

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: JOSIP PREDOVAN

Broj indeksa: _____

Vrijeme: od _____ do _____ ♣B

Broj bodova:

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (17.5) Odredi inverz matrice:

$$A = \begin{bmatrix} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{bmatrix}$$

2. (17.5) Riješi u skupu \mathbb{C} jednadžbu:

$$z^3 = \frac{(1+i)^4}{i^{312}}$$

3. (15) Odredi asimptote sljedeće funkcije:

$$f(x) = x \cdot e^{\frac{1}{x-2}}$$

4. (12.5+12.5)

- a) Deriviraj funkciju:

$$f(x) = \frac{\operatorname{tg} x - \sin x}{x^3}$$

- b) Odredi domenu funkcije:

$$f(x) = \operatorname{arctg}\left(\frac{1}{x^2}\right)$$

5. (25) Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \ln \frac{x+3}{1-x}$$

② $z^3 = \frac{(1+i)^4}{i^0}$ $312 : 4 = 78$

$z^3 = \frac{(1+i)^4}{1}$ ③ $f(x) = x \cdot e^{\frac{1}{x-2}}$

$z^3 = \frac{1^4 + i^4}{1}$ Df: $\{2\}$

$z^3 = \frac{1+1}{1}$ V.A

$z = \sqrt[3]{2}$ $\lim_{x \rightarrow 2^-} x \cdot e^{\frac{1}{x-2}} = 2 \cdot e^{\frac{1}{2-2}} = \text{N/P}$

NEMA VERTIKALNE ASIMPTOTE

H.A $\lim_{x \rightarrow +\infty} x \cdot e^{\frac{1}{x-2}} = \lim_{x \rightarrow +\infty} x \cdot e^{\frac{1}{x-2} \cdot \frac{1}{x}} = \lim_{x \rightarrow +\infty} x \cdot e^{\frac{1}{x(x-2)}} = x \cdot e^{\frac{0}{1-0}} = x \cdot e^0 = x$

$\lim_{x \rightarrow +\infty} -x \cdot e^{\frac{1}{x-2}} = \lim_{x \rightarrow +\infty} -x \cdot e^{\frac{1}{x-2} \cdot \frac{1}{x}} = \lim_{x \rightarrow +\infty} -x \cdot e^{\frac{1}{x(x-2)}} = -x \cdot e^{\frac{0}{1-0}} = -x$

IHA H.A
• KOSE NEMA

① DERIVATAJ FUNKCUSA

a) $f(x) = \frac{\text{tg } x - \text{ctg } x}{x^3}$

$f'(x) = \frac{(\text{tg } x - \text{ctg } x)' \cdot x^3 + (\text{tg } x - \text{ctg } x) \cdot (x^3)'}{(x^3)^2}$

$f'(x) = \frac{(\frac{1}{\cos^2 x} - \cos x) \cdot x^3 + (\text{tg } x - \text{ctg } x) \cdot 3x^2}{(x^3)^2}$

$f'(x) = \frac{(\frac{x^3}{\cos^2 x} - \cos x^4)' + (3\text{tg } x^3 - 3\text{ctg } x^3)}{x^6}$

12.5

b) DOMENA

$f(x) = \text{arc tg} \left(\frac{1}{x^2} \right)$

$\text{arc tg} \left(\frac{1}{x^2} \right) \neq 0$

12.5

$D_f \mathbb{R} \setminus \{0\}$

① ODREDI INVERZ MATRICE

$A = \begin{bmatrix} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 15 \\ 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{bmatrix} \rightarrow 0 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 11 & 15 \\ 2 & 1 & 2 \\ 5 & 2 & 5 \end{vmatrix} + 0 \cdot (-1)^{2+1} \begin{vmatrix} 0 & 9 & 4 \\ 2 & 1 & 2 \\ 5 & 2 & 5 \end{vmatrix} + 3 \cdot (-1)^{3+1} \begin{vmatrix} 0 & 0 & 4 \\ 0 & 11 & 15 \\ 5 & 2 & 5 \end{vmatrix}$

$7 \cdot (-1)^{4+1} \begin{vmatrix} 0 & 9 & 4 \\ 0 & 11 & 15 \\ 2 & 1 & 2 \end{vmatrix}$

$\text{Det } A = 0 \cdot D_1 + 0 \cdot D_2 + 3 \cdot D_3 - 7 \cdot D_4$
 $\text{Det } A = 0 \cdot 85 + 0 \cdot (-4) + 3 \cdot 455 - 7 \cdot 182 = 91$
 Inverza $A^{-1} = \frac{1}{91}$

$D_1 = \begin{vmatrix} 0 & 11 & 15 & 0 & 11 \\ 2 & 1 & 2 & 2 & 1 \\ 5 & 2 & 5 & 5 & 2 \end{vmatrix} \rightarrow 0 + 110 + 60 - 75 - 0 - 10 = 85$

$D_2 = \begin{vmatrix} 0 & 9 & 4 & 0 & 9 \\ 2 & 1 & 2 & 2 & 1 \\ 5 & 2 & 5 & 5 & 2 \end{vmatrix} \rightarrow 0 + 90 + 16 - 20 - 0 - 90 = -4$

$D_3 = \begin{vmatrix} 0 & 0 & 4 & 0 & 9 \\ 0 & 11 & 15 & 0 & 11 \\ 5 & 2 & 5 & 5 & 2 \end{vmatrix} \rightarrow 0 + 675 + 0 - 220 - 0 - 0 = 455$

$D_4 = \begin{vmatrix} 0 & 9 & 4 & 0 & 9 \\ 0 & 11 & 15 & 0 & 11 \\ 2 & 1 & 2 & 2 & 1 \end{vmatrix} \rightarrow 0 + 270 + 0 - 88 - 0 - 0 = 182$

25/100

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: LOVRE KEREŠ

Broj indeksa: 57933

Vrijeme: od 8³⁰ do ♣B

Broj bodova:

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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4. (12.5+12.5)

- a) Deriviraj funkciju:

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$$4. f(x) = \frac{\operatorname{tg} x - \sin x}{x^3}$$

$$a) f'(x) = \frac{(\operatorname{tg} x - \sin x)' \cdot x^3 - (\operatorname{tg} x - \sin x) \cdot (x^3)'}{(x^3)^2}$$

$$f'(x) = \frac{\left(\frac{1}{\cos^2 x} - \cos x\right) \cdot x^3 - (\operatorname{tg} x - \sin x) \cdot 3x^2}{(x^3)^2}$$

✓ 12.5

$$b) f(x) = \operatorname{arctag} \left(\frac{1}{x^2} \right)$$

$$\operatorname{arctag} = \mathbb{R}$$

$$D(f) = \mathbb{R} \setminus \{0\}$$

✓ 12.5

$$\frac{1}{x^2} \Rightarrow x^2 \neq 0 \quad | \sqrt{\quad}$$

$$x \neq 0$$

$$5. f(x) = \ln \left(\frac{x+3}{1-x} \right)$$

12.5

• Doménia

$$x \geq 0$$

$$1-x \neq 0$$

$$-x = -1 \quad | \cdot (-1)$$

$$x = 1$$

$$D(f) = \langle 1, +\infty \rangle$$

↑ ↑

$$2 - \frac{15}{7} = \frac{14-15}{7} = -\frac{1}{7}$$

$$\frac{5}{7} - \frac{10}{77} = \frac{55-10}{77} = \frac{45}{77}$$

$$1 - \frac{6}{7} = \frac{7-6}{7} = \frac{1}{7}$$

$$1 + \frac{5}{11} = \frac{11+5}{11} = \frac{16}{11}$$

$$4 - \frac{45}{11} = \frac{44-45}{11} = -\frac{1}{11}$$

$$\frac{2}{7} \cdot \frac{5}{11} = \frac{10}{77}$$

⊛B

$$1. \left[\begin{array}{cccc} 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \end{array} \right] \sim \left[\begin{array}{cccc} 7 & 5 & 2 & 5 \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 0 & 0 & 9 & 4 \end{array} \right] \begin{array}{l} :7R1 \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & 0 & 11 & 5 \\ 3 & 2 & 1 & 2 \\ 0 & 0 & 9 & 4 \end{array} \right] \begin{array}{l} \\ R3-3R1 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & 0 & 11 & 5 \\ 0 & -\frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & 9 & 4 \end{array} \right] \sim \left[\begin{array}{cccc} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & -\frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & 11 & 5 \\ 0 & 0 & 9 & 4 \end{array} \right] \begin{array}{l} \\ \cdot (-7)R2 \\ \\ \end{array} \sim \left[\begin{array}{cccc} 1 & \frac{5}{7} & \frac{2}{7} & \frac{5}{7} \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 11 & 5 \\ 0 & 0 & 9 & 4 \end{array} \right] \begin{array}{l} R1 - \frac{5}{7}R2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & \frac{2}{7} & \frac{5}{7} \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 11 & 5 \\ 0 & 0 & 9 & 4 \end{array} \right] \begin{array}{l} \\ \\ :11R3 \\ \end{array} \sim \left[\begin{array}{cccc} 1 & 0 & \frac{2}{7} & \frac{5}{7} \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & \frac{5}{11} \\ 0 & 0 & 9 & 4 \end{array} \right] \begin{array}{l} R1 - \frac{2}{7}R3 \\ R2 + R3 \\ \\ R4 - 9R3 \end{array} \sim \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{45}{77} \\ 0 & 1 & 0 & \frac{16}{11} \\ 0 & 0 & 1 & \frac{5}{11} \\ 0 & 0 & 0 & -\frac{4}{11} \end{array} \right] \begin{array}{l} \\ \\ \cdot (-11)R4 \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & \frac{45}{77} \\ 0 & 1 & 0 & \frac{16}{11} \\ 0 & 0 & 1 & \frac{5}{11} \\ 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} R1 - \frac{45}{77}R4 \\ R2 - \frac{16}{11}R4 \\ R3 - \frac{5}{11}R4 \\ \end{array} \sim \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] = I$$

?

3.

 $(\frac{1}{x-2})' = -1$

$$f(x) = x \cdot e^{\frac{1}{x-2}}$$

$$x \in \mathbb{R}$$

$$D(x) = \mathbb{R} \setminus \{2\}$$

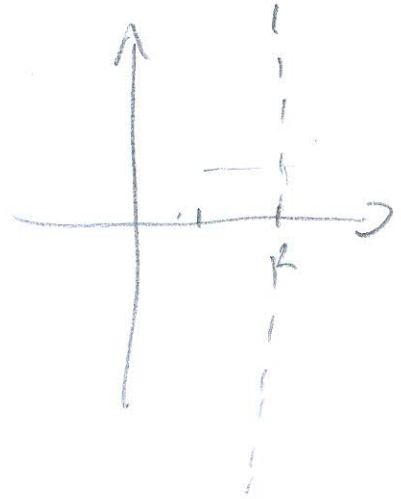
$$e^{\frac{1}{x-2}} \rightarrow x-2 \neq 0$$

$$x \neq 2$$

V.A.

$$\lim_{x \rightarrow 2^+} x \cdot e^{\frac{1}{x-2}} = +\infty$$

$$\lim_{x \rightarrow 2^-} x \cdot e^{\frac{1}{x-2}} = -\infty$$

V.A. $x=2$ 

H.A.

$$\lim_{x \rightarrow +\infty} x \cdot e^{\frac{1}{x-2}} = +\infty$$

$$\lim_{x \rightarrow -\infty} x \cdot e^{\frac{1}{x-2}} = -\infty$$

Nema H.A.

D.K.A.

$$L_0 = \frac{f(x)}{x} = \frac{x \cdot e^{\frac{1}{x-2}}}{x} = \lim_{x \rightarrow \infty} e^{\frac{1}{x-2}} = +\infty$$

Nema K.A.

