USMENI : MATE KOSOR

25 / 100

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: JOSIP PREDOVAN **Broj indeksa:** _____

Vrijeme: od _____ **do** _____ **♣B** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Odredi inverz matrice:

$$A = \begin{bmatrix} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{bmatrix}.$$

2. *(17.5)* Riješi u skupu $\mathbb{C}$ jednadžbu:

$$z^3 = \frac{(1+i)^4}{i^{312}}$$

3. *(15)* Odredi asimptote sljedeće funkcije:

$$f(x) = x \cdot e^{\frac{1}{x-2}}$$

4. *(12.5+12.5)*

a) Deriviraj funkciju:

$$f(x) = \frac{tgx - \sin x}{x^3}$$

b) Odredi domenu funkcije:

$$f(x) = arctg(\frac{1}{x^2})$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \ln \frac{x+3}{1-x}$$

(2) $z^3 = \frac{(1+i)^4}{i^0}$ $312:4=78$ 0

$z^3 = \frac{(1+i)^4}{1}$

$z^3 = \frac{1^4+i^4}{1}$ $i^1 \cdot i^1 = -1$

$z^3 = \frac{1-1}{1}$

$z = \sqrt[3]{2}$

(3) $f(x) = x \cdot e^{\frac{1}{x-2}}$

$Df \setminus \{2\}$

• V.A

$\lim_{x \to 2^-} x \cdot e^{\frac{1}{x-2}} = 2 \cdot e^{\frac{1}{2-2}} = N/P$???

NEMA VERTIKALNE ASIMPTOTE

• H.A

$\lim_{x \to +\infty} x \cdot e^{\frac{1}{x-2}} = \lim_{x \to +\infty} x \cdot e^{\frac{1:x}{x-2:x}} = \lim_{x \to +\infty} x \cdot e^{\frac{\frac{1}{x}}{\frac{x}{x}-\frac{2}{x}}} = x \cdot e^{\frac{0}{1-0}} = x \cdot e^0 = x$

$\lim_{x \to +\infty} -x \cdot \frac{1}{e^{x-2}} = \lim_{x \to +\infty} -x \cdot e^{\frac{1:x}{-x-2:x}} = \lim_{x \to +\infty} -x \cdot e^{\frac{0}{-1}} = -x$

IMA H.A

• KOSE NEMA

④ DERIVATA FUNZIONE

a) $f(x) = \underbrace{\frac{tgx - \sin x}{x^3}}_{y}$

$$f'(x) = \frac{(tgx - \sin x)' \cdot x^3 + (tgx - \sin x)(x^3)'}{(x^3)^2}$$

$$f'(x) = \frac{\left(\frac{1}{\cos^2 x} - \cos x\right) \cdot x^3 + (tgx - \sin x) \cdot 3x^2}{x^6}$$

$$f'(x) = \frac{\left(\frac{x^3}{\cos^2 x} - \cos x^4\right) x^6 + (3tgx^3 - 3\sin x^3)}{x^6}$$

12.5

b) DOMINIO

$$f(x) = \arctg\left(\frac{1}{x^2}\right)$$

$$\arctg\left(\frac{1}{x^2}\right) \neq 0$$

$$Df\ \mathbb{R} \setminus \{0\}$$

12.5

① ORDINA INVERSA MATRICE

$$A\begin{bmatrix} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 15 \\ 0 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{bmatrix} + 0 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 11 & 15 \\ 2 & 1 & 2 \\ 5 & 2 & 5 \end{vmatrix} + 0 \cdot (-1)^{2+1} \begin{vmatrix} 0 & 9 & 4 \\ 2 & 1 & 2 \\ 5 & 2 & 5 \end{vmatrix} + 3 \cdot (-1)^{3+1} \begin{vmatrix} 0 & 9 & 4 \\ 0 & 11 & 15 \\ 5 & 2 & 5 \end{vmatrix}$$

$$7 \cdot (-1)^{4+1} \begin{vmatrix} 0 & 9 & 4 \\ 0 & 11 & 15 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\det A = 0 \cdot D_1 + 0 \cdot D_2 + 3 \cdot D_3 - 7 \cdot D_4$$

$$\det A = 0 \cdot 85 + 0 \cdot (-4) + 3 \cdot 455 - 7 \cdot 182 = 91$$

INVERSA $A^{-1} = \frac{1}{91}$

2

$$D_1 = \begin{vmatrix} 0 & 11 & 15 & 0 & 11 \\ 2 & 1 & 2 & 2 & 1 \\ 5 & 2 & 5 & 5 & 2 \end{vmatrix} \quad 0 + 110 + 60 - 75 - 0 - 10 = 85$$

$$D_2 \begin{vmatrix} 0 & 9 & 4 & 0 & 9 \\ 2 & 1 & 2 & 2 & 1 \\ 5 & 2 & 5 & 5 & 2 \end{vmatrix} \quad 0 + 90 + 16 - 20 - 0 - 90 = -4$$

$$D_3 \begin{vmatrix} 0 & 9 & 4 & 0 & 9 \\ 0 & 11 & 15 & 0 & 11 \\ 5 & 2 & 5 & 5 & 2 \end{vmatrix} \quad 0 + 675 + 0 - 220 - 0 - 0 = 455$$

$$D_4 \begin{vmatrix} 0 & 9 & 4 & 0 & 9 \\ 0 & 11 & 15 & 0 & 11 \\ 2 & 1 & 2 & 2 & 1 \end{vmatrix} \quad 0 + 270 + 0 - 88 - 0 - 0 = 182$$

25|100

MATEMATIKA 1
7. veljače 2013.

Ime i prezime: LOVRE KEREŠ **Broj indeksa:** 57933

Vrijeme: od 8:30 **do** _____ ♣B **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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4. *(12.5+12.5)*

a) Deriviraj funkciju:

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b) Odredi domenu funkcije:

$$f(x) = arctg(\frac{1}{x^2})$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \ln \frac{x+3}{1-x}$$

4. $f(x) = \frac{tgx - \sin x}{x^3}$

a) $f'(x) = \frac{(tgx - \sin x)' \cdot x^3 - (tgx - \sin x) \cdot (x^3)'}{(x^3)^2}$

$f'(x) = \frac{\left(\frac{1}{\cos^2 x} - \cos x\right) \cdot x^3 - (tgx - \sin x) \cdot 3x^2}{(x^3)^2}$

✓ (12.)

b)

$f(x) = \operatorname{arctag}\left(\frac{1}{x^2}\right)$

$\operatorname{arctag} = \mathbb{R}$

$\frac{1}{x^2} \Rightarrow x^2 \neq 0 \quad / \sqrt{\ }$

$x \neq 0$

$D(f) = \mathbb{R} \setminus \{0\}$ / (12.)

5. $f(x) = \ln\left(\frac{x+3}{1-x}\right)$ D

• Domena

$\ln > 0$

$1 - x \neq 0$

$-x = -1 \quad / \cdot(-1)$

$x = 1$

$D(f) = \langle 1, +\infty \rangle$

$2 - \frac{15}{7} = \frac{14-15}{7} = \frac{-1}{7}$ $\quad \frac{5}{7} - \frac{10}{77} = \frac{35-10}{77} = \frac{25}{77}$

$1 - \frac{6}{7} = \frac{7-6}{7} = \frac{1}{7}$ $\quad 1 + \frac{5}{11} = \frac{11+5}{11} = \frac{16}{11}$

$4 - \frac{45}{11} = \frac{44-45}{11} = \frac{-1}{11}$

$\frac{2}{7} \cdot \frac{5}{11} = \frac{10}{77}$

♣B

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1. $\begin{bmatrix} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{bmatrix} \sim \begin{bmatrix} 7 & 5 & 2 & 5 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 0 & 0 & 9 & 4 \end{bmatrix} :7R1 \sim \begin{bmatrix} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 0 & 0 & 9 & 4 \end{bmatrix} R3-3R1 \sim$

$\begin{bmatrix} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & 0 & 11 & 5 \\ 0 & -\frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & 9 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & -\frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & 11 & 5 \\ 0 & 0 & 9 & 4 \end{bmatrix} \cdot(-7)R2 \sim \begin{bmatrix} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 11 & 5 \\ 0 & 0 & 9 & 4 \end{bmatrix} R1-\frac{5}{7}R2 \sim$

$\begin{bmatrix} 1 & 0 & \frac{2}{7} & \frac{5}{7} \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 11 & 5 \\ 0 & 0 & 9 & 4 \end{bmatrix} :11R3 \sim \begin{bmatrix} 1 & 0 & \frac{2}{7} & \frac{5}{7} \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & \frac{5}{11} \\ 0 & 0 & 9 & 4 \end{bmatrix} \begin{matrix} R1-\frac{2}{7}R3 \\ R2+R3 \\ \sim \\ R4-9R3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \frac{45}{77} \\ 0 & 1 & 0 & \frac{16}{11} \\ 0 & 0 & 1 & \frac{5}{11} \\ 0 & 0 & 0 & -\frac{9}{11} \end{bmatrix} \cdot(-11)R4 \sim$

$\begin{bmatrix} 1 & 0 & 0 & \frac{45}{77} \\ 0 & 1 & 0 & \frac{16}{11} \\ 0 & 0 & 1 & \frac{5}{11} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} R1-\frac{45}{77}R4 \\ R2-\frac{16}{11}R4 \\ R3-\frac{5}{11}R4 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$?

3.

$f(x) = x \cdot e^{\frac{1}{x-2}}$

$D(f) = \mathbb{R} \setminus \{2\}$

$x = \mathbb{R}$

$e^{\frac{1}{x-2}} \rightarrow x - 2 \neq 0$

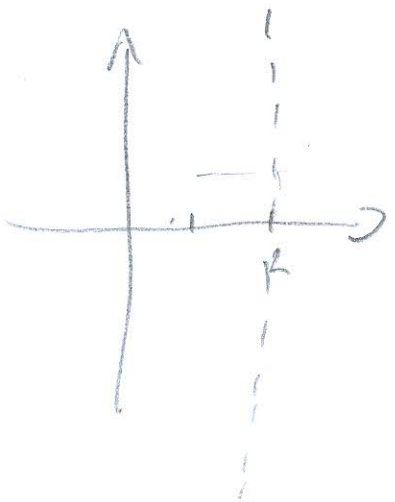
$x \neq 2$

V. A

$\lim_{x \to 2^+} x \cdot e^{\frac{1}{x-2}} = +\infty$

$\lim_{x \to 2^-} x \cdot e^{\frac{1}{x-2}} = -\infty$

V. A. $x = 2$



H. A.

$\lim_{x \to +\infty} x \cdot e^{\frac{1}{x-2}} = +\infty$

$\lim_{x \to -\infty} x \cdot e^{\frac{1}{x-2}} = -\infty$

Nema H. A.

D. K. A.

$k = \frac{f(x)}{x} = \frac{x \cdot e^{\frac{1}{x-2}}}{x} = \lim_{x \to +\infty} e^{\frac{1}{x-2}} = +\infty$

Nema K. A.

- Nul točke

$f(x) = \ln \frac{x+3}{1-x}$

$\ln 1 = 0$

$\ln 0 - N/D$

♣2

$N(1,0)$

$x + 3 \neq 0$

$\boxed{x \neq -3}$

- Rast - pad inflekcija

$f(x) = \ln \frac{x+3}{1-x}$

$f'(x) = \frac{1}{\frac{x-3}{1-x}} \cdot \left(\frac{x-3}{1-x}\right)' = \frac{x-3}{1-x} \cdot \left(\frac{(x-3)' \cdot (1-x) - (x-3) \cdot (1-x)'}{(1-x)^2}\right)$

$= \frac{x-3}{1-x} \cdot \frac{1 \cdot (1-x) - (x-3) \cdot (-1)}{(1-x)^2} = \frac{x-3}{1-x} \cdot \frac{1 - x + x - 3}{(1-x)^2}$

$= \frac{x-3}{1-x} \cdot \frac{-2}{(1-x)^2} = \frac{-2x+6}{1-x \cdot (1-x)^2}$

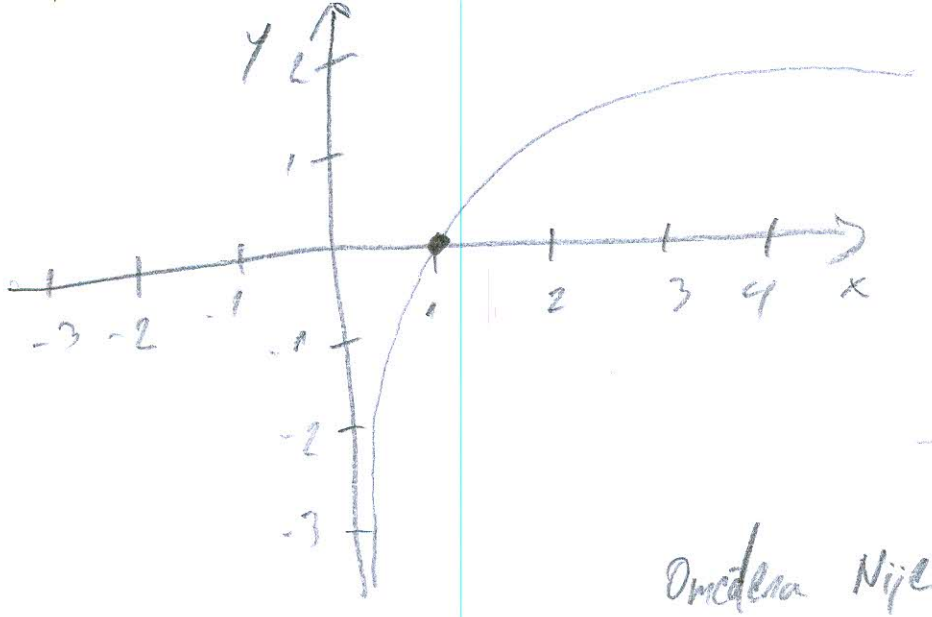
$-2x + 6 = 0$

$-2x = -6 \mid (-1)$

$2x = 6$

$\boxed{x = \frac{6}{2} = 3}$

	1		2	3	4	$+\infty$
$f'(x)$		+		−		
$f(x)$	∄	↗		↘		



Skica grafa

Omčdena Nije odozdo
Ni odozgo,

- Asimptote

V.A.

$$\lim_{x\to 1^+} \ln\frac{x+3}{1-x} = N/D$$

$$\lim_{x\to 1^-} \ln\frac{x+3}{1-x} = 8.26$$

} Nema V.A.

H.A.

$$\lim_{x\to+\infty} \ln\frac{x+3}{1-x} \;\overset{1:x}{=}\; \lim_{x\to+\infty} \ln\frac{\frac{x}{x}+\frac{3}{x}}{\frac{1}{x}-\frac{x}{x}} = \ln\frac{1}{-1} = -1$$

$$\ln -1 = N/D$$

Nema H.A.

D.K.A

$$k = \lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{\ln\frac{x+3}{1-x}}{x}$$

Nema K.A.

- Parnost - neparnost

$f(x) = \ln\frac{-x+3}{1+x} \neq f(x)$ → Nije parna

$-f(x) = -\ln\frac{-x+3}{1+x} \neq f(x)$ → Nije neparna

20/100

MATEMATIKA 1
7. veljače 2013.

Ime i prezime: GABRIJELA JORDAN **Broj indeksa:** 17-2-0118-20M

Vrijeme: od 08 **do** _____ **♣B** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

USMENI: prof. Uglešić

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1. $\begin{bmatrix} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{bmatrix} = 0 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 11 & 5 \\ 2 & 1 & 2 \\ 5 & 2 & 5 \end{vmatrix} + 0 \cdot (-1)^{1+2} \begin{vmatrix} 0 & 11 & 5 \\ 3 & 1 & 2 \\ 7 & 2 & 5 \end{vmatrix} + 9 \cdot (-1)^{1+3} \begin{vmatrix} 0 & 0 & 5 \\ 3 & 2 & 2 \\ 7 & 5 & 5 \end{vmatrix}$

$+ 4 \cdot (-1)^{1+4} \begin{vmatrix} 0 & 0 & 11 \\ 3 & 2 & 1 \\ 7 & 5 & 2 \end{vmatrix} = 9 \cdot 1 \cdot 5 + (-4) \cdot 11 = 45 - 44 = 1$

ima inverz

$\begin{vmatrix} 0 & 0 & 5 \\ 3 & 2 & 2 \\ 7 & 5 & 5 \end{vmatrix} \begin{matrix} 0 & 0 \\ 3 & 2 \\ 7 & 5 \end{matrix} = 0 + 0 + 75 - 70 - 0 - 0 = 5$

$\begin{vmatrix} 0 & 0 & 11 \\ 3 & 2 & 1 \\ 7 & 5 & 2 \end{vmatrix} \begin{matrix} 0 & 0 \\ 3 & 2 \\ 7 & 5 \end{matrix} = 0 + 0 + 165 - 154 - 0 - 0 = 11$

$$\begin{vmatrix} 0&0&9&4\\ 0&0&11&5\\ 3&2&1&2\\ 7&5&2&5 \end{vmatrix} \uparrow I \quad \begin{vmatrix} 3&2&1&2\\ 0&0&11&5\\ 0&0&9&4\\ 7&5&2&5 \end{vmatrix} :3 \quad \begin{vmatrix} 1&\frac{3}{2}&\frac{1}{3}&\frac{2}{3}\\ 0&0&11&5\\ 0&0&9&4\\ 7&5&2&5 \end{vmatrix} \text{I}\cdot(-7)+\text{IV} \quad \begin{vmatrix} 1&\frac{3}{2}&\frac{1}{3}&\frac{2}{3}\\ 0&0&11&5\\ 0&0&9&4\\ 0&-5.5&-0.3&0.3 \end{vmatrix} \uparrow II \quad \begin{vmatrix} 1&1.5&0.3&0.6\\ 0&-5.5&-0.3&0.3\\ 0&0&9&4\\ 0&0&11&5 \end{vmatrix} :(-5.5)$$

$$\begin{vmatrix} 1&1.5&0.3&0.6\\ 0&1&-0.54&0.54\\ 0&0&9&4\\ 0&0&11&5 \end{vmatrix} :9 \quad \begin{vmatrix} 1&1.5&0.3&0.6\\ 0&1&-0.54&0.54\\ 0&0&1&0.4\\ 0&0&11&5 \end{vmatrix} \text{III}\cdot(-11)+\text{IV} \quad \begin{vmatrix} 1&1.5&0.3&0.6\\ 0&1&-0.54&0.54\\ 0&0&1&0.4\\ 0&0&0&0.6 \end{vmatrix} :0.6$$

$$\begin{vmatrix} 1&1.5&0.3&0.6\\ 0&1&-0.54&0.54\\ 0&0&1&0.4\\ 0&0&0&1 \end{vmatrix} \begin{matrix} \text{IV}\cdot(-0.6)+\text{I}\\ \text{IV}\cdot(-0.54)+\text{II}\\ \text{IV}\cdot(-0.4)+\text{III} \end{matrix} \quad \begin{vmatrix} 1&1.5&0.3&0\\ 0&1&-0.54&0\\ 0&0&1&0\\ 0&0&0&1 \end{vmatrix} \begin{matrix} \text{III}\cdot(-0.3)+\text{I}\\ \text{III}\cdot 0.54+\text{II} \end{matrix} \quad \begin{vmatrix} 1&1.5&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{vmatrix} \text{II}\cdot(-1.5)+\text{I}$$

$$\begin{vmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{vmatrix}$$

② $z^3 = \dfrac{(1+i)^4}{i^{3+2}}$

$z^3 = -(1+i)^4$

③ $f(x) = x \cdot e^{\frac{1}{x-2}}$

u: $x-2 \neq 0$

$x \neq 2$

$Df \in \mathbb{R} \setminus \{2\}$

$\lim\limits_{x\to+\infty} x \cdot e^{\frac{1}{x-2}} = \infty \cdot e^{0}$ NEMA

$\lim\limits_{x\to-\infty} x \cdot e^{\frac{1}{x-2}} = -\infty$ NEMA

$\lim\limits_{x\to+\infty} \dfrac{x \cdot e^{\frac{1}{x-2}}}{x} = e^{\frac{1}{x-2}} = 1$

$\lim\limits_{x\to-\infty} = \left|\begin{matrix} -x\to x \\ -\infty\to\infty \end{matrix}\right| = \dfrac{(-x)\cdot e^{\frac{1}{-x-2}}}{(-x)} = 1$

V.A.

$\lim\limits_{x\to 2^-} x \cdot e^{\frac{1}{x-2}} =$ NEMA

$\lim\limits_{x\to 2^+} x \cdot e^{\frac{1}{x-2}}$ NEMA

$k\left(\lim\limits_{x\to+\infty} f(x) - kx\right) = x \cdot e^{\frac{1}{x-2}} - x$

♣B

4. $f(x)=\frac{tgx-\sin x}{x^3}=\frac{(tgx-\sin x)'x^3-(x^3)'(tgx-\sin x)}{(x^3)^2}=\frac{\left(\frac{1}{\sin^2 x}-\cos x\right)x^3-}{}$

a) $\frac{3x^2(tgx-\sin x)}{x^5}$

✓ 12.5

b) $f(x)=\operatorname{arctg}\left(\frac{1}{x^2}\right)$

U: $x^2\neq 0 \quad / ?$ $D\in R\setminus\{0\}$

$x=0$

✓ 12.5

5. $f(x)=\ln\frac{x+3}{1-x}$

U:

$\frac{x+3}{1-x}>0 \qquad 1-x=0$

$-x\neq -1 \quad /\cdot(-1)$

$x\neq 1$

	$-\infty$		-3		1		$+\infty$
$x+3$		−	0	+		+	
$1-x$		+		+	0	−	
		−	0	+	∄	−	

$D_f\langle -3, 1\rangle$

$\lim_{x\to}$