r\cdot(2)+1r$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 17/8 & 0 & 3/4 & -1/4 & 1/8 \\ 0 & 1 & 0 & -5/4 & 0 & -1/2 & 1/2 & -1/4 \\ 0 & 0 & 1 & 7/8 & 0 & 1/4 & -3/4 & 7/8 \\ 0 & 0 & 0 & -13/8 & 1 & -3/4 & 1/4 & -5/8 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 17/8 & 0 & 3/4 & -1/4 & 1/8 \\ 0 & 1 & 0 & -5/4 & 0 & -1/2 & 1/2 & -1/4 \\ 0 & 0 & 1 & 7/8 & 0 & 1/4 & -3/4 & 7/8 \\ 0 & 0 & 0 & 1 & -8/13 & 6/13 & -2/13 & 5/13 \end{array}\right]$$

$4r\cdot(-7/8)+3r$ $4r\cdot 5/4+2$ $4r\cdot(-17/8)+1r$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 17/13 & -3/13 & 1/13 & -9/13 \\ 0 & 1 & 0 & 0 & -10/13 & 1/13 & 4/13 & 3/13 \\ 0 & 0 & 1 & 0 & 7/13 & -2/13 & -8/13 & 7/13 \\ 0 & 0 & 0 & 1 & -8/13 & 6/13 & -2/13 & 5/13 \end{array}\right]$$

$$A\left[\begin{array}{cccc} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{array}\right] \qquad A^{-1}\left[\begin{array}{cccc} 17/13 & -3/13 & 1/13 & -9/13 \\ -10/13 & 1/13 & 4/13 & 3/13 \\ 7/13 & -2/13 & -8/13 & 7/13 \\ -8/13 & 6/13 & -2/13 & 5/13 \end{array}\right]$$

♣c

$$\det A = \begin{vmatrix} 2 & 2 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 3 & 5 & 1 & 1 \\ 2 & 4 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 2 & 1 & 0 \end{vmatrix}$$

$1.\cdot(-1)+3.$
$+4.$
$+2.$

$$= (-1)^{4+1} \cdot 1 \begin{vmatrix} 0 & -1 & -1 \\ 1 & 3 & 0 \\ 0 & 2 & 1 \end{vmatrix} = -1 \cdot (-1)^{1+2} \cdot 1 \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (-1 \cdot 1) - 2 \cdot (-1) = 1 \qquad 1 \neq 0 \text{ Matrica ima inverz}$$

$$\left[\begin{array}{cccc|cccc} 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 2 & 4 & 2 & 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \end{array}\right]$$

$4. \leftrightarrow 1.$ $\qquad 1. : 2)$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1/2 & 0 & 0 & 0 & 1/2 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 5 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & -3 & -2 & 2 & 0 & 1 & 0 & -1 \\ 0 & -1 & -2 & -1/2 & 0 & 0 & 1 & -3/2 \\ 0 & -2 & -1 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$1.\cdot(-2)+2.$
$1.\cdot(-3)+3.$
$1.\cdot(-2)+4.$

$3. \leftrightarrow 2.$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & -1 & -2 & -1/2 & 0 & 0 & 1 & -3/2 \\ 0 & -3 & -2 & 2 & 0 & 1 & 0 & -1 \\ 0 & -2 & -1 & 0 & 1 & 0 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 2 & 1/2 & 0 & 0 & -1 & 3/2 \\ 0 & -3 & -2 & 2 & 0 & 1 & 0 & -1 \\ 0 & -2 & -1 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$2. : (-1)$